

# Matrices

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## Question 1.

State, whether the following statements are true or false. If false, give a reason.

- (i) If A and B are two matrices of orders  $3 \times 2$  and  $2 \times 3$  respectively; then their sum  $A + B$  is possible.
- (ii) The matrices  $A_{2 \times 3}$  and  $B_{2 \times 3}$  are conformable for subtraction.
- (iii) Transpose of a  $2 \times 1$  matrix is a  $2 \times 1$  matrix.
- (iv) Transpose of a square matrix is a square matrix.
- (v) A column matrix has many columns and one row.

## Solution:

- (i) False

The sum  $A + B$  is possible when the order of both the matrices A and B are same.

- (ii) True

- (iii) False

Transpose of a  $2 \times 1$  matrix is a  $1 \times 2$  matrix.

- (iv) True

- (v) False

A column matrix has only one column and many rows.

## Question 2.

Given:  $\begin{bmatrix} x & y+2 \\ 3 & z-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$ , find x, y and z.

## Solution:

If two matrices are equal, then their corresponding elements are also equal. Therefore, we have:

$$x = 3,$$

$$y + 2 = 1 \Rightarrow y = -1$$

$$z - 1 = 2 \Rightarrow z = 3$$

## Question 3.

Solve for a, b and c if

$$(i) \begin{bmatrix} -4 & a+5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} b+4 & 2 \\ 3 & c-1 \end{bmatrix}$$

$$(ii) \begin{bmatrix} a & a-b \\ b+c & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$$

**Solution:**

If two matrices are equal, then their corresponding elements are also equal.

(i)

$$a + 5 = 2 \Rightarrow a = -3$$

$$-4 = b + 4 \Rightarrow b = -8$$

$$2 = c - 1 \Rightarrow c = 3$$

(ii)  $a = 3$

$$a - b = -1$$

$$\Rightarrow b = a + 1 = 4$$

$$b + c = 2$$

$$\Rightarrow c = 2 - b = 2 - 4 = -2$$

**Question 4.**

If  $A = \begin{bmatrix} 8 & -3 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & -5 \end{bmatrix}$ ; find: (i)  $A + B$  (ii)  $B - A$

**Solution:**

$$(i) A + B = \begin{bmatrix} 8 & -3 \end{bmatrix} + \begin{bmatrix} 4 & -5 \end{bmatrix} = \begin{bmatrix} 8 + 4 & -3 - 5 \end{bmatrix} = \begin{bmatrix} 12 & -8 \end{bmatrix}$$

$$(ii) B - A = \begin{bmatrix} 4 & -5 \end{bmatrix} - \begin{bmatrix} 8 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 8 & -5 + 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -2 \end{bmatrix}$$

**Question 5.**

If  $A = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$  and  $C = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$ ; find:

(i)  $B + C$  (ii)  $A - C$

(iii)  $A + B - C$  (iv)  $A - B + C$

**Solution:**

$$(i) B + C = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 + 6 \\ 4 - 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$(ii) A - C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 - 6 \\ 5 + 2 \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \end{bmatrix}$$

$$(iii) A + B - C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1-6 \\ 5+4+2 \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \end{bmatrix}$$

$$(iv) A - B + C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1+6 \\ 5-4-2 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

### Question 6.

Wherever possible, write each of the following as a single matrix.

$$(i) \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 1 & -7 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 3 \\ 6 & -1 & 0 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 0 & 1 & 2 \\ 4 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$$

### Solution:

$$(i) \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 1 & -7 \end{bmatrix} = \begin{bmatrix} 1-1 & 2-2 \\ 3+1 & 4-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 4 & -3 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 3 \\ 6 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2-0 & 3-2 & 4-3 \\ 5-6 & 6+1 & 7-0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 7 & 7 \end{bmatrix}$$

(iii) Addition is not possible, because both matrices are not of same order.

### Question 7.

Find, x and y from the following equations :

$$(i) \begin{bmatrix} 5 & 2 \\ -1 & y-1 \end{bmatrix} - \begin{bmatrix} 1 & x-1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$$

$$(ii) \begin{bmatrix} -8 & x \\ y & -2 \end{bmatrix} + \begin{bmatrix} y & -2 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \end{bmatrix}$$

**Solution:**

(i)

$$\begin{bmatrix} 5 & 2 \\ -1 & y-1 \end{bmatrix} - \begin{bmatrix} 1 & x-1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5-1 & 2-x+1 \\ -1-2 & y-1+3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 3-x \\ -3 & y+2 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$$

Equating the corresponding elements, we get,

$$3-x=7 \text{ and } y+2=2$$

Thus, we get,  $x = -4$  and  $y = 0$ .

(ii)

$$\begin{bmatrix} -8 & x \\ y & -2 \end{bmatrix} + \begin{bmatrix} y & -2 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -8+y & x-2 \\ -3+y & 0 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -3 & 2 \end{bmatrix}$$

Equating the corresponding elements, we get,

$$-8+y = -3 \text{ and } x-2 = 2$$

Thus, we get,  $x = 4$  and  $y = 5$ .

**Question 8.**

Given:  $M = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$ , find its transpose matrix  $M^t$ . If possible, find:

(i)  $M + M^t$  (ii)  $M^t - M$

**Solution:**

$$M = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$$

$$M^t = \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix}$$

$$(i) M + M^t = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 5+5 & -3-2 \\ -2-3 & 4+4 \end{bmatrix} = \begin{bmatrix} 10 & -5 \\ -5 & 8 \end{bmatrix}$$

$$(ii) M^t - M = \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 5-5 & -2+3 \\ -3+2 & 4-4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

**Question 9.**

Write the additive inverse of matrices A, B and C:

Where  $A = \begin{bmatrix} 6 & -5 \end{bmatrix}$ ;  $B = \begin{bmatrix} -2 & 0 \\ 4 & -1 \end{bmatrix}$  and  $C = \begin{bmatrix} -7 \\ 4 \end{bmatrix}$

**Solution:**

We know additive inverse of a matrix is its negative.

Additive inverse of  $A = -A = -\begin{bmatrix} 6 & -5 \end{bmatrix} = \begin{bmatrix} -6 & 5 \end{bmatrix}$

Additive inverse of  $B = -B = -\begin{bmatrix} -2 & 0 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -4 & 1 \end{bmatrix}$

Additive inverse of  $C = -C = -\begin{bmatrix} -7 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$

**Question 10.**

Given  $A = \begin{bmatrix} 2 & -3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 2 \end{bmatrix}$  and  $C = \begin{bmatrix} -1 & 4 \end{bmatrix}$ ; find the matrix X in each of the following:

(i)  $X + B = C - A$

(ii)  $A - X = B + C$

**Solution:**

(i)  $X + B = C - A$

$$X + \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -3 \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} -1-2 & 4+3 \end{bmatrix} = \begin{bmatrix} -3 & 7 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} -3-0 & 7-2 \end{bmatrix} = \begin{bmatrix} -3 & 5 \end{bmatrix}$$

(ii)  $A - X = B + C$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - X = \begin{bmatrix} 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - X = \begin{bmatrix} 0-1 & 2+4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - X = \begin{bmatrix} -1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 6 \end{bmatrix} = X$$

$$X = \begin{bmatrix} 2+1 & -3-6 \end{bmatrix} = \begin{bmatrix} 3 & -9 \end{bmatrix}$$

**Question 11.**

Given  $A = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix}$ ; find the matrix  $X$  in each of the following:

(i)  $A + X = B$

(ii)  $A - X = B$

(iii)  $X - B = A$

**Solution:**

(i)  $A + X = B$

$$X = B - A$$

$$X = \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3+1 & -3-0 \\ -2-2 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -4 & 4 \end{bmatrix}$$

(ii)  $A - X = B$

$$X = A - B$$

$$X = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -1-3 & 0+3 \\ 2+2 & -4-0 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 4 & -4 \end{bmatrix}$$

(iii)  $X - B = A$

$$X = A + B$$

$$X = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -1+3 & 0-3 \\ 2-2 & -4+0 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 0 & -4 \end{bmatrix}$$

**Exercise 9B****Question 1.**

Evaluate:

(i)  $3 \begin{bmatrix} 5 & -2 \end{bmatrix}$

(ii)  $7 \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

(iii)  $2 \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix}$

(iv)  $6 \begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2 \begin{bmatrix} -8 \\ 1 \end{bmatrix}$

**Solution:**

$$(i) 3 \begin{bmatrix} 5 & -2 \end{bmatrix} = \begin{bmatrix} 15 & -6 \end{bmatrix}$$

$$(ii) 7 \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 14 \\ 0 & 7 \end{bmatrix}$$

$$(iii) 2 \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 4 & -6 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} -2+3 & 0+3 \\ 4+5 & -6+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix}$$

$$(iv) 6 \begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2 \begin{bmatrix} -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 18 \\ -12 \end{bmatrix} - \begin{bmatrix} -16 \\ 2 \end{bmatrix} = \begin{bmatrix} 18+16 \\ -12-2 \end{bmatrix} = \begin{bmatrix} 34 \\ -14 \end{bmatrix}$$

**Question 2.**

Find x and y if:

$$(i) 3 \begin{bmatrix} 4 & x \end{bmatrix} + 2 \begin{bmatrix} y & -3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$$

$$(ii) x \begin{bmatrix} -1 \\ 2 \end{bmatrix} - 4 \begin{bmatrix} -2 \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$$

**Solution:**

$$(i) 3 \begin{bmatrix} 4 & x \end{bmatrix} + 2 \begin{bmatrix} y & -3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 3x \end{bmatrix} + \begin{bmatrix} 2y & -6 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 12+2y & 3x-6 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$12 + 2y = 10 \text{ and } 3x - 6 = 0$$

Simplifying, we get,  $y = -1$  and  $x = 2$ .

$$(ii) x \begin{bmatrix} -1 \\ 2 \end{bmatrix} - 4 \begin{bmatrix} -2 \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} -x \\ 2x \end{bmatrix} - \begin{bmatrix} -8 \\ 4y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} -x+8 \\ 2x-4y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$$

Comparing corresponding the elements, we get,

$$-x + 8 = 7 \text{ and } 2x - 4y = -8$$

Simplifying, we get,

$$x = 1 \text{ and } y = \frac{5}{2} = 2.5$$

**Question 3.**

Given  $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$  and  $C = \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$ ; find:

(i)  $2A - 3B + C$

(ii)  $A + 2C - B$

**Solution:**

(i)  $2A - 3B + C$

$$= 2 \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} - 3 \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix} + \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 3 \\ 15 & 6 \end{bmatrix} + \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3-3 & 2-3-1 \\ 6-15+0 & 0-6+0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ -9 & -6 \end{bmatrix}$$

(ii)  $A + 2C - B$

$$= \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} + 2 \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} -6 & -2 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6-1 & 1-2-1 \\ 3+0-5 & 0+0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 \\ -2 & -2 \end{bmatrix}$$

**Question 4.**

If  $\begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + 3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}$ ; find A.

**Solution:**

$$\begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + 3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}$$

$$3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix}$$

$$3A = \begin{bmatrix} -2-4 & -2+2 \\ 1-4 & -3-0 \end{bmatrix}$$

$$3A = \begin{bmatrix} -6 & 0 \\ -3 & -3 \end{bmatrix}$$

$$A = \frac{1}{3} \begin{bmatrix} -6 & 0 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix}$$

**Question 5.**

Given  $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$

(i) find the matrix  $2A + B$

(ii) find the matrix  $C$  such that:

$$C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

**Solution:**

$$(i) 2 \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 2-4 & 8-1 \\ 4-3 & 6-2 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 1 & 4 \end{bmatrix}$$

$$(ii) C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 0+4 & 0+1 \\ 0+3 & 0+2 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

**Question 6.**

If  $2 \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} + 3 \begin{bmatrix} 1 & 3 \\ y & 2 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$ ; find the values of  $x, y$  and  $z$ .

**Solution:**

$$2\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 3 \\ y & 2 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2x \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 9 \\ 3y & 6 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 2x+9 \\ 3y & 8 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$2x + 9 = -7 \Rightarrow 2x = -16 \Rightarrow x = -8$$

$$3y = 15 \Rightarrow y = 5$$

$$z = 9$$

**Question 7.**

Given  $A = \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$  and  $A^t$  is its transpose matrix. Find:

(i)  $2A + 3A^t$  (ii)  $2A^t - 3A$

(iii)  $\frac{1}{2}A - \frac{1}{3}A^t$  (iv)  $A^t - \frac{1}{3}A$

**Solution:**

$$A = \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$$

$$A^t = \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix}$$

(i)  $2A + 3A^t$

$$= 2\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix} + 3\begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 12 \\ 0 & -18 \end{bmatrix} + \begin{bmatrix} -9 & 0 \\ 18 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 12 \\ 18 & -45 \end{bmatrix}$$

$$(ii) 2A^t - 3A$$

$$= 2 \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix} - 3 \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 0 \\ 12 & -18 \end{bmatrix} - \begin{bmatrix} -9 & 18 \\ 0 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -18 \\ 12 & 9 \end{bmatrix}$$

$$(iii) \frac{1}{2}A - \frac{1}{3}A^t$$

$$= \frac{1}{2} \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-3}{2} & 3 \\ 0 & \frac{-9}{2} \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{2} & 3 \\ -2 & \frac{-3}{2} \end{bmatrix}$$

$$(iv) A^t - \frac{1}{3}A$$

$$= \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ 6 & -6 \end{bmatrix}$$

### Question 8.

$$\text{Given } A = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

Solve for matrix X:

$$(i) X + 2A = B$$

$$(ii) 3X + B + 2A = O$$

$$(iii) 3A - 2X = X - 2B.$$

**Solution:**

$$(i) X + 2A = B$$

$$X = B - 2A$$

$$X = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ -4 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & -3 \\ 5 & 1 \end{bmatrix}$$

$$(ii) 3X + B + 2A = O$$

$$3X = -2A - B$$

$$3X = -2 \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$3X = \begin{bmatrix} -2 & -2 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$3X = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{-4}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} \end{bmatrix}$$

$$(iii) 3A - 2X = X - 2B$$

$$3A + 2B = X + 2X$$

$$3X = 3A + 2B$$

$$3X = 3 \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} + 2 \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$3X = \begin{bmatrix} 3 & 3 \\ -6 & 0 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 2 & 2 \end{bmatrix}$$

$$3X = \begin{bmatrix} 7 & 1 \\ -4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{7}{3} & \frac{1}{3} \\ \frac{-4}{3} & \frac{2}{3} \end{bmatrix}$$

**Question 9.**

If  $M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$  and  $N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ , show that:

$$3M + 5N = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

**Solution:**

$$\begin{aligned} 3M + 5N &= 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 5 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ 3 \end{bmatrix} \end{aligned}$$

**Question 10.**

If  $I$  is the unit matrix of order  $2 \times 2$ ; find the matrix  $M$ , such that:

$$(i) M - 2I = 3 \begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$(ii) 5M + 3I = 4 \begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix}$$

**Solution:**

$$\begin{aligned} (i) M - 2I &= 3 \begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} \\ M &= 3 \begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} + 2I \\ M &= 3 \begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ M &= \begin{bmatrix} -3 & 0 \\ 12 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ M &= \begin{bmatrix} -1 & 0 \\ 12 & 5 \end{bmatrix} \end{aligned}$$

$$(ii) 5M + 3I = 4 \begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix}$$

$$5M = 4 \begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix} - 3I$$

$$5M = 4 \begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$5M = \begin{bmatrix} 8 & -20 \\ 0 & -12 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$5M = \begin{bmatrix} 5 & -20 \\ 0 & -15 \end{bmatrix}$$

$$M = \frac{1}{5} \begin{bmatrix} 5 & -20 \\ 0 & -15 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 0 & -3 \end{bmatrix}$$

**Question 11.**

If  $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3 \begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$ , find the matrix M

**Solution:**

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3 \begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$$

$$\Rightarrow 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$$

## Exercise 9C

### Question 1.

Evaluate, if possible:

$$(i) \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \end{bmatrix}$$

### Solution:

$$(i) \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = [6 + 0] = [6]$$

$$(ii) \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix} = [-2 + 2 \quad 3 - 8] = [0 \quad -5]$$

$$(iii) \begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -6 + 12 \\ -3 - 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \end{bmatrix}$$

The number of columns in the first matrix is not equal to the number of rows in the second matrix. Thus, the product is not possible.

### Question 2.

If  $A = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$  and  $I$  is a unit matrix of order  $2 \times 2$ , find:

(i)  $AB$  (ii)  $BA$  (iii)  $AI$

(iv)  $IB$  (v)  $A^2$  (vi)  $B^2A$

### Solution:

$$\begin{aligned}
 \text{(i) } AB &= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 0+6 & 0+4 \\ 5-6 & -5-4 \end{bmatrix} \\
 &= \begin{bmatrix} 6 & 4 \\ -1 & -9 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } BA &= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 0-5 & 2+2 \\ 0+10 & 6-4 \end{bmatrix} \\
 &= \begin{bmatrix} -5 & 4 \\ 10 & 2 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) } AI &= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 0+0 & 0+2 \\ 5-0 & 0-2 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} = A
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } IB &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1+0 & -1+0 \\ 0+3 & 0+2 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} = B
 \end{aligned}$$

$$\begin{aligned}
 (v)A^2 &= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 0+10 & 0-4 \\ 0-10 & 10+4 \end{bmatrix} \\
 &= \begin{bmatrix} 10 & -4 \\ -10 & 14 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 (vi)B^2 &= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1-3 & -1-2 \\ 3+6 & -3+4 \end{bmatrix} \\
 &= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 B^2A &= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 0-15 & -4+6 \\ 0+5 & 18-2 \end{bmatrix} \\
 &= \begin{bmatrix} -15 & 2 \\ 5 & 16 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 (v)A^2 &= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 0+10 & 0-4 \\ 0-10 & 10+4 \end{bmatrix} \\
 &= \begin{bmatrix} 10 & -4 \\ -10 & 14 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 (vi)B^2 &= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1-3 & -1-2 \\ 3+6 & -3+4 \end{bmatrix} \\
 &= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 B^2A &= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 0-15 & -4+6 \\ 0+5 & 18-2 \end{bmatrix} \\
 &= \begin{bmatrix} -15 & 2 \\ 5 & 16 \end{bmatrix}
 \end{aligned}$$

**Question 3.**

If  $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$ , find  $x$  and  $y$  when  $A^2 = B$ .

**Solution:**

Given :  $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$  and  $A^2 = B$

Now,  $A^2 = A \times A$

$$\begin{aligned} &= \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 3x + x \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} \end{aligned}$$

We have  $A^2 = B$

Two matrices are equal if each and every corresponding element is equal.

$$\Rightarrow \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

$$\Rightarrow 4x = 16 \text{ and } 1 = -y$$

$$\Rightarrow x = 4 \text{ and } y = -1$$

**Question 4.**

Find  $x$  and  $y$ , if:

$$(i) \begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$$

$$(ii) \begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

**Solution:**

$$(i) \begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 20 + 3x \\ 5x - 2 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$5x - 2 = 8 \Rightarrow x = 2$$

$$20 + 3x = y \Rightarrow y = 20 + 6 = 26$$

$$(ii) \begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x+0 & x+0 \\ -3+0 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x & x \\ -3 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$x = 2$$

$$-3 + y = -2 \Rightarrow y = 1$$

**Question 5.**

If  $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$  and  $C = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ , find:

(i)  $(AB)C$  (ii)  $A(BC)$

Is  $A(BC) = (AB)C$ ?

**Solution:**

$$(i) AB = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1+12 & 2+9 \\ 2+16 & 4+12 \end{bmatrix} = \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 52+11 & 39+22 \\ 72+16 & 54+32 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$$

$$(ii) BC = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+2 & 3+4 \\ 16+3 & 12+6 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix} = \begin{bmatrix} 6+57 & 7+54 \\ 12+76 & 14+72 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$$

Hence,  $A(BC) = (AB)C$ .

**Question 6.**

Given  $A = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}$ , find; if possible:

(i)  $AB$  (ii)  $BA$  (iii)  $A^2$

**Solution:**

$$\begin{aligned} \text{(i) } AB &= \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix} \\ &= \begin{bmatrix} 0-4-30 & 0+8-36 \\ 0-0+5 & 3+0+6 \end{bmatrix} \\ &= \begin{bmatrix} -34 & -28 \\ 5 & 9 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{(ii) } BA &= \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix} \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 0+3 & 0+0 & 0-1 \\ 0+6 & -4+0 & -6-2 \\ 0-18 & -20-0 & -30+6 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 0 & -1 \\ 6 & -4 & -8 \\ -18 & -20 & -24 \end{bmatrix} \end{aligned}$$

(iii) Product  $AA (=A^2)$  is not possible as the number of columns of matrix  $A$  is not equal to its number of rows.

**Question 7.**

Let  $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$  and

$C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$ . Find  $A^2 + AC - 5B$ .

**Solution:**

$$\text{Given: } A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \text{ and } C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$$

Now,

$$A^2 = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix}$$

$$5B = 5 \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$\begin{aligned} \therefore A^2 + AC - 5B &= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix} \\ &= \begin{bmatrix} 4-7-20 & 0+8-5 \\ 0+2+15 & 4-8+10 \end{bmatrix} \\ &= \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix} \end{aligned}$$

**Question 8.**

If  $M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$  and  $I$  is a unit matrix of the same order as that of  $M$ ; show that:

$$M^2 = 2M + 3I$$

**Solution:**

$$\begin{aligned}
M^2 &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} \\
&= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \\
2M + 3I &= 2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}
\end{aligned}$$

Hence,  $M^2 = 2M + 3I$ .

#### Question 9.

If  $A = \begin{bmatrix} a & 0 \\ 0 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & -b \\ 1 & 0 \end{bmatrix}$ ,  $M = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$  and  $BA = M^2$ , find the values of  $a$  and  $b$ .

**Solution:**

$$\begin{aligned}
BA &= \begin{bmatrix} 0 & -b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & 2 \end{bmatrix} \\
&= \begin{bmatrix} 0+0 & 0-2b \\ a+0 & 0+0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & -2b \\ a & 0 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
M^2 &= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1-1 & -1-1 \\ 1+1 & -1+1 \end{bmatrix}
\end{aligned}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

Given,  $BA = M^2$

$$\begin{bmatrix} 0 & -2b \\ a & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$a = 2$$

$$-2b = -2 \Rightarrow b = 1$$

#### Question 10.

Given  $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ , find:

(i)  $A - B$  (ii)  $A^2$

(iii)  $AB$  (iv)  $A^2 - AB + 2B$

**Solution:**

$$(i) A - B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\begin{aligned} (ii) A^2 &= \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 16+2 & 4+3 \\ 8+6 & 2+9 \end{bmatrix} \\ &= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} (iii) AB &= \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4-2 & 0+1 \\ 2-6 & 0+3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
 & \text{(iv)} A^2 - AB + 2B \\
 &= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 16 & 6 \\ 18 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 18 & 6 \\ 14 & 10 \end{bmatrix}
 \end{aligned}$$

**Question 11.**

If  $A = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ ; find:

- (i)  $(A+B)^2$  (ii)  $A^2 + B^2$   
 (iii) Is  $(A+B)^2 = A^2 + B^2$ ?

**Solution:**

$$(i) A+B = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix}$$

$$\begin{aligned}
 (A+B)^2 &= \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix} \\
 &= \begin{bmatrix} 4+0 & 12-24 \\ 0+0 & 0+16 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & -12 \\ 0 & 16 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 (ii) A^2 &= \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} \\
 &= \begin{bmatrix} 1+4 & 4-12 \\ 1-3 & 4+9 \end{bmatrix} \\
 &= \begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 B^2 &= \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} 1-2 & 2-2 \\ -1+1 & -2+1 \end{bmatrix} \\
 &= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \\
 A^2 + B^2 &= \begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & -8 \\ -2 & 12 \end{bmatrix}
 \end{aligned}$$

(iii) Clearly,  $(A+B)^2 \neq A^2 + B^2$

### Question 12.

Find the matrix A, if  $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$  and  $B^2 = B + \frac{1}{2}A$ .

**Solution:**

$$B^2 = B + \frac{1}{2}A$$

$$\frac{1}{2}A = B^2 - B$$

$$A = 2(B^2 - B)$$

$$\begin{aligned}
 B^2 &= \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 4+0 & 2+1 \\ 0+0 & 0+1 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 B^2 - B &= \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}
 \end{aligned}$$

$$\therefore A = 2(B^2 - B)$$

$$= 2 \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}$$

**Question 13.**

If  $A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$  and  $A^2 = I$ ; find  $a$  and  $b$ .

**Solution:**

$$A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$$

$$\begin{aligned} A^2 &= \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix} \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix} \\ &= \begin{bmatrix} 1+a & -1+b \\ -a+ab & a+b^2 \end{bmatrix} \end{aligned}$$

It is given that  $A^2 = I$ .

$$\therefore \begin{bmatrix} 1+a & -1+b \\ -a+ab & a+b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$1+a = 1$$

Therefore,  $a = 0$

$$-1+b = 0$$

Therefore,  $b = 1$

**Question 14.**

If  $A = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$ ; then show that:

(i)  $A(B + C) = AB + AC$

(ii)  $(B - A)C = BC - AC$ .

**Solution:**

$$(i) B + C = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 4 & 3 \end{bmatrix}$$

$$\begin{aligned} A(B + C) &= \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 6+4 & 14+3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 17 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$AB = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4+4 & 6+1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 0 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 8 & 7 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 17 \\ 0 & 0 \end{bmatrix}$$

Hence,  $A(B + C) = AB + AC$

$$(ii) B - A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 4 & 1 \end{bmatrix}$$

$$\begin{aligned} (B - A)C &= \begin{bmatrix} 0 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0+4 \\ 4+0 & 16+2 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 18 \end{bmatrix} \end{aligned}$$

$$BC = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+6 \\ 4+0 & 16+2 \end{bmatrix} = \begin{bmatrix} 2 & 14 \\ 4 & 18 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix}$$

$$BC - AC = \begin{bmatrix} 2 & 14 \\ 4 & 18 \end{bmatrix} - \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 18 \end{bmatrix}$$

Hence,  $(B - A)C = BC - AC$

**Question 15.**

If  $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ , simplify:

$A^2 + BC$ .

**Solution:**

$$A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$$

$$BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$

**Question 16(i).**

Solve for x and y:

$$\begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \end{bmatrix}$$

**Solution:**

$$\begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 5y \\ 5x + 2y \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$2x + 5y = -7 \dots(1)$$

$$5x + 2y = 14 \dots(2)$$

Multiplying (1) with 2 and (2) with 5, we get,

$$4x + 10y = -14 \dots(3)$$

$$25x + 10y = 70 \dots(4)$$

Subtracting (3) from (4), we get,

$$21x = 84 \Rightarrow x = 4$$

$$\text{From (2), } 2y = 14 - 5x = 14 - 20 = -6 \Rightarrow y = -3$$

**Question 16(ii).**

Solve for x and y:

$$\begin{bmatrix} x+y & x-4 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$$

**Solution:**

$$\begin{aligned} [x+y \ x-4] \begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix} &= [-7 \ -11] \\ [-x-y+2x-8 \ -2x-2y+2x-8] &= [-7 \ -11] \\ [-y+x-8 \ -2y-8] &= [-7 \ -11] \end{aligned}$$

Comparing the corresponding elements, we get,

$$\begin{aligned} -2y - 8 &= -11 \Rightarrow -2y = -3 \Rightarrow y = \frac{3}{2} \\ -y + x - 8 &= -7 \end{aligned}$$

$$\Rightarrow -\frac{3}{2} + x - 8 = -7$$

$$\Rightarrow x = 1 + \frac{3}{2} = \frac{5}{2}$$

**Question 16(iii).**

Solve for x and y:

$$\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} y \\ 3 \end{bmatrix}.$$

**Solution:**

$$\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} y \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+0 \\ -3+2x \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \\ -3+2x \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-6 \\ -3+2x+3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 \\ 2x \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow 2y = -4 \text{ and } 2x = 6$$

$$\Rightarrow y = -2 \text{ and } x = 3$$

Thus, the values of x and y are: 3, -2

**Question 17.**

In each case given below, find:

(a) The order of matrix M.

(b) The matrix M.

$$(i) M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times M = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

**Solution:**

We know, the product of two matrices is defined only when the number of columns of first matrix is equal to the number of rows of the second matrix.

(i) Let the order of matrix M be  $a \times b$ .

$$M_{a \times b} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}_{2 \times 2} = \begin{bmatrix} 1 & 2 \end{bmatrix}_{1 \times 2}$$

Clearly, the order of matrix M is  $1 \times 2$ .

Let  $M = \begin{bmatrix} a & b \end{bmatrix}$

$$M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a+0 & a+2b \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$a = 1 \text{ and } a + 2b = 2 \Rightarrow 2b = 2 - 1 = 1 \Rightarrow b = \frac{1}{2}$$

$$\therefore M = \begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix}$$

(ii) Let the order of matrix M be  $a \times b$ .

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}_{2 \times 2} \times M_{a \times b} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}_{2 \times 1}$$

Clearly, the order of matrix M is  $2 \times 1$ .

$$\text{Let } M = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times M = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} a + 4b \\ 2a + b \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$a + 4b = 13 \dots (1)$$

$$2a + b = 5 \dots (2)$$

Multiplying (2) by 4, we get,

$$8a + 4b = 20 \dots (3)$$

Subtracting (1) from (3), we get,

$$7a = 7 \Rightarrow a = 1$$

From (2), we get,

$$b = 5 - 2a = 5 - 2 = 3$$

$$\therefore M = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

#### Question 18.

If  $A = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$ ; find the value of x, given that:  $A^2 = B$ .

**Solution:**

$$A^2 = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 2x+x \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix}$$

Given,  $A^2 = B$

$$\begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$$

Comparing the two matrices, we get,

$$3x = 36 \Rightarrow x = 12$$

**Question 19.**

$$\text{If } A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}.$$

Find:  $AB - 5C$ .

**Solution:**

$$\text{Given: } A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

Now,

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3 \times 0 + 7 \times 5 & 3 \times 2 + 7 \times 3 \\ 2 \times 0 + 4 \times 5 & 2 \times 2 + 4 \times 3 \end{bmatrix} \\ &= \begin{bmatrix} 0 + 35 & 6 + 21 \\ 0 + 20 & 4 + 12 \end{bmatrix} \\ &= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} \end{aligned}$$

$$5C = 5 \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$\therefore AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

**Question 20.**

If A and B are any two  $2 \times 2$  matrices such that  $AB = BA = B$  and B is not a zero matrix, what can you say about the matrix A?

**Solution:**

$$AB = BA = B$$

We know that  $AI = IA = I$ , where I is the identity matrix.

Hence, B is the identity matrix.

**Question 21.**

Given  $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$  and that  $AB = A + B$ ; find the values of  $a$ ,  $b$  and  $c$ .

**Solution:**

$$AB = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3a+0 & 3b+0 \\ 0+0 & 0+4c \end{bmatrix} = \begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3+a & b \\ 0 & 4+c \end{bmatrix}$$

Given,  $AB = A + B$

$$\therefore \begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix} = \begin{bmatrix} 3+a & b \\ 0 & 4+c \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$3a = 3 + a$$

$$\Rightarrow 2a = 3$$

$$\Rightarrow a = \frac{3}{2}$$

$$3b = b \Rightarrow b = 0$$

$$4c = 4 + c \Rightarrow 3c = 4 \Rightarrow c = \frac{4}{3}$$

**Question 22.**

If  $P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$  and  $Q = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ , then compute:

(i)  $P^2 - Q^2$  (ii)  $(P + Q)(P - Q)$

Is  $(P + Q)(P - Q) = P^2 - Q^2$  true for matrix algebra?

**Solution:**

$$(i) P^2 = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1+4 & 2-2 \\ 2-2 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$Q^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ 2+2 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$P^2 - Q^2 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -4 & 4 \end{bmatrix}$$

$$P + Q = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 0 \end{bmatrix}$$

$$P - Q = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & -2 \end{bmatrix}$$

$$(P + Q)(P - Q) = \begin{bmatrix} 2 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0+0 & 4-4 \\ 0+0 & 8-0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 8 \end{bmatrix}$$

Clearly, it can be said that:

$(P + Q)(P - Q) = P^2 - Q^2$  not true for matrix algebra.

### Question 23.

Given the matrices:

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \text{ and } C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}. \text{ Find:}$$

(i)  $ABC$  (ii)  $ACB$ .

State whether  $ABC = ACB$ .

**Solution:**

$$(i) AB = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 6-1 & 8-2 \\ 12-2 & 16-4 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix}$$

$$ABC = \begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -15+0 & 5-12 \\ -30+0 & 10-24 \end{bmatrix} = \begin{bmatrix} -15 & -7 \\ -30 & -14 \end{bmatrix}$$

$$(ii) AC = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -6+0 & 2-2 \\ -12+0 & 4-4 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix}$$

$$ACB = \begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -18-0 & -24-0 \\ -36-0 & -48-0 \end{bmatrix} = \begin{bmatrix} -18 & -24 \\ -36 & -48 \end{bmatrix}$$

Hence,  $ABC \neq ACB$ .

**Question 24.**

If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix}$  and  $C = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}$ ; find each of the following and state if they are equal:

(i)  $CA + B$  (ii)  $A + CB$

**Solution:**

$$(i) CA = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2-9 & -4-12 \\ 0+3 & 0+4 \end{bmatrix} = \begin{bmatrix} -11 & -16 \\ 3 & 4 \end{bmatrix}$$

$$CA + B = \begin{bmatrix} -11 & -16 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -15 \\ 4 & 5 \end{bmatrix}$$

$$(ii) CB = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -12-3 & -2-3 \\ 0+1 & 0+1 \end{bmatrix} = \begin{bmatrix} -15 & -5 \\ 1 & 1 \end{bmatrix}$$

$$A + CB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -14 & -3 \\ 4 & 5 \end{bmatrix}$$

Thus,  $CA + B \neq A + CB$

**Question 25.**

If  $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$ ; find the matrix  $X$  such that  $AX = B$ .

**Solution:**

Let the order of the matrix  $X$  be  $a \times b$ .

$$AX = B$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}_{2 \times 2} \times X_{a \times b} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}_{2 \times 1}$$

Clearly, the order of matrix  $X$  is  $2 \times 1$ .

$$\text{Let } X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$$

$$\begin{bmatrix} 2x + y \\ x + 3y \end{bmatrix} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$$

Comparing the two matrices, we get,

$$2x + y = 3 \dots (1)$$

$$x + 3y = -11 \dots (2)$$

Multiplying (1) with 3, we get,

$$6x + 3y = 9 \dots (3)$$

Subtracting (2) from (3), we get,

$$5x = 20$$

$$x = 4$$

From (1), we have:

$$y = 3 - 2x = 3 - 8 = -5$$

$$\therefore X = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

**Question 26.**

If  $A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$ , find  $(A - 2I)(A - 3I)$ .

**Solution:**

$$A - 2I = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}$$

$$A - 3I = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$$\begin{aligned} (A - 2I)(A - 3I) &= \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 2+2 & 4-4 \\ 1-1 & 2+2 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \end{aligned}$$

**Question 27.**

If  $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$ , find:

(i)  $A^t \cdot A$  (ii)  $A \cdot A^t$

Where  $A^t$  is the transpose of matrix  $A$ .

**Solution:**

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ -1 & -2 \end{bmatrix}$$

$$\begin{aligned} \text{(i)} A^t \cdot A &= \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 4+0 & 2+0 & -2-0 \\ 2+0 & 1+1 & -1-2 \\ -2-0 & -1-2 & 1+4 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 5 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{(ii)} A \cdot A^t &= \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ -1 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 4+1+1 & 0+1+2 \\ 0+1+2 & 0+1+4 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 3 \\ 3 & 5 \end{bmatrix} \end{aligned}$$

**Question 28.**

If  $M = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$ , show that:  $6M - M^2 = 9I$ ; where  $I$  is a  $2 \times 2$  unit matrix.

**Solution:**

$$M^2 = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 16-1 & 4+2 \\ -4-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$6M - M^2 = 6 \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$= 9 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9I$$

Hence, proved.

**Question 29.**

If  $P = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$  and  $Q = \begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix}$ ; find x and y such that  $PQ = \text{null matrix}$ .

**Solution:**

$$PQ = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix} = \begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix}$$

$PQ = \text{Null matrix}$

$$\therefore \begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$2x + 12 = 0$$

$$\text{Therefore } x = -6$$

$$6 + 6y = 0$$

$$\text{Therefore } y = -1$$

**Question 30.**

Evaluate without using tables:

$$\begin{bmatrix} 2 \cos 60^\circ & -2 \sin 30^\circ \\ -\tan 45^\circ & \cos 0^\circ \end{bmatrix} \begin{bmatrix} \cot 45^\circ & \operatorname{cosec} 30^\circ \\ \sec 60^\circ & \sin 90^\circ \end{bmatrix}$$

**Solution:**

$$\begin{aligned} & \begin{bmatrix} 2\cos 60^\circ & -2\sin 30^\circ \\ -\tan 45^\circ & \cos 0^\circ \end{bmatrix} \begin{bmatrix} \cot 45^\circ & \operatorname{cosec} 30^\circ \\ \sec 60^\circ & \sin 90^\circ \end{bmatrix} \\ &= \begin{bmatrix} 2 \times \frac{1}{2} & -2 \times \frac{1}{2} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1-2 & 2-1 \\ -1+2 & -2+1 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \end{aligned}$$

**Question 31.**

State, with reason, whether the following are true or false. A, B and C are matrices of order  $2 \times 2$ .

- (i)  $A + B = B + A$
- (ii)  $A - B = B - A$
- (iii)  $(B \cdot C) \cdot A = B \cdot (C \cdot A)$
- (iv)  $(A + B) \cdot C = A \cdot C + B \cdot C$
- (v)  $A \cdot (B - C) = A \cdot B - A \cdot C$
- (vi)  $(A - B) \cdot C = A \cdot C - B \cdot C$
- (vii)  $A^2 - B^2 = (A + B)(A - B)$
- (viii)  $(A - B)^2 = A^2 - 2A \cdot B + B^2$

**Solution:**

- (i) True.  
Addition of matrices is commutative.
- (ii) False.  
Subtraction of matrices is commutative.
- (iii) True.  
Multiplication of matrices is associative.
- (iv) True.  
Multiplication of matrices is distributive over addition.
- (v) True.  
Multiplication of matrices is distributive over subtraction.
- (vi) True.  
Multiplication of matrices is distributive over subtraction.
- (vii) False.  
Laws of algebra for factorization and expansion are not applicable to matrices.

(viii) False.

Laws of algebra for factorization and expansion are not applicable to matrices.

## Exercise 9D

### Question 1.

Find  $x$  and  $y$ , if:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ y \end{bmatrix}$$

**Solution:**

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ y \end{bmatrix}$$

$$\begin{bmatrix} 6x - 2 \\ -2x + 4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x - 10 \\ -2x + 14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$6x - 10 = 8$$

$$\Rightarrow 6x = 18$$

$$\Rightarrow x = 3$$

$$-2x + 14 = 4y$$

$$\Rightarrow 4y = -6 + 14 = 8$$

$$\Rightarrow y = 2$$

### Question 2.

Find  $x$  and  $y$ , if:

$$\begin{bmatrix} 3x & 8 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3 \begin{bmatrix} 2 & -7 \end{bmatrix} = 5 \begin{bmatrix} 3 & 2y \end{bmatrix}$$

**Solution:**

$$[3x \ 8] \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3[2 \ -7] = 5[3 \ 2y]$$

$$[3x + 24 \ 12x + 56] - [6 \ -21] = [15 \ 10y]$$

$$[3x + 24 - 6 \ 12x + 56 + 21] = [15 \ 10y]$$

$$[3x + 18 \ 12x + 77] = [15 \ 10y]$$

Comparing the corresponding elements, we get,

$$3x + 18 = 15$$

$$\Rightarrow 3x = -3$$

$$\Rightarrow x = -1$$

$$12x + 77 = 10y$$

$$\Rightarrow 10y = -12 + 77 = 65$$

$$\Rightarrow y = 6.5$$

**Question 3.**

If  $\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [25]$  and  $\begin{bmatrix} -x & y \end{bmatrix} \begin{bmatrix} 2x \\ y \end{bmatrix} = [-2]$ ; find  $x$  and  $y$ , if:

(i)  $x, y \in W$  (whole numbers)

(ii)  $x, y \in Z$  (integers)

**Solution:**

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [25]$$

$$x^2 + y^2 = 25$$

and

$$-2x^2 + y^2 = -2$$

(i)  $x, y \in W$  (whole numbers)

It can be observed that the above two equations are satisfied when  $x = 3$  and  $y = 4$ .

(ii)  $x, y \in Z$  (integers)

It can be observed that the above two equations are satisfied when  $x = \pm 3$  and  $y = \pm 4$ .

**Question 4.**

Given  $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \times X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ . Write

- (i) the order of matrix  $X$ .
- (ii) the matrix  $X$ .

**Solution:**

(i)

Let the order of matrix  $X$  be  $a \times b$

$$\therefore \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}_{2 \times 2} \times X_{a \times b} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}_{2 \times 1}$$

$$\Rightarrow a = 2 \text{ and } b = 1$$

$$\therefore \text{The order of the matrix } X = a \times b = 2 \times 1$$

(ii)

$$\text{Let } X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x+y \\ -3x+4y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$\Rightarrow 2x + y = 7 \text{ and } -3x + 4y = 6$$

On solving the above simultaneous equations in  $x$  and  $y$ , we have,  $x = 2$  and  $y = 3$

$$\therefore \text{The matrix } X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

**Question 5.**

Evaluate:

$$\begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2} \cos 0^\circ & \sin 0^\circ \end{bmatrix} \begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix}$$

**Solution:**

$$\begin{aligned}& \begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2} \cos 0^\circ & \sin 0^\circ \end{bmatrix} \begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix} \\&= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 1 & 1 \end{bmatrix} \\&= \begin{bmatrix} \frac{1}{2} + \frac{1}{2} & 0 + \frac{1}{2} \\ 1 + 0 & 0 + 0 \end{bmatrix} \\&= \begin{bmatrix} 1 & 0.5 \\ 1 & 0 \end{bmatrix}\end{aligned}$$

**Question 6.**

If  $A = \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}$ ,  $B = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$  and  $3A \times M = 2B$ ; find matrix M.

**Solution:**

Let the order of matrix M be  $a \times b$ .

$$3A \times M = 2B$$

$$3 \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}_{2 \times 2} \times M_{a \times b} = 2 \begin{bmatrix} -5 \\ 6 \end{bmatrix}_{2 \times 1}$$

Clearly, the order of matrix M is  $2 \times 1$ .

$$\text{Let } M = \begin{bmatrix} x \\ y \end{bmatrix}$$

Then,

$$3 \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 \\ 12 & -9 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 0 - 3y \\ 12x - 9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} -3y \\ 12x - 9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

Comparing the corresponding elements, we get,  
 $-3y = -10$

$$\Rightarrow y = \frac{10}{3}$$

$$12x - 9y = 12$$

$$\Rightarrow 12x - 30 = 12$$

$$\Rightarrow 12x = 42$$

$$\Rightarrow x = \frac{7}{2}$$

$$\therefore M = \begin{bmatrix} \frac{7}{2} \\ \frac{10}{3} \end{bmatrix}$$

#### Question 7.

If  $\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$ , find the values of a, b and c.

**Solution:**

$$\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a+1 & 2+b \\ 7 & -1-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$a+1=5 \Rightarrow a=4$$

$$2+b=0 \Rightarrow b=-2$$

$$-1-c=3 \Rightarrow c=-4$$

**Question 8.**

If  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ ; find:

(i)  $A(BA)$

(ii)  $(AB).B$

**Solution:**

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

(i)

$$\begin{aligned} BA &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A(BA) &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix} \\ &= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix} \end{aligned}$$

(ii)

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 2+2 & 1+4 \\ 4+1 & 2+2 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} (AB)B &= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 8+5 & 4+10 \\ 10+4 & 5+8 \end{bmatrix} \\ &= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix} \end{aligned}$$

**Question 9.**

Find x and y, if:  $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

**Solution:**

$$\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$5x = 5 \Rightarrow x = 1$$

$$6y = 12 \Rightarrow y = 2$$

**Question 10.**

If matrix  $X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$  and  $2X - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$ ; find the matrix 'X' and 'Y'.

**Solution:**

$$X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 - 8 \\ 4 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} -14 \\ 10 \end{bmatrix}$$

$$\text{Given, } 2X - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$$

$$2 \begin{bmatrix} -14 \\ 10 \end{bmatrix} - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$$

$$3Y = 2 \begin{bmatrix} -14 \\ 10 \end{bmatrix} - \begin{bmatrix} 10 \\ -8 \end{bmatrix}$$

$$3Y = \begin{bmatrix} -28 \\ 20 \end{bmatrix} - \begin{bmatrix} 10 \\ -8 \end{bmatrix}$$

$$3Y = \begin{bmatrix} -38 \\ 28 \end{bmatrix}$$

$$Y = \frac{1}{3} \begin{bmatrix} -38 \\ 28 \end{bmatrix}$$

**Question 11.**

Given  $A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ ; find the matrix X such that:

$$A + X = 2B + C$$

**Solution:**

$$\text{Given, } A + X = 2B + C$$

$$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = 2 \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = \begin{bmatrix} -5 & 4 \\ 8 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -5 & 4 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} -7 & 5 \\ 6 & 2 \end{bmatrix}$$

**Question 12.**

Find the value of x, given that  $A^2 = B$ ,

$$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

**Solution:**

$$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 24+12 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$$

$$\text{Given, } A^2 = B$$

$$\therefore \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements, we get,  
 $x = 36$

**Question 13.**

If  $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ , and  $I$  is identity matrix of the same order and  $A^t$  is the transpose of matrix  $A$ , find  $A^t \cdot B + B \cdot I$

**Solution:**

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$\therefore A^t = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$\begin{aligned} A^t \cdot B &= \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 4 + 1 \times (-1) & 2 \times (-2) + 1 \times 3 \\ 5 \times 4 + 3 \times (-1) & 5 \times (-2) + 3 \times 3 \end{bmatrix} \\ &= \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} B \cdot I &= \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}\therefore A^t \cdot B + BI &= \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}\end{aligned}$$

**Question 14.**

Given  $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$  and  $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ .

Find the matrix  $X$  such that  $A + 2X = 2B + C$ .

**Solution:**

Given :  $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$  and  $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Now,  $A + 2X = 2B + C$

$$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2 \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -6+4 & 4+0 \\ 8+0 & 0+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$$

**Question 15.**

Let  $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$  and  $C = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$ . Find  $A^2 - A + BC$ .

**Solution:**

$$A^2 = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 16-12 & -8+6 \\ 24-18 & -12+9 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$$

$$BC = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0+2 & 0-2 \\ -2-1 & 3+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

$$\begin{aligned} A^2 - A + BC &= \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} \end{aligned}$$

**Question 16.**

Let  $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ . Find  $A^2 + AB + B^2$ .

**Solution:**

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$\begin{aligned} A^2 &= A \times A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 1 + 0 \times 2 & 1 \times 0 + 0 \times 1 \\ 2 \times 1 + 1 \times 2 & 2 \times 0 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \end{aligned}$$

$$AB = A \times B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$\begin{aligned}
&= \begin{bmatrix} 1 \times 2 + 0 \times (-1) & 1 \times 3 + 0 \times 0 \\ 2 \times 2 + 1 \times (-1) & 2 \times 3 + 1 \times 0 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} \\
B^2 &= B \times B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 2 \times 2 + 3 \times (-1) & 2 \times 3 + 3 \times 0 \\ (-1) \times 2 + 0 \times (-1) & -1 \times 3 + 0 \times 0 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} \\
\therefore A^2 + AB + B^2 &= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} \\
&= \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}
\end{aligned}$$

**Question 17.**

If  $A = \begin{bmatrix} 3 & a \\ -4 & 8 \end{bmatrix}$ ,  $B = \begin{bmatrix} c & 4 \\ -3 & 0 \end{bmatrix}$ ,  $C = \begin{bmatrix} -1 & 4 \\ 3 & b \end{bmatrix}$  and  $3A - 2C = 6B$ , find the values of  $a$ ,  $b$  and  $c$ .

**Solution:**

$$3A - 2C = 6B$$

$$\begin{aligned}
3 \begin{bmatrix} 3 & a \\ -4 & 8 \end{bmatrix} - 2 \begin{bmatrix} -1 & 4 \\ 3 & b \end{bmatrix} &= 6 \begin{bmatrix} c & 4 \\ -3 & 0 \end{bmatrix} \\
\begin{bmatrix} 9 & 3a \\ -12 & 24 \end{bmatrix} - \begin{bmatrix} -2 & 8 \\ 6 & 2b \end{bmatrix} &= \begin{bmatrix} 6c & 24 \\ -18 & 0 \end{bmatrix} \\
\begin{bmatrix} 11 & 3a - 8 \\ -18 & 24 - 2b \end{bmatrix} &= \begin{bmatrix} 6c & 24 \\ -18 & 0 \end{bmatrix}
\end{aligned}$$

Comparing the corresponding elements, we get,

$$3a - 8 = 24 \Rightarrow 3a = 32 \Rightarrow a = \frac{32}{3} = 10\frac{2}{3}$$

$$24 - 2b = 0 \Rightarrow 2b = 24 \Rightarrow b = 12$$

$$11 = 6c \Rightarrow c = \frac{11}{6} = 1\frac{5}{6}$$

**Question 18.**

Given  $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$ ,  $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$  and  $BA = C^2$ .

Find the values of p and q.

**Solution:**

$$A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$$BA = C^2 \Rightarrow \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

By comparing,

$$-2q = -8 \Rightarrow q = 4$$

And  $p = 8$

**Question 19.**

Given  $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ ,  $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$  and  $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ . Find  $AB + 2C - 4D$ .

**Solution:**

$$AB = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 18-2 \\ -6+4 \end{bmatrix} = \begin{bmatrix} 16 \\ -2 \end{bmatrix}$$

$$\therefore AB + 2C - 4D = \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

**Question 20.**

Evaluate:

$$\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

**Solution:**

$$\begin{aligned}& \begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\&= \begin{bmatrix} 4 \times \frac{1}{2} & 2 \times \frac{1}{2} \\ 1 & 2 \times 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\&= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix} \\&= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}\end{aligned}$$

**Question 21.**

$$\text{If } A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ find } A^2 - 5A + 7I$$

**Solution:**

$$\text{Given that } A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

We need to find  $A^2 - 5A + 7I$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I = 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
&= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\
A^2 - 5A + 7I &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\
&= \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 + 5 + 0 & 3 - 10 + 7 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned}$$

**Question 22.**

Given  $A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $A^2 = 9A + mI$ . Find  $m$ .

**Solution:**

$$A^2 = 9A + mI$$

$$\Rightarrow A^2 - 9A = mI \dots (1)$$

$$\text{Now, } A^2 = AA$$

$$= \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$$

Substituting  $A^2$  in (1), we have

$$A^2 - 9A = mI$$

$$\Rightarrow \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} - 9 \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} = m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} - \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$$\Rightarrow m = -14$$

**Question 23.**

Given matrix  $A = \begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix}$  and  $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ . If  $AX = B$ .

- (i) Write the order of matrix X.
- (ii) Find the matrix 'X'

**Solution:**

Given,  $A = \begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix}$  and  $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

(i) Let the order of matrix  $X = m \times n$

Order of matrix  $A = 2 \times 2$

Order of matrix  $B = 2 \times 1$

Now,  $AX = B$

$$\Rightarrow A_{2 \times 2} \cdot X_{m \times n} = B_{2 \times 1}$$

$\therefore m = 2$  and  $n = 1$

Thus, order of matrix  $X = m \times n = 2 \times 1$

(ii) Let the matrix  $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$$AX = B$$

$$\Rightarrow \begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4\left(\frac{1}{2}\right) & 1 \\ 1 & 4\left(\frac{1}{2}\right) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x + y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow 2x + y = 4 \quad \dots (1)$$

$$x + 2y = 5 \quad \dots (2)$$

Multiplying (1) by 2, we get

$$4x + 2y = 8 \quad \dots (3)$$

Subtracting (2) from (3), we get

$$3x = 3$$

$$\Rightarrow x = 1$$

Substituting the value of  $x$  in (1), we get

$$2(1) + y = 4$$

$$\Rightarrow 2 + y = 4$$

$$\Rightarrow y = 2$$

Hence, the matrix  $X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

#### Question 24.

$$\text{If } A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \text{ and } A^2 - 5B^2 = 5C.$$

Find the matrix  $C$  where  $C$  is a 2 by 2 matrix.

**Solution:**

$$\begin{aligned} A^2 &= A \times A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 1 + 3 \times 3 & 1 \times 3 + 3 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 3 + 4 \times 4 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} \\ B^2 &= B \times B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -2 \times -2 + 1 \times -3 & -2 \times 1 + 1 \times 2 \\ -3 \times -2 + 2 \times -3 & -3 \times 1 + 2 \times 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$\text{Given: } A^2 - 5B^2 = 5C$$

$$\Rightarrow \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 5C$$

$$\Rightarrow \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 5C$$

$$\Rightarrow \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5C$$

$$\Rightarrow 5 \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = 5C$$

$$\Rightarrow C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

**Question 25.**

Given matrix  $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$ . Find the matrix X if,  $X = B^2 - 4B$ .

Hence, solve for a and b given  $X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$ .

**Solution:**

$$B^2 = B \times B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + 1 \times 8 & 1 \times 1 + 1 \times 3 \\ 8 \times 1 + 3 \times 8 & 8 \times 1 + 3 \times 3 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix}$$

$$4B = 4 \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$\text{Given : } X = B^2 - 4B$$

$$\Rightarrow X = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

To find: a and b

$$X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix} \quad \dots \text{given}$$

$$\Rightarrow \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

$$\Rightarrow 5 \begin{bmatrix} a \\ b \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

$$\Rightarrow a = 1 \text{ and } b = 10$$