

Geometric Progression

Question 1.

Find, which of the following sequence form a G.P. :

(i) 8, 24, 72, 216,

(ii) $\frac{1}{8}, \frac{1}{24}, \frac{1}{72}, \frac{1}{216}, \dots$

(iii) 9, 12, 16, 24,

Solution 1(i).

Given sequence: 8, 24, 72, 216.....

Now,

$$\frac{24}{8} = 3, \quad \frac{72}{24} = 3, \quad \frac{216}{72} = 3$$

Since $\frac{24}{8} = \frac{72}{24} = \frac{216}{72} = \dots = 3$, the given sequence is a G.P.

with common ratio 3.

Solution 1(ii).

Given sequence: $\frac{1}{8}, \frac{1}{24}, \frac{1}{72}, \frac{1}{216}, \dots$

Now,

$$\frac{\frac{1}{24}}{\frac{1}{8}} = \frac{1}{3}, \quad \frac{\frac{1}{72}}{\frac{1}{24}} = \frac{1}{3}, \quad \frac{\frac{1}{216}}{\frac{1}{72}} = \frac{1}{3}$$

Since $\frac{\frac{1}{24}}{\frac{1}{8}} = \frac{\frac{1}{72}}{\frac{1}{24}} = \frac{\frac{1}{216}}{\frac{1}{72}} = \dots = \frac{1}{3}$, the given sequence is a G.P.

with common ratio $\frac{1}{3}$.

Solution 1(iii).

Given sequence: 9, 12, 16, 24.....

Now,

$$\frac{12}{9} = \frac{4}{3}, \quad \frac{16}{12} = \frac{4}{3}, \quad \frac{24}{16} = \frac{3}{2}$$

Since $\frac{24}{9} \neq \frac{72}{24}$, the given sequence is not a G.P.

Question 2.

Find the 9th term of the series :

1, 4, 16, 64

Solution:

Given sequence: 1, 4, 16, 64.....

Now,

$$\frac{4}{1} = 4, \quad \frac{16}{4} = 4, \quad \frac{64}{16} = 4$$

Since $\frac{4}{1} = \frac{16}{4} = \frac{64}{16} = \dots = 4$, the given sequence is a G.P.

with first term, $a = 1$ and common ratio, $r = 4$.

Now, $t_n = ar^{n-1}$

$$\Rightarrow t_9 = 1 \times 4^8 = 65536$$

Question 3.

Find the seventh term of the G.P. :

1, $\sqrt{3}$, 3, $3\sqrt{3}$

Solution:

Given G.P.: 1, $\sqrt{3}$, 3, $3\sqrt{3}$,

Here,

First term, $a = 1$

Common ratio, $r = \frac{\sqrt{3}}{1} = \sqrt{3}$

Now, $t_n = ar^{n-1}$

$$\Rightarrow t_7 = 1 \times (\sqrt{3})^6 = 27$$

Question 4.

Find the 8th term of the sequence :

$\frac{3}{4}$, $1\frac{1}{2}$, 3,

Solution:

Given sequence: $\frac{3}{4}, 1\frac{1}{2}, 3, \dots$

i.e. $\frac{3}{4}, \frac{3}{2}, 3, \dots$

Now,

$$\frac{\frac{3}{2}}{\frac{3}{4}} = 2, \quad \frac{3}{\frac{3}{2}} = 2,$$

Since $\frac{\frac{3}{2}}{\frac{3}{4}} = \frac{3}{\frac{3}{2}} = \dots = 2$, the given sequence is a G.P.

with first term, $a = \frac{3}{4}$ and common ratio, $r = 2$.

Now, $t_n = ar^{n-1}$

$$\Rightarrow t_8 = \frac{3}{4} \times 2^7 = \frac{3}{4} \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 3 \times 2^5 = 96$$

Question 5.

Find the 10th term of the G.P. :

Solution:

Given G.P.: $12, 4, 1\frac{1}{3}, \dots$

Here,

First term, $a = 12$

Common ratio, $r = \frac{4}{12} = \frac{1}{3}$

Now, $t_n = ar^{n-1}$

$$\Rightarrow t_{10} = 12 \times \left(\frac{1}{3}\right)^9 = 12 \times \frac{1}{19683} = \frac{4}{6561}$$

Question 6.

Find the nth term of the series :

Solution:

Given series: 1, 2, 4, 8,

Now,

$$\frac{2}{1} = 2, \quad \frac{4}{2} = 2, \quad \frac{8}{4} = 2$$

Since $\frac{2}{1} = \frac{4}{2} = \frac{8}{4} = \dots = 2$, the given sequence is a G.P.

with first term, $a = 1$ and common ratio, $r = 2$.

Now, $t_n = ar^{n-1}$

$$\Rightarrow t_n = 1 \times 2^{n-1} = 2^{n-1}$$

Question 7.

Find the next three terms of the sequence :

$\sqrt{5}, 5, 5\sqrt{5}, \dots$

Solution:

Given sequence: $\sqrt{5}, 5, 5\sqrt{5}, \dots$

Now,

$$\frac{5}{\sqrt{5}} = \sqrt{5}, \quad \frac{5\sqrt{5}}{5} = \sqrt{5}$$

Since $\frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \dots = \sqrt{5}$, the given sequence is a G.P.

with first term, $a = \sqrt{5}$ and common ratio, $r = \sqrt{5}$.

Now, $t_n = ar^{n-1}$

$\therefore$ Next three terms:

$$4^{\text{th}} \text{ term} = \sqrt{5} \times (\sqrt{5})^3 = \sqrt{5} \times 5\sqrt{5} = 25$$

$$5^{\text{th}} \text{ term} = \sqrt{5} \times (\sqrt{5})^4 = \sqrt{5} \times 25 = 25\sqrt{5}$$

$$6^{\text{th}} \text{ term} = \sqrt{5} \times (\sqrt{5})^5 = \sqrt{5} \times 25\sqrt{5} = 125$$

Question 8.

Find the sixth term of the series :

$2^2, 2^3, 2^4, \dots$

Solution:

Given sequence: $2^2, 2^3, 2^4, \dots$

Now,

$$\frac{2^3}{2^2} = 2, \quad \frac{2^4}{2^3} = 2$$

Since $\frac{2^3}{2^2} = \frac{2^4}{2^3} = \dots = 2$, the given sequence is a G.P.

with first term, $a = 2^2 = 4$ and common ratio, $r = 2$.

Now, $t_n = ar^{n-1}$

$$\therefore t_6 = 4 \times (2)^5 = 4 \times 32 = 128$$

Question 9.

Find the seventh term of the G.P. :

$[\text{late}] \sqrt{3}+1, 1, \frac{\sqrt{3}-1}{2} [\text{late}]$,

Solution:

Given G.P.: $\sqrt{3} + 1, 1, \frac{\sqrt{3}-1}{2}, \dots$

Here,

First term, $a = \sqrt{3} + 1$

Common ratio, $r = \frac{1}{\sqrt{3} + 1}$

Now, $t_n = ar^{n-1}$

$$\Rightarrow t_7 = (\sqrt{3} + 1) \times \left(\frac{1}{\sqrt{3} + 1} \right)^6$$

$$= \left(\frac{1}{\sqrt{3} + 1} \right)^5$$

$$= \left(\frac{1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} \right)^5$$

$$= \left(\frac{\sqrt{3} - 1}{2} \right)^5$$

$$= \frac{1}{32} (\sqrt{3} - 1)^5$$

Question 10.

Find the G.P. whose first term is 64 and next term is 32.

Solution:

First term, $a = 64$

Second term, $t_2 = 32$

$$\Rightarrow ar = 32$$

$$\Rightarrow 64 \times r = 32$$

$$\Rightarrow r = \frac{32}{64} = \frac{1}{2}$$

$\therefore$ Required G.P. = $a, ar, ar^{n-1}, ar^{n-2}, \dots$

$$\begin{aligned} &= 64, 32, 64 \times \left(\frac{1}{2}\right)^2, 64 \times \left(\frac{1}{2}\right)^3, \dots \\ &= 64, 32, 16, 8, \dots \end{aligned}$$

Question 11.

Find the next three terms of the series:

$$\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \dots$$

Solution:

Given sequence: $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \dots$

Now,

$$\frac{\frac{2}{9}}{\frac{2}{27}} = 3, \quad \frac{\frac{2}{3}}{\frac{2}{9}} = 3$$

Since $\frac{\frac{2}{9}}{\frac{2}{27}} = \frac{\frac{2}{3}}{\frac{2}{9}} = \dots = 3$, the given sequence is a G.P.

with first term, $a = \frac{2}{27}$ and common ratio, $r = 3$.

Now, $t_n = ar^{n-1}$

$\therefore$ Next three terms:

$$4^{\text{th}} \text{ term} = \frac{2}{27} \times (3)^3 = \frac{2}{27} \times 27 = 2$$

$$5^{\text{th}} \text{ term} = \frac{2}{27} \times (3)^4 = \frac{2}{27} \times 27 \times 3 = 6$$

$$6^{\text{th}} \text{ term} = \frac{2}{27} \times (3)^5 = \frac{2}{27} \times 27 \times 9 = 18$$

Question 12.

Find the next two terms of the series

$2 - 6 + 18 - 54 \dots\dots\dots$

Solution:

Given series: $2 - 6 + 18 - 54 \dots\dots\dots$

Now,

$$\frac{-6}{2} = -3, \quad \frac{18}{-6} = -3, \quad \frac{-54}{18} = -3$$

Since $\frac{-6}{2} = \frac{18}{-6} = \frac{-54}{18} = \dots\dots\dots = -3$, the given sequence is a G.P.

with first term, $a = 2$ and common ratio, $r = -3$.

Now, $t_n = ar^{n-1}$

$\therefore$ Next two terms:

$$5^{\text{th}} \text{ term} = 2 \times (-3)^4 = 2 \times 81 = 162$$

$$6^{\text{th}} \text{ term} = 2 \times (-3)^5 = 2 \times (-243) = -486$$

Exercise 11B

Question 1.

Which term of the G.P. :

$$-10, \frac{5}{\sqrt{3}}, -\frac{5}{6}, \dots\dots\dots \text{ is } -\frac{5}{72} ?$$

Solution:

For the given G.P.:

First term, $a = -10$

$$\text{Common ratio, } r = \frac{5/\sqrt{3}}{-10} = -\frac{1}{2\sqrt{3}}$$

If $-\frac{5}{72}$ is the n^{th} term of the given G.P., then

$$-\frac{5}{72} = ar^{n-1}$$

$$\Rightarrow -\frac{5}{72} = -10 \times \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow \frac{1}{144} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow \frac{1}{2 \times 2 \times 2 \times 2 \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3}} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow \left(\frac{1}{2\sqrt{3}}\right)^4 = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow n - 1 = 4$$

$$\Rightarrow n = 5$$

Question 2.

The fifth term of a G.P. is 81 and its second term is 24. Find the geometric progression.

Solution:

Let the first term of the G.P. be a and its common ratio be r .

$$5^{\text{th}} \text{ term} = 81 \Rightarrow ar^4 = 81$$

$$2^{\text{nd}} \text{ term} = 24 \Rightarrow ar = 24$$

$$\text{Now, } \frac{ar^4}{ar} = \frac{81}{24}$$

$$\Rightarrow r^3 = \frac{27}{8}$$

$$\Rightarrow r = \frac{3}{2}$$

$$ar = 24$$

$$\Rightarrow a = 16$$

$$\begin{aligned}\therefore \text{G.P.} &= a, ar, ar^2, ar^3, \dots \\ &= 16, 24, 16 \times \left(\frac{3}{2}\right)^2, 16 \times \left(\frac{3}{2}\right)^3, \dots \\ &= 16, 24, 36, 54, \dots\end{aligned}$$

Question 3.

Fourth and seventh terms of a G.P. are $\frac{1}{18}$ and $-\frac{1}{486}$ respectively. Find the GP.

Solution:

Let the first term of the G.P. be a and its common ratio be r .

$$4^{\text{th}} \text{ term} = \frac{1}{18} \Rightarrow ar^3 = \frac{1}{18}$$

$$7^{\text{th}} \text{ term} = -\frac{1}{486} \Rightarrow ar^6 = -\frac{1}{486}$$

$$\text{Now, } \frac{ar^6}{ar^3} = \frac{-\frac{1}{486}}{\frac{1}{18}}$$

$$\Rightarrow r^3 = -\frac{1}{27}$$

$$\Rightarrow r = -\frac{1}{3}$$

$$ar^3 = \frac{1}{18}$$

$$\Rightarrow a \times \left(-\frac{1}{3}\right)^3 = \frac{1}{18}$$

$$\Rightarrow a = -\frac{27}{18} = -\frac{3}{2}$$

$$\begin{aligned}\therefore \text{G.P.} &= a, ar, ar^2, ar^3, \dots \\ &= -\frac{3}{2}, -\frac{3}{2} \times \left(-\frac{1}{3}\right), -\frac{3}{2} \times \left(-\frac{1}{3}\right)^2, \frac{1}{18}, \dots \\ &= -\frac{3}{2}, \frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, \dots\end{aligned}$$

Question 4.

If the first and the third terms of a G.P. are 2 and 8 respectively, find its second term.

Solution:

Let the first term of the G.P. be a and its common ratio be r .

$$\therefore 1^{\text{st}} \text{ term} = a = 2$$

$$\text{And, } 3^{\text{rd}} \text{ term} = 8 \Rightarrow ar^2 = 8$$

$$\text{Now, } \frac{ar^2}{a} = \frac{8}{2}$$

$$\Rightarrow r^2 = 4$$

$$\Rightarrow r = \pm 2$$

When $a = 2$ and $r = 2$

$$2^{\text{nd}} \text{ term} = ar = 2 \times 2 = 4$$

When $a = 2$ and $r = -2$

$$2^{\text{nd}} \text{ term} = ar = 2 \times (-2) = -4$$

Question 5.

The product of 3rd and 8th terms of a G.P. is 243. If its 4th term is 3, find its 7th term.

Solution:

Let the first term of the G.P. be a and its common ratio be r .

Now,

$$t_3 \times t_8 = 243$$

$$\Rightarrow ar^2 \times ar^7 = 243$$

$$\Rightarrow a^2 r^9 = 243 \quad \dots (i)$$

Also,

$$t_4 = 3$$

$$\Rightarrow ar^3 = 3$$

$$\Rightarrow a = \frac{3}{r^3}$$

Substituting the value of a in (i), we get

$$\left(\frac{3}{r^3}\right)^2 \times r^9 = 243$$

$$\Rightarrow \frac{9}{r^6} \times r^9 = 243$$

$$\Rightarrow r^3 = 27$$

$$\Rightarrow r = 3$$

$$\Rightarrow a = \frac{3}{3^3} = \frac{3}{27} = \frac{1}{9}$$

$$\therefore 7^{\text{th}} \text{ term} = t_7 = ar^6 = \frac{1}{9} \times (3)^6 = 81$$

Question 6.

Find the geometric progression with 4th term = 54 and 7th term = 1458.

Solution:

Let the first term of the G.P. be a and its common ratio be r .

$$4^{\text{th}} \text{ term} = 54 \Rightarrow ar^3 = 54$$

$$7^{\text{th}} \text{ term} = 1458 \Rightarrow ar^6 = 1458$$

$$\text{Now, } \frac{ar^6}{ar^3} = \frac{1458}{54}$$

$$\Rightarrow r^3 = 27$$

$$\Rightarrow r = 3$$

$$ar^3 = 54$$

$$\Rightarrow a \times (3)^3 = 54$$

$$\Rightarrow a = \frac{54}{27} = 2$$

$$\begin{aligned} \therefore \text{G.P.} &= a, ar, ar^2, ar^3, \dots \\ &= 2, 2 \times 3, 2 \times (3)^2, 54, \dots \\ &= 2, 6, 18, 54, \dots \end{aligned}$$

Question 7.

Second term of a geometric progression is 6 and its fifth term is 9 times of its third term. Find the geometric progression. Consider that each term of the G.P. is positive.

Solution:

Let the first term of the G.P. be a and its common ratio be r .

$$\text{Now, } 2^{\text{nd}} \text{ term} = t_2 = 6 \Rightarrow ar = 6$$

$$\text{Also, } t_5 = 9 \times t_3$$

$$\Rightarrow ar^4 = 9 \times ar^2$$

$$\Rightarrow r^2 = 9$$

$$\Rightarrow r = \pm 3$$

Since, each term of a G.P. is positive, we have $r = 3$

$$ar = 6$$

$$\Rightarrow a \times 3 = 6 \Rightarrow a = 2$$

$$\begin{aligned}\therefore \text{G.P.} &= a, ar, ar^2, ar^3, \dots \\ &= 2, 6, 2 \times (3)^2, 2 \times (3)^3, \dots \\ &= 2, 6, 18, 54, \dots\end{aligned}$$

Question 8.

The fourth term, the seventh term and the last term of a geometric progression are 10, 80 and 2560 respectively. Find its first term, common ratio and number of terms.

Solution:

Let the first term of the G.P. be a and its common ratio be r .

Now,

$$4^{\text{th}} \text{ term} = t_4 = 10 \Rightarrow ar^3 = 10$$

$$7^{\text{th}} \text{ term} = t_7 = 80 \Rightarrow ar^6 = 80$$

$$\frac{ar^6}{ar^3} = \frac{80}{10}$$

$$\Rightarrow r^3 = 8$$

$$\Rightarrow r = 2$$

$$ar^3 = 10$$

$$\Rightarrow a \times (2)^3 = 10$$

$$\Rightarrow a = \frac{10}{8} = \frac{5}{4}$$

Last term = $l = 2560$

Let there be n terms in given G.P.

$$\Rightarrow t_n = 2560$$

$$\Rightarrow ar^{n-1} = 2560$$

$$\Rightarrow \frac{5}{4} \times (2)^{n-1} = 2560$$

$$\Rightarrow (2)^{n-1} = 2048$$

$$\Rightarrow (2)^{n-1} = (2)^{11}$$

$$\Rightarrow n - 1 = 11$$

$$\Rightarrow n = 12$$

Thus, we have

First term = $\frac{5}{4}$, Common ratio = 2 and Number of terms = 12

Question 9.

If the 4th and 9th terms of a G.P. are 54 and 13122 respectively, find the GP. Also, find its general term.

Solution:

Let the first term of the G.P. be a and its common ratio be r .

Now,

$$4^{\text{th}} \text{ term} = t_4 = 54 \Rightarrow ar^3 = 54$$

$$9^{\text{th}} \text{ term} = t_9 = 13122 \Rightarrow ar^8 = 13122$$

$$\frac{ar^8}{ar^3} = \frac{13122}{54}$$

$$\Rightarrow r^5 = 243$$

$$\Rightarrow r = 3$$

$$ar^3 = 54$$

$$\Rightarrow a \times (3)^3 = 54$$

$$\Rightarrow a = \frac{54}{27} = 2$$

$$\begin{aligned} \therefore \text{Required G.P.} &= a, ar, ar^2, ar^3, \dots \\ &= 2, 2 \times 3, 2 \times (3)^2, 54 \\ &= 2, 6, 18, 54 \end{aligned}$$

$$\text{General term} = t_n = ar^{n-1} = 2 \times (3)^{n-1}$$

Question 10.

The fifth, eight and eleventh terms of a geometric progression are p, q and r respectively. Show that : $q^2 = pr$.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

$$5^{\text{th}} \text{ term} = t_5 = p$$

$$\Rightarrow ar^4 = p$$

$$8^{\text{th}} \text{ term} = t_8 = q$$

$$\Rightarrow ar^7 = q$$

$$11^{\text{th}} \text{ term} = t_{11} = r$$

$$\Rightarrow ar^{10} = r$$

Now,

$$pr = ar^4 \times ar^{10} = a^2 \times r^{14} = (a \times r^7)^2 = q^2$$

$$\Rightarrow q^2 = pr$$

Exercise 11C**Question 1.**

Find the seventh term from the end of the series : $\sqrt{2}, 2, 2\sqrt{2}, \dots, 32$.

Solution:

Given series: $\sqrt{2}, 2, 2\sqrt{2}, \dots, 32$

$$\text{Now, } \frac{2}{\sqrt{2}} = \sqrt{2}, \quad \frac{2\sqrt{2}}{2} = \sqrt{2}$$

So, the given series is a G.P. with common ratio, $r = \sqrt{2}$

Here, last term, $l = 32$

$$\therefore 7^{\text{th}} \text{ term from an end} = \frac{l}{r^6} = \frac{32}{(\sqrt{2})^6} = \frac{32}{8} = 4$$

Question 2.

Find the third term from the end of the GP.

$$\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \dots, 162$$

Solution:

$$\text{Given G.P.: } \frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \dots, 162$$

Here,

$$\text{Common ratio, } r = \frac{\frac{2}{9}}{\frac{2}{27}} = 3$$

$$\text{Last term, } l = 162$$

$$\therefore 3^{\text{rd}} \text{ term from an end} = \frac{l}{r^2} = \frac{162}{(3)^2} = \frac{162}{9} = 18$$

Question 3.

For the $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \dots, 81$;

find the product of fourth term from the beginning and the fourth term from the end.

Solution:

$$\text{Given G.P.: } \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \dots, 81$$

Here,

$$\text{Common ratio, } r = \frac{\frac{1}{9}}{\frac{1}{27}} = 3$$

$$\text{First term, } a = \frac{1}{27} \text{ and Last term, } l = 81$$

$$\therefore 4^{\text{th}} \text{ term from the beginning} = ar^3 = \frac{1}{27} \times (3)^3 = \frac{1}{27} \times 27 = 1$$

$$\text{And, } 4^{\text{th}} \text{ term from an end} = \frac{l}{r^3} = \frac{81}{(3)^3} = \frac{81}{27} = 3$$

$$\text{Thus, required product} = 1 \times 3 = 3$$

Question 4.

If for a G.P., p^{th} , q^{th} and r^{th} terms are a , b and c respectively ; prove that :
 $(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$

Solution:

Let the first term of the G.P. be A and its common ratio be R .

Then,

$$p^{\text{th}} \text{ term} = a \Rightarrow AR^{p-1} = a$$

$$q^{\text{th}} \text{ term} = b \Rightarrow AR^{q-1} = b$$

$$r^{\text{th}} \text{ term} = c \Rightarrow AR^{r-1} = c$$

Now,

$$\begin{aligned} a^{q-r} \times b^{r-p} \times c^{p-q} &= (AR^{p-1})^{q-r} \times (AR^{q-1})^{r-p} \times (AR^{r-1})^{p-q} \\ &= A^{q-r} \cdot R^{(p-1)(q-r)} \times A^{r-p} \cdot R^{(q-1)(r-p)} \times A^{p-q} \cdot R^{(r-1)(p-q)} \\ &= A^{q-r+r-p+p-q} \times R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)} \\ &= A^0 \times R^0 \\ &= 1 \end{aligned}$$

Taking log on both the sides, we get

$$\log(a^{q-r} \times b^{r-p} \times c^{p-q}) = \log 1$$

$$\Rightarrow (q - r) \log a + (r - p) \log b + (p - q) \log c = 0 \quad \dots(\text{proved})$$

Question 5.

If a , b and c in G.P., prove that : $\log a^n$, $\log b^n$ and $\log c^n$ are in A.P.

Solution:

Here, a , b , c are in G.P.

$$\Rightarrow b^2 = ac$$

Taking log on both sides, we get

$$\log(b^2) = \log(ac)$$

$$\Rightarrow 2 \log b = \log a + \log c$$

$$\Rightarrow \log b + \log b = \log a + \log c$$

$$\Rightarrow \log b - \log a = \log c - \log b$$

$$\Rightarrow \log a, \log b \text{ and } \log c \text{ are in A.P.}$$

Question 6.

If each term of a G.P. is raised to the power x , show that the resulting sequence is also a G.P.

Solution:

Let $a_1, a_2, a_3, \dots, a_n, \dots$ be a G.P. with common ratio r .

$$\Rightarrow \frac{a_{n+1}}{a_n} = r \text{ for all } n \in \mathbb{N}$$

If each term of a G.P. is raised to the power x , we get the sequence

$$a_1^x, a_2^x, a_3^x, \dots, a_n^x, \dots$$

$$\text{Now, } \frac{(a_{n+1})^x}{(a_n)^x} = \left(\frac{a_{n+1}}{a_n} \right)^x = r^x \text{ for all } n \in \mathbb{N}$$

Hence, $a_1^x, a_2^x, a_3^x, \dots, a_n^x, \dots$ is also a G.P.

Question 7.

If a, b and c are in A.P. a, x, b are in G.P. whereas b, y and c are also in G.P. Show that : x^2, b^2, y^2 are in A.P.

Solution:

a, b and c are in A.P.

$$\Rightarrow 2b = a + c$$

a, x and b are in G.P.

$$\Rightarrow x^2 = ab$$

b, y and c are in G.P.

$$\Rightarrow y^2 = bc$$

Now,

$$x^2 + y^2 = ab + bc$$

$$= b(a + c)$$

$$= b \times 2b$$

$$= 2b^2$$

$$\Rightarrow x^2, b^2 \text{ and } y^2 \text{ are in A.P.}$$

Question 8.

If a, b, c are in G.P. and a, x, b, y, c are in A.P., prove that :

$$(i) \frac{1}{x} + \frac{1}{y} = \frac{2}{b} \quad (ii) \frac{a}{x} + \frac{c}{y} = 2$$

Solution 8(i).

a, b and c are in G.P.

$$\Rightarrow b^2 = ac$$

a, x, b, y and c are in A.P.

$$\Rightarrow 2x = a + b \Rightarrow x = \frac{a+b}{2}$$

$$2b = x + y \Rightarrow b = \frac{x+y}{2}$$

$$2y = b + c \Rightarrow y = \frac{b+c}{2}$$

Now,

$$\begin{aligned} \frac{1}{x} + \frac{1}{y} &= \frac{2}{a+b} + \frac{2}{b+c} \\ &= \frac{2b + 2c + 2a + 2b}{ab + ac + b^2 + bc} \\ &= \frac{2a + 2c + 4b}{ab + b^2 + b^2 + bc} \\ &= \frac{2a + 2c + 4b}{ab + 2b^2 + bc} \\ &= \frac{2(a + c + 2b)}{b(a + 2b + c)} \\ &= \frac{2}{b} \end{aligned}$$

Solution 8(ii).

a, b and c are in G.P.

$$\Rightarrow b^2 = ac$$

a, x, b, y and c are in A.P.

$$\Rightarrow 2x = a + b \Rightarrow x = \frac{a+b}{2}$$

$$2b = x + y \Rightarrow b = \frac{x+y}{2}$$

$$2y = b + c \Rightarrow y = \frac{b+c}{2}$$

Now,

$$\begin{aligned}\frac{a}{x} + \frac{c}{y} &= \frac{2a}{a+b} + \frac{2c}{b+c} \\&= \frac{2a(b+c) + 2c(a+b)}{(a+b)(b+c)} \\&= \frac{2ab + 2ac + 2ac + 2bc}{ab + ac + b^2 + bc} \\&= \frac{2ab + 4ac + 2bc}{ab + b^2 + b^2 + bc} \\&= \frac{2(ab + 2ac + bc)}{ab + 2b^2 + bc} \\&= \frac{2(ab + 2ac + bc)}{ab + 2ac + bc} \\&= 2\end{aligned}$$

Question 9.

If a , b and c are in A.P. and also in G.P., show that: $a = b = c$.

Solution:

a , b and c are in A.P.

$$\Rightarrow 2b = a + c$$

$$\Rightarrow b = \frac{a+c}{2}$$

a , b and c are also in G.P.

$$\Rightarrow b^2 = ac$$

$$\Rightarrow \left(\frac{a+c}{2}\right)^2 = ac$$

$$\Rightarrow \frac{a^2 + c^2 + 2ac}{4} = ac$$

$$\Rightarrow a^2 + c^2 + 2ac = 4ac$$

$$\Rightarrow a^2 + c^2 - 2ac = 0$$

$$\Rightarrow (a-c)^2 = 0$$

$$\Rightarrow a - c = 0$$

$$\Rightarrow a = c$$

Now, $2b = a + c$

$$\Rightarrow 2b = a + a$$

$$\Rightarrow 2b = 2a$$

$$\Rightarrow b = a$$

Thus, we have $a = b = c$

Question 10.

The first term of a G.P. is a and its n^{th} term is b , where n is an even number. If the product of first n numbers of this G.P. is P ; prove that : $p^2 = (ab)^n$.

Solution:

For a G.P.,

First term = a

Let the common ratio = r

n^{th} term = b

$$\Rightarrow ar^{n-1} = b$$

P = Product of first n numbers of the given G.P.

$$\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \dots \times ar^{n-1}$$

$$\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \dots \times b$$

$$\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \dots \times \frac{b}{r^2} \times \frac{b}{r} \times b$$

$$\Rightarrow P = (ab) \times \left(ar \times \frac{b}{r} \right) \times \left(ar^2 \times \frac{b}{r^2} \right) \times \dots \times \frac{n}{2} \text{ terms}$$

$$\Rightarrow P = (ab) \times (ab) \times (ab) \times \dots \times \frac{n}{2} \text{ terms}$$

$$\Rightarrow P = (ab)^{\frac{n}{2}}$$

$$\Rightarrow P = \sqrt{ab^n}$$

$$\Rightarrow p^2 = ab^n$$

Question 11.

If a, b, c and d are consecutive terms of a G.P. ; prove that : $(a^2 + b^2), (b^2 + c^2)$ and $(c^2 + d^2)$ are in GP.

Solution:

Let r be the common ratio of this G.P.

Given: a, b, c, d are in G.P.

$$\Rightarrow 1^{\text{st}} = a,$$

$$2^{\text{nd}} \text{ term} = b = ar,$$

$$3^{\text{rd}} \text{ term} = c = ar^2$$

$$4^{\text{th}} \text{ term} = d = ar^3$$

$$\begin{aligned}\text{Now, } (b^2 + c^2)^2 &= [(ar)^2 + (ar^2)^2]^2 \\ &= [a^2r^2 + a^2r^4]^2 \\ &= [a^2r^2(1 + r^2)]^2 \\ &= a^4r^4(1 + r^2)^2\end{aligned}$$

$$\begin{aligned}\text{And, } (a^2 + b^2) \times (c^2 + d^2) &= [a^2 + (ar)^2] \times [(ar^2)^2 + (ar^3)^2] \\ &= [a^2 + a^2r^2] \times [a^2r^4 + a^2r^6] \\ &= a^2(1 + r^2) \times a^2r^4(1 + r^2) \\ &= a^4r^4(1 + r^2)^2\end{aligned}$$

$$\Rightarrow (b^2 + c^2)^2 = (a^2 + b^2) \times (c^2 + d^2)$$

$$\text{i.e. } \frac{b^2 + c^2}{a^2 + b^2} = \frac{c^2 + d^2}{b^2 + c^2}$$

Hence, $(a^2 + b^2)$, $(b^2 + c^2)$ and $(c^2 + d^2)$ are in G.P.

Question 12.

If a, b, c and d are consecutive terms of a G.P. To prove:

$\frac{1}{a^2 + b^2}$, $\frac{1}{b^2 + c^2}$ and $\frac{1}{c^2 + d^2}$ are in G.P.

Solution:

Let r be the common ratio of this G.P.

Given: a, b, c, d are in G.P.

$$\Rightarrow 1^{\text{st}} = a,$$

$$2^{\text{nd}} \text{ term} = b = ar,$$

$$3^{\text{rd}} \text{ term} = c = ar^2$$

$$4^{\text{th}} \text{ term} = d = ar^3$$

$$\begin{aligned}\text{Now, } \left(\frac{1}{b^2 + c^2} \right)^2 &= \left[\frac{1}{(ar)^2 + (ar^2)^2} \right]^2 \\ &= \left[\frac{1}{a^2r^2 + a^2r^4} \right]^2 \\ &= \frac{1}{a^4r^4} \left[\frac{1}{1+r^2} \right]^2 \\ &= \frac{1}{a^4r^4} \times \frac{1}{(1+r^2)^2}\end{aligned}$$

$$\begin{aligned}\text{And, } \left(\frac{1}{a^2 + b^2} \right) \times \left(\frac{1}{c^2 + d^2} \right) &= \left[\frac{1}{a^2 + (ar)^2} \right] \times \left[\frac{1}{(ar^2)^2 + (ar^3)^2} \right] \\ &= \left[\frac{1}{a^2 + a^2r^2} \right] \times \left[\frac{1}{a^2r^4 + a^2r^6} \right] \\ &= \frac{1}{a^2} \left(\frac{1}{1+r^2} \right) \times \frac{1}{a^2r^4} \left(\frac{1}{1+r^2} \right) \\ &= \frac{1}{a^4r^4} \times \frac{1}{(1+r^2)^2}\end{aligned}$$

$$\Rightarrow \left(\frac{1}{b^2 + c^2} \right)^2 = \left(\frac{1}{a^2 + b^2} \right) \times \left(\frac{1}{c^2 + d^2} \right)$$

Hence, $\frac{1}{a^2 + b^2}, \frac{1}{b^2 + c^2}$ and $\frac{1}{c^2 + d^2}$ are in G.P.

Exercise 11D**Question 1.**

Find the sum of G.P. :

(i) $1 + 3 + 9 + 27 + \dots$ to 12 terms.

(ii) $0.3 + 0.03 + 0.003 + 0.0003 + \dots$ to 8 terms.

(iii) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$ to 9 terms.

(iv) $1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots$ to n terms.

(v) $\frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \dots$ upto n terms.

(vi) $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \dots$ to n terms.

Solution 1(i).

Given GP. : $1 + 3 + 9 + 27 + \dots$

Here,

first term, $a = 1$

common ratio, $r = \frac{3}{1} = 3$ ($r > 1$)

number of terms to be added, $n = 12$

$$\therefore S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_{12} = \frac{1(3^{12} - 1)}{3 - 1} = \frac{3^{12} - 1}{2} = \frac{531441 - 1}{2} = \frac{531440}{2} = 265720$$

Solution 1(ii).

Given GP. : $0.3 + 0.03 + 0.003 + 0.0003 + \dots$

Here,

first term, $a = 0.3$

common ratio, $r = \frac{0.03}{0.3} = 0.1$ ($r < 1$)

number of terms to be added, $n = 8$

$$\therefore S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\Rightarrow S_8 = \frac{0.3(1 - (0.1)^8)}{1 - 0.1} = \frac{0.3(1 - (0.1)^8)}{0.9} = \frac{1 - (0.1)^8}{3} = \frac{1}{3} \left(1 - \frac{1}{10^8} \right)$$

Solution 1(iii).

Given GP.: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$

Here,

first term, $a = 1$

common ratio, $r = \frac{-1/2}{1} = -\frac{1}{2}$ ($r < 1$)

number of terms to be added, $n = 9$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_8 = \frac{1 \left(1 - \left(-\frac{1}{2} \right)^9 \right)}{1 - \left(-\frac{1}{2} \right)}$$

$$= \frac{1 - \left(-\frac{1}{2} \right)^9}{1 + \frac{1}{2}}$$

$$= \frac{1 + \frac{1}{2^9}}{\frac{3}{2}}$$

$$= \frac{2}{3} \left(1 + \frac{1}{2^9} \right)$$

$$= \frac{2}{3} \left(1 + \frac{1}{512} \right)$$

$$= \frac{2}{3} \times \frac{513}{512}$$

$$= \frac{171}{256}$$

Solution 1(iv).

Given G.P.: $1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots$ upto n terms

Here,

first term, $a = 1$

common ratio, $r = \frac{-\frac{1}{3}}{1} = -\frac{1}{3}$ ($r < 1$)

number of terms to be added = n

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n = \frac{1 \left(1 - \left(-\frac{1}{3} \right)^n \right)}{1 - \left(-\frac{1}{3} \right)}$$

$$= \frac{1 \left(1 - \left(-\frac{1}{3} \right)^n \right)}{1 + \frac{1}{3}}$$

$$= \frac{\left[1 - \left(-\frac{1}{3} \right)^n \right]}{\frac{4}{3}}$$

$$= \frac{3}{4} \left[1 - \left(-\frac{1}{3} \right)^n \right]$$

Solution 1(v).

Given G.P.: $\frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \dots$ upto n terms

Here,

$$\text{first term, } a = \frac{x+y}{x-y}$$

$$\text{common ratio, } r = \frac{1}{\frac{x+y}{x-y}} = \frac{x-y}{x+y} \quad (r < 1)$$

number of terms to be added = n

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n = \frac{\frac{x+y}{x-y} \left(1 - \left(\frac{x-y}{x+y} \right)^n \right)}{1 - \left(\frac{x-y}{x+y} \right)}$$

$$= \frac{\frac{x+y}{x-y} \left(1 - \left(\frac{x-y}{x+y} \right)^n \right)}{\frac{x+y - x+y}{x+y}}$$

$$= \frac{\frac{x+y}{x-y} \left(1 - \left(\frac{x-y}{x+y} \right)^n \right)}{\frac{2y}{x+y}}$$

$$= \frac{(x+y)^2 \left(1 - \left(\frac{x-y}{x+y} \right)^n \right)}{2y(x-y)}$$

Solution 1(vi).

Given G.P.: $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \dots$ upto n terms

Here,

first term, $a = \sqrt{3}$

common ratio, $r = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}$ ($r < 1$)

number of terms to be added = n

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n = \frac{\sqrt{3} \left(1 - \left(\frac{1}{3} \right)^n \right)}{1 - \frac{1}{3}}$$

$$= \frac{\sqrt{3} \left(1 - \frac{1}{3^n} \right)}{\frac{2}{3}}$$

$$= \frac{3\sqrt{3}}{2} \left(1 - \frac{1}{3^n} \right)$$

Question 2.

How many terms of the geometric progression $1+4+16+64+\dots$ must be added to get sum equal to 5461?

Solution:

Given G.P.: $1 + 4 + 16 + 64 + \dots$

Here,

first term, $a = 1$

common ratio, $r = \frac{4}{1} = 4$ ($r > 1$)

Let the number of terms to be added = n

Then, $S_n = 5461$

$$\Rightarrow \frac{a(r^n - 1)}{r - 1} = 5461$$

$$\Rightarrow \frac{1(4^n - 1)}{4 - 1} = 5461$$

$$\Rightarrow \frac{4^n - 1}{3} = 5461$$

$$\Rightarrow 4^n - 1 = 16383$$

$$\Rightarrow 4^n = 16384$$

$$\Rightarrow 4^n = 4^7$$

$$\Rightarrow n = 7$$

Hence, required number of terms = 7

Question 3.

The first term of a G.P. is 27 and its 8th term is $\frac{1}{81}$. Find the sum of its first 10 terms.

Solution:

Given,

First term, $a = 27$

$$8^{\text{th}} \text{ term} = ar^7 = \frac{1}{81}$$

$n = 10$

Now,

$$\frac{ar^7}{a} = \frac{1/81}{27}$$

$$\Rightarrow r^7 = \frac{1}{2187}$$

$$\Rightarrow r^7 = \left(\frac{1}{3}\right)^7$$

$$\Rightarrow r = \frac{1}{3} \quad (r < 1)$$

$$\therefore S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\Rightarrow S_{10} = \frac{27 \left(1 - \left(\frac{1}{3} \right)^{10} \right)}{1 - \frac{1}{3}}$$

$$= \frac{27 \left(1 - \frac{1}{3^{10}} \right)}{\frac{2}{3}}$$

$$\frac{81}{2} \left(1 - \frac{1}{3^{10}} \right)$$

Question 4.

A boy spends ₹ 10 on first day, ₹ 20 on second day, ₹ 40 on third day and so on. Find how much, in all, will he spend in 12 days?

Solution:

Amount spent on 1st day = Rs. 10

Amount spent on 2nd day = Rs. 20

Amount spent on 3rd day = Rs. 40 and so on

$$\text{Now, } \frac{20}{10} = 2, \frac{40}{20} = 2,$$

Thus, 10, 20, 40, is a G.P. with first term, $a = 10$

and common ratio, $r = 2$ ($r > 1$)

∴ Total amount spent in 12 days = S_{12}

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_{12} = \frac{10(2^{12} - 1)}{2 - 1} = 10(2^{12} - 1) = 10(4096 - 1) = 10 \times 4095 = 40950$$

Hence, the total amount spent in 12 days is Rs. 40950.

Question 5.

The 4th and the 7th terms of a G.P. are $\frac{1}{27}$ and $\frac{1}{729}$ respectively. Find the sum of n terms of this G.P.

Solution:

For a G.P.,

$$4^{\text{th}} \text{ term} = ar^3 = \frac{1}{27}$$

$$7^{\text{th}} \text{ term} = ar^6 = \frac{1}{729}$$

$$\text{Now, } \frac{ar^6}{ar^3} = \frac{\frac{1}{729}}{\frac{1}{27}}$$

$$\Rightarrow r^3 = \frac{1}{27} = \left(\frac{1}{3}\right)^3$$

$$\Rightarrow r = \frac{1}{3} \quad (r < 1)$$

$$\Rightarrow a \times \frac{1}{27} = \frac{1}{27}$$

$$\Rightarrow a = 1$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n = \frac{1 \left(1 - \left(\frac{1}{3} \right)^n \right)}{1 - \frac{1}{3}}$$

$$= \frac{\left(1 - \frac{1}{3^n} \right)}{\frac{2}{3}}$$

$$= \frac{3}{2} \left(1 - \frac{1}{3^n} \right)$$

Question 6.

A geometric progression has common ratio = 3 and last term = 486. If the sum of its terms is 728 ; find its first term.

Solution:

For a G.P.,

Common ratio, $r = 3$ ($r > 1$)

Last term, $l = 486$

$S = 728$

$$\Rightarrow \frac{l r - a}{r - 1} = 728$$

$$\Rightarrow \frac{486 \times 3 - a}{3 - 1} = 728$$

$$\Rightarrow \frac{1458 - a}{2} = 728$$

$$\Rightarrow 1458 - a = 1456$$

Hence, the first term is 2.

Question 7.

Find the sum of G.P. : 3, 6, 12, 1536.

Solution:

Given G.P. : 3, 6, 12,, 1536

Here,

First term, $a = 3$

Common ratio, $r = \frac{6}{3} = 2$ ($r > 1$)

Last term, $l = 1536$

$$\begin{aligned}\therefore \text{ Required sum} &= \frac{lr - a}{r - 1} \\ &= \frac{1536 \times 2 - 3}{2 - 1} \\ &= 3072 - 3 \\ &= 3069\end{aligned}$$

Question 8.

How many terms of the series $2 + 6 + 18 + \dots$ must be taken to make the sum equal to 728 ?

Solution:

Given series: $2 + 6 + 18 + \dots$

$$\text{Now, } \frac{6}{2} = 3, \frac{18}{6} = 3$$

Thus, given series is a G.P. with first term, $a = 2$
and common ratio, $r = 3$ ($r > 1$)

Let the number of terms to be added = n

$$\text{Then, } S_n = 728$$

$$\Rightarrow \frac{a(r^n - 1)}{r - 1} = 728$$

$$\Rightarrow \frac{2(3^n - 1)}{3 - 1} = 728$$

$$\Rightarrow 3^n - 1 = 728$$

$$\Rightarrow 3^n = 729$$

$$\Rightarrow 3^n = 3^6$$

$$\Rightarrow n = 6$$

Hence, required number of terms = 6

Question 9.

In a G.P., the ratio between the sum of first three terms and that of the first six terms is 125 : 152.

Find its common ratio.

Solution:

Let a be the first term and r be the common ratio of given G.P.

$$\text{Now, sum of first three terms} = S_3 = \frac{a(r^3 - 1)}{r - 1}$$

$$\text{Now, sum of first six terms} = S_6 = \frac{a(r^6 - 1)}{r - 1}$$

It is given that

$$\frac{\frac{a(r^3 - 1)}{r - 1}}{\frac{a(r^6 - 1)}{r - 1}} = \frac{125}{152}$$

$$\Rightarrow \frac{r^3 - 1}{r^6 - 1} = \frac{125}{152}$$

$$\Rightarrow \frac{r^3 - 1}{(r^3)^2 - (1)^2} = \frac{125}{152}$$

$$\Rightarrow \frac{r^3 - 1}{(r^3 - 1)(r^3 + 1)} = \frac{125}{152}$$

$$\Rightarrow \frac{1}{r^3 + 1} = \frac{125}{152}$$

$$\Rightarrow r^3 + 1 = \frac{152}{125}$$

$$\Rightarrow r^3 = \frac{152}{125} - 1 = \frac{152 - 125}{125} = \frac{27}{125}$$

$$\Rightarrow r^3 = \left(\frac{3}{5}\right)^3$$

$$\Rightarrow r = \frac{3}{5}$$

Hence, the common ratio is $\frac{3}{5}$.

Question 10.

Find how many terms of G.P. $\frac{2}{9} - \frac{1}{3} + \frac{1}{2} \dots\dots\dots$ must be added to get the sum equal to $\frac{55}{72}$?

Solution:

Given G.P.: $\frac{2}{9} - \frac{1}{3} + \frac{1}{2} \dots\dots\dots$

Here,

First term, $a = \frac{2}{9}$

Common ratio, $r = \frac{-1/3}{2/9} = -\frac{3}{2} < 1$

Let required number of terms be n .

$$\Rightarrow S_n = \frac{55}{72}$$

$$\Rightarrow \frac{a(1-r^n)}{1-r} = \frac{55}{72}$$

$$\Rightarrow \frac{\frac{2}{9}\left(1 - \left(-\frac{3}{2}\right)^n\right)}{1 - \left(-\frac{3}{2}\right)} = \frac{55}{72}$$

$$\Rightarrow \frac{\frac{2}{9}\left(1 - \left(-\frac{3}{2}\right)^n\right)}{\frac{5}{2}} = \frac{55}{72}$$

$$\Rightarrow \frac{2}{9}\left(1 - \left(-\frac{3}{2}\right)^n\right) = \frac{55}{72} \times \frac{5}{2}$$

$$\Rightarrow 1 - \left(-\frac{3}{2}\right)^n = \frac{55}{72} \times \frac{5}{2} \times \frac{9}{2}$$

$$\Rightarrow 1 - \left(-\frac{3}{2}\right)^n = \frac{275}{32}$$

$$\Rightarrow 1 - \frac{275}{32} = \left(-\frac{3}{2}\right)^n$$

$$\Rightarrow -\frac{243}{32} = \left(-\frac{3}{2}\right)^n$$

$$\Rightarrow \left(-\frac{3}{2}\right)^5 = \left(-\frac{3}{2}\right)^n$$

$$\Rightarrow n = 5$$

$\therefore$ Required number of terms = 5

Question 11.

If the sum $1 + 2 + 2^2 + \dots + 2^{n-1}$ is 255, find the value of n .

Solution:

Required series: $1 + 2 + 2^2 + \dots + 2^{n-1}$

$$\text{Now, } \frac{2}{1} = 2, \frac{2^2}{2} = 2$$

Thus, given series is a G.P. with

first term, $a = 1$

common ratio, $r = 2$ ($r > 1$)

Last term, $l = 2^{n-1}$

Let there be n terms in the series.

Then, $S_n = 255$

$$\Rightarrow \frac{l r - a}{r - 1} = 255$$

$$\Rightarrow \frac{2^{n-1} \times 2 - 1}{2 - 1} = 255$$

$$\Rightarrow 2^{n-1} \times 2 - 1 = 255$$

$$\Rightarrow 2^{n-1} \times 2 = 256$$

$$\Rightarrow 2^{n-1} = 128$$

$$\Rightarrow 2^{n-1} = 2^7$$

$$\Rightarrow n - 1 = 7$$

$$\Rightarrow n = 8$$

Question 12.

Find the geometric mean between :

(i) $\frac{4}{9}$ and $\frac{9}{4}$

(ii) 14 and $\frac{7}{32}$

(iii) $2a$ and $8a^3$

Solution 12(i).

$$\text{Geometric mean between } \frac{4}{9} \text{ and } \frac{9}{4} = \sqrt{\frac{4}{9} \times \frac{9}{4}} = \sqrt{1} = 1$$

Solution 12(ii).

$$\text{Geometric mean between } 14 \text{ and } \frac{7}{32} = \sqrt{14 \times \frac{7}{32}} = \sqrt{\frac{49}{16}} = \frac{7}{4} = 1\frac{3}{4}$$

Solution 12(iii).

Geometric mean between $2a$ and $8a^3 = \sqrt{2a \times 8a^3} = \sqrt{16 \times a^4} = 4a^2$

Question 13.

The sum of three numbers in G.P. is $\frac{39}{10}$ and their product is 1. Find the numbers.

Solution:

Let the numbers be $\frac{a}{r}$, a and ar .

$$\Rightarrow \frac{a}{r} \times a \times ar = 1$$

$$\Rightarrow a^3 = 1$$

$$\Rightarrow a = 1$$

$$\text{Now, } \frac{a}{r} + a + ar = \frac{39}{10}$$

$$\Rightarrow \frac{1}{r} + 1 + r = \frac{39}{10}$$

$$\Rightarrow \frac{1+r+r^2}{r} = \frac{39}{10}$$

$$\Rightarrow 10 + 10r + 10r^2 = 39r$$

$$\Rightarrow 10r^2 - 29r + 10 = 0$$

$$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$$

$$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$$

$$\Rightarrow (2r - 5)(5r - 2) = 0$$

$$\Rightarrow r = \frac{5}{2} \text{ or } r = \frac{2}{5}$$

Thus, required terms are:

$$\begin{aligned} \frac{a}{r}, a, ar &= \frac{1}{\frac{5}{2}}, 1, 1 \times \frac{5}{2} \quad \text{OR} \quad \frac{1}{\frac{2}{5}}, 1, 1 \times \frac{2}{5} \\ &= \frac{2}{5}, 1, \frac{5}{2} \quad \text{OR} \quad \frac{5}{2}, 1, \frac{2}{5} \end{aligned}$$

Question 14.

The first term of a G.P. is -3 and the square of the second term is equal to its 4th term. Find its 7th term.

Solution:

For a G.P.,

First term, $a = -3$

It is given that,

$$(2^{\text{nd}} \text{ term})^2 = 4^{\text{th}} \text{ term}$$

$$\Rightarrow (ar)^2 = ar^3$$

$$\Rightarrow a^2 r^2 = ar^3$$

$$\Rightarrow a = r$$

$$\Rightarrow r = -3$$

$$\text{Now, } 7^{\text{th}} \text{ term} = ar^6 = -3 \times (-3)^6 = -3 \times 729 = -2187$$

Question 15.

Find the 5th term of the G.P. $\frac{5}{2}, 1, \dots$

Solution:

$$\text{First term (a)} = \frac{5}{2}$$

$$\text{And, common ratio (r)} = \frac{1}{\frac{5}{2}} = \frac{2}{5}$$

$$\text{Now, } t_n = ar^{n-1}$$

$$\Rightarrow 5^{\text{th}} \text{ term} = t_5 = \frac{5}{2} \times \left(\frac{5}{2}\right)^{5-1} = \frac{5}{2} \times \left(\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$$

Question 16.

The first two terms of a G.P. are 125 and 25 respectively. Find the 5th and the 6th terms of the G.P.

Solution:

$$\text{First term (a)} = 125$$

$$\text{And, common ratio (r)} = \frac{25}{125} = \frac{1}{5}$$

$$\text{Now, } t_n = ar^{n-1}$$

$$\Rightarrow 5^{\text{th}} \text{ term} = t_5 = 125 \times \left(\frac{1}{5}\right)^{5-1} = 125 \times \left(\frac{1}{5}\right)^4 = 125 \times \frac{1}{625} = \frac{1}{5}$$

$$\Rightarrow 6^{\text{th}} \text{ term} = t_6 = 125 \times \left(\frac{1}{5}\right)^{6-1} = 125 \times \left(\frac{1}{5}\right)^5 = 125 \times \frac{1}{3125} = \frac{1}{25}$$

Question 17.

Find the sum of the sequence $-\frac{1}{3}, 1, -3, 9, \dots$ upto 8 terms.

Solution:

Here,

$$\frac{1}{-\frac{1}{3}} = \frac{-3}{1} = \frac{9}{-3} = -3$$

Thus, the given sequence is a G.P. with first term $(a) = -\frac{1}{3}$ and common ratio $(r) = -3$ ($r < 1$).

Number of terms to be added, $n = 8$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_8 = \frac{-\frac{1}{3}(1-(-3)^8)}{1+3} = \frac{-1+3^8}{12} = \frac{1}{12}(3^8-1)$$

Question 18.

The first term of a G.P. is 27. If the 8th term be $\frac{1}{81}$, what will be the sum of 10 terms?

Solution:

Given,

First term, $a = 27$

$$8^{\text{th}} \text{ term} = ar^7 = \frac{1}{81}$$

$n = 10$

Now,

$$\frac{ar^7}{a} = \frac{1/81}{27}$$

$$\Rightarrow r^7 = \frac{1}{2187}$$

$$\Rightarrow r^7 = \left(\frac{1}{3}\right)^7$$

$$\Rightarrow r = \frac{1}{3} \quad (r < 1)$$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\begin{aligned}
 \Rightarrow S_{10} &= \frac{27 \left(1 - \left(\frac{1}{3} \right)^{10} \right)}{1 - \frac{1}{3}} \\
 &= \frac{27 \left(1 - \frac{1}{3^{10}} \right)}{\frac{2}{3}} \\
 &= \frac{81}{2} \left(1 - \frac{1}{3^{10}} \right) \\
 &= \frac{81}{2} (1 - 3^{-10})
 \end{aligned}$$

Question 19.

Find a G.P. for which the sum of first two terms is -4 and the fifth term is 4 times the third term.

Solution:

Let the five terms of the given G.P. be

$$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$$

Given, sum of first two terms = -4

$$\frac{a}{r^2} + \frac{a}{r} = -4$$

$$\Rightarrow \frac{a + ar}{r^2} = -4$$

$$\Rightarrow a + ar = -4r^2$$

$$\Rightarrow a(1 + r) = -4r^2$$

$$\Rightarrow a = -\frac{4r^2}{1+r}$$

And, 5th term = 4(3rd term)

$$\Rightarrow ar^2 = 4(a)$$

$$\Rightarrow r^2 = 4$$

$$\Rightarrow r = \pm 2$$

When $r = +2$,

$$a = -\frac{4(2)^2}{1+2} = -\frac{16}{3}$$

When $r = -2$,

$$a = -\frac{4(-2)^2}{1-2} = 16$$

Thus, the required terms are $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

$$\text{i.e. } -\frac{16}{4}, -\frac{16}{2}, -\frac{16}{3}, -\frac{16}{3} \times 2, -\frac{16}{3} \times 4 \text{ OR } \frac{16}{4}, \frac{16}{-2}, 16, 16(-2), 16 \times 4$$

$$\text{i.e. } -\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}, -\frac{32}{3}, -\frac{64}{3} \text{ OR } 4, -8, 16, -32, 64$$

Additional Questions

Question 1.

Find the sum of n terms of the series :

(i) $4 + 44 + 444 + \dots$

(ii) $0.8 + 0.88 + 0.888 + \dots$

Solution 1(i).

Required sum = $4 + 44 + 444 + \dots$ upto n terms

$$= 4(1 + 11 + 111 + \dots \text{ upto } n \text{ terms})$$

$$= \frac{4}{9}(9 + 99 + 999 + \dots \text{ upto } n \text{ terms})$$

$$= \frac{4}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \dots \text{ upto } n \text{ terms}]$$

$$= \frac{4}{9} \left[(10 + 10^2 + 10^3 + \dots \text{ upto } n \text{ terms}) - (1 + 1 + 1 + \dots \text{ upto } n \text{ terms}) \right]$$

$$= \frac{4}{9} \left[\frac{10(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{4}{9} \left[\frac{10}{9}(10^n - 1) - n \right]$$

Solution 1(ii).

$$\begin{aligned}
\text{Required sum} &= 0.8 + 0.88 + 0.888 + \dots \text{ upto } n \text{ terms} \\
&= 8(0.1 + 0.11 + 0.111 + \dots \text{ upto } n \text{ terms}) \\
&= \frac{8}{9}(0.9 + 0.99 + 0.999 + \dots \text{ upto } n \text{ terms}) \\
&= \frac{8}{9}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \dots \text{ upto } n \text{ terms}] \\
&= \frac{8}{9} \left[(1 + 1 + 1 + \dots \text{ upto } n \text{ terms}) \right. \\
&\quad \left. - (0.1 + 0.01 + 0.001 + \dots \text{ upto } n \text{ terms}) \right] \\
&= \frac{8}{9} \left[(1 + 1 + 1 + \dots \text{ upto } n \text{ terms}) \right. \\
&\quad \left. - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{ upto } n \text{ terms} \right) \right] \\
&= \frac{8}{9} \left[n - \frac{\frac{1}{10} \left(1 - \frac{1}{10^n} \right)}{1 - \frac{1}{10}} \right] \quad \left[\because r = \frac{1}{10} < 1 \right] \\
&= \frac{8}{9} \left[n - \frac{10}{9} \times \frac{1}{10} \left(1 - \frac{1}{10^n} \right) \right] \\
&= \frac{8}{9} \left[n - \frac{1}{9} \left(1 - \frac{1}{10^n} \right) \right]
\end{aligned}$$

Question 2.

Find the sum of infinite terms of each of the following geometric progression:

(i) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

(ii) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$

(iii) $\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots$

(iv) $\sqrt{2} - \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}} + \dots$

(v) $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \frac{1}{9\sqrt{3}} + \dots$

Solution 2(i).

Given GP. : $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

Here,

First term, $a = 1$

Common ratio, $r = \frac{\frac{1}{3}}{1} = \frac{1}{3} \left(|r| = \left| \frac{1}{3} \right| = \frac{1}{3} < 1 \right)$

$\therefore$ Required sum $= \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

Solution 2(ii).

Given GP. : $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$

Here,

First term, $a = 1$

Common ratio, $r = \frac{-\frac{1}{2}}{1} = -\frac{1}{2} \left(|r| = \left| -\frac{1}{2} \right| = \frac{1}{2} < 1 \right)$

$\therefore$ Required sum $= \frac{a}{1-r} = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$

Solution 2(iii).

Given GP. : $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots$

Here,

First term, $a = \frac{1}{3}$

Common ratio, $r = \frac{\frac{1}{3^2}}{\frac{1}{3}} = \frac{1}{3} \left(|r| = \left| \frac{1}{3} \right| = \frac{1}{3} < 1 \right)$

$\therefore$ Required sum $= \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

Solution 2(iv).

$$\text{Given G.P. : } \sqrt{2} - \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}} + \dots$$

Here,

$$\text{First term, } a = \sqrt{2}$$

$$\text{Common ratio, } r = \frac{-\frac{1}{\sqrt{2}}}{\sqrt{2}} = -\frac{1}{2} \left(|r| = \left| -\frac{1}{2} \right| = \frac{1}{2} < 1 \right)$$

$$\therefore \text{ Required sum} = \frac{a}{1-r} = \frac{\sqrt{2}}{1-\left(-\frac{1}{2}\right)} = \frac{\sqrt{2}}{1+\frac{1}{2}} = \frac{2\sqrt{2}}{3}$$

Solution 2(v).

$$\text{Given G.P. : } \sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \frac{1}{9\sqrt{3}} + \dots$$

Here,

$$\text{First term, } a = \sqrt{3}$$

$$\text{Common ratio, } r = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3} \left(|r| = \left| \frac{1}{3} \right| = \frac{1}{3} < 1 \right)$$

$$\therefore \text{ Required sum} = \frac{a}{1-r} = \frac{\sqrt{3}}{1-\frac{1}{3}} = \frac{\sqrt{3}}{\frac{2}{3}} = \frac{3\sqrt{3}}{2}$$

Question 3.

The second term of a G.P. is 9 and sum of its infinite terms is 48. Find its first three terms.

Solution:

Let a be the first term and r be the common ratio of a G.P.

$$2^{\text{nd}} \text{ term, } t_2 = ar = 9$$

$$\Rightarrow r = \frac{9}{a}$$

$$\text{Sum of its infinite terms, } S = 48$$

$$\Rightarrow \frac{a}{1-r} = 48$$

$$\Rightarrow \frac{a}{1 - \frac{9}{a}} = 48$$

$$\Rightarrow \frac{a^2}{a - 9} = 48$$

$$\Rightarrow a^2 = 48a - 432$$

$$\Rightarrow a^2 - 48a + 432 = 0$$

$$\Rightarrow a^2 - 36a - 12a + 432 = 0$$

$$\Rightarrow a(a - 36) - 12(a - 36) = 0$$

$$\Rightarrow (a - 36)(a - 12) = 0$$

$$\Rightarrow a = 36 \text{ or } a = 12$$

$$\text{When } a = 36, r = \frac{9}{36} = \frac{1}{4}$$

$$\Rightarrow 1^{\text{st}} \text{ term} = 36,$$

$$2^{\text{nd}} \text{ term} = ar = 36 \times \frac{1}{4} = 9$$

$$3^{\text{rd}} \text{ term} = ar^2 = 36 \times \frac{1}{16} = \frac{9}{4}$$

$$\text{When } a = 12, r = \frac{9}{12} = \frac{3}{4}$$

$$\Rightarrow 1^{\text{st}} \text{ term} = 12,$$

$$2^{\text{nd}} \text{ term} = ar = 12 \times \frac{3}{4} = 9$$

$$3^{\text{rd}} \text{ term} = ar^2 = 12 \times \frac{9}{16} = \frac{27}{4}$$

Question 4.

Find three geometric means between $\frac{1}{3}$ and 432.

Solution:

Let G_1, G_2, G_3 be three geometric means between $a = \frac{1}{3}$ and $b = 432$.

Then, $\frac{1}{3}, G_1, G_2, G_3, 432$ is a G.P.

Thus, we have

$$\text{First term} = a = \frac{1}{3}$$

$$5^{\text{th}} \text{ term of the G.P.} = ar^4 = 432$$

$$\Rightarrow \frac{1}{3} \times r^4 = 432$$

$$\Rightarrow r^4 = 1296$$

$$\Rightarrow r^4 = 6^4$$

$$\Rightarrow r = 6$$

$$\therefore G_1 = ar = \frac{1}{3} \times 6 = 2$$

$$G_2 = ar^2 = \frac{1}{3} \times 6 \times 6 = 12$$

$$G_3 = ar^3 = \frac{1}{3} \times 6 \times 6 \times 6 = 72$$

Question 5.

Find :

(i) two geometric means between 2 and 16

(ii) four geometric means between 3 and 96.

(iii) five geometric means between $3\frac{5}{9}$ and $40\frac{1}{2}$

Solution 5(i).

Let G_1, G_2 be two geometric means between $a = 2$ and $b = 16$.

Then, $2, G_1, G_2, 16$ is a G.P.

Thus, we have

$$\text{First term} = a = 2$$

$$4^{\text{th}} \text{ term of the G.P.} = ar^3 = 16$$

$$\Rightarrow 2 \times r^3 = 16$$

$$\Rightarrow r^3 = 8$$

$$\Rightarrow r^3 = 2^3$$

$$\Rightarrow r = 2$$

$$\therefore G_1 = ar = 2 \times 2 = 4$$

$$G_2 = ar^2 = 2 \times 2 \times 2 = 8$$

Solution 5(ii).

Let G_1, G_2, G_3, G_4 be four geometric means between $a = 3$ and $b = 96$.

Then, $3, G_1, G_2, G_3, G_4, 96$ is a G.P.

Thus, we have

First term = $a = 3$

6th term of the G.P. = $ar^5 = 96$

$$\Rightarrow 3 \times r^5 = 96$$

$$\Rightarrow r^5 = 32$$

$$\Rightarrow r^5 = 2^5$$

$$\Rightarrow r = 2$$

$$\therefore G_1 = ar = 3 \times 2 = 6$$

$$G_2 = ar^2 = 3 \times 4 = 12$$

$$G_3 = ar^3 = 3 \times 8 = 24$$

$$G_4 = ar^4 = 3 \times 16 = 48$$

Solution 5(iii).

Let G_1, G_2, G_3, G_4, G_5 be five geometric means between

$$a = 3\frac{5}{9} = \frac{32}{9} \text{ and } b = 40\frac{1}{2} = \frac{81}{2}.$$

Then, $\frac{32}{9}, G_1, G_2, G_3, G_4, G_5, \frac{81}{2}$ is a G.P.

Thus, we have

$$\text{First term} = a = \frac{32}{9}$$

$$7^{\text{th}} \text{ term of the G.P.} = ar^6 = \frac{81}{2}$$

$$\Rightarrow \frac{32}{9} \times r^6 = \frac{81}{2}$$

$$\Rightarrow r^6 = \frac{81}{2} \times \frac{9}{32}$$

$$\Rightarrow r^6 = \frac{729}{64}$$

$$\Rightarrow r^6 = \left(\frac{3}{2}\right)^6$$

$$\Rightarrow r = \frac{3}{2}$$

$$\therefore G_1 = ar = \frac{32}{9} \times \frac{3}{2} = \frac{16}{3}$$

$$G_2 = ar^2 = \frac{32}{9} \times \frac{9}{4} = 8$$

$$G_3 = ar^3 = \frac{32}{9} \times \frac{27}{8} = 12$$

$$G_4 = ar^4 = \frac{32}{9} \times \frac{81}{16} = 18$$

$$G_5 = ar^5 = \frac{32}{9} \times \frac{243}{32} = 27$$

Question 6.

The sum of three numbers in G.P. is $\frac{39}{10}$ and their product is 1. Find the numbers.

Solution:

Sum of three numbers in G.P. = $\frac{39}{10}$ and their product = 1

Let number be $\frac{a}{r}$, a , ar , then

$$\frac{a}{r} \times a \times ar = 1 \Rightarrow a^3 = 1 = (1)^3$$

$$\therefore a = 1$$

$$\text{and } \frac{a}{r} + a + ar = \frac{39}{10}$$

$$\Rightarrow a \left(\frac{1}{r} + 1 + r \right) = \frac{39}{10}$$

$$\frac{1}{r} + 1 + r = \frac{39}{10} \times 1 = \frac{39}{10}$$

$$r + \frac{1}{r} = \frac{39}{10} - 1 = \frac{39-10}{10} = \frac{29}{10}$$

$$r^2 + 1 = \frac{29}{10}r$$

$$\begin{aligned}
10r^2 + 10 &= 29r \Rightarrow 10r^2 - 29r + 10 = 0 \\
\Rightarrow 10r^2 - 4r - 25r + 10 &= 0 \\
\Rightarrow 2r(5r - 2) - 5(5r - 2) &= 0 \\
\Rightarrow (5r - 2)(2r - 5) &= 0
\end{aligned}$$

$$\text{Either } 5r - 2 = 0, \text{ then } r = \frac{2}{5}$$

$$\text{or } 2r - 5 = 0, \text{ then } r = \frac{5}{2}$$

$$\therefore \text{Numbers are } \frac{2}{5}, 1, \frac{4}{25}, \text{ or } \frac{5}{2}, 1, \frac{25}{4}$$

Question 7.

Find the numbers in G.P. whose sum is 52 and the sum of whose product in pairs is 624.

Solution:

Let the numbers be a , ar and ar^2 .

$$\Rightarrow a + ar + ar^2 = 52 \quad \dots(i)$$

$$\text{And, } (a \times ar) + (ar \times ar^2) + (ar^2 \times a) = 624$$

$$\Rightarrow a^2r + a^2r^3 + a^2r^2 = 624$$

$$\Rightarrow ar(a + ar^2 + ar) = 624$$

$$\Rightarrow ar \times 52 = 624 \quad \dots[\text{From (i)}]$$

$$\Rightarrow ar = 12$$

$$\Rightarrow a = \frac{12}{r}$$

Substituting in (i), we get

$$\frac{12}{r} + \frac{12}{r} \times r + \frac{12}{r} \times r^2 = 52$$

$$\Rightarrow \frac{12}{r} + 12 + 12r = 52$$

$$\Rightarrow \frac{12 + 12r + 12r^2}{r} = 52$$

$$\Rightarrow 12 + 12r + 12r^2 = 52r$$

$$\Rightarrow 12r^2 - 40r + 12 = 0$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow 3r^2 - 9r - r + 3 = 0$$

$$\Rightarrow 3r(r - 3) - 1(r - 3) = 0$$

$$\Rightarrow (3r - 1)(r - 3) = 0$$

$$\Rightarrow r = \frac{1}{3} \text{ or } r = 3$$

$$\Rightarrow a = \frac{12}{\frac{1}{3}} = 36 \text{ or } 4$$

Thus, required terms are:

$$\begin{aligned} a, ar, ar^2 &= 36, 36 \times \frac{1}{3}, 36 \times \frac{1}{9} \quad \text{OR} \quad 4, 4 \times 3, 4 \times 9 \\ &= 36, 12, 4 \quad \text{OR} \quad 4, 12, 36 \end{aligned}$$

Question 8.

The sum of three numbers in G.P. is 21 and the sum of their squares is 189. Find the numbers.

Solution:

Let the numbers be a , ar and ar^2 .

$$\Rightarrow (a)^2 + (ar)^2 + (ar^2)^2 = 189$$

$$\Rightarrow a^2 + a^2r^2 + a^2r^4 = 189$$

$$\text{And, } a + ar + ar^2 = 21$$

$$\Rightarrow (a + ar + ar^2)^2 = 21^2$$

$$\Rightarrow a^2 + a^2r^2 + a^2r^4 + 2a^2r + 2a^2r^3 + 2a^2r^2 = 441$$

$$\Rightarrow 189 + 2ar(a + ar^2 + ar) = 441$$

$$\Rightarrow 2ar \times 21 = 441 - 189$$

$$\Rightarrow 42ar = 252$$

$$\Rightarrow ar = 6$$

$$\Rightarrow r = \frac{6}{a}$$

$$\text{Now, } a + ar + ar^2 = 21$$

$$\Rightarrow a + a \times \frac{6}{a} + a \times \frac{36}{a^2} = 21$$

$$\Rightarrow a + 6 + \frac{36}{a} = 21$$

$$\Rightarrow a^2 + 6a + 36 = 21a$$

$$\Rightarrow a^2 - 15a + 36 = 0$$

$$\Rightarrow a^2 - 12a - 3a + 36 = 0$$

$$\Rightarrow a(a-12) - 3(a-12) = 0$$

$$\Rightarrow (a-12)(a-3) = 0$$

$$\Rightarrow a = 12 \text{ or } a = 3$$

$$\Rightarrow r = \frac{6}{12} = \frac{1}{2} \text{ or } r \frac{6}{3} = 2$$

Thus, required terms are:

$$\begin{aligned} a, ar, ar^2 &= 12, 12 \times \frac{1}{2}, 12 \times \frac{1}{4} \quad \text{OR} \quad 3, 3 \times 2, 3 \times 4 \\ &= 12, 6, 3 \quad \text{OR} \quad 3, 6, 12 \end{aligned}$$