

Section and Mid-Point Formula

Exercise 13A

Question 1.

Calculate the co-ordinates of the point P which divides the line segment joining:

(i) A (1, 3) and B (5, 9) in the ratio 1: 2.

(ii) A (-4, 6) and B (3, -5) in the ratio 3: 2.

Solution:

(i) Let the co-ordinates of the point P be (x, y).

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times 5 + 2 \times 1}{1 + 2} = \frac{7}{3}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1 \times 9 + 2 \times 3}{1 + 2} = \frac{15}{3} = 5$$

Thus, the co-ordinates of point P are $\left(\frac{7}{3}, 5\right)$.

(ii) Let the co-ordinates of the point P be (x, y).

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{3 \times 3 + 2 \times (-4)}{3 + 2} = \frac{1}{5}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{3 \times (-5) + 2 \times 6}{3 + 2} = \frac{-3}{5}$$

Thus, the co-ordinates of point P are $\left(\frac{1}{5}, -\frac{3}{5}\right)$.

Question 2.

In what ratio is the line joining (2, -3) and (5, 6) divided by the x-axis.

Solution:

Let the line joining points A (2, -3) and B (5, 6) be divided by point P (x, 0) in the ratio k: 1.

$$y = \frac{ky_2 + y_1}{k + 1}$$

$$0 = \frac{k \times 6 + 1 \times (-3)}{k + 1}$$

$$0 = 6k - 3$$

$$k = \frac{1}{2}$$

Thus, the required ratio is 1: 2.

Question 3.

In what ratio is the line joining (2, -4) and (-3, 6) divided by the y-axis.

Solution:

Let the line joining points A (2, -4) and B (-3, 6) be divided by point P (0, y) in the ratio k: 1.

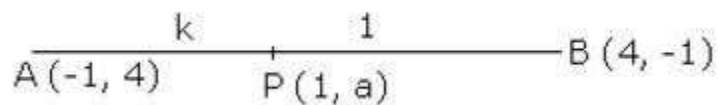
$$\begin{aligned}x &= \frac{kx_2 + x_1}{k + 1} \\0 &= \frac{k \times (-3) + 1 \times 2}{k + 1} \\0 &= -3k + 2 \\k &= \frac{2}{3}\end{aligned}$$

Thus, the required ratio is 2: 3.

Question 4.

In what ratio does the point (1, a) divided the join of (-1, 4) and (4, -1)? Also, find the value of a.

Solution:



Let the point P (1, a) divides the line segment AB in the ratio k: 1.
Using section formula, we have:

$$\begin{aligned}1 &= \frac{4k - 1}{k + 1}, \\ \Rightarrow k + 1 &= 4k - 1 \\ \Rightarrow 2 &= 3k \\ \Rightarrow k &= \frac{2}{3} \quad \dots(1) \\ \Rightarrow a &= \frac{-k + 4}{k + 1}\end{aligned}$$

$$\Rightarrow a = \frac{\frac{-2}{3} + 4}{\frac{2}{3} + 1} \quad (\text{from (1)})$$

$$\Rightarrow a = \frac{10}{5} = 2$$

Hence, the required ratio is 2 : 3 and the value of a is 2.

Question 5.

In what ratio does the point (a, 6) divide the join of (-4, 3) and (2, 8)? Also, find the value of a.

Solution:

Let the point P (a, 6) divides the line segment joining A (-4, 3) and B (2, 8) in the ratio k : 1.

Using section formula, we have:

$$6 = \frac{8k + 3}{k + 1},$$

$$\Rightarrow 6k + 6 = 8k + 3$$

$$\Rightarrow 3 = 2k$$

$$\Rightarrow k = \frac{3}{2} \quad \dots (1)$$

$$\Rightarrow a = \frac{2k - 4}{k + 1}$$

$$\Rightarrow a = \frac{2 \times \frac{3}{2} - 4}{\frac{3}{2} + 1} \quad (\text{from (1)})$$

$$\Rightarrow a = -\frac{2}{5}$$

Hence, the required ratio is 3 : 2 and the value of a is $-\frac{2}{5}$.

Question 6.

In what ratio is the join of (4, 3) and (2, -6) divided by the x-axis. Also, find the co-ordinates of the point of intersection.

Solution:

Let the point P (x, 0) on x-axis divides the line segment joining A (4, 3) and B (2, -6) in the ratio k: 1.

Using section formula, we have:

$$0 = \frac{-6k + 3}{k + 1}$$

$$0 = -6k + 3$$

$$k = \frac{1}{2}$$

Thus, the required ratio is 1: 2.

Also, we have:

$$x = \frac{2k + 4}{k + 1}$$

$$= \frac{2 \times \frac{1}{2} + 4}{\frac{1}{2} + 1}$$

$$= \frac{10}{3}$$

Thus, the required co-ordinates of the point of intersection are $\left(\frac{10}{3}, 0\right)$.

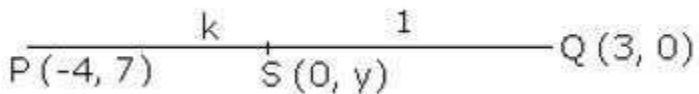
Question 7.

Find the ratio in which the join of $(-4, 7)$ and $(3, 0)$ is divided by the y-axis. Also, find the coordinates of the point of intersection.

Solution:

Let $S(0, y)$ be the point on y -axis which divides the line segment PQ in the ratio $k:1$.

Using section formula, we have:



$$0 = \frac{3k - 4}{k + 1}$$

$$\Rightarrow 3k = 4$$

$$k = \frac{4}{3} \quad \dots (1)$$

$$y = \frac{0+7}{k+1}$$

$$y = \frac{7}{\frac{4}{3}+1} \quad (\text{from (1)})$$

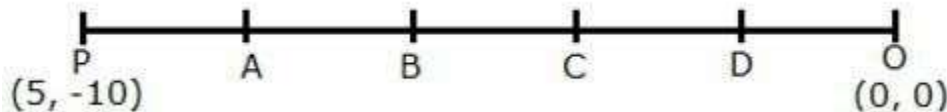
$$y = 3$$

Hence, the required ratio is 4:3 and the required point is S(0, 3).

Question 8.

Points A, B, C and D divide the line segment joining the point (5, -10) and the origin in five equal parts. Find the co-ordinates of A, B, C and D.

Solution:



Point A divides PO in the ratio 1:4.

Co-ordinates of point A are:

$$\left(\frac{1 \times 0 + 4 \times 5}{1 + 4}, \frac{1 \times 0 + 4 \times (-10)}{1 + 4} \right) = \left(\frac{20}{5}, \frac{-40}{5} \right) = (4, -8)$$

Point B divides PO in the ratio 2:3.

Co-ordinates of point B are:

$$\left(\frac{2 \times 0 + 3 \times 5}{2 + 3}, \frac{2 \times 0 + 3 \times (-10)}{2 + 3} \right) = \left(\frac{15}{5}, \frac{-30}{5} \right) = (3, -6)$$

Point C divides PO in the ratio 3:2.

Co-ordinates of point C are:

$$\left(\frac{3 \times 0 + 2 \times 5}{3 + 2}, \frac{3 \times 0 + 2 \times (-10)}{3 + 2} \right) = \left(\frac{10}{5}, \frac{-20}{5} \right) = (2, -4)$$

Point D divides PO in the ratio 4:1.

Co-ordinates of point D are:

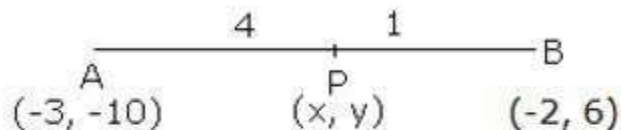
$$\left(\frac{4 \times 0 + 1 \times 5}{4 + 1}, \frac{4 \times 0 + 1 \times (-10)}{4 + 1} \right) = \left(\frac{5}{5}, \frac{-10}{5} \right) = (1, -2)$$

Question 9.

The line joining the points A (-3, -10) and B (-2, 6) is divided by the point P such that $\frac{PB}{AB} = \frac{1}{5}$. Find the co-ordinates of P.

Solution:

Let the co-ordinates of point P are (x, y).



Given : $PB : AB = 1 : 5$

$\therefore PB : PA = 1 : 4$

Coordinates of P are

$$(x, y) = \left(\frac{4 \times (-2) + 1 \times (-3)}{5}, \frac{4 \times 6 + 1 \times (-10)}{5} \right) = \left(-\frac{11}{5}, \frac{14}{5} \right)$$

Question 10.

P is a point on the line joining A (4, 3) and B (-2, 6) such that $5AP = 2BP$. Find the co-ordinates of P.

Solution:

$$5AP = 2BP$$

$$\frac{AP}{BP} = \frac{2}{5}$$

The co-ordinates of the point P are

$$\left(\frac{2 \times (-2) + 5 \times 4}{2+5}, \frac{2 \times 6 + 5 \times 3}{2+5} \right)$$

$$\left(\frac{16}{7}, \frac{27}{7} \right)$$

Question 11.

Calculate the ratio in which the line joining the points (-3, -1) and (5, 7) is divided by the line $x = 2$. Also, find the co-ordinates of the point of intersection.

Solution:

The co-ordinates of every point on the line $x = 2$ will be of the type (2, y).

Using section formula, we have:

$$x = \frac{m_1 \times 5 + m_2 \times (-3)}{m_1 + m_2}$$

$$2 = \frac{5m_1 - 3m_2}{m_1 + m_2}$$

$$2m_1 + 2m_2 = 5m_1 - 3m_2$$

$$5m_2 = 3m_1$$

$$\frac{m_1}{m_2} = \frac{5}{3}$$

Thus, the required ratio is 5:3.

$$y = \frac{m_1 \times 7 + m_2 \times (-1)}{m_1 + m_2}$$

$$y = \frac{5 \times 7 + 3 \times (-1)}{5 + 3}$$

$$y = \frac{35 - 3}{8}$$

$$y = \frac{32}{8} = 4$$

Thus, the required co-ordinates of the point of intersection are (2, 4).

Question 12.

Calculate the ratio in which the line joining A (6, 5) and B (4, -3) is divided by the line $y = 2$.

Solution:

The co-ordinates of every point on the line $y = 2$ will be of the type $(x, 2)$.

Using section formula, we have:

$$y = \frac{m_1 \times (-3) + m_2 \times 5}{m_1 + m_2}$$

$$2 = \frac{-3m_1 + 5m_2}{m_1 + m_2}$$

$$2m_1 + 2m_2 = -3m_1 + 5m_2$$

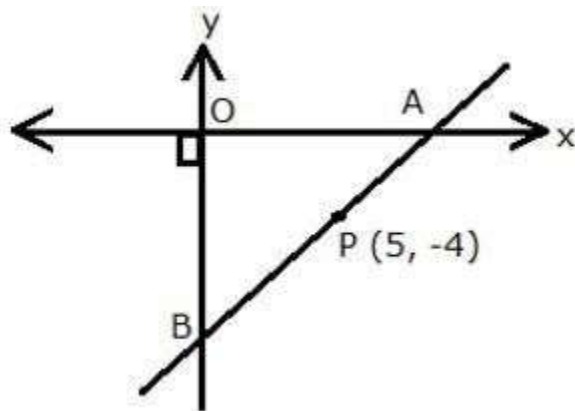
$$5m_1 = 3m_2$$

$$\frac{m_1}{m_2} = \frac{3}{5}$$

Thus, the required ratio is 3:5.

Question 13.

The point P (5, -4) divides the line segment AB, as shown in the figure, in the ratio 2: 5. Find the co-ordinates of points A and B.

**Solution:**

Point A lies on x-axis. So, let the co-ordinates of A be (x, 0).

Point B lies on y-axis. So, let the co-ordinates of B be (0, y).

P divides AB in the ratio 2: 5.

We have:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$5 = \frac{2 \times 0 + 5 \times x}{2 + 5}$$

$$5 = \frac{5x}{7}$$

$$x = 7$$

Thus, the co-ordinates of point A are (7, 0).

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$-4 = \frac{2 \times y + 5 \times 0}{2 + 5}$$

$$-4 = \frac{2y}{7}$$

$$-2 = \frac{y}{7}$$

$$y = -14$$

Thus, the co-ordinates of point B are (0, -14).

Question 14.

Find the co-ordinates of the points of trisection of the line joining the points $(-3, 0)$ and $(6, 6)$.

Solution:

Let P and Q be the point of trisection of the line segment joining the points A $(-3, 0)$ and B $(6, 6)$.

So, $AP = PQ = QB$

We have $AP:PB = 1:2$

Co-ordinates of the point P are

$$\left(\frac{1 \times 6 + 2 \times (-3)}{1 + 2}, \frac{1 \times 6 + 2 \times 0}{1 + 2} \right)$$

$$= \left(\frac{6 - 6}{3}, \frac{6}{3} \right)$$

$$= (0, 2)$$

We have $AQ:QB = 2:1$

Co-ordinates of the point Q are

$$\left(\frac{2 \times 6 + 1 \times (-3)}{2 + 1}, \frac{2 \times 6 + 1 \times 0}{2 + 1} \right)$$

$$= \left(\frac{9}{3}, \frac{12}{3} \right)$$

$$= (3, 4)$$

Question 15.

Show that the line segment joining the points $(-5, 8)$ and $(10, -4)$ is trisected by the co-ordinate axes.

Solution:

Let P and Q be the point of trisection of the line segment joining the points A $(-5, 8)$ and B $(10, -4)$.

So, $AP = PQ = QB$

We have AP: PB = 1: 2

Co-ordinates of the point P are

$$\begin{aligned} & \left(\frac{1 \times 10 + 2 \times (-5)}{1 + 2}, \frac{1 \times (-4) + 2 \times 8}{1 + 2} \right) \\ &= \left(\frac{10 - 10}{3}, \frac{12}{3} \right) \\ &= (0, 4) \end{aligned}$$

So, point P lies on the y-axis.

We have AQ: QB = 2: 1

Co-ordinates of the point Q are

$$\begin{aligned} & \left(\frac{2 \times 10 + 1 \times (-5)}{2 + 1}, \frac{2 \times (-4) + 1 \times 8}{2 + 1} \right) \\ &= \left(\frac{20 - 5}{3}, \frac{-8 + 8}{3} \right) \\ &= (5, 0) \end{aligned}$$

So, point Q lies on the x-axis.

Hence, the line segment joining the given points A and B is trisected by the co-ordinate axes.

Question 16.

Show that A (3, -2) is a point of trisection of the line-segment joining the points (2, 1) and (5, -8). Also, find the co-ordinates of the other point of trisection.

Solution:

Let A and B be the point of trisection of the line segment joining the points P (2, 1) and Q (5, -8).

So, PA = AB = BQ

We have PA: AQ = 1: 2

Co-ordinates of the point A are

$$\begin{aligned} & \left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, \frac{1 \times (-8) + 2 \times 1}{1 + 2} \right) \\ &= \left(\frac{9}{3}, \frac{-6}{3} \right) \\ &= (3, -2) \end{aligned}$$

Hence, A (3, -2) is a point of trisection of PQ.

We have PB: BQ = 2: 1

Co-ordinates of the point B are

$$\begin{aligned} & \left(\frac{2 \times 5 + 1 \times 2}{2 + 1}, \frac{2 \times (-8) + 1 \times 1}{2 + 1} \right) \\ &= \left(\frac{10 + 2}{3}, \frac{-16 + 1}{3} \right) \\ &= (4, -5) \end{aligned}$$

Question 17.

If A = (-4, 3) and B = (8, -6)

(i) Find the length of AB.

(ii) In what ratio is the line joining A and B, divided by the x-axis?

Solution:

(i) A (-4, 3) and B (8, -6)

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 + 4)^2 + (-6 - 3)^2} \\ &= \sqrt{144 + 81} \\ &= \sqrt{225} \\ &= 15 \text{ units} \end{aligned}$$

(ii) Let P be the point, which divides AB on the x-axis in the ratio k : 1.

Therefore, y-co-ordinate of P = 0.

$$\Rightarrow \frac{-6k + 3}{k + 1} = 0$$

$$\Rightarrow -6k + 3 = 0$$

$$\Rightarrow k = \frac{1}{2}$$

$\therefore$ Required ratio is 1: 2.

Question 18.

The line segment joining the points M (5, 7) and N (-3, 2) is intersected by the y-axis at point L. Write down the abscissa of L. Hence, find the ratio in which L divides MN. Also, find the co-ordinates of L.

Solution:

Since, point L lies on y-axis, its abscissa is 0.

Let the co-ordinates of point L be (0, y). Let L divides MN in the ratio k: 1.

Using section formula, we have:

$$x = \frac{k \times (-3) + 1 \times 5}{k + 1}$$

$$0 = \frac{-3k + 5}{k + 1}$$

$$-3k + 5 = 0$$

$$k = \frac{5}{3}$$

Thus, the required ratio is 5: 3.

$$\text{Now, } y = \frac{k \times 2 + 1 \times 7}{k + 1}$$

$$= \frac{\frac{5}{3} \times 2 + 7}{\frac{5}{3} + 1}$$

$$= \frac{10 + 21}{5 + 3}$$

$$= \frac{31}{8}$$

Question 19.

A (2, 5), B (-1, 2) and C (5, 8) are the co-ordinates of the vertices of the triangle ABC. Points P and Q lie on AB and AC respectively, such that AP: PB = AQ: QC = 1: 2.

- (i) Calculate the co-ordinates of P and Q.
- (ii) Show that PQ = 1/3 BC.

Solution:

(i) Co-ordinates of P are

$$\left(\frac{1 \times (-1) + 2 \times 2}{1 + 2}, \frac{1 \times 2 + 2 \times 5}{1 + 2} \right)$$

$$= \left(\frac{3}{3}, \frac{12}{3} \right)$$

$$= (1, 4)$$

Co-ordinates of Q are

$$\left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, \frac{1 \times 8 + 2 \times 5}{1 + 2} \right)$$

$$= \left(\frac{9}{3}, \frac{18}{3} \right)$$

$$= (3, 6)$$

(ii) Using distance formula, we have:

$$BC = \sqrt{(5 + 1)^2 + (8 - 2)^2} = \sqrt{36 + 36} = 6\sqrt{2}$$

$$PQ = \sqrt{(3 - 1)^2 + (6 - 4)^2} = \sqrt{4 + 4} = 2\sqrt{2}$$

$$\text{Hence, } PQ = \frac{1}{3} BC.$$

Question 20.

A (-3, 4), B (3, -1) and C (-2, 4) are the vertices of a triangle ABC. Find the length of line segment AP, where point P lies inside BC, such that BP: PC = 2: 3.

Solution:

$$BP: PC = 2: 3$$

Co-ordinates of P are

$$\left(\frac{2 \times (-2) + 3 \times 3}{2 + 3}, \frac{2 \times 4 + 3 \times (-1)}{2 + 3} \right)$$

$$= \left(\frac{-4 + 9}{5}, \frac{8 - 3}{5} \right)$$

$$= (1, 1)$$

Using distance formula, we have:

$$AP = \sqrt{(1 + 3)^2 + (1 - 4)^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ units}$$

Question 21.

The line segment joining A (2, 3) and B (6, -5) is intercepted by x-axis at the point K. Write down the ordinate of the point K. Hence, find the ratio in which K divides AB. Also, find the co-ordinates of the point K.

Solution:

Since, point K lies on x-axis, its ordinate is 0.

Let the point K (x, 0) divides AB in the ratio k: 1.

We have,

$$y = \frac{k \times (-5) + 1 \times 3}{k + 1}$$

$$0 = \frac{-5k + 3}{k + 1}$$

$$k = \frac{3}{5}$$

Thus, K divides AB in the ratio 3: 5.

Also, we have:

$$x = \frac{k \times 6 + 1 \times 2}{k + 1}$$

$$x = \frac{\frac{3}{5} \times 6 + 2}{\frac{3}{5} + 1}$$

$$x = \frac{18 + 10}{3 + 5}$$

$$x = \frac{28}{8} = \frac{7}{2} = 3\frac{1}{2}$$

Thus, the co-ordinates of the point K are $\left(3\frac{1}{2}, 0\right)$.

Question 22.

The line segment joining A (4, 7) and B (-6, -2) is intercepted by the y-axis at the point K. Write down the abscissa of the point K. Hence, find the ratio in which K divides AB. Also, find the co-ordinates of the point K.

Solution:

Since, point K lies on y-axis, its abscissa is 0.

Let the point K (0, y) divides AB in the ratio k: 1.

We have,

$$x = \frac{k \times (-6) + 1 \times 4}{k + 1}$$

$$0 = \frac{-6k + 4}{k + 1}$$

$$k = \frac{4}{6} = \frac{2}{3}$$

Thus, K divides AB in the ratio 2: 3.

Also, we have:

$$y = \frac{k \times (-2) + 1 \times 7}{k + 1}$$

$$y = \frac{-2k + 7}{k + 1}$$

$$y = \frac{-2 \times \frac{2}{3} + 7}{\frac{2}{3} + 1}$$

$$y = \frac{-4 + 21}{2 + 3}$$

$$y = \frac{17}{5}$$

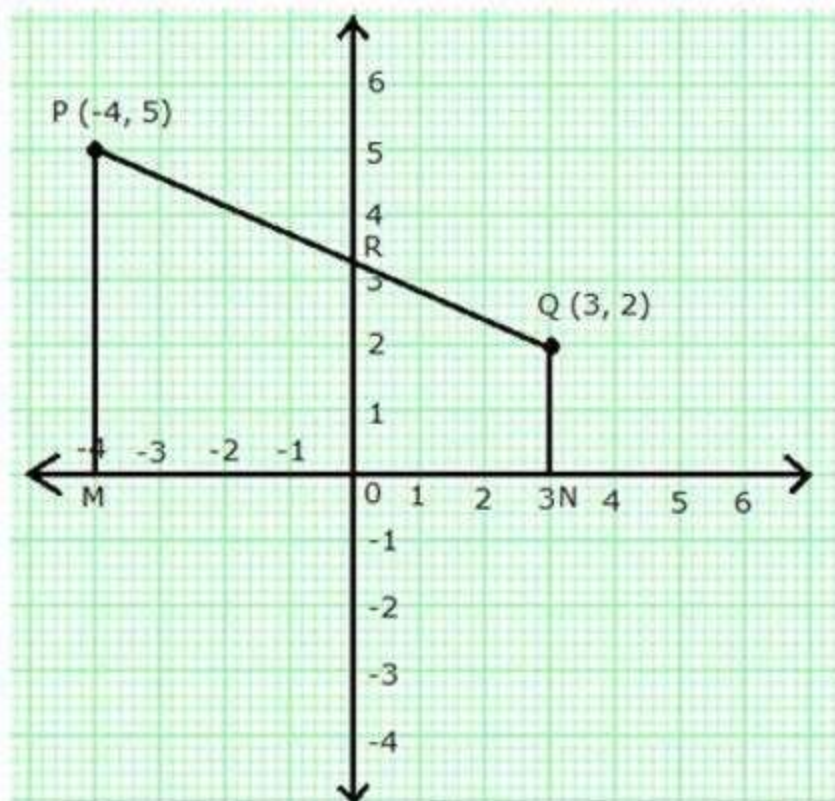
Thus, the co-ordinates of the point K are $\left(0, \frac{17}{5}\right)$.

Question 23.

The line joining P (-4, 5) and Q (3, 2) intersects the y-axis at point R. PM and QN are perpendiculars from P and Q on the x-axis. Find:

- (i) the ratio PR: RQ.
- (ii) the co-ordinates of R.
- (iii) the area of the quadrilateral PMNQ.

Solution:



(i) Let point R (0, y) divides PQ in the ratio k: 1.

We have:

$$x = \frac{k \times 3 + 1 \times (-4)}{k + 1}$$

$$0 = \frac{3k - 4}{k + 1}$$

$$0 = 3k - 4$$

$$k = \frac{4}{3}$$

Thus, PR: RQ = 4:3

(ii) Also, we have:

$$y = \frac{k \times 2 + 1 \times 5}{k + 1}$$

$$y = \frac{2k + 5}{k + 1}$$

$$y = \frac{2 \times \frac{4}{3} + 5}{\frac{4}{3} + 1}$$

$$y = \frac{8+15}{4+3}$$

$$y = \frac{23}{7}$$

Thus, the co-ordinates of point R are $\left(0, \frac{23}{7}\right)$.

(iii) Area of quadrilateral PMNQ

$$= \frac{1}{2} \times (PM + QN) \times MN$$

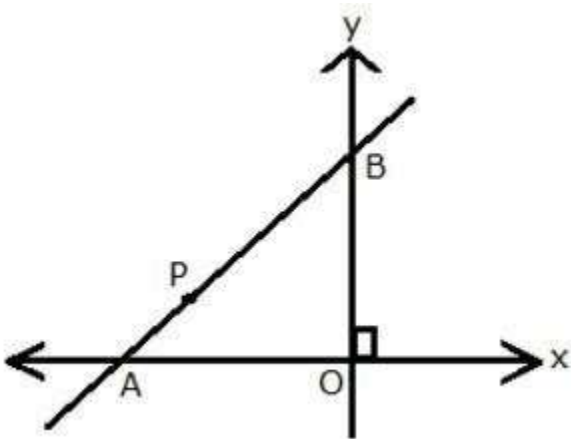
$$= \frac{1}{2} \times (5 + 2) \times 7$$

$$= \frac{1}{2} \times 7 \times 7$$

$$= 24.5 \text{ sq units}$$

Question 24.

In the given figure, line APB meets the x-axis at point A and y-axis at point B. P is the point $(-4, 2)$ and $AP: PB = 1: 2$. Find the co-ordinates of A and B.



Solution:

Given, A lies on x-axis and B lies on y-axis.

Let the co-ordinates of A and B be $(x, 0)$ and $(0, y)$ respectively.

Given, P is the point $(-4, 2)$ and $AP: PB = 1: 2$.

Using section formula, we have:

$$-4 = \frac{1 \times 0 + 2 \times x}{1 + 2}$$

$$-4 = \frac{2x}{3}$$

$$x = \frac{-4 \times 3}{2} = -6$$

Also,

$$2 = \frac{1 \times y + 2 \times 0}{1 + 2}$$

$$2 = \frac{y}{3}$$

$$y = 6$$

Thus, the co-ordinates of points A and B are (-6, 0) and (0, 6) respectively.

Question 25.

Given a line segment AB joining the points A(-4, 6) and B(8, -3). Find:

- (i) the ratio in which AB is divided by the y-axis
- (ii) find the coordinates of the point of intersection
- (iii) the length of AB

Solution:

(i)

Let the required ratio be $m_1 : m_2$

Consider A (-4, 6) = (x_1, y_1) ; B (8, -3) = (x_2, y_2) and let

P (x, y) be the point of intersection of the line segment and the y-axis.

By section formula, we have,

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{8m_1 - 4m_2}{m_1 + m_2}, \quad y = \frac{-3m_1 + 6m_2}{m_1 + m_2}$$

The equation of the y-axis is $x = 0$

$$\Rightarrow x = \frac{8m_1 - 4m_2}{m_1 + m_2} = 0$$

$$\Rightarrow 8m_1 - 4m_2 = 0$$

$$\Rightarrow 8m_1 = 4m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{4}{8}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{1}{2}$$

(ii)

From the previous subpart, we have,

$$\frac{m_1}{m_2} = \frac{1}{2}$$

$\Rightarrow m_1 = k$ and $m_2 = 2k$, where k is any constant.

Also, we have,

$$x = \frac{8m_1 - 4m_2}{m_1 + m_2}, \quad y = \frac{-3m_1 + 6m_2}{m_1 + m_2}$$

$$\Rightarrow x = \frac{8 \times k - 4 \times 2k}{k + 2k}, \quad y = \frac{-3 \times k + 6 \times 2k}{k + 2k}$$

$$\Rightarrow x = \frac{8k - 8k}{3k}, \quad y = \frac{-3k + 12k}{3k}$$

$$\Rightarrow x = \frac{0}{3k}, \quad y = \frac{9k}{3k}$$

$$\Rightarrow x = 0, \quad y = 3$$

Thus, the point of intersection is $P(0, 3)$

(iii)

The length of AB = Distance between two points A and B.

The distance between two given points

A (x_1, y_1) and B (x_2, y_2) is given by,

$$\begin{aligned} \text{Distance AB} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 + 4)^2 + (-3 - 6)^2} \\ &= \sqrt{(12)^2 + (9)^2} \\ &= \sqrt{144 + 81} \\ &= \sqrt{225} \\ &= 15 \text{ units} \end{aligned}$$

Question 26.

If $P(-b, 9a - 2)$ divides the line segment joining the points $A(-3, 3a + 1)$ and $B(5, 8a)$ in the ratio 3: 1, find the values of a and b .

Solution:

Take $(x_1, y_1) = (-3, 3a + 1)$; $(x_2, y_2) = B(5, 8a)$ and $(x, y) = (-b, 9a - 2)$

Here $m_1 = 3$ and $m_2 = 1$

$$\therefore \text{Coordinate of } P(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

$$\Rightarrow x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \text{ and } y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow -b = \frac{3 \times 5 + 1 \times (-3)}{3 + 1} \text{ and } 9a - 2 = \frac{3 \times 8a + 1 \times (3a + 1)}{3 + 1}$$

$$\Rightarrow -b = \frac{15 - 3}{4} \text{ and } 9a - 2 = \frac{24a + 3a + 1}{4}$$

$$\Rightarrow -4b = 12 \text{ and } 36a - 8 = 27a + 1$$

$$\Rightarrow b = -3 \text{ and } 9a = 9$$

$$\therefore a = 1 \text{ and } b = -3$$

Exercise 13B**Question 1.**

Find the mid-point of the line segment joining the points:

(i) $(-6, 7)$ and $(3, 5)$

(ii) $(5, -3)$ and $(-1, 7)$

Solution:

(i) $A(-6, 7)$ and $B(3, 5)$

$$\text{Mid-point of } AB = \left(\frac{-6 + 3}{2}, \frac{7 + 5}{2} \right) = \left(\frac{-3}{2}, 6 \right)$$

(ii) $A(5, -3)$ and $B(-1, 7)$

$$\text{Mid-point of } AB = \left(\frac{5 - 1}{2}, \frac{-3 + 7}{2} \right) = (2, 2)$$

Question 2.

Points A and B have co-ordinates (3, 5) and (x, y) respectively. The mid-point of AB is (2, 3). Find the values of x and y.

Solution:

Mid-point of AB = (2, 3)

$$\therefore \left(\frac{3+x}{2}, \frac{5+y}{2} \right) = (2, 3)$$

$$\Rightarrow \frac{3+x}{2} = 2 \quad \text{and} \quad \frac{5+y}{2} = 3$$

$$\Rightarrow 3+x = 4 \quad \text{and} \quad 5+y = 6$$

$$\Rightarrow x = 1 \quad \text{and} \quad y = 1$$

Question 3.

A (5, 3), B (-1, 1) and C (7, -3) are the vertices of triangle ABC. If L is the mid-point of AB and M is the mid-point of AC, show that $LM = \frac{1}{2} BC$.

Solution:

Given, L is the mid-point of AB and M is the mid-point of AC.

Co-ordinates of L are

$$\left(\frac{5-1}{2}, \frac{3+1}{2} \right) = (2, 2)$$

Co-ordinates of M are

$$\left(\frac{5+7}{2}, \frac{3-3}{2} \right) = (6, 0)$$

Using distance formula, we have:

$$BC = \sqrt{(7+1)^2 + (-3-1)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$LM = \sqrt{(6-2)^2 + (0-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Hence, } LM = \frac{1}{2} BC$$

Question 4.

Given M is the mid-point of AB, find the co-ordinates of:

(i) A; if M = (1, 7) and B = (-5, 10)

(ii) B; if A = (3, -1) and M = (-1, 3).

Solution:

(i) Let the co-ordinates of A be (x, y) .

$$\therefore (1, 7) = \left(\frac{x-5}{2}, \frac{y+10}{2} \right)$$

$$\Rightarrow 1 = \frac{x-5}{2} \quad \text{and} \quad 7 = \frac{y+10}{2}$$

$$\Rightarrow 2 = x-5 \quad \text{and} \quad 14 = y+10$$

$$\Rightarrow x = 7 \quad \text{and} \quad y = 4$$

Hence, the co-ordinates of A are $(7, 4)$.

(ii) Let the co-ordinates of B be (x, y) .

$$\therefore (-1, 3) = \left(\frac{3+x}{2}, \frac{-1+y}{2} \right)$$

$$\Rightarrow -1 = \frac{3+x}{2} \quad \text{and} \quad 3 = \frac{-1+y}{2}$$

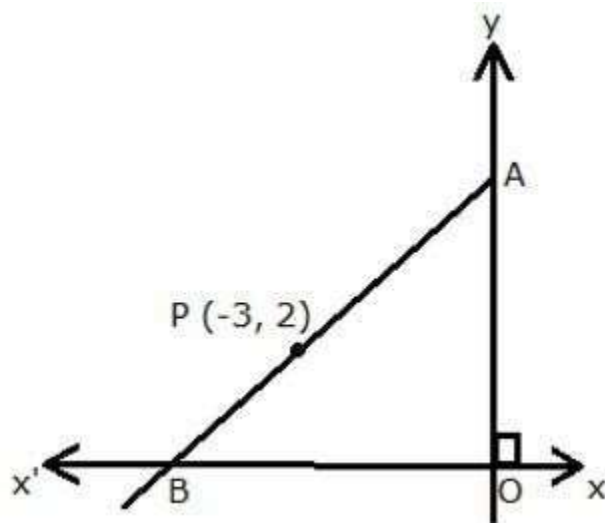
$$\Rightarrow -2 = 3+x \quad \text{and} \quad 6 = -1+y$$

$$\Rightarrow x = -5 \quad \text{and} \quad y = 7$$

Hence, the co-ordinates of B are $(-5, 7)$.

Question 5.

P $(-3, 2)$ is the mid-point of line segment AB as shown in the given figure. Find the co-ordinates of points A and B.



Solution:

Point A lies on y-axis, so let its co-ordinates be $(0, y)$.

Point B lies on x-axis, so let its co-ordinates be $(x, 0)$.

P $(-3, 2)$ is the mid-point of line segment AB.

$$\therefore (-3, 2) = \left(\frac{0+x}{2}, \frac{y+0}{2} \right)$$

$$\Rightarrow (-3, 2) = \left(\frac{x}{2}, \frac{y}{2} \right)$$

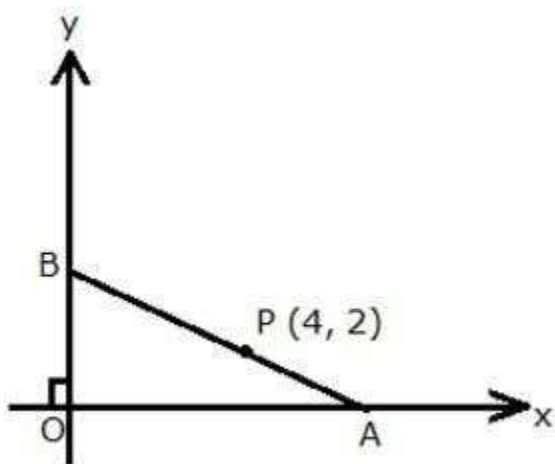
$$\Rightarrow -3 = \frac{x}{2} \quad \text{and} \quad 2 = \frac{y}{2}$$

$$\Rightarrow -6 = x \quad \text{and} \quad 4 = y$$

Thus, the co-ordinates of points A and B are $(0, 4)$ and $(-6, 0)$ respectively.

Question 6.

In the given figure, P $(4, 2)$ is mid-point of line segment AB. Find the co-ordinates of A and B.

**Solution:**

Point A lies on x-axis, so let its co-ordinates be $(x, 0)$.

Point B lies on y-axis, so let its co-ordinates be $(0, y)$.

P $(4, 2)$ is mid-point of line segment AB.

$$\therefore (4, 2) = \left(\frac{x+0}{2}, \frac{0+y}{2} \right)$$

$$\Rightarrow 4 = \frac{x}{2} \quad \text{and} \quad 2 = \frac{y}{2}$$

$$\Rightarrow 8 = x \quad \text{and} \quad 4 = y$$

Hence, the co-ordinates of points A and B are $(8, 0)$ and $(0, 4)$ respectively.

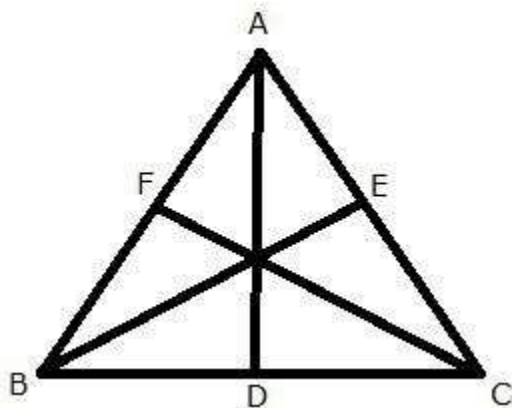
Question 7.

$(-5, 2)$, $(3, -6)$ and $(7, 4)$ are the vertices of a triangle. Find the lengths of its median through the vertex $(3, -6)$

Solution:

Let $A(-5, 2)$, $B(3, -6)$ and $C(7, 4)$ be the vertices of the given triangle.

Let AD be the median through A , BE be the median through B and CF be the median through C .



We know that median of a triangle bisects the opposite side.

Co-ordinates of point F are

$$\left(\frac{-5+3}{2}, \frac{2-6}{2} \right) = \left(\frac{-2}{2}, \frac{-4}{2} \right) = (-1, -2)$$

Co-ordinates of point D are

$$\left(\frac{3+7}{2}, \frac{-6+4}{2} \right) = \left(\frac{10}{2}, \frac{-2}{2} \right) = (5, -1)$$

Co-ordinates of point E are

$$\left(\frac{-5+7}{2}, \frac{2+4}{2} \right) = \left(\frac{2}{2}, \frac{6}{2} \right) = (1, 3)$$

The median of the triangle through the vertex $B(3, -6)$ is BE

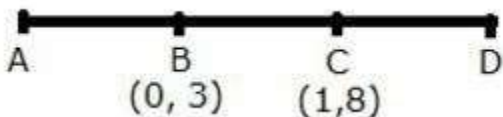
Using distance formula,

$$BE = \sqrt{(1-3)^2 + (3+6)^2} = \sqrt{4+81} = \sqrt{85} = 9.22$$

Question 8.

Given a line ABCD in which $AB = BC = CD$, $B = (0, 3)$ and $C = (1, 8)$. Find the co-ordinates of A and D.

Solution:



Given, $AB = BC = CD$

So, B is the mid-point of AC. Let the co-ordinates of point A be (x, y) .

$$\therefore (0, 3) = \left(\frac{x+1}{2}, \frac{y+8}{2} \right)$$

$$\Rightarrow 0 = \frac{x+1}{2} \quad \text{and} \quad 3 = \frac{y+8}{2}$$

$$\Rightarrow 0 = x+1 \quad \text{and} \quad 6 = y+8$$

$$\Rightarrow -1 = x \quad \text{and} \quad -2 = y$$

Thus, the co-ordinates of point A are $(-1, -2)$.

Also, C is the mid-point of BD. Let the co-ordinates of point D be (p, q) .

$$\therefore (1, 8) = \left(\frac{0+p}{2}, \frac{3+q}{2} \right)$$

$$\Rightarrow 1 = \frac{0+p}{2} \quad \text{and} \quad 8 = \frac{3+q}{2}$$

$$\Rightarrow 2 = 0+p \quad \text{and} \quad 16 = 3+q$$

$$\Rightarrow 2 = p \quad \text{and} \quad 13 = q$$

Thus, the co-ordinates of point D are $(2, 13)$.

Question 9.

One end of the diameter of a circle is $(-2, 5)$. Find the co-ordinates of the other end of it, if the centre of the circle is $(2, -1)$.

Solution:

We know that the centre is the mid-point of diameter.

Let the required co-ordinates of the other end of mid-point be (x, y) .

$$\therefore (2, -1) = \left(\frac{-2+x}{2}, \frac{5+y}{2} \right)$$

$$\Rightarrow 2 = \frac{-2+x}{2} \text{ and } -1 = \frac{5+y}{2}$$

$$\Rightarrow 4 = -2+x \text{ and } -2 = 5+y$$

$$\Rightarrow 6 = x \text{ and } -7 = y$$

Thus, the required co-ordinates are $(6, -7)$.

Question 10.

A $(2, 5)$, B $(1, 0)$, C $(-4, 3)$ and D $(-3, 8)$ are the vertices of a quadrilateral ABCD. Find the co-ordinates of the mid-points of AC and BD.

Give a special name to the quadrilateral.

Solution:

Co-ordinates of the mid-point of AC are

$$\left(\frac{2-4}{2}, \frac{5+3}{2} \right) = \left(\frac{-2}{2}, \frac{8}{2} \right) = (-1, 4)$$

Co-ordinates of the mid-point of BD are

$$\left(\frac{1-3}{2}, \frac{0+8}{2} \right) = \left(\frac{-2}{2}, \frac{8}{2} \right) = (-1, 4)$$

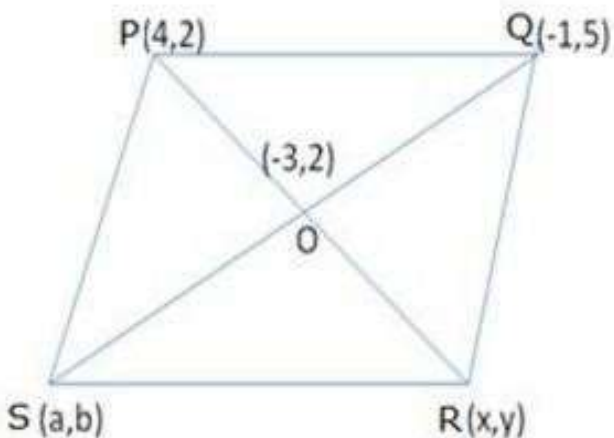
Since, mid-point of AC = mid-point of BD

Hence, ABCD is a parallelogram.

Question 11.

P $(4, 2)$ and Q $(-1, 5)$ are the vertices of a parallelogram PQRS and $(-3, 2)$ are the co-ordinates of the points of intersection of its diagonals. Find the coordinates of R and S.

Solution:



Let the coordinates of R and S be (x,y) and (a,b) respectively.
Mid-point of PR is O.

$$\therefore O(-3,2) = O\left(\frac{4+x}{2}, \frac{2+y}{2}\right)$$

$$-3 = \frac{4+x}{2}, 2 = \frac{2+y}{2}$$

$$-6 = 4+x, 4 = 2+y$$

$$x = -10, y = 2$$

Hence, $R = (-10, 2)$

Similarly, the mid-point of SQ is O.

$$\therefore O(-3,2) = O\left(\frac{a-1}{2}, \frac{b+5}{2}\right)$$

$$-3 = \frac{a-1}{2}, 2 = \frac{b+5}{2}$$

$$-6 = a-1, 4 = b+5$$

$$a = -5, b = -1$$

Hence, $S = (-5, -1)$

Thus, the coordinates of the point R and S are $(-10, 2)$ and $(-5, -1)$.

Question 12.

A $(-1, 0)$, B $(1, 3)$ and D $(3, 5)$ are the vertices of a parallelogram ABCD. Find the coordinates of vertex C.

Solution:

Let the co-ordinates of vertex C be (x, y) .

ABCD is a parallelogram.

$\therefore$ Mid-point of AC = Mid-point of BD

$$\left(\frac{-1+x}{2}, \frac{0+y}{2} \right) = \left(\frac{1+3}{2}, \frac{3+5}{2} \right)$$

$$\left(\frac{-1+x}{2}, \frac{y}{2} \right) = (2, 4)$$

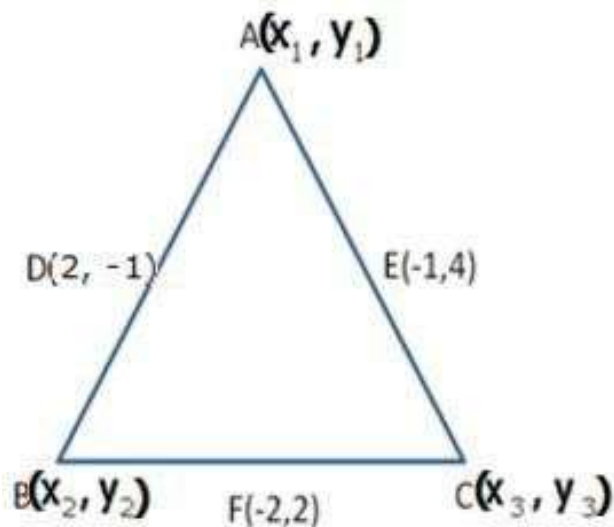
$$\frac{-1+x}{2} = 2 \quad \text{and} \quad \frac{y}{2} = 4$$

$$x = 5 \quad \text{and} \quad y = 8$$

Thus, the co-ordinates of vertex C is $(5, 8)$.

Question 13.

The points $(2, -1)$, $(-1, 4)$ and $(-2, 2)$ are mid-points of the sides of a triangle. Find its vertices.

Solution:

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the co-ordinates of the vertices of ΔABC .

Midpoint of AB, i.e. D

$$D(2, -1) = D\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$2 = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} = -1$$

$$x_1 + x_2 = 4 \quad \text{--- (1)} \quad y_1 + y_2 = -2 \quad \text{--- (2)}$$

Similarly,

$$x_1 + x_3 = -2 \quad \text{--- (3)} \quad y_1 + y_3 = 8 \quad \text{--- (4)}$$

$$x_2 + x_3 = -4 \quad \text{--- (5)} \quad y_2 + y_3 = 4 \quad \text{--- (6)}$$

Adding (1), (3) and (5), we get,

$$2(x_1 + x_2 + x_3) = -2$$

$$x_1 + x_2 + x_3 = -1$$

$$4 + x_3 = -1 \text{ [from (1)]}$$

$$x_3 = -5$$

From (3)

$$x_1 - 5 = -2$$

$$x_1 = 3$$

From (5)

$$x_2 - 5 = -4$$

$$x_2 = 1$$

Adding (2), (4) and (6), we get,

$$2(y_1 + y_2 + y_3) = 10$$

$$y_1 + y_2 + y_3 = 5$$

$$-2 + y_3 = 5 \text{ [from (2)]}$$

$$y_3 = 7$$

From (4)

$$y_1 + 7 = 8$$

$$y_1 = 1$$

From (6)

$$y_2 + 7 = 4$$

$$y_2 = -3$$

Thus, the co-ordinates of the vertices of ΔABC are (3, 1), (1, -3) and (-5, 7).

Question 14.

Points A (-5, x), B (y, 7) and C (1, -3) are collinear (i.e., lie on the same straight line) such that AB = BC. Calculate the values of x and y.

Solution:

Given, AB = BC, i.e., B is the mid-point of AC.

$$\therefore (y, 7) = \left(\frac{-5+1}{2}, \frac{x-3}{2} \right)$$

$$(y, 7) = \left(-2, \frac{x-3}{2} \right)$$

$$\Rightarrow y = -2 \quad \text{and} \quad 7 = \frac{x-3}{2}$$

$$\Rightarrow y = -2 \quad \text{and} \quad x = 17$$

Question 15.

Points P (a, -4), Q (-2, b) and R (0, 2) are collinear. If Q lies between P and R, such that PR = 2QR, calculate the values of a and b.

Solution:

Given, PR = 2QR

Now, Q lies between P and R, so, PR = PQ + QR

$$\therefore PQ + QR = 2QR$$

$$\Rightarrow PQ = QR$$

$\Rightarrow$ Q is the mid-point of PR.

$$\therefore (-2, b) = \left(\frac{a+0}{2}, \frac{-4+2}{2} \right)$$

$$(-2, b) = \left(\frac{a}{2}, -1 \right)$$

$$\Rightarrow a = -4, \quad b = -1$$

Question 16.

Calculate the co-ordinates of the centroid of a triangle ABC, if A = (7, -2), B = (0, 1) and C = (-1, 4).

Solution:

Co-ordinates of the centroid of triangle ABC are

$$\begin{aligned} & \left(\frac{7+0-1}{3}, \frac{-2+1+4}{3} \right) \\ &= \left(\frac{6}{3}, \frac{3}{3} \right) \\ &= (2, 1) \end{aligned}$$

Question 17.

The co-ordinates of the centroid of a PQR are (2, -5). If Q = (-6, 5) and R = (11, 8); calculate the co-ordinates of vertex P.

Solution:

Let G be the centroid of DPQR whose coordinates are (2, -5) and let (x,y) be the coordinates of vertex P.

Coordinates of G are,

$$G(2, -5) = G\left(\frac{x - 6 + 11}{3}, \frac{y + 5 + 8}{3}\right)$$

$$2 = \frac{x+5}{3}, \quad -5 = \frac{y+13}{3}$$

$$6 = x+5, \quad -15 = y+13$$

$$x = 1, y = -28$$

Coordinates of vertex P are (1, -28)

Question 18.

A (5, x), B (-4, 3) and C (y, -2) are the vertices of the triangle ABC whose centroid is the origin. Calculate the values of x and y.

Solution:

Given, centroid of triangle ABC is the origin.

$$\therefore (0, 0) = \left(\frac{5 - 4 + y}{3}, \frac{x + 3 - 2}{3} \right)$$

$$(0, 0) = \left(\frac{1 + y}{3}, \frac{x + 1}{3} \right)$$

$$0 = \frac{1 + y}{3} \quad \text{and} \quad 0 = \frac{x + 1}{3}$$

$$y = -1 \quad \text{and} \quad x = -1$$

Exercise 13C

Question 1.

Given a triangle ABC in which A = (4, -4), B = (0, 5) and C = (5, 10). A point P lies on BC such that BP: PC = 3: 2. Find the length of line segment AP.

Solution:

Given, BP: PC = 3: 2

Using section formula, the co-ordinates of point P are

$$\begin{aligned} & \left(\frac{3 \times 5 + 2 \times 0}{3 + 2}, \frac{3 \times 10 + 2 \times 5}{3 + 2} \right) \\ &= \left(\frac{15}{5}, \frac{40}{5} \right) \\ &= (3, 8) \end{aligned}$$

Using distance formula, we have:

$$AP = \sqrt{(3 - 4)^2 + (8 + 4)^2} = \sqrt{1 + 144} = \sqrt{145} = 12.04$$

Question 2.

A (20, 0) and B (10, -20) are two fixed points. Find the co-ordinates of a point P in AB such that: 3PB = AB. Also, find the co-ordinates of some other point Q in AB such that AB = 6AQ.

Solution:

Given, 3PB = AB

$$\begin{aligned} \Rightarrow \frac{AB}{PB} &= \frac{3}{1} \\ \Rightarrow \frac{AB - PB}{PB} &= \frac{3 - 1}{1} \\ \Rightarrow \frac{AP}{PB} &= \frac{2}{1} \end{aligned}$$

Using section formula,
Coordinates of P are

$$P(x, y) = P \left(\frac{2 \times 10 + 1 \times 20}{2 + 1}, \frac{2 \times (-20) + 1 \times 0}{2 + 1} \right)$$

$$=P\left(\frac{40}{3}, -\frac{40}{3}\right)$$

Given, $AB = 6AQ$

$$\Rightarrow \frac{AQ}{AB} = \frac{1}{6}$$

$$\Rightarrow \frac{AQ}{AB - AQ} = \frac{1}{6 - 1}$$

$$\Rightarrow \frac{AQ}{QB} = \frac{1}{5}$$

Using section formula,

Coordinates of Q are

$$\begin{aligned} Q(x, y) &= Q\left(\frac{1 \times 10 + 5 \times 20}{1 + 5}, \frac{1 \times (-20) + 5 \times 0}{1 + 5}\right) \\ &= Q\left(\frac{110}{6}, -\frac{20}{6}\right) \\ &= Q\left(\frac{55}{3}, -\frac{10}{3}\right) \end{aligned}$$

Question 3.

A (-8, 0), B (0, 16) and C (0, 0) are the vertices of a triangle ABC. Point P lies on AB and Q lies on AC such that AP: PB = 3: 5 and AQ: QC = 3: 5. Show that: PQ = $\frac{3}{8}$ BC.

Solution:

Given that, point P lies on AB such that AP: PB = 3: 5.

The co-ordinates of point P are

$$\begin{aligned} &\left(\frac{3 \times 0 + 5 \times (-8)}{3 + 5}, \frac{3 \times 16 + 5 \times 0}{3 + 5}\right) \\ &= \left(\frac{-40}{8}, \frac{48}{8}\right) \\ &= (-5, 6) \end{aligned}$$

Also, given that, point Q lies on AB such that AQ: QC = 3: 5.

The co-ordinates of point Q are

$$\begin{aligned} &\left(\frac{3 \times 0 + 5 \times (-8)}{3 + 5}, \frac{3 \times 0 + 5 \times 0}{3 + 5}\right) \\ &= \left(\frac{-40}{8}, \frac{0}{8}\right) \end{aligned}$$

$$= (-5, 0)$$

Using distance formula,

$$PQ = \sqrt{(-5+5)^2 + (0-6)^2} = \sqrt{0+36} = 6$$

$$BC = \sqrt{(0-0)^2 + (0-16)^2} = \sqrt{0+(16)^2} = 16$$

$$\text{Now, } \frac{3}{8}BC = \frac{3}{8} \times 16 = 6 = PQ$$

Hence, proved.

Question 4.

Find the co-ordinates of points of trisection of the line segment joining the point (6, -9) and the origin.

Solution:

Let P and Q be the points of trisection of the line segment joining A (6, -9) and B (0, 0).

P divides AB in the ratio 1: 2. Therefore, the co-ordinates of point P are

$$\left(\frac{1 \times 0 + 2 \times 6}{1+2}, \frac{1 \times 0 + 2 \times (-9)}{1+2} \right)$$

$$= \left(\frac{12}{3}, \frac{-18}{3} \right)$$

$$= (4, -6)$$

Q divides AB in the ratio 2: 1. Therefore, the co-ordinates of point Q are

$$\left(\frac{2 \times 0 + 1 \times 6}{2+1}, \frac{2 \times 0 + 1 \times (-9)}{2+1} \right)$$

$$= \left(\frac{6}{3}, \frac{-9}{3} \right)$$

$$= (2, -3)$$

Thus, the required points are (4, -6) and (2, -3).

Question 5.

A line segment joining A(-1, 5/3) and B (a, 5) is divided in the ratio 1: 3 at P, point where the line segment AB intersects the y-axis.

(i) Calculate the value of 'a'.

(ii) Calculate the co-ordinates of 'P'.

Solution:

Since, the line segment AB intersects the y-axis at point P, let the co-ordinates of point P

be $(0, y)$.

P divides AB in the ratio 1: 3.

$$\therefore (0, y) = \left(\frac{1 \times a + 3 \times (-1)}{1 + 3}, \frac{1 \times 5 + 3 \times \frac{5}{3}}{1 + 3} \right)$$

$$(0, y) = \left(\frac{a - 3}{4}, \frac{10}{4} \right)$$

$$0 = \frac{a - 3}{4} \quad \text{and} \quad y = \frac{10}{4}$$

$$a = 3 \quad \text{and} \quad y = \frac{5}{2} = 2\frac{1}{2}$$

Thus, the value of a is 3 and the co-ordinates of point P are $\left(0, 2\frac{1}{2}\right)$.

Question 6.

In what ratio is the line joining A (0, 3) and B (4, -1) divided by the x-axis? Write the co-ordinates of the point where AB intersects the x-axis.

Solution:

Let the line segment AB intersects the x-axis by point P $(x, 0)$ in the ratio $k: 1$.

$$\therefore (x, 0) = \left(\frac{k \times 4 + 1 \times 0}{k + 1}, \frac{k \times (-1) + 1 \times 3}{k + 1} \right)$$

$$(x, 0) = \left(\frac{4k}{k + 1}, \frac{-k + 3}{k + 1} \right)$$

$$\Rightarrow 0 = \frac{-k + 3}{k + 1}$$

$$\Rightarrow k = 3$$

Thus, the required ratio in which P divides AB is 3: 1.

Also, we have:

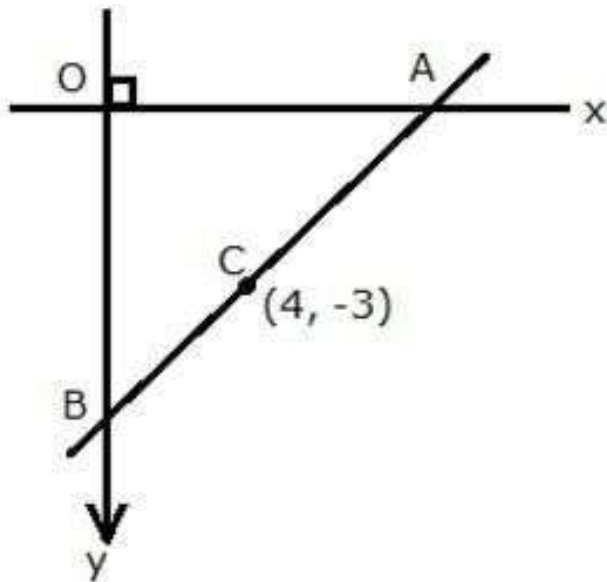
$$x = \frac{4k}{k + 1}$$

$$\Rightarrow x = \frac{4 \times 3}{3 + 1} = \frac{12}{4} = 3$$

Thus, the co-ordinates of point P are (3, 0).

Question 7.

The mid-point of the segment AB, as shown in diagram, is C (4, -3). Write down the co-ordinates of A and B.

**Solution:**

Since, point A lies on x-axis, let the co-ordinates of point A be (x, 0).

Since, point B lies on y-axis, let the co-ordinates of point B be (0, y).

Given, mid-point of AB is C (4, -3).

$$\therefore (4, -3) = \left(\frac{x+0}{2}, \frac{0+y}{2} \right)$$

$$\Rightarrow (4, -3) = \left(\frac{x}{2}, \frac{y}{2} \right)$$

$$\Rightarrow 4 = \frac{x}{2} \quad \text{and} \quad -3 = \frac{y}{2}$$

$$\Rightarrow x = 8 \quad \text{and} \quad y = -6$$

Thus, the co-ordinates of point A are (8, 0) and the co-ordinates of point B are (0, -6).

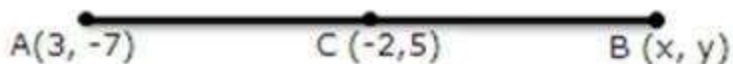
Question 8.

AB is a diameter of a circle with centre C = (-2, 5). If A = (3, -7), find

(i) the length of radius AC

(ii) the coordinates of B.

Solution:



$$\begin{aligned}\text{(i) Radius AC} &= \sqrt{(3+2)^2 + (-7-5)^2} \\ &= \sqrt{5^2 + (-12)^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \text{ units}\end{aligned}$$

(ii) Let the coordinates of B be (x, y).

Using mid – point formula, we have

$$-2 = \frac{3+x}{2} \quad \text{and} \quad 5 = \frac{-7+y}{2}$$

$$\Rightarrow -4 = 3+x \quad \text{and} \quad 10 = -7+y$$

$$\Rightarrow x = -7 \quad \text{and} \quad y = 17$$

Thus, the coordinates of B are (-7, 17).

Question 9.

Find the co-ordinates of the centroid of a triangle ABC whose vertices are:

A (-1, 3), B (1, -1) and C (5, 1)

Solution:

Co-ordinates of the centroid of triangle ABC are

$$\begin{aligned}&\left(\frac{-1+1+5}{3}, \frac{3-1+1}{3} \right) \\ &= \left(\frac{5}{3}, 1 \right)\end{aligned}$$

Question 10.

The mid-point of the line-segment joining (4a, 2b – 3) and (-4, 3b) is (2, -2a). Find the values of a and b.

Solution:

It is given that the mid-point of the line-segment joining (4a, 2b – 3) and (-4, 3b) is (2, -

2a).

$$\therefore (2, -2a) = \left(\frac{4a - 4}{2}, \frac{2b - 3 + 3b}{2} \right)$$

$$\Rightarrow 2 = \frac{4a - 4}{2}$$

$$\Rightarrow 4a - 4 = 4$$

$$\Rightarrow 4a = 8$$

$$\Rightarrow a = 2$$

Also,

$$-2a = \frac{2b - 3 + 3b}{2}$$

$$\Rightarrow -2 \times 2 = \frac{5b - 3}{2}$$

$$\Rightarrow 5b - 3 = -8$$

$$\Rightarrow 5b = -5$$

$$\Rightarrow b = -1$$

Question 11.

The mid-point of the line segment joining $(2a, 4)$ and $(-2, 2b)$ is $(1, 2a + 1)$. Find the value of a and b .

Solution:

Mid-point of $(2a, 4)$ and $(-2, 2b)$ is $(1, 2a + 1)$, therefore using mid-point formula, we have:

$$x = \frac{x_1 + x_2}{2} \quad y = \frac{y_1 + y_2}{2}$$

$$1 = \frac{2a - 2}{2} \quad 2a + 1 = \frac{4 + 2b}{2}$$

$$1 = a - 1$$

$$\therefore a = 2 \quad 2a + 1 = 2 + b$$

Putting, $a = 2$ in $2a + 1 = 2 + b$, we get,

$$5 - 2 = b \Rightarrow b = 3$$

Therefore, $a = 2$, $b = 3$.

Question 12.

- (i) Write down the co-ordinates of the point P that divides the line joining A (-4, 1) and B (17, 10) in the ratio 1: 2.
(ii) Calculate the distance OP, where O is the origin.
(iii) In what ratio does the y-axis divide the line AB?

Solution:

(i) Co-ordinates of point P are

$$\begin{aligned} & \left(\frac{1 \times 17 + 2 \times (-4)}{1 + 2}, \frac{1 \times 10 + 2 \times 1}{1 + 2} \right) \\ &= \left(\frac{17 - 8}{3}, \frac{10 + 2}{3} \right) \\ &= \left(\frac{9}{3}, \frac{12}{3} \right) \\ &= (3, 4) \end{aligned}$$

$$(ii) OP = \sqrt{(0 - 3)^2 + (0 - 4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units}$$

(iii) Let AB be divided by the point P (0, y) lying on y-axis in the ratio k: 1.

$$\begin{aligned} \therefore (0, y) &= \left(\frac{k \times 17 + 1 \times (-4)}{k + 1}, \frac{k \times 10 + 1 \times 1}{k + 1} \right) \\ \Rightarrow (0, y) &= \left(\frac{17k - 4}{k + 1}, \frac{10k + 1}{k + 1} \right) \\ \Rightarrow 0 &= \frac{17k - 4}{k + 1} \\ \Rightarrow 17k - 4 &= 0 \\ \Rightarrow k &= \frac{4}{17} \end{aligned}$$

Thus, the ratio in which the y-axis divide the line AB is 4: 17.

Question 13.

Prove that the points A (-5, 4), B (-1, -2) and C (5, 2) are the vertices of an isosceles right-angled triangle. Find the co-ordinates of D so that ABCD is a square.

Solution:

We have:

$$AB = \sqrt{(-1+5)^2 + (-2-4)^2} = \sqrt{16+36} = \sqrt{52}$$

$$BC = \sqrt{(-1-5)^2 + (-2-2)^2} = \sqrt{36+16} = \sqrt{52}$$

$$AC = \sqrt{(5+5)^2 + (2-4)^2} = \sqrt{100+4} = \sqrt{104}$$

$$AB^2 + BC^2 = 52 + 52 = 104$$

$$AC^2 = 104$$

$$\therefore AB = BC \text{ and } AB^2 + BC^2 = AC^2$$

$\therefore$ ABC is an isosceles right-angled triangle.

Let the coordinates of D be (x, y).

If ABCD is a square, then,

Mid-point of AC = Mid-point of BD

$$\left(\frac{-5+5}{2}, \frac{4+2}{2} \right) = \left(\frac{x-1}{2}, \frac{y-2}{2} \right)$$

$$0 = \frac{x-1}{2}, 3 = \frac{y-2}{2}$$

$$x = 1, y = 8$$

Thus, the co-ordinates of point D are (1, 8).

Question 14.

M is the mid-point of the line segment joining the points A (-3, 7) and B (9, -1). Find the co-ordinates of point M. Further, if R (2, 2) divides the line segment joining M and the origin in the ratio p: q, find the ratio p: q.

Solution:

Given, M is the mid-point of the line segment joining the points A (-3, 7) and B (9, -1).

The co-ordinates of point M are

$$\left(\frac{-3+9}{2}, \frac{7-1}{2} \right)$$

$$= \left(\frac{6}{2}, \frac{6}{2} \right)$$

$$= (3, 3)$$

Also, given that, R (2, 2) divides the line segment joining M and the origin in the ratio p: q.

$$\therefore (2, 2) = \left(\frac{p \times 0 + q \times 3}{p + q}, \frac{p \times 0 + q \times 3}{p + q} \right)$$

$$\Rightarrow \frac{p \times 0 + q \times 3}{p + q} = 2$$

$$\Rightarrow \frac{3q}{p + q} = 2$$

$$\Rightarrow 3q = 2p + 2q$$

$$\Rightarrow 3q - 2q = 2p$$

$$\Rightarrow q = 2p$$

$$\Rightarrow \frac{p}{q} = \frac{1}{2}$$

Thus, the ratio p: q is 1: 2.

Question 15.

Calculate the ratio in which the line joining A(-4, 2) and B(3, 6) is divided by point P(x, 3).

Also, find

(a) x

(b) length of AP.

Solution:

Let P(x, 3) divides the line segment joining the points

A(-4, 2) and B(3, 6) in the ratio k : 1.

Thus, we have

$$\frac{3k - 4}{k + 1} = x; \quad \frac{6k + 2}{k + 1} = 3$$

For

$$6k + 2 = 3(k + 1)$$

$$\Rightarrow 6k + 2 = 3k + 3$$

$$\Rightarrow 3k = 3 - 2$$

$$\Rightarrow 3k = 1 \Rightarrow k = \frac{1}{3}$$

$\therefore$ Required ratio is 1 : 3.

a. Now consider the equation $\frac{3k-4}{k+1} = x$.

Substituting the value of k in the above equation, we have

$$\frac{3 \times \frac{1}{3} - 4}{\frac{1}{3} + 1} = x \Rightarrow \frac{-3}{\frac{4}{3}} = x \Rightarrow \frac{-9}{4} = x$$

$$\therefore x = -\frac{9}{4}$$

$$\begin{aligned} \text{b. } AP &= \sqrt{\left(-\frac{9}{4} + 4\right)^2 + (3-2)^2} = \sqrt{\frac{49}{16} + 1} = \sqrt{\frac{49+16}{16}} = \sqrt{\frac{65}{16}} \\ &\Rightarrow AP = \frac{\sqrt{65}}{4} \text{ units} \end{aligned}$$

Question 16.

Find the ratio in which the line $2x + y = 4$ divides the line segment joining the points $P(2, -2)$ and $Q(3, 7)$.

Solution:

Let the line $2x + y = 4$ divides the line segment joining the points $P(2, -2)$ and $Q(3, 7)$ in the ratio $k : 1$.

Then, we have

$$(x, y) = \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1} \right)$$

Since $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1} \right)$ lies on line $2x + y = 4$, we have

$$2\left(\frac{3k+2}{k+1}\right) + \frac{7k-2}{k+1} = 4$$

$$\Rightarrow 6k + 4 + 7k - 2 = 4k + 4$$

$$\Rightarrow 13k + 2 = 4k + 4$$

$$\Rightarrow 9k = 2$$

$$\Rightarrow k = \frac{2}{9}$$

Hence, required ratio is $2 : 9$.

Question 17.

If the abscissa of a point P is 2. Find the ratio in which this point divides the line segment joining the point (-4, 3) and (6, 3). Also, find the co-ordinates of point P.

Solution:

Let the point P divides the line segment joining the points A(-4, 3) and B(6, 3) in the ratio $k : 1$.

Then, we have

$$(2, y) = \left(\frac{6k - 4}{k + 1}, \frac{3k + 3}{k + 1} \right)$$

$$\Rightarrow \frac{6k - 4}{k + 1} = 2$$

$$\Rightarrow 6k - 4 = 2k + 2$$

$$\Rightarrow 4k = 6$$

$$\Rightarrow k = \frac{3}{2}$$

$\therefore$ Required ratio is 3 : 2.

$$\text{Also, } \frac{3k + 3}{k + 1} = y$$

$$\Rightarrow \frac{3 \times \frac{3}{2} + 3}{\frac{3}{2} + 1} = y$$

$$\Rightarrow \frac{15/2}{5/2} = y$$

$$\Rightarrow y = 3$$

Hence, coordinates of point P are (2, 3).

Question 18.

The line joining the points (2, 1) and (5, -8) is trisected at the points P and Q, point P lies on the line $2x - y + k = 0$, find the value of k. Also, find the co-ordinates of point Q.

Solution:

Let $A(2, 1)$ and $B(5, -8)$ be the given points trisected by the points P and Q .

$$\Rightarrow AP = PQ = QB$$

For P :

$$m_1 : m_2 = AP : PB = 1 : 2$$

$$(x_1, y_1) = (2, 1) \text{ and } (x_2, y_2) = (5, -8)$$

$$\therefore x = \frac{1 \times 5 + 2 \times 2}{1 + 2} = \frac{5 + 4}{3} = \frac{9}{3} = 3$$

$$y = \frac{1 \times (-8) + 2 \times 1}{1 + 2} = \frac{-8 + 2}{3} = \frac{-6}{3} = -2$$

$\therefore$ Coordinates of P are $(3, -2)$.

Since point P lies on the line $2x - y + k = 0$, we have

$$2(3) - (-2) + k = 0$$

$$\Rightarrow 6 + 2 + k = 0$$

$$\Rightarrow k = -8$$

For Q :

$$m_1 : m_2 = AQ : QB = 2 : 1$$

$$(x_1, y_1) = (2, 1) \text{ and } (x_2, y_2) = (5, -8)$$

$$\therefore x = \frac{2 \times 5 + 1 \times 2}{2 + 1} = \frac{10 + 2}{3} = \frac{12}{3} = 4$$

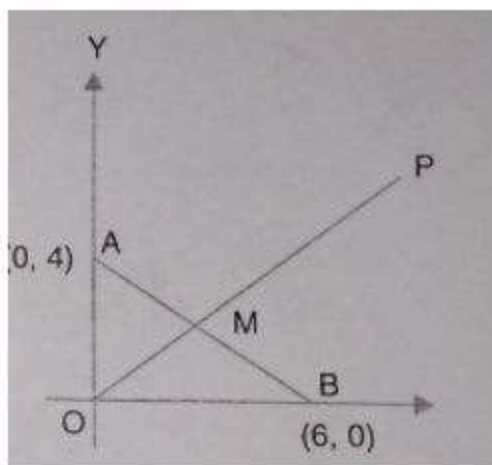
$$y = \frac{2 \times (-8) + 1 \times 1}{2 + 1} = \frac{-16 + 1}{3} = \frac{-15}{3} = -5$$

$\therefore$ Coordinates of Q are $(4, -5)$.

Question 19.

M is the mid-point of the line segment joining the points $A(0, 4)$ and $B(6, 0)$. M also divides the line segment OP in the ratio $1 : 3$. Find :

- (a) co-ordinates of M
- (b) co-ordinates of P
- (c) length of BP



Solution:

a. M is the mid-point of line segment joining points A(0, 4) and B(6, 0).

$$\therefore M = \left(\frac{0+6}{2}, \frac{4+0}{2} \right) = (3, 2)$$

b. M divides OP in the ratio 1 : 3

Let the coordinates of P be (x, y).

$$\therefore M = \left(\frac{x+0}{1+3}, \frac{y+0}{1+3} \right) = \left(\frac{x}{4}, \frac{y}{4} \right)$$

But, M = (3, 2)

$$\therefore \frac{x}{4} = 3 \text{ and } \frac{y}{4} = 2$$

$$\Rightarrow x = 12 \text{ and } y = 8$$

$\therefore$ Coordinates of P are (12, 8).

$$c. BP = \sqrt{(12-6)^2 + (8-0)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ units}$$

Question 20.

Find the image of the point A(5, -3) under reflection in the point P(-1, 3).

Solution:

Let the image be $B(x, y)$.

Since A is reflected in P, P is the mid-point of AB.

Using Mid-point Formula, we get

$$\Rightarrow P(-1, 3) = \left(\frac{x+5}{2}, \frac{y-3}{2} \right)$$

$$\Rightarrow \frac{x+5}{2} = -1 \text{ and } \frac{y-3}{2} = 3$$

$$\Rightarrow x+5 = -2 \text{ and } y-3 = 6$$

$$\Rightarrow x = -7 \text{ and } y = 9$$

So, the image of A in P is $B(-7, 9)$.

Question 21.

$A(-4, 2)$, $B(0, 2)$ and $C(-2, -4)$ are the vertices of a triangle ABC. P, Q and R are mid-points of sides BC, CA and AB respectively. Show that the centroid of ΔPQR is the same as the centroid of ΔABC .

Solution:

$A(-4, 2)$, $B(0, 2)$ and $C(-2, -4)$ are the vertices of ΔABC .

$$\therefore \text{Centroid of } \Delta ABC = \left(\frac{-4+0-2}{3}, \frac{2+2-4}{3} \right) = \left(\frac{-6}{3}, \frac{0}{3} \right) = (-2, 0)$$

P, Q and R are the mid-points of sides BC, CA and AB respectively.

$$\therefore \text{Coordinates of P} = \left(\frac{0-2}{2}, \frac{2-4}{2} \right) = \left(\frac{-2}{2}, \frac{-2}{2} \right) = (-1, -1)$$

$$\text{Coordinates of Q} = \left(\frac{-2-4}{2}, \frac{-4+2}{2} \right) = \left(\frac{-6}{2}, \frac{-2}{2} \right) = (-3, -1)$$

$$\text{Coordinates of R} = \left(\frac{-4+0}{2}, \frac{2+2}{2} \right) = \left(\frac{-4}{2}, \frac{4}{2} \right) = (-2, 2)$$

$$\therefore \text{Centroid of } \Delta PQR = \left(\frac{-1-3-2}{3}, \frac{-1-1+2}{3} \right) = \left(\frac{-6}{3}, \frac{0}{3} \right) = (-2, 0)$$

$$\Rightarrow \text{Centroid of } \Delta ABC = \text{Centroid of } \Delta PQR.$$

Question 22.

$$\text{Centroid of } \triangle ABC = \left(\frac{3 + y + 1}{3}, \frac{1 + 4 + x}{3} \right) = \left(\frac{4 + y}{3}, \frac{5 + x}{3} \right) \quad \dots (i)$$

P, Q and R are the mid points of the sides BC, CA and AB.

By mid - point formula, we get

$$\Rightarrow P = \left(\frac{y + 1}{2}, \frac{4 + x}{2} \right), Q = \left(\frac{4}{2}, \frac{1 + x}{2} \right) \text{ and } R = \left(\frac{3 + y}{2}, \frac{5}{2} \right)$$

$$\begin{aligned} \therefore \text{Centroid of a } \triangle PQR &= \left(\frac{\frac{y + 1}{2} + \frac{4}{2} + \frac{3 + y}{2}}{3}, \frac{\frac{4 + x}{2} + \frac{1 + x}{2} + \frac{5}{2}}{3} \right) \\ &= \left(\frac{\frac{y + 1 + 4 + 3 + y}{2}}{3}, \frac{\frac{4 + x + 1 + x + 5}{2}}{3} \right) \\ &= \left(\frac{8 + 2y}{6}, \frac{10 + 2x}{6} \right) \\ &= \left(\frac{4 + y}{3}, \frac{5 + x}{3} \right) \quad \dots\dots (ii) \end{aligned}$$

From (i) and (ii), we get

Centroid of a $\triangle ABC$ = Centroid of a $\triangle PQR$