

Trigonometrical Identities

Exercise 21A

Question 1.

Prove:

$$\frac{\sec A - 1}{\sec A + 1} = \frac{1 - \cos A}{1 + \cos A}$$

Solution:

$$\begin{aligned}\text{LHS} &= \frac{\sec A - 1}{\sec A + 1} = \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1} \\ &= \frac{1 - \cos A}{1 + \cos A} = \text{RHS}\end{aligned}$$

Question 2.

Prove:

$$\frac{1 + \sin A}{1 - \sin A} = \frac{\csc A + 1}{\csc A - 1}$$

Solution:

$$\begin{aligned}\text{LHS} &= \frac{1 + \sin A}{1 - \sin A} \\ \text{RHS} &= \frac{\csc A + 1}{\csc A - 1} = \frac{\frac{1}{\sin A} + 1}{\frac{1}{\sin A} - 1} \\ &= \frac{1 + \sin A}{1 - \sin A}\end{aligned}$$

Question 3.

Prove:

$$\frac{1}{\tan A + \cot A} = \sin A \cos A$$

Solution:

$$\begin{aligned}\frac{1}{\tan A + \cot A} &= \sin A \cos A \\ \text{LHS} &= \frac{1}{\tan A + \cot A} \\ &= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} \\ &= \frac{1}{1} \left(\because \sin^2 A + \cos^2 A = 1 \right) \\ &= \frac{1}{\sin A \cos A} \\ &= \sin A \cos A = \text{RHS}\end{aligned}$$

Question 4.

Prove:

$$\tan A - \cot A = \frac{1 - 2 \cos^2 A}{\sin A \cos A}$$

Solution:

$$\begin{aligned}\tan A - \cot A &= \frac{\sin A}{\cos A} - \frac{\cos A}{\sin A} \\ &= \frac{\sin^2 A - \cos^2 A}{\sin A \cos A} \\ &= \frac{1 - \cos^2 A - \cos^2 A}{\sin A \cos A} \left(\because \sin^2 A = 1 - \cos^2 A \right) \\ &= \frac{1 - 2 \cos^2 A}{\sin A \cos A}\end{aligned}$$

Question 5.

Prove:

$$\sin^4 A - \cos^4 A = 2 \sin^2 A - 1$$

Solution:

$$\begin{aligned}\sin^4 A - \cos^4 A &= (\sin^2 A)^2 - (\cos^2 A)^2 \\ &= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A) \\ &= \sin^2 A - \cos^2 A \\ &= \sin^2 A - (1 - \sin^2 A) \\ &= 2 \sin^2 A - 1\end{aligned}$$

Question 6.**Prove:**

$$(1 - \tan A)^2 + (1 + \tan A)^2 = 2\sec^2 A$$

Solution:

$$\begin{aligned} & (1 - \tan A)^2 + (1 + \tan A)^2 \\ &= (1 + \tan^2 A - 2\tan A) + (1 + \tan^2 A + 2\tan A) \\ &= 2(1 + \tan^2 A) \\ &= 2\sec^2 A \end{aligned}$$

Question 7.**Prove:**

$$\operatorname{cosec}^4 A - \operatorname{cosec}^2 A = \cot^4 A + \cot^2 A$$

Solution:

$$\begin{aligned} \text{LHS} &= \operatorname{cosec}^4 A - \operatorname{cosec}^2 A \\ &= \operatorname{cosec}^2 A (\operatorname{cosec}^2 A - 1) \\ \text{RHS} &= \cot^4 A + \cot^2 A \\ &= \cot^2 A (\cot^2 A + 1) \\ &= (\operatorname{cosec}^2 A - 1) \operatorname{cosec}^2 A \\ \text{Thus, LHS} &= \text{RHS} \end{aligned}$$

Question 8.**Prove:**

$$\sec A(1 - \sin A)(\sec A + \tan A) = 1$$

Solution:

$$\begin{aligned} \text{LHS} &= \sec A(1 - \sin A)(\sec A + \tan A) \\ &= \frac{1}{\cos A}(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) \\ &= \frac{(1 - \sin A)\left(\frac{1 + \sin A}{\cos A}\right)}{\cos A} = \left(\frac{1 - \sin^2 A}{\cos^2 A}\right) \\ &= \left(\frac{\cos^2 A}{\cos^2 A}\right) = 1 = \text{RHS} \end{aligned}$$

Question 9.**Prove:**

$$\operatorname{cosec} A(1 + \cos A)(\operatorname{cosec} A - \cot A) = 1$$

Solution:

$$\begin{aligned}\text{LHS} &= \operatorname{cosec} A(1 + \cos A)(\operatorname{cosec} A - \cot A) \\&= \frac{1}{\sin A}(1 + \cos A)\left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right) \\&= \frac{(1 + \cos A)}{\sin A}\left(\frac{1 - \cos A}{\sin A}\right) \\&= \frac{1 - \cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1 = \text{RHS}\end{aligned}$$

Question 10.**Prove:**

$$\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \operatorname{cosec}^2 A$$

Solution:

$$\begin{aligned}\text{LHS} &= \sec^2 A + \operatorname{cosec}^2 A \\&= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\cos^2 A \cdot \sin^2 A} \\&= \frac{1}{\cos^2 A \cdot \sin^2 A} = \sec^2 A \operatorname{cosec}^2 A = \text{RHS}\end{aligned}$$

Question 11.**Prove:**

$$\frac{(1 + \tan^2 A) \cot A}{\operatorname{cosec}^2 A} = \tan A$$

Solution:

$$\begin{aligned}& \frac{(1 + \tan^2 A) \cot A}{\operatorname{cosec}^2 A} \\&= \frac{\sec^2 A \cot A}{\operatorname{cosec}^2 A} \left(\because \sec^2 A = 1 + \tan^2 A \right) \\&= \frac{\frac{1}{\cos^2 A} \times \frac{\cos A}{\sin A}}{\frac{1}{\sin^2 A}} = \frac{\frac{1}{\cos A \sin A}}{\frac{1}{\sin^2 A}} \\&= \frac{\sin A}{\cos A} = \tan A\end{aligned}$$

Question 12.

Prove:

$$\tan^2 A - \sin^2 A = \tan^2 A \cdot \sin^2 A$$

Solution:

$$\begin{aligned}\text{LHS} &= \tan^2 A - \sin^2 A \\&= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A} \\&= \frac{\sin^2 A}{\cos^2 A} \cdot \sin^2 A = \tan^2 A \cdot \sin^2 A = \text{RHS}\end{aligned}$$

Question 13.

Prove:

$$\cot^2 A - \cos^2 A = \cos^2 A \cdot \cot^2 A$$

Solution:

$$\begin{aligned}\text{LHS} &= \cot^2 A - \cos^2 A \\&= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A = \frac{\cos^2 A(1 - \sin^2 A)}{\sin^2 A} \\&= \cos^2 A \frac{\cos^2 A}{\sin^2 A} = \cos^2 A \cdot \cot^2 A = \text{RHS}\end{aligned}$$

Question 14.**Prove:**

$$(\operatorname{cosec} A + \sin A)(\operatorname{cosec} A - \sin A) = \cot^2 A + \cos^2 A$$

Solution:

$$\begin{aligned} & (\operatorname{cosec} A + \sin A)(\operatorname{cosec} A - \sin A) \\ &= \operatorname{cosec}^2 A - \sin^2 A \\ &= (1 + \cot^2 A) - (1 - \cos^2 A) \\ &= \cot^2 A + \cos^2 A \end{aligned}$$

Question 15.**Prove:**

$$(\sec A - \cos A)(\sec A + \cos A) = \sin^2 A + \tan^2 A$$

Solution:

$$\begin{aligned} & (\sec A - \cos A)(\sec A + \cos A) \\ &= \sec^2 A - \cos^2 A \\ &= (1 + \tan^2 A) - (1 - \sin^2 A) \\ &= \sin^2 A + \tan^2 A \end{aligned}$$

Question 16.**Prove:**

$$(\cos A + \sin A)^2 + (\cos A - \sin A)^2 = 2$$

Solution:

$$\begin{aligned} \text{LHS} &= (\cos A + \sin A)^2 + (\cos A - \sin A)^2 \\ &= \cos^2 A + \sin^2 A + 2\cos A \cdot \sin A + \cos^2 A + \sin^2 A - 2\cos A \cdot \sin A \\ &= 2(\cos^2 A + \sin^2 A) = 2 = \text{RHS} \end{aligned}$$

Question 17.**Prove:**

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$$

Solution:

$$\begin{aligned}\text{LHS} &= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) \\&= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\tan A + \frac{1}{\tan A}\right) \\&= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right) \\&= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right) \\&= 1\end{aligned}$$

Question 18.

Prove:

$$\frac{1}{\sec A + \tan A} = \sec A - \tan A$$

Solution:

$$\begin{aligned}&\frac{1}{\sec A + \tan A} \\&= \frac{1}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A} \\&= \frac{\sec A - \tan A}{\sec^2 A - \tan^2 A} \\&= \sec A - \tan A\end{aligned}$$

Question 19.

Prove:

$$\operatorname{cosec} A + \cot A = \frac{1}{\operatorname{cosec} A - \cot A}$$

Solution:

$$\begin{aligned}&\operatorname{cosec} A + \cot A \\&= \frac{\operatorname{cosec} A + \cot A}{1} \times \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A - \cot A} \\&= \frac{\operatorname{cosec}^2 A - \cot^2 A}{\operatorname{cosec} A - \cot A} = \frac{1 + \cot^2 A - \cot^2 A}{\operatorname{cosec} A - \cot A} \\&= \frac{1}{\operatorname{cosec} A - \cot A}\end{aligned}$$

Question 20.**Prove:**

$$\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2 \sec A \tan A + 2 \tan^2 A$$

Solution:

$$\begin{aligned} & \frac{\sec A - \tan A}{\sec A + \tan A} \\ &= \frac{\sec A - \tan A}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A} \\ &= \frac{(\sec A - \tan A)^2}{\sec^2 A - \tan^2 A} \\ &= \frac{\sec^2 A + \tan^2 A - 2 \sec A \tan A}{1} \\ &= 1 + \tan^2 A + \tan^2 A - 2 \sec A \tan A \\ &= 1 - 2 \sec A \tan A + 2 \tan^2 A \end{aligned}$$

Question 21.**Prove:**

$$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

Solution:

$$\begin{aligned} & (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\ &= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A \\ &= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2 + 2 \\ &= 1 + \operatorname{cosec}^2 A + \sec^2 A + 4 \\ &= (1 + \cot^2 A) + (1 + \tan^2 A) + 5 \\ &= 7 + \tan^2 A + \cot^2 A \end{aligned}$$

Question 22.**Prove:**

$$\sec^2 A \cdot \operatorname{cosec}^2 A = \tan^2 A + \cot^2 A + 2$$

Solution:

$$\text{LHS} = \sec^2 A \cdot \operatorname{cosec}^2 A = \frac{1}{\cos^2 A \cdot \sin^2 A}$$

$$\text{RHS} = \tan^2 A + \cot^2 A + 2 = \tan^2 A + \cot^2 A + 2 \tan A \cdot \cot A$$

$$= (\tan A + \cot A)^2 = \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)^2$$

$$= \left(\frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A} \right)^2 = \frac{1}{\cos^2 A \cdot \sin^2 A}$$

Hence, LHS = RHS

Question 23.

Prove:

$$\frac{1}{1 + \cos A} + \frac{1}{1 - \cos A} = 2 \operatorname{cosec}^2 A$$

Solution:

$$\begin{aligned} & \frac{1}{1 + \cos A} + \frac{1}{1 - \cos A} \\ &= \frac{1 - \cos A + 1 + \cos A}{(1 + \cos A)(1 - \cos A)} \\ &= \frac{2}{1 - \cos^2 A} \\ &= \frac{2}{\sin^2 A} \\ &= 2 \operatorname{cosec}^2 A \end{aligned}$$

Question 24.

Prove:

$$\frac{1}{1 - \sin A} + \frac{1}{1 + \sin A} = 2 \sec^2 A$$

Solution:

$$\begin{aligned} & \frac{1}{1 - \sin A} + \frac{1}{1 + \sin A} \\ &= \frac{1 + \sin A + 1 - \sin A}{(1 - \sin A)(1 + \sin A)} \\ &= \frac{2}{1 - \sin^2 A} \end{aligned}$$

$$\begin{aligned}
 &= \frac{2}{\cos^2 A} \\
 &= 2 \sec^2 A
 \end{aligned}$$

Question 25.

Prove:

$$\frac{\cos \operatorname{csc} A}{\operatorname{cosec} A - 1} + \frac{\cos \operatorname{csc} A}{\operatorname{cosec} A + 1} = 2 \sec^2 A$$

Solution:

$$\begin{aligned}
 &\frac{\cos \operatorname{csc} A}{\operatorname{cosec} A - 1} + \frac{\cos \operatorname{csc} A}{\operatorname{cosec} A + 1} \\
 &= \frac{\cos \operatorname{csc}^2 A + \cos \operatorname{csc} A + \cos \operatorname{csc}^2 A - \cos \operatorname{csc} A}{\operatorname{cosec}^2 A - 1} \\
 &= \frac{2 \cos \operatorname{csc}^2 A}{\cot^2 A} \left(\because \operatorname{cosec}^2 A - 1 = \cot^2 A \right) \\
 &= \frac{2}{\frac{\sin^2 A}{\cos^2 A}} = \frac{2}{\cos^2 A} = 2 \sec^2 A
 \end{aligned}$$

Question 26.

Prove:

$$\frac{\sec A}{\sec A + 1} + \frac{\sec A}{\sec A - 1} = 2 \operatorname{cosec}^2 A$$

Solution:

$$\begin{aligned}
 &\frac{\sec A}{\sec A + 1} + \frac{\sec A}{\sec A - 1} \\
 &= \frac{\sec^2 A - \sec A + \sec^2 A + \sec A}{\sec^2 A - 1} \\
 &= \frac{2 \sec^2 A}{\tan^2 A} \left(\because \sec^2 A - 1 = \tan^2 A \right) \\
 &= \frac{2}{\frac{\cos^2 A}{\sin^2 A}} = \frac{2}{\sin^2 A} = 2 \operatorname{cosec}^2 A
 \end{aligned}$$

Question 27.**Prove:**

$$\frac{1 + \cos A}{1 - \cos A} = \frac{\tan^2 A}{(\sec A - 1)^2}$$

Solution:

$$\begin{aligned} & \frac{1 + \cos A}{1 - \cos A} \\ &= \frac{1 + \frac{1}{\sec A}}{1 - \frac{1}{\sec A}} = \frac{\sec A + 1}{\sec A - 1} \\ &= \frac{\sec A + 1}{\sec A - 1} \times \frac{\sec A - 1}{\sec A - 1} \\ &= \frac{\sec^2 A - 1}{(\sec A - 1)^2} = \frac{\tan^2 A}{(\sec A - 1)^2} \quad (\because \sec^2 A - 1 = \tan^2 A) \end{aligned}$$

Question 28.**Prove:**

$$\frac{\cot^2 A}{(\operatorname{cosec} A + 1)^2} = \frac{1 - \sin A}{1 + \sin A}$$

Solution:

$$\begin{aligned} \text{R.H.S} &= \frac{1 - \sin A}{1 + \sin A} \\ &= \frac{1 - \frac{1}{\operatorname{cosec} A}}{1 + \frac{1}{\operatorname{cosec} A}} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \\ &= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \times \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A + 1} \\ &= \frac{\operatorname{cosec}^2 A - 1}{(\operatorname{cosec} A + 1)^2} = \frac{\cot^2 A}{(\operatorname{cosec} A + 1)^2} \quad (\because \operatorname{cosec}^2 A - 1 = \cot^2 A) \\ &= \text{L.H.S} \end{aligned}$$

Question 29.**Prove:**

$$\frac{1 + \sin A}{\cos A} + \frac{\cos A}{1 + \sin A} = 2 \sec A$$

Solution:

$$\begin{aligned} & \frac{1 + \sin A}{\cos A} + \frac{\cos A}{1 + \sin A} \\ &= \frac{(1 + \sin A)^2 + \cos^2 A}{\cos A(1 + \sin A)} \\ &= \frac{1 + \sin^2 A + 2 \sin A + \cos^2 A}{\cos A(1 + \sin A)} \\ &= \frac{1 + 2 \sin A + 1}{\cos A(1 + \sin A)} \\ &= \frac{2(1 + \sin A)}{\cos A(1 + \sin A)} \\ &= 2 \sec A \end{aligned}$$

Question 30.**Prove:**

$$\frac{1 - \sin A}{1 + \sin A} = (\sec A - \tan A)^2$$

Solution:

$$\begin{aligned} & \frac{1 - \sin A}{1 + \sin A} \\ &= \frac{1 - \sin A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A} \\ &= \frac{(1 - \sin A)^2}{1 - \sin^2 A} \\ &= \frac{(1 - \sin A)^2}{\cos^2 A} \\ &= \left(\frac{1 - \sin A}{\cos A} \right)^2 \\ &= (\sec A - \tan A)^2 \end{aligned}$$

Question 31.**Prove:**

$$(\cot A - \operatorname{cosec} A)^2 = \frac{1 - \cos A}{1 + \cos A}$$

Solution:

$$\begin{aligned} R.H.S. &= \frac{1 - \cos A}{1 + \cos A} \\ &= \frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A} \\ &= \frac{(1 - \cos A)^2}{1 - \cos^2 A} \\ &= \frac{(1 - \cos A)^2}{\sin^2 A} \\ &= \left(\frac{1 - \cos A}{\sin A} \right)^2 \\ &= (\operatorname{cosec} A - \cot A)^2 \\ &= (\cot A - \operatorname{cosec} A)^2 \\ &= L.H.S \end{aligned}$$

Question 32.**Prove:**

$$\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \left(\frac{\cos A}{1 + \sin A} \right)^2$$

Solution:

$$\begin{aligned} &\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \\ &= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \times \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A + 1} \\ &= \frac{\operatorname{cosec}^2 A - 1}{(\operatorname{cosec} A + 1)^2} \\ &= \frac{\cot^2 A}{(\operatorname{cosec} A + 1)^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{\frac{\cos^2 A}{\sin^2 A}}{\left(\frac{1}{\sin A} + 1\right)^2} \\
&= \left(\frac{\cos A}{1 + \sin A}\right)^2
\end{aligned}$$

Question 33.

Prove:

$$\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$$

Solution:

$$\begin{aligned}
&\tan^2 A - \tan^2 B \\
&= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B} \\
&= \frac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B} \\
&= \frac{\sin^2 A(1 - \sin^2 B) - \sin^2 B(1 - \sin^2 A)}{\cos^2 A \cos^2 B} \\
&= \frac{\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B}{\cos^2 A \cos^2 B} \\
&= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}
\end{aligned}$$

Question 34.

Prove:

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$$

Solution:

$$\begin{aligned}
& \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} \\
&= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)} \\
&= \frac{\sin A(\sin^2 A + \cos^2 A - 2\sin^2 A)}{\cos A(2\cos^2 A - \sin^2 A - \cos^2 A)} \\
&= \frac{\sin A(\cos^2 A - \sin^2 A)}{\cos A(\cos^2 A - \sin^2 A)} \\
&= \frac{\sin A}{\cos A} \\
&= \tan A
\end{aligned}$$

Question 35.

Prove:

$$\frac{\sin A}{1 + \cos A} = \operatorname{cosec} A - \cot A$$

Solution:

$$\begin{aligned}
& \frac{\sin A}{1 + \cos A} \\
&= \frac{\sin A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A} \\
&= \frac{\sin A(1 - \cos A)}{1 - \cos^2 A} \\
&= \frac{\sin A(1 - \cos A)}{\sin^2 A} \\
&= \frac{1 - \cos A}{\sin A} \\
&= \frac{1}{\sin A} - \frac{\cos A}{\sin A} \\
&= \operatorname{cosec} A - \cot A
\end{aligned}$$

Question 36.

Prove:

$$\frac{\cos A}{1 - \sin A} = \sec A + \tan A$$

Solution:

$$\text{LHS} = \frac{\cos A}{1 - \sin A}$$

$$\text{RHS} = \sec A + \tan A$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \frac{1 + \sin A}{\cos A}$$

$$= \frac{1 + \sin A}{\cos A} \left(\frac{1 - \sin A}{1 - \sin A} \right) = \left(\frac{1 - \sin^2 A}{\cos A(1 - \sin A)} \right)$$

$$= \frac{\cos^2 A}{\cos A(1 - \sin A)} = \frac{\cos A}{(1 - \sin A)} = \text{LHS}$$

Question 37.

Prove:

$$\frac{\sin A \tan A}{1 - \cos A} = 1 + \sec A$$

Solution:

$$\frac{\sin A \tan A}{1 - \cos A}$$

$$= \frac{\sin A \tan A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A}$$

$$= \frac{\sin A \tan A(1 + \cos A)}{1 - \cos^2 A}$$

$$= \frac{\sin A \frac{\sin A}{\cos A} (1 + \cos A)}{\sin^2 A}$$

$$= \frac{1 + \cos A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\cos A}{\cos A}$$

$$= \sec A + 1$$

Question 38.

Prove: $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

Solution:

$$\begin{aligned}& (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) \\&= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right) \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) \\&= \left(\frac{\sin A + \cos A - 1}{\sin A}\right) \left(\frac{\cos A + \sin A + 1}{\cos A}\right) \\&= \frac{(\sin A + \cos A - 1)(\sin A + \cos A + 1)}{\sin A \cos A} \\&= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cos A} \\&= \frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A - 1}{\sin A \cos A} \\&= \frac{1 + 2 \sin A \cos A - 1}{\sin A \cos A} \\&= \frac{2 \sin A \cos A}{\sin A \cos A} = 2\end{aligned}$$

Question 39.

Prove:

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

Solution:

$$\begin{aligned}& \sqrt{\frac{1 + \sin A}{1 - \sin A}} \\&= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A}} \\&= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} = \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \\&= \frac{1 + \sin A}{\cos A} \\&= \sec A + \tan A\end{aligned}$$

Question 40.**Prove:**

$$\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$$

Solution:

$$\begin{aligned} & \sqrt{\frac{1 - \cos A}{1 + \cos A}} \\ &= \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A}} \\ &= \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} \\ &= \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} \\ &= \frac{1 - \cos A}{\sin A} \\ &= \operatorname{cosec} A - \cot A \end{aligned}$$

Question 41.**Prove:**

$$\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$$

Solution:

$$\begin{aligned} & \sqrt{\frac{1 - \cos A}{1 + \cos A}} \\ &= \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 + \cos A}{1 + \cos A}} \\ &= \sqrt{\frac{1 - \cos^2 A}{(1 + \cos A)^2}} \\ &= \sqrt{\frac{\sin^2 A}{(1 + \cos A)^2}} \\ &= \frac{\sin A}{1 + \cos A} \end{aligned}$$

Question 42.

Prove:

$$\frac{\sqrt{1 - \sin A}}{\sqrt{1 + \sin A}} = \frac{\cos A}{1 + \sin A}$$

Solution:

$$\begin{aligned} & \frac{\sqrt{1 - \sin A}}{\sqrt{1 + \sin A}} \\ &= \frac{\sqrt{1 - \sin A} \times \sqrt{1 + \sin A}}{\sqrt{1 + \sin A} \times \sqrt{1 + \sin A}} \\ &= \frac{\sqrt{1 - \sin^2 A}}{\sqrt{(1 + \sin A)^2}} \\ &= \frac{\sqrt{\cos^2 A}}{\sqrt{(1 + \sin A)^2}} \\ &= \frac{\cos A}{1 + \sin A} \end{aligned}$$

Question 43.

Prove:

$$1 - \frac{\cos^2 A}{1 + \sin A} = \sin A$$

Solution:

$$\begin{aligned} & 1 - \frac{\cos^2 A}{1 + \sin A} \\ &= \frac{1 + \sin A - \cos^2 A}{1 + \sin A} \\ &= \frac{\sin A + \sin^2 A}{1 + \sin A} \\ &= \frac{\sin A(1 + \sin A)}{1 + \sin A} \\ &= \sin A \end{aligned}$$

Question 44.**Prove:**

$$\frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A} = \frac{2 \sin A}{1 - 2 \cos^2 A}$$

Solution:

$$\begin{aligned} & \frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A} \\ &= \frac{\sin A - \cos A + \sin A + \cos A}{\sin^2 A - \cos^2 A} \\ &= \frac{2 \sin A}{1 - \cos^2 A - \cos^2 A} = \frac{2 \sin A}{1 - 2 \cos^2 A} \end{aligned}$$

Question 45.**Prove:**

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2 \sin^2 A - 1}$$

Solution:

$$\begin{aligned} & \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} \\ &= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A + \cos A)(\sin A - \cos A)} \\ &= \frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A}{\sin^2 A - \cos^2 A} \\ &= \frac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A} \\ &= \frac{2}{\sin^2 A - \cos^2 A} \quad [\sin^2 A + \cos^2 A = 1] \\ &= \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{\sin^2 A - (1 - \sin^2 A)} \\ &\Rightarrow \frac{2}{2 \sin^2 A - 1} \end{aligned}$$

Question 46.**Prove:**

$$\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A}$$

Solution:

$$\begin{aligned} & \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} \\ &= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1} \quad [\operatorname{cosec}^2 A - \cot^2 A = 1] \\ &= \frac{\cot A + \operatorname{cosec} A - [(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)]}{\cot A - \operatorname{cosec} A + 1} \\ &= \frac{\cot A + \operatorname{cosec} A [1 - \operatorname{cosec} A + \cot A]}{\cot A - \operatorname{cosec} A + 1} \\ &= \cot A + \operatorname{cosec} A \\ &= \frac{\cos A}{\sin A} + \frac{1}{\sin A} \\ &= \frac{1 + \cos A}{\sin A} \end{aligned}$$

Question 47.**Prove:**

$$\frac{\sin \theta \tan \theta}{1 - \cos \theta} = 1 + \sec \theta$$

Solution:

$$\begin{aligned} & \frac{\sin \theta \tan \theta}{1 - \cos \theta} \\ &= \frac{\sin \theta \tan \theta}{1 - \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta} \\ &= \frac{\sin \theta \tan \theta (1 + \cos \theta)}{1 - \cos^2 \theta} \\ &= \frac{\sin \theta \frac{\sin \theta}{\cos \theta} (1 + \cos \theta)}{\sin^2 \theta} \\ &= \frac{(1 + \cos \theta)}{\cos \theta} \\ &= \frac{1}{\cos \theta} + 1 \\ &= \sec \theta + 1 \end{aligned}$$

Question 48.**Prove:**

$$\frac{\cos \theta \cot \theta}{1 + \sin \theta} = \operatorname{cosec} \theta - 1$$

Solution:

$$\begin{aligned} & \frac{\cos \theta \cot \theta}{1 + \sin \theta} \\ &= \frac{\cos \theta \cot \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} \\ &= \frac{\cos \theta \cot \theta (1 - \sin \theta)}{1 - \sin^2 \theta} \\ &= \frac{\cos \theta \frac{\cos \theta}{\sin \theta} (1 - \sin \theta)}{\cos^2 \theta} \\ &= \frac{(1 - \sin \theta)}{\sin \theta} \\ &= \frac{1}{\sin \theta} - 1 \\ &= \operatorname{cosec} \theta - 1 \end{aligned}$$

Exercise 21 B**Question 1.****Prove that:**

$$(i) \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

$$(ii) \frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = 2$$

$$(iii) \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = \sec A \operatorname{cosec} A + 1$$

$$(iv) \left(\tan A + \frac{1}{\cos A} \right)^2 + \left(\tan A - \frac{1}{\cos A} \right)^2 = 2 \left(\frac{1 + \sin^2 A}{1 - \sin^2 A} \right)$$

$$(v) 2\sin^2 A + \cos^4 A = 1 + \sin^4 A$$

$$(vi) \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$$

$$(vii) (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$(viii) (1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$$

$$(ix) \frac{1}{\cos A + \sin A - 1} + \frac{1}{\cos A + \sin A + 1} = \operatorname{cosec} A + \sec A$$

Solution:

(i)

$$\begin{aligned}\text{LHS} &= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} \\&= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}} = \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}} \\&= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} = \frac{\cos^2 A - \sin^2 A}{(\cos A - \sin A)} \\&= \sin A + \cos A = \text{RHS}\end{aligned}$$

(ii)

$$\begin{aligned}&\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} \\&= \frac{(\cos^3 A + \sin^3 A)(\cos A - \sin A) + (\cos^3 A - \sin^3 A)(\cos A + \sin A)}{\cos^2 A - \sin^2 A} \\&= \frac{\cos^4 A - \cos^3 A \sin A + \sin^3 A \cos A - \sin^4 A}{\cos^2 A - \sin^2 A} \\&= \frac{+\cos^4 A + \cos^3 A \sin A - \sin^3 A \cos A - \sin^4 A}{\cos^2 A - \sin^2 A} \\&= \frac{2(\cos^4 A - \sin^4 A)}{\cos^2 A - \sin^2 A} \\&= \frac{2(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)}{(\cos^2 A - \sin^2 A)} \\&= 2(\cos^2 A + \sin^2 A) \\&= 2 \left(\because \cos^2 A + \sin^2 A = 1 \right)\end{aligned}$$

(iii)

$$\begin{aligned}&\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} \\&= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1 - \tan A} \\&= \frac{\tan^2 A}{\tan A - 1} + \frac{1}{\tan A(1 - \tan A)}\end{aligned}$$

$$= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)}$$

$$= \frac{(\tan A - 1)(\tan^2 A + 1 + \tan A)}{\tan A(\tan A - 1)}$$

$$= \frac{\sec^2 A + \tan A}{\tan A}$$

$$= \frac{1}{\frac{\cos^2 A}{\sin A}} + 1$$

$$= \frac{1}{\sin A \cos A} + 1$$

$$= \sec A \operatorname{cosec} A + 1$$

(iv)

$$\left(\tan A + \frac{1}{\cos A} \right)^2 + \left(\tan A - \frac{1}{\cos A} \right)^2$$

$$= \left(\frac{\sin A + 1}{\cos A} \right)^2 + \left(\frac{\sin A - 1}{\cos A} \right)^2$$

$$= \frac{\sin^2 A + 1 + 2 \sin A + \sin^2 A + 1 - 2 \sin A}{\cos^2 A}$$

$$= \frac{2 + 2 \sin^2 A}{\cos^2 A}$$

$$= 2 \left(\frac{1 + \sin^2 A}{1 - \sin^2 A} \right)$$

(v)

$$2 \sin^2 A + \cos^4 A$$

$$= 2 \sin^2 A + (1 - \sin^2 A)^2$$

$$= 2 \sin^2 A + 1 + \sin^4 A - 2 \sin^2 A$$

$$= 1 + \sin^4 A$$

(vi)

$$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1-1}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= 0$$

(vii)

LHS

$$= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

$$= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right)$$

$$= \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right)$$

$$= \sin A \cos A$$

$$\text{RHS} = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \sin A \cos A$$

LHS = RHS

(viii)

$$(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2$$

$$= 1 + \tan^2 A \tan^2 B + 2 \tan A \tan B + \tan^2 A + \tan^2 B - 2 \tan A \tan B$$

$$= 1 + \tan^2 A + \tan^2 B + \tan^2 A \tan^2 B$$

$$= \sec^2 A + \tan^2 B (1 + \tan^2 A)$$

$$= \sec^2 A + \tan^2 B \sec^2 A$$

$$= \sec^2 A (1 + \tan^2 B)$$

$$= \sec^2 A \sec^2 B$$

(ix)

$$\begin{aligned}
& \frac{1}{(\cos A + \sin A) - 1} + \frac{1}{(\cos A + \sin A) + 1} \\
&= \frac{\cos A + \sin A + 1 + \cos A + \sin A - 1}{(\cos A + \sin A)^2 - 1} \\
&= \frac{2(\cos A + \sin A)}{\cos^2 A + \sin^2 A + 2\cos A \sin A - 1} \\
&= \frac{2(\cos A + \sin A)}{1 + 2\cos A \sin A - 1} = \frac{\cos A + \sin A}{\cos A \sin A} \\
&= \frac{\cos A}{\cos A \sin A} + \frac{\sin A}{\cos A \sin A} \\
&= \frac{1}{\sin A} + \frac{1}{\cos A} \\
&= \operatorname{cosec} A + \sec A
\end{aligned}$$

Question 2.

If $x \cos A + y \sin A = m$ and $x \sin A - y \cos A = n$, then prove that $x^2 + y^2 = m^2 + n^2$.

Solution:

$$\begin{aligned}
& m^2 + n^2 \\
&= (x \cos A + y \sin A)^2 + (x \sin A - y \cos A)^2 \\
&= x^2 \cos^2 A + y^2 \sin^2 A + 2xy \sin A \cos A \\
&\quad + x^2 \sin^2 A + y^2 \cos^2 A - 2xy \sin A \cos A \\
&= x^2 (\cos^2 A + \sin^2 A) + y^2 (\cos^2 A + \sin^2 A) \\
&= x^2 + y^2 \\
&\text{Hence, } x^2 + y^2 = m^2 + n^2.
\end{aligned}$$

Question 3.

If $m = a \sec A + b \tan A$ and $n = a \tan A + b \sec A$, prove that $m^2 - n^2 = a^2 - b^2$

Solution:

Given,

$$m = a \sec A + b \tan A \text{ and } n = a \tan A + b \sec A$$

$$m^2 - n^2 = (a \sec A + b \tan A)^2 - (a \tan A + b \sec A)^2$$

$$= a^2 \sec^2 A + b^2 \tan^2 A + 2ab \sec A \tan A$$

$$- (a^2 \tan^2 A + b^2 \sec^2 A + 2ab \sec A \tan A)$$

$$= \sec^2 A (a^2 - b^2) + \tan^2 A (b^2 - a^2)$$

$$= (a^2 - b^2) [\sec^2 A - \tan^2 A]$$

$$= (a^2 - b^2) \text{ [Since } \sec^2 A - \tan^2 A = 1]$$

$$\text{Hence, } m^2 - n^2 = a^2 - b^2$$

Question 4.

If $x = r \sin A \cos B$, $y = r \sin A \sin B$ and $z = r \cos A$, prove that $x^2 + y^2 + z^2 = r^2$

Solution:

$$\text{LHS} = (r \sin A \cos B)^2 + (r \sin A \sin B)^2 + (r \cos A)^2$$

$$= r^2 \sin^2 A \cos^2 B + r^2 \sin^2 A \sin^2 B + r^2 \cos^2 A$$

$$= r^2 \sin^2 A (\cos^2 B + \sin^2 B) + r^2 \cos^2 A$$

$$= r^2 (\sin^2 A + \cos^2 A) = r^2 = \text{RHS}$$

Question 5.

If $\sin A + \cos A = m$ and $\sec A + \csc A = n$, prove that $n(m^2 - 1) = 2m$.

Solution:

Given:

$$\sin A + \cos A = m$$

and

$$\sec A + \csc A = n$$

$$\text{Consider L.H.S} = n(m^2 - 1)$$

$$= (\sec A + \csc A) [(\sin A + \cos A)^2 - 1]$$

$$= \left(\frac{1}{\cos A} + \frac{1}{\sin A} \right) [\sin^2 A + \cos^2 A + 2 \sin A \cos A - 1]$$

$$= \left(\frac{\cos A + \sin A}{\sin A \cos A} \right) (1 + 2 \sin A \cos A - 1)$$

$$= \frac{(\cos A + \sin A)}{\sin A \cos A} (2 \sin A \cos A)$$

$$= 2(\sin A + \cos A)$$

$$= 2m = \text{R.H.S.}$$

Question 6.

If $x = r \cos A \cos B$, $y = r \cos A \sin B$ and $z = r \sin A$, prove that $x^2 + y^2 + z^2 = r^2$

Solution:

$$\begin{aligned}
 \text{LHS} &= (r \cos A \cos B)^2 + (r \cos A \sin B)^2 + (r \sin A)^2 \\
 &= r^2 \cos^2 A \cos^2 B + r^2 \cos^2 A \sin^2 B + r^2 \sin^2 A \\
 &= r^2 \cos^2 A (\cos^2 B + \sin^2 B) + r^2 \sin^2 A \\
 &= r^2 (\cos^2 A + \sin^2 A) = r^2 = \text{RHS}
 \end{aligned}$$

Question 7.

If $\frac{\cos A}{\cos B} = m$ and $\frac{\cos A}{\sin B} = n$, show that $(m^2 + n^2) \cos^2 B = n^2$.

Solution:

$$\begin{aligned}
 \text{LHS} &= (m^2 + n^2) \cos^2 B \\
 &= \left(\frac{\cos^2 A}{\cos^2 B} + \frac{\cos^2 A}{\sin^2 B} \right) \cos^2 B \\
 &= \left(\frac{\cos^2 A \sin^2 B + \cos^2 A \cos^2 B}{\cos^2 B \sin^2 B} \right) \cos^2 B \\
 &= \left(\frac{\cos^2 A \sin^2 B + \cos^2 A \cos^2 B}{\sin^2 B} \right) \\
 &= \frac{\cos^2 A (\sin^2 B + \cos^2 B)}{\sin^2 B} \\
 &= \frac{\cos^2 A}{\sin^2 B} \\
 &= n^2
 \end{aligned}$$

Hence, $(m^2 + n^2) \cos^2 B = n^2$.

Exercise 21 C**Question 1.**

Without using trigonometric tables, show that:

- (i) $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ = 1$
- (ii) $\sin 42^\circ \sec 48^\circ + \cos 42^\circ \csc 48^\circ = 2$
- (iii) $\frac{\sin 26^\circ}{\sec 64^\circ} + \frac{\cos 26^\circ}{\csc 64^\circ} = 1$

Solution:

(i)

$$\begin{aligned}
 & \tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ \\
 &= \tan(90^\circ - 80^\circ) \tan(90^\circ - 75^\circ) \tan 75^\circ \tan 80^\circ \\
 &= \cot 80^\circ \cot 75^\circ \tan 75^\circ \tan 80^\circ \\
 &= 1 \text{ [As } \tan \theta \cot \theta = 1]
 \end{aligned}$$

(ii)

$$\begin{aligned}
 & \sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ = 2 \\
 & \text{Consider } \sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ \\
 & \Rightarrow \sin 42^\circ \sec(90^\circ - 42^\circ) + \cos 42^\circ \operatorname{cosec}(90^\circ - 42^\circ) \\
 & \Rightarrow \sin 42^\circ \cdot \operatorname{cosec} 42^\circ + \cos 42^\circ \sec 42^\circ \\
 & \Rightarrow \sin 42^\circ \cdot \frac{1}{\sin 42^\circ} + \cos 42^\circ \frac{1}{\cos 42^\circ} \\
 & \Rightarrow 1 + 1 = 2
 \end{aligned}$$

(iii)

$$\begin{aligned}
 & \frac{\sin 26^\circ}{\sec 64^\circ} + \frac{\cos 26^\circ}{\operatorname{cosec} 64^\circ} \\
 &= \frac{\sin 26^\circ}{\sec(90^\circ - 26^\circ)} + \frac{\cos 26^\circ}{\operatorname{cosec}(90^\circ - 26^\circ)} \\
 &= \frac{\sin 26^\circ}{\operatorname{cosec} 26^\circ} + \frac{\cos 26^\circ}{\sec 26^\circ} \\
 &= \sin^2 26^\circ + \cos^2 26^\circ \\
 &= 1
 \end{aligned}$$

Question 2.Express each of the following in terms of angles between 0° and 45° :

(i) $\sin 59^\circ + \tan 63^\circ$

(ii) $\operatorname{cosec} 68^\circ + \cot 72^\circ$

(iii) $\cos 74^\circ + \sec 67^\circ$

Solution:

$$\begin{aligned}
 & \text{(i) } \sin 59^\circ + \tan 63^\circ \\
 &= \sin(90 - 31)^\circ + \tan(90 - 27)^\circ \\
 &= \cos 31^\circ + \cot 27^\circ \\
 & \text{(ii) } \operatorname{cosec} 68^\circ + \cot 72^\circ \\
 &= \operatorname{cosec}(90 - 22)^\circ + \cot(90 - 18)^\circ \\
 &= \sec 22^\circ + \tan 18^\circ \\
 & \text{(iii) } \cos 74^\circ + \sec 67^\circ \\
 &= \cos(90 - 16)^\circ + \sec(90 - 23)^\circ \\
 &= \sin 16^\circ + \operatorname{cosec} 23^\circ
 \end{aligned}$$

Question 3.**Show that:**

$$(i) \frac{\sin A}{\sin(90^\circ - A)} + \frac{\cos A}{\cos(90^\circ - A)} = \sec A \operatorname{cosec} A$$

$$(ii) \sin A \cos A - \frac{\sin A \cos(90^\circ - A) \cos A}{\sec(90^\circ - A)} - \frac{\cos A \sin(90^\circ - A) \sin A}{\operatorname{cosec}(90^\circ - A)} = 0$$

Solution:

$$(i) \frac{\sin A}{\sin(90^\circ - A)} + \frac{\cos A}{\cos(90^\circ - A)}$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$$

$$= \frac{1}{\cos A \sin A}$$

$$= \sec A \operatorname{cosec} A$$

$$(ii) \sin A \cos A - \frac{\sin A \cos(90^\circ - A) \cos A}{\sec(90^\circ - A)} - \frac{\cos A \sin(90^\circ - A) \sin A}{\operatorname{cosec}(90^\circ - A)}$$

$$= \sin A \cos A - \frac{\sin A \sin A \cos A}{\operatorname{cosec} A} - \frac{\cos A \cos A \sin A}{\sec A}$$

$$= \sin A \cos A - \sin^3 A \cos A - \cos^3 A \sin A$$

$$= \sin A \cos A - \sin A \cos A (\sin^2 A + \cos^2 A)$$

$$= \sin A \cos A - \sin A \cos A (1)$$

$$= 0$$

Question 4.**For triangle ABC, show that:**

$$(i) \sin\left(\frac{A+B}{2}\right) = \cos \frac{C}{2}$$

$$(ii) \tan\left(\frac{B+C}{2}\right) = \cot \frac{A}{2}$$

Solution:

(i) We know that for a triangle $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\frac{\angle B + \angle A}{2} = 90^\circ - \frac{\angle C}{2}$$

$$\begin{aligned}\sin\left(\frac{A+B}{2}\right) &= \sin\left(90^\circ - \frac{C}{2}\right) \\ &= \cos\left(\frac{C}{2}\right)\end{aligned}$$

(ii) We know that for a triangle $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle C = 180^\circ - \angle A$$

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$$\begin{aligned}\tan\left(\frac{B+C}{2}\right) &= \tan\left(90^\circ - \frac{A}{2}\right) \\ &= \cot\left(\frac{A}{2}\right)\end{aligned}$$

Question 5.

Evaluate:

(i) $3 \frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(ii) $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$

(iii) $\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$

(iv) $\tan(55^\circ - A) - \cot(35^\circ + A)$

(v) $\operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A)$

(vi) $2 \frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2} \cos 45^\circ$

(vii) $\frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2 \frac{\sin^2 75^\circ}{\cos^2 15^\circ}$

(viii) $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ$

(ix) $14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ$

Solution:

(i)

$$\begin{aligned}
 & 3 \frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ} \\
 &= 3 \frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ} \\
 &= 3 \frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} = 3 - 1 = 2
 \end{aligned}$$

(ii) $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$

$$\begin{aligned}
 &= 3 \cos(90^\circ - 10^\circ) \operatorname{cosec} 10^\circ + 2 \cos(90^\circ - 31^\circ) \operatorname{cosec} 31^\circ \\
 &= 3 \sin 10^\circ \operatorname{cosec} 10^\circ + 2 \sin 31^\circ \operatorname{cosec} 31^\circ \\
 &= 3 + 2 = 5
 \end{aligned}$$

(iii) $\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$

$$\begin{aligned}
 &= \frac{\sin(90^\circ - 10^\circ)}{\cos 10^\circ} + \sin(90^\circ - 31^\circ) \sec 31^\circ \\
 &= \frac{\cos 10^\circ}{\cos 10^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ} \\
 &= 1 + 1 = 2
 \end{aligned}$$

(iv) $\tan(55^\circ - A) - \cot(35^\circ + A)$

$$\begin{aligned}
 &= \tan[90^\circ - (35^\circ + A)] - \cot(35^\circ + A) \\
 &= \cot(35^\circ + A) - \cot(35^\circ + A) \\
 &= 0
 \end{aligned}$$

(v) $\operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A)$

$$\begin{aligned}
 &= \operatorname{cosec}[90^\circ - (25^\circ - A)] - \sec(25^\circ - A) \\
 &= \sec(25^\circ - A) - \sec(25^\circ - A) \\
 &= 0
 \end{aligned}$$

(vi) $2 \frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2} \cos 45^\circ$

$$\begin{aligned}
 &= 2 \frac{\tan(90^\circ - 33^\circ)}{\cot 33^\circ} - \frac{\cot(90^\circ - 20^\circ)}{\tan 20^\circ} - \sqrt{2} \left(\frac{1}{\sqrt{2}} \right) \\
 &= 2 \frac{\cot 33^\circ}{\cot 33^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 1 \\
 &= 2 - 1 - 1 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned} \text{(vii)} \quad & \frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2 \frac{\sin^2 75^\circ}{\cos^2 15^\circ} \\ &= \frac{[\cot(90^\circ - 49^\circ)]^2}{\tan^2 49^\circ} - 2 \frac{[\sin(90^\circ - 15^\circ)]^2}{\cos^2 15^\circ} \end{aligned}$$

$$= \frac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2 \frac{\cos^2 15^\circ}{\cos^2 15^\circ}$$

$$= 1 - 2 = -1$$

$$\text{(viii)} \quad \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ$$

$$= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8 \left(\frac{1}{2}\right)^2$$

$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 2$$

$$= 1 + 1 - 2 = 0$$

$$\text{(ix)} \quad 14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ$$

$$= 14 \left(\frac{1}{2}\right) + 6 \left(\frac{1}{2}\right) - 5(1)$$

$$= 7 + 3 - 5 = 5$$

Question 6.

A triangle ABC is right angled at B; find the value of $\frac{\sec A \cdot \operatorname{cosec} C - \tan A \cdot \cot C}{\sin B}$

Solution:

Since, ABC is a right angled triangle, right angled at B.

So, $A + C = 90^\circ$

$$\frac{\sec A \cdot \operatorname{cosec} C - \tan A \cdot \cot C}{\sin B}$$

$$= \frac{\sec(90^\circ - C) \cdot \operatorname{cosec} C - \tan(90^\circ - C) \cdot \cot C}{\sin 90^\circ}$$

$$= \frac{\operatorname{cosec} C \cdot \operatorname{cosec} C - \cot C \cdot \cot C}{1}$$

$$= 1 \quad [\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1]$$

Question 7.

Find (in each case, given below) the value of x if:

(i) $\sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

(ii) $\sin x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

(iii) $\cos x = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

(iv) $\tan x = \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$

(v) $\sin 2x = 2 \sin 45^\circ \cos 45^\circ$

(vi) $\sin 3x = 2 \sin 30^\circ \cos 30^\circ$

(vii) $\cos(2x - 6^\circ) = \cos^2 30^\circ - \cos^2 60^\circ$

Solution:

(i) $\sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$$\sin x = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\sin x = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \sin 30^\circ$$

Hence, $x = 30^\circ$

(ii) $\sin x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$$\sin x = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\sin x = \frac{3}{4} + \frac{1}{4} = 1 = \sin 90^\circ$$

Hence, $x = 90^\circ$

(iii) $\cos x = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$\cos x = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$\cos x = 0 = \cos 90^\circ$$

Hence, $x = 90^\circ$

(iv) $\tan x = \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$

$$\tan x = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}}$$

$$(i) \sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$$

$$\sin x = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\sin x = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \sin 30^\circ$$

Hence, $x = 30^\circ$

$$(ii) \sin x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$\sin x = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\sin x = \frac{3}{4} + \frac{1}{4} = 1 = \sin 90^\circ$$

Hence, $x = 90^\circ$

$$(iii) \cos x = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$$

$$\cos x = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$\cos x = 0 = \cos 90^\circ$$

Hence, $x = 90^\circ$

$$(iv) \tan x = \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$$

$$\tan x = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}}$$

$$\tan x = \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Hence, $x = 30^\circ$

$$(v) \sin 2x = 2 \sin 45^\circ \cos 45^\circ$$

$$\sin 2x = 2 \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right)$$

$$\sin 2x = 1 = \sin 90^\circ$$

$$2x = 90^\circ$$

$$\text{Hence, } x = 45^\circ$$

$$(vi) \sin 3x = 2 \sin 30^\circ \cos 30^\circ$$

$$\sin 3x = 2 \left(\frac{1}{2} \right) \left(\frac{\sqrt{3}}{2} \right)$$

$$\sin 3x = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$$3x = 60^\circ$$

$$\text{Hence, } x = 20^\circ$$

$$(vii) \cos(2x - 6^\circ) = \cos^2 30^\circ - \cos^2 60^\circ$$

$$\cos(2x - 6) = \cos^2(90^\circ - 60^\circ) - \cos^2 60^\circ$$

$$\cos(2x - 6) = \sin^2 60^\circ - \cos^2 60^\circ$$

$$\cos(2x - 6) = 1 - 2 \cos^2 60^\circ = 1 - 2 \left(\frac{1}{2} \right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\cos(2x - 6) = \frac{1}{2}$$

$$\cos(2x - 6) = \cos 60^\circ$$

$$(2x - 6) = 60^\circ$$

$$2x = 66^\circ$$

$$\text{Hence, } x = 33^\circ$$

Question 8.

In each case, given below, find the value of angle A, where $0^\circ \leq A \leq 90^\circ$.

$$(i) \sin(90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$$

$$(ii) \cos(90^\circ - A) \cdot \sec 77^\circ = 1$$

Solution:

$$(i) \sin(90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$$

$$\cos 3A \cdot \frac{1}{\sin 42^\circ} = 1$$

$$\cos 3A = \sin 42^\circ = \sin(90^\circ - 48^\circ) = \cos 48^\circ$$

$$3A = 48^\circ$$

$$A = 16^\circ$$

$$(ii) \cos(90^\circ - A) \cdot \sec 77^\circ = 1$$

$$\cos(90^\circ - A) \cdot \sec 77^\circ = 1$$

$$\sin A \cdot \frac{1}{\cos 77^\circ} = 1$$

$$\sin A = \cos 77^\circ = \cos(90^\circ - 13^\circ) = \sin 13^\circ$$

$$A = 13^\circ$$

Question 9.**Prove that:**

$$(i) \frac{\cos(90^\circ - \theta) \cos \theta}{\cot \theta} = 1 - \cos^2 \theta$$

$$(ii) \frac{\sin \theta \sin(90^\circ - \theta)}{\cot(90^\circ - \theta)} = 1 - \sin^2 \theta$$

Solution:

(i)

$$\text{LHS} = \frac{\cos(90^\circ - \theta) \cos \theta}{\cot \theta} = \frac{\sin \theta \cos \theta}{\frac{\cos \theta}{\sin \theta}} = \sin^2 \theta = 1 - \cos^2 \theta$$

(ii)

$$\text{LHS} = \frac{\sin \theta \sin(90^\circ - \theta)}{\cot(90^\circ - \theta)} = \frac{\sin \theta \cos \theta}{\tan \theta} = \frac{\sin \theta \cos \theta}{\frac{\sin \theta}{\cos \theta}} = \cos^2 \theta = 1 - \sin^2 \theta$$

Question 10.**Evaluate :**

$$\frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ}$$

Solution:

$$\begin{aligned}& \frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ} \\&= \frac{\sin 35^\circ \cdot \cos (90^\circ - 35^\circ) + \cos 35^\circ \cdot \sin (90^\circ - 35^\circ)}{\operatorname{cosec}^2 (90^\circ - 80^\circ) - \tan^2 80^\circ} \\&= \frac{\sin 35^\circ \cdot \sin 35^\circ + \cos 35^\circ \cdot \cos 35^\circ}{\sec^2 80^\circ - \tan^2 80^\circ} \\&= \frac{\sin^2 35^\circ + \cos^2 35^\circ}{\sec^2 80^\circ - \tan^2 80^\circ} = \frac{1}{1} = 1\end{aligned}$$

Question 11.

Evaluate

$$\sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$$

Solution:

$$\begin{aligned}& \sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ \\&= \sin^2 34^\circ + \sin^2 (90^\circ - 34^\circ) + 2 \tan 18^\circ \tan (90^\circ - 72^\circ) - \cot^2 30^\circ \\&= \sin^2 34^\circ + \cos^2 34^\circ + 2 \tan 18^\circ \cot 18^\circ - \cot^2 30^\circ \\&= (\sin^2 34^\circ + \cos^2 34^\circ) + 2 \tan 18^\circ \times \frac{1}{\tan 18^\circ} - \cot^2 30^\circ \\&= 1 + 2 \times 1 - (\sqrt{3})^2 \\&= 1 + 2 - 3 \\&= 3 - 3 \\&= 0\end{aligned}$$

Question 12.

Without using trigonometrical tables, evaluate:

$$\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ + \cos 44^\circ \operatorname{cosec} 46^\circ - \sqrt{2} \cos 45^\circ - \tan^2 60^\circ$$

Solution:

$$\begin{aligned}
& \operatorname{cosec}^2 57^\circ - \tan^2 33^\circ + \cos 44^\circ \operatorname{cosec} 46^\circ - \sqrt{2} \cos 45^\circ - \tan^2 60^\circ \\
&= \operatorname{cosec}^2 (90^\circ - 33^\circ) - \tan^2 33^\circ + \cos 44^\circ \operatorname{cosec} (90^\circ - 44^\circ) - \sqrt{2} \cos 45^\circ - \tan^2 60^\circ \\
&= \sec^2 33^\circ - \tan^2 33^\circ + \cos 44^\circ \sec 44^\circ - \sqrt{2} \cos 45^\circ - \tan^2 60^\circ \\
&= 1 + 1 - \sqrt{2} \cos 45^\circ - \tan^2 60^\circ \\
&= 1 + 1 - \sqrt{2} \left(\frac{1}{\sqrt{2}} \right) - (\sqrt{3})^2 \\
&= 2 - 1 - 3 \\
&= -2
\end{aligned}$$

Exercise 21 D

Question 1.

Use tables to find sine of:

- (i) 21°
- (ii) $34^\circ 42'$
- (iii) $47^\circ 32'$
- (iv) $62^\circ 57'$
- (v) $10^\circ 20' + 20^\circ 45'$

Solution:

- (i) $\sin 21^\circ = 0.3584$
- (ii) $\sin 34^\circ 42' = 0.5693$
- (iii) $\sin 47^\circ 32' = \sin (47^\circ 30' + 2') = 0.7373 + 0.0004 = 0.7377$
- (iv) $\sin 62^\circ 57' = \sin (62^\circ 54' + 3') = 0.8902 + 0.0004 = 0.8906$
- (v) $\sin (10^\circ 20' + 20^\circ 45') = \sin 30^\circ 65' = \sin 31^\circ 5' = 0.5150 + 0.0012 = 0.5162$

Question 2.

Use tables to find cosine of:

- (i) $2^\circ 4'$
- (ii) $8^\circ 12'$
- (iii) $26^\circ 32'$
- (iv) $65^\circ 41'$
- (v) $9^\circ 23' + 15^\circ 54'$

Solution:

- (i) $\cos 2^\circ 4' = 0.9994 - 0.0001 = 0.9993$
- (ii) $\cos 8^\circ 12' = \cos 0.9898$
- (iii) $\cos 26^\circ 32' = \cos (26^\circ 30' + 2') = 0.8949 - 0.0003 = 0.8946$
- (iv) $\cos 65^\circ 41' = \cos (65^\circ 36' + 5') = 0.4131 - 0.0013 = 0.4118$
- (v) $\cos (9^\circ 23' + 15^\circ 54') = \cos 24^\circ 77' = \cos 25^\circ 17' = \cos (25^\circ 12' + 5') = 0.9048 - 0.0006 = 0.9042$

Question 3.

Use trigonometrical tables to find tangent of:

(i) 37°

(ii) $42^\circ 18'$

(iii) $17^\circ 27'$

Solution:

(i) $\tan 37^\circ = 0.7536$

(ii) $\tan 42^\circ 18' = 0.9099$

(iii) $\tan 17^\circ 27' = \tan (17^\circ 24' + 3') = 0.3134 + 0.0010 = 0.3144$

Question 4.

Use tables to find the acute angle θ , if the value of $\sin \theta$ is:

(i) 0.4848

(ii) 0.3827

(iii) 0.6525

Solution:

(i) From the tables, it is clear that $\sin 29^\circ = 0.4848$

Hence, $\theta = 29^\circ$

(ii) From the tables, it is clear that $\sin 22^\circ 30' = 0.3827$

Hence, $\theta = 22^\circ 30'$

(iii) From the tables, it is clear that $\sin 40^\circ 42' = 0.6521$

$\sin \theta - \sin 40^\circ 42' = 0.6525 - 0.6521 = 0.0004$

From the tables, diff of $2' = 0.0004$

Hence, $\theta = 40^\circ 42' + 2' = 40^\circ 44'$

Question 5.

Use tables to find the acute angle θ , if the value of $\cos \theta$ is:

(i) 0.9848

(ii) 0.9574

(iii) 0.6885

Solution:

(i) From the tables, it is clear that $\cos 10^\circ = 0.9848$

Hence, $\theta = 10^\circ$

(ii) From the tables, it is clear that $\cos 16^\circ 48' = 0.9573$

$$\cos \theta - \cos 16^\circ 48' = 0.9574 - 0.9573 = 0.0001$$

From the tables, diff of $1' = 0.0001$

$$\text{Hence, } \theta = 16^\circ 48' - 1' = 16^\circ 47'$$

(iii) From the tables, it is clear that $\cos 46^\circ 30' = 0.6884$

$$\cos q - \cos 46^\circ 30' = 0.6885 - 0.6884 = 0.0001$$

From the tables, diff of $1' = 0.0002$

$$\text{Hence, } \theta = 46^\circ 30' - 1' = 46^\circ 29'$$

Question 6.

Use tables to find the acute angle θ , if the value of $\tan q$ is:

(i) 0.2419

(ii) 0.4741

(iii) 0.7391

Solution:

(i) From the tables, it is clear that $\tan 13^\circ 36' = 0.2419$

$$\text{Hence, } \theta = 13^\circ 36'$$

(ii) From the tables, it is clear that $\tan 25^\circ 18' = 0.4727$

$$\tan \theta - \tan 25^\circ 18' = 0.4741 - 0.4727 = 0.0014$$

From the tables, diff of $4' = 0.0014$

$$\text{Hence, } \theta = 25^\circ 18' + 4' = 25^\circ 22'$$

(iii) From the tables, it is clear that $\tan 36^\circ 24' = 0.7373$

$$\tan \theta - \tan 36^\circ 24' = 0.7391 - 0.7373 = 0.0018$$

From the tables, diff of $4' = 0.0018$

$$\text{Hence, } \theta = 36^\circ 24' + 4' = 36^\circ 28'$$

Exercise 21 E**Question 1.**

Prove the following identities:

$$(i) \frac{1}{\cos A + \sin A} + \frac{1}{\cos A - \sin A} = \frac{2 \cos A}{2 \cos^2 A - 1}$$

$$(ii) \operatorname{cosec} A - \cot A = \frac{\sin A}{1 + \cos A}$$

$$(iii) 1 - \frac{\sin^2 A}{1 + \cos A} = \cos A$$

$$(iv) \frac{1 - \cos A}{\sin A} + \frac{\sin A}{1 - \cos A} = 2 \operatorname{cosec} A$$

$$(v) \frac{\cot A}{1 - \tan A} + \frac{\tan A}{1 - \cot A} = 1 + \tan A + \cot A$$

$$(vi) \frac{\cos A}{1 + \sin A} + \tan A = \sec A$$

$$(vii) \frac{\sin A}{1 - \cos A} - \cot A = \operatorname{cosec} A$$

$$(viii) \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{\cos A}{1 - \sin A}$$

$$(ix) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{\cos A}{1 - \sin A}$$

$$(x) \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$$

$$(xi) \frac{1 + (\sec A - \tan A)^2}{\operatorname{cosec} A (\sec A - \tan A)} = 2 \tan A$$

$$(xii) \frac{(\operatorname{cosec} A - \cot A)^2 + 1}{\sec A (\operatorname{cosec} A - \cot A)} = 2 \cot A$$

$$(xiii) \cot^2 A \left(\frac{\sec A - 1}{1 + \sin A} \right) + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) = 0$$

$$(xiv) \frac{(1 - 2 \sin^2 A)^2}{\cos^4 A - \sin^4 A} = 2 \cos^2 A - 1$$

$$(xv) \sec^4 A (1 - \sin^4 A) - 2 \tan^2 A = 1$$

$$(xvi) \operatorname{cosec}^4 A (1 - \cos^4 A) - 2 \cot^2 A = 1$$

$$(xvii) (1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A) = 2$$

Solution:

$$\begin{aligned} (i) & \frac{1}{\cos A + \sin A} + \frac{1}{\cos A - \sin A} \\ &= \frac{\cos A + \sin A + \cos A - \sin A}{(\cos A + \sin A)(\cos A - \sin A)} \\ &= \frac{2 \cos A}{\cos^2 A - \sin^2 A} \\ &= \frac{2 \cos A}{\cos^2 A - (1 - \cos^2 A)} \\ &= \frac{2 \cos A}{2 \cos^2 A - 1} \end{aligned}$$

$$(ii) \operatorname{cosec} A - \cot A$$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= \frac{1 - \cos A}{\sin A}$$

$$= \frac{1 - \cos A}{\sin A} \times \frac{1 + \cos A}{1 + \cos A}$$

$$= \frac{1 - \cos^2 A}{\sin A(1 + \cos A)}$$

$$= \frac{\sin^2 A}{\sin A(1 + \cos A)}$$

$$= \frac{\sin A}{1 + \cos A}$$

$$(iii) 1 - \frac{\sin^2 A}{1 + \cos A}$$

$$= \frac{1 + \cos A - \sin^2 A}{1 + \cos A}$$

$$= \frac{\cos A + \cos^2 A}{1 + \cos A}$$

$$= \frac{\cos A(1 + \cos A)}{1 + \cos A}$$

$$= \cos A$$

$$(iv) \frac{1 - \cos A}{\sin A} + \frac{\sin A}{1 - \cos A}$$

$$= \frac{(1 - \cos A)^2 + \sin^2 A}{\sin A(1 - \cos A)}$$

$$= \frac{1 + \cos^2 A - 2\cos A + \sin^2 A}{\sin A(1 - \cos A)}$$

$$= \frac{2 - 2\cos A}{\sin A(1 - \cos A)}$$

$$= \frac{2(1 - \cos A)}{\sin A(1 - \cos A)}$$

$$= 2\operatorname{cosec} A$$

$$\begin{aligned}
\text{(v)} \quad & \frac{\cot A}{1 - \tan A} + \frac{\tan A}{1 - \cot A} \\
&= \frac{\frac{1}{\tan A}}{1 - \tan A} + \frac{\tan A}{1 - \frac{1}{\tan A}} \\
&= \frac{1}{\tan A(1 - \tan A)} + \frac{\tan^2 A}{\tan A - 1} \\
&= \frac{1 - \tan^3 A}{\tan A(1 - \tan A)} \\
&= \frac{(1 - \tan A)(1 + \tan A + \tan^2 A)}{\tan A(1 - \tan A)} \\
&= \frac{1 + \tan A + \tan^2 A}{\tan A} \\
&= \cot A + 1 + \tan A
\end{aligned}$$

$$\begin{aligned}
\text{(vi)} \quad & \frac{\cos A}{1 + \sin A} + \tan A \\
&= \frac{\cos A}{1 + \sin A} + \frac{\sin A}{\cos A} \\
&= \frac{\cos^2 A + \sin A + \sin^2 A}{(1 + \sin A)\cos A} \\
&= \frac{1 + \sin A}{(1 + \sin A)\cos A} \\
&= \frac{1}{\cos A} \\
&= \sec A
\end{aligned}$$

$$\begin{aligned}
\text{(vii)} \quad & \frac{\sin A}{1 - \cos A} - \cot A \\
&= \frac{\sin A}{1 - \cos A} - \frac{\cos A}{\sin A} \\
&= \frac{\sin^2 A - \cos A + \cos^2 A}{(1 - \cos A)\sin A} \\
&= \frac{1 - \cos A}{(1 - \cos A)\sin A}
\end{aligned}$$

$$= \frac{1}{\sin A}$$

$$= \operatorname{cosec} A$$

$$\begin{aligned} \text{(viii)} \quad & \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \\ &= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{\sin A - (\cos A - 1)}{\sin A - (\cos A - 1)} \\ &= \frac{(\sin A - \cos A + 1)^2}{\sin^2 A - (\cos A - 1)^2} \\ &= \frac{\sin^2 A + \cos^2 A + 1 - 2\sin A \cos A - 2\cos A + 2\sin A}{\sin^2 A - \cos^2 A - 1 + 2\cos A} \\ &= \frac{1 + 1 - 2\sin A \cos A - 2\cos A + 2\sin A}{-\cos^2 A - \cos^2 A + 2\cos A} \\ &= \frac{2(1 - \cos A) + 2\sin A(1 - \cos A)}{2\cos A(1 - \cos A)} \\ &= \frac{1 + \sin A}{\cos A} \\ &= \frac{1 + \sin A}{\cos A} \times \frac{1 - \sin A}{1 - \sin A} \\ &= \frac{\cos^2 A}{\cos A(1 - \sin A)} \\ &= \frac{\cos A}{1 - \sin A} \end{aligned}$$

$$\begin{aligned}
 \text{(ix)} \quad & \sqrt{\frac{1+\sin A}{1-\sin A}} \\
 &= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}} \\
 &= \sqrt{\frac{1-\sin^2 A}{(1-\sin A)^2}} \\
 &= \sqrt{\frac{\cos^2 A}{(1-\sin A)^2}} \\
 &= \frac{\cos A}{1-\sin A}
 \end{aligned}$$

$$\begin{aligned}
 \text{(x)} \quad & \sqrt{\frac{1-\cos A}{1+\cos A}} \\
 &= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}} \\
 &= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}} \\
 &= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}} \\
 &= \frac{\sin A}{1+\cos A}
 \end{aligned}$$

$$\begin{aligned}
 \text{(xi)} \quad & \frac{1+(\sec A - \tan A)^2}{\operatorname{cosec} A (\sec A - \tan A)} \\
 &= \frac{(\sec^2 A - \tan^2 A) + (\sec A - \tan A)^2}{\operatorname{cosec} A (\sec A - \tan A)} \\
 &= \frac{(\sec A - \tan A)(\sec A + \tan A) + (\sec A - \tan A)^2}{\operatorname{cosec} A (\sec A - \tan A)} \\
 &= \frac{(\sec A + \tan A) + (\sec A - \tan A)}{\operatorname{cosec} A} \\
 &= \frac{2\sec A}{\operatorname{cosec} A} \\
 &= 2 \frac{1}{\frac{1}{\sin A}} \\
 &= 2 \tan A
 \end{aligned}$$

$$\begin{aligned}
& \text{(xii)} \quad \frac{(\operatorname{cosec} A - \cot A)^2 + 1}{\sec A(\operatorname{cosec} A - \cot A)} \\
&= \frac{(\operatorname{cosec} A - \cot A)^2 + (\operatorname{cosec}^2 A - \cot^2 A)}{\sec A(\operatorname{cosec} A - \cot A)} \\
&= \frac{(\operatorname{cosec} A - \cot A)^2 + (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)}{\sec A(\operatorname{cosec} A - \cot A)} \\
&= \frac{(\operatorname{cosec} A - \cot A) + (\operatorname{cosec} A + \cot A)}{\sec A} \\
&= \frac{2 \operatorname{cosec} A}{\sec A} \\
&= 2 \cot A
\end{aligned}$$

$$\begin{aligned}
& \text{(xiii)} \quad \cot^2 A \left(\frac{\sec A - 1}{1 + \sin A} \right) + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) \\
&= \cot^2 A \left(\frac{\sec A - 1}{1 + \sin A} \times \frac{\sec A + 1}{\sec A + 1} \right) + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) \\
&= \cot^2 A \left[\frac{\sec^2 A - 1}{(1 + \sin A)(\sec A + 1)} \right] + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) \\
&= \cot^2 A \left[\frac{\tan^2 A}{(1 + \sin A)(\sec A + 1)} \right] + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) \\
&= \frac{1}{(1 + \sin A)(\sec A + 1)} + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) \\
&= \frac{1 + \sec^2 A(\sin A - 1)(1 + \sin A)}{(1 + \sin A)(\sec A + 1)} \\
&= \frac{1 + \sec^2 A(\sin^2 A - 1)}{(1 + \sin A)(\sec A + 1)} \\
&= \frac{1 + \sec^2 A(-\cos^2 A)}{(1 + \sin A)(\sec A + 1)} \\
&= \frac{1 - 1}{(1 + \sin A)(\sec A + 1)} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
& \text{(xiv)} \quad \frac{(1 - 2\sin^2 A)^2}{\cos^4 A - \sin^4 A} \\
&= \frac{(1 - 2\sin^2 A)^2}{(\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)} \\
&= \frac{(1 - 2\sin^2 A)^2}{1 - \sin^2 A - \sin^2 A} \\
&= \frac{(1 - 2\sin^2 A)^2}{1 - 2\sin^2 A} \\
&= 1 - 2\sin^2 A \\
&= 1 - 2(1 - \cos^2 A) \\
&= 2\cos^2 A - 1
\end{aligned}$$

$$\begin{aligned}
& \text{(xv)} \quad \sec^4 A(1 - \sin^4 A) - 2\tan^2 A \\
&= \sec^4 A(1 - \sin^2 A)(1 + \sin^2 A) - 2\tan^2 A \\
&= \sec^4 A(\cos^2 A)(1 + \sin^2 A) - 2\tan^2 A \\
&= \sec^2 A + \frac{\sin^2 A}{\cos^2 A} - 2\tan^2 A \\
&= \sec^2 A + \tan^2 A - 2\tan^2 A \\
&= \sec^2 A - \tan^2 A \\
&= 1
\end{aligned}$$

$$\begin{aligned}
& \text{(xvi)} \quad \operatorname{cosec}^4 A(1 - \cos^4 A) - 2\cot^2 A \\
&= \operatorname{cosec}^4 A(1 - \cos^2 A)(1 + \cos^2 A) - 2\cot^2 A \\
&= \operatorname{cosec}^4 A(\sin^2 A)(1 + \cos^2 A) - 2\cot^2 A \\
&= \operatorname{cosec}^2 A(1 + \cos^2 A) - 2\cot^2 A \\
&= \operatorname{cosec}^2 A + \frac{\cos^2 A}{\sin^2 A} - 2\cot^2 A \\
&= \operatorname{cosec}^2 A + \cot^2 A - 2\cot^2 A \\
&= \operatorname{cosec}^2 A - \cot^2 A \\
&= 1
\end{aligned}$$

$$(xvii) (1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A)$$

$$= 1 + \cot A - \operatorname{cosec} A + \tan A + 1 - \sec A +$$

$$\sec A + \operatorname{cosec} A - \operatorname{cosec} A \sec A$$

$$= 2 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A} - \frac{1}{\sin A \cos A}$$

$$= 2 + \frac{\cos^2 A + \sin^2 A}{\sin A \cos A} - \frac{1}{\sin A \cos A}$$

$$= 2 + \frac{1}{\sin A \cos A} - \frac{1}{\sin A \cos A}$$

$$= 2$$

Question 2.

If $\sin A + \cos A = p$ and $\sec A + \operatorname{cosec} A = q$, then prove that:

$$q(p^2 - 1) = 2p$$

Solution:

$$q(p^2 - 1) = (\sec A + \operatorname{cosec} A) [(\sin A + \cos A)^2 - 1]$$

$$= (\sec A + \operatorname{cosec} A) [(\sin^2 A + \cos^2 A + 2\sin A \cos A) - 1]$$

$$= (\sec A + \operatorname{cosec} A) [(1 + 2\sin A \cos A) - 1]$$

$$= (\sec A + \operatorname{cosec} A) (2\sin A \cos A)$$

$$= 2\sin A + 2\cos A$$

$$= 2p$$

Question 3.

If $x = a \cos \theta$ and $y = b \cot \theta$, show that:

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

Solution:

$$\begin{aligned}
& \frac{a^2}{x^2} - \frac{b^2}{y^2} \\
&= \frac{a^2}{a^2 \cos^2 \theta} - \frac{b^2}{b^2 \cot^2 \theta} \\
&= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \\
&= \frac{1 - \sin^2 \theta}{\cos^2 \theta} \\
&= \frac{\cos^2 \theta}{\cos^2 \theta} \\
&= 1
\end{aligned}$$

Question 4.

If $\sec A + \tan A = p$, show that:

$$\sin A = \frac{p^2 - 1}{p^2 + 1}$$

Solution:

$$\begin{aligned}
& \frac{p^2 - 1}{p^2 + 1} \\
&= \frac{(\sec A + \tan A)^2 - 1}{(\sec A + \tan A)^2 + 1} \\
&= \frac{\sec^2 A + \tan^2 A + 2 \tan A \sec A - 1}{\sec^2 A + \tan^2 A + 2 \tan A \sec A + 1} \\
&= \frac{\tan^2 A + \tan^2 A + 2 \tan A \sec A}{\sec^2 A + \sec^2 A + 2 \tan A \sec A} \\
&= \frac{2 \tan^2 A + 2 \tan A \sec A}{2 \sec^2 A + 2 \tan A \sec A} \\
&= \frac{2 \tan A (\tan A + \sec A)}{2 \sec A (\tan A + \sec A)} \\
&= \sin A
\end{aligned}$$

Question 5.

If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that:

$$\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

Solution:

Given that, $\tan A = n \tan B$ and $\sin A = m \sin B$.

$$\Rightarrow n = \frac{\tan A}{\tan B} \text{ and } m = \frac{\sin A}{\sin B}$$

$$\therefore \frac{m^2 - 1}{n^2 - 1}$$

$$= \frac{\left(\frac{\sin A}{\sin B}\right)^2 - 1}{\left(\frac{\tan A}{\tan B}\right)^2 - 1}$$

$$= \frac{\left(\frac{\sin A}{\sin B}\right)^2 - 1}{\left(\frac{\tan A}{\tan B}\right)^2 - 1}$$

$$= \frac{\tan^2 B (\sin^2 A - \sin^2 B)}{\sin^2 B (\tan^2 A - \tan^2 B)}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 B \left(\frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B} \right)}$$

$$= \frac{\cos^2 A (\sin^2 A - \sin^2 B)}{\sin^2 A \cos^2 B - (1 - \cos^2 B) \cos^2 A}$$

$$= \frac{\cos^2 A (1 - \cos^2 A - 1 + \cos^2 B)}{\cos^2 B (\sin^2 A + \cos^2 A) - \cos^2 A}$$

$$= \frac{\cos^2 A (\cos^2 B - \cos^2 A)}{\cos^2 B - \cos^2 A}$$

$$= \cos^2 A$$

Question 6.

(i) If $2 \sin A - 1 = 0$, show that:

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

(ii) If $4 \cos^2 A - 3 = 0$, show that:

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

Solution:

(i) $2 \sin A - 1 = 0$

$$\Rightarrow \sin A = \frac{1}{2}$$

We know $\sin 30^\circ = \frac{1}{2}$

So, $A = 30^\circ$

$$\text{LHS} = \sin 3A = \sin 90^\circ = 1$$

$$\begin{aligned}\text{RHS} &= 3 \sin A - 4 \sin^3 A \\ &= 3 \sin 30^\circ - 4 \sin^3 30^\circ \\ &= 3 \left(\frac{1}{2} \right) - 4 \left(\frac{1}{2} \right)^3 \\ &= \frac{3}{2} - \frac{1}{2} = 1\end{aligned}$$

$$\text{LHS} = \text{RHS}$$

(ii)

$$4 \cos^2 A - 3 = 0$$

$$\Rightarrow 4 \cos^2 A = 3$$

$$\Rightarrow \cos^2 A = \frac{3}{4}$$

$$\Rightarrow \cos A = \frac{\sqrt{3}}{2}$$

We know $\cos 30^\circ = \frac{\sqrt{3}}{2}$

So, $A = 30^\circ$

$$\text{LHS} = \cos 3A = \cos 90^\circ = 0$$

$$\begin{aligned}\text{RHS} &= 4 \cos^3 A - 3 \cos A \\ &= 4 \cos^3 30^\circ - 3 \cos 30^\circ \\ &= 4 \left(\frac{\sqrt{3}}{2} \right)^3 - 3 \left(\frac{\sqrt{3}}{2} \right) \\ &= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0\end{aligned}$$

$$\text{LHS} = \text{RHS}$$

Question 7.**Evaluate:**

$$(i) 2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$$

$$(ii) \sec 26^\circ \sin 64^\circ + \frac{\operatorname{cosec} 33^\circ}{\sec 57^\circ}$$

$$(iii) \frac{5 \sin 66^\circ}{\cos 24^\circ} - \frac{2 \cot 85^\circ}{\tan 5^\circ}$$

$$(iv) \cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ$$

$$(v) \sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$$

$$(vi) \frac{3 \sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$

$$(vii) 3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$$

$$(viii) \frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$$

Solution:

$$\begin{aligned} (i) & 2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right) \\ &= 2\left(\frac{\tan(90^\circ - 55^\circ)}{\cot 55^\circ}\right)^2 + \left(\frac{\cot(90^\circ - 35^\circ)}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec(90^\circ - 50^\circ)}{\operatorname{cosec} 50^\circ}\right) \\ &= 2\left(\frac{\cot 55^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\tan 35^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\operatorname{cosec} 50^\circ}{\operatorname{cosec} 50^\circ}\right) \\ &= 2(1)^2 + 1^2 - 3 \\ &= 2 + 1 - 3 \\ &= 0 \end{aligned}$$

$$\begin{aligned} (ii) & \sec 26^\circ \sin 64^\circ + \frac{\operatorname{cosec} 33^\circ}{\sec 57^\circ} \\ &= \sec(90^\circ - 64^\circ) \sin 64^\circ + \frac{\operatorname{cosec} \sec(90^\circ - 57^\circ)}{\sec 57^\circ} \\ &= \operatorname{cosec} 64^\circ \sin 64^\circ + \frac{\sec 57^\circ}{\sec 57^\circ} \\ &= 1 + 1 = 2 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \frac{5 \sin 66^\circ}{\cos 24^\circ} - \frac{2 \cot 85^\circ}{\tan 5^\circ} \\
 &= \frac{5 \sin(90^\circ - 24^\circ)}{\cos 24^\circ} - \frac{2 \cot(90^\circ - 5^\circ)}{\tan 5^\circ} \\
 &= \frac{5 \cos 24^\circ}{\cos 24^\circ} - \frac{2 \tan 5^\circ}{\tan 5^\circ} \\
 &= 5 - 2 = 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & \cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ \\
 &= \cos(90^\circ - 50^\circ) \operatorname{cosec} 50^\circ + \sin(90^\circ - 40^\circ) \sec 40^\circ \\
 &= \sin 50^\circ \operatorname{cosec} 50^\circ + \cos 40^\circ \sec 40^\circ \\
 &= 1 + 1 = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & \sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ \\
 &= \sin(90^\circ - 63^\circ) \sin 63^\circ - \cos 63^\circ \cos(90^\circ - 63^\circ) \\
 &= \cos 63^\circ \sin 63^\circ - \cos 63^\circ \sin 63^\circ \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad & \frac{3 \sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ} \\
 &= \frac{3 \sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ} \\
 &= \frac{3 \cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} \\
 &= 3 - 1 = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad & 3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ \\
 &= 3 \cos(90^\circ - 10^\circ) \operatorname{cosec} 10^\circ + 2 \cos(90^\circ - 31^\circ) \operatorname{cosec} 31^\circ \\
 &= 3 \sin 10^\circ \operatorname{cosec} 10^\circ + 2 \sin 31^\circ \operatorname{cosec} 31^\circ \\
 &= 3 + 2 = 5
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad & \frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ} \\
 &= \frac{\cos(90^\circ - 15^\circ)}{\sin 15^\circ} + \frac{\sin(90^\circ - 78^\circ)}{\cos 78^\circ} - \frac{\cos(90^\circ - 72^\circ)}{\sin 72^\circ} \\
 &= \frac{\sin 15^\circ}{\sin 15^\circ} + \frac{\cos 78^\circ}{\cos 78^\circ} - \frac{\sin 72^\circ}{\sin 72^\circ} \\
 &= 1 + 1 - 1 = 1
 \end{aligned}$$

Question 8.

Prove that:

$$\text{(i)} \quad \tan(55^\circ + x) = \cot(35^\circ - x)$$

$$\text{(ii)} \quad \sec(70^\circ - \theta) = \operatorname{cosec}(20^\circ + \theta)$$

$$\text{(iii)} \quad \sin(28^\circ + A) = \cos(62^\circ - A)$$

$$\text{(iv)} \quad \frac{1}{1 + \cos(90^\circ - A)} + \frac{1}{1 - \cos(90^\circ - A)} = 2 \operatorname{cosec}^2(90^\circ - A)$$

$$\text{(v)} \quad \frac{1}{1 + \sin(90^\circ - A)} + \frac{1}{1 - \sin(90^\circ - A)} = 2 \sec^2(90^\circ - A)$$

Solution:

$$\text{(i)} \quad \tan(55^\circ + x) = \tan[90^\circ - (35^\circ - x)] = \cot(35^\circ - x)$$

$$\text{(ii)} \quad \sec(70^\circ - \theta) = \sec[90^\circ - (20^\circ + \theta)] = \operatorname{cosec}(20^\circ + \theta)$$

$$\text{(iii)} \quad \sin(28^\circ + A) = \sin[90^\circ - (62^\circ - A)] = \cos(62^\circ - A)$$

$$\text{(iv)} \quad \frac{1}{1 + \cos(90^\circ - A)} + \frac{1}{1 - \cos(90^\circ - A)}$$

$$= \frac{1}{1 + \sin A} + \frac{1}{1 - \sin A}$$

$$= \frac{1 - \sin A + 1 + \sin A}{(1 + \sin A)(1 - \sin A)}$$

$$= \frac{2}{1 - \sin^2 A}$$

$$= \frac{2}{\cos^2 A}$$

$$= 2 \sec^2 A$$

$$= 2 \operatorname{cosec}^2(90^\circ - A)$$

$$\begin{aligned}
 \text{(v)} \quad & \frac{1}{1 + \sin(90^\circ - A)} + \frac{1}{1 - \sin(90^\circ - A)} \\
 &= \frac{1}{1 + \cos A} + \frac{1}{1 - \cos A} \\
 &= \frac{1 - \cos A + 1 + \cos A}{(1 + \cos A)(1 - \cos A)} \\
 &= \frac{2}{1 - \cos^2 A} \\
 &= 2 \operatorname{cosec}^2 A \\
 &= 2 \sec^2(90^\circ - A)
 \end{aligned}$$

Question 9.

If A and B are complementary angles, prove that:

- (i) $\cot B + \cos B = \sec A \cos B(1 + \sin B)$
- (ii) $\cot A \cot B - \sin A \cos B - \cos A \sin B = 0$
- (iii) $\operatorname{cosec}^2 A + \operatorname{cosec}^2 B = \operatorname{cosec}^2 A \operatorname{cosec}^2 B$
- (iv) $\frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos B - \cos A}{\cos B + \cos A} = \frac{2}{2 \sin^2 A - 1}$

Solution:

Since, A and B are complementary angles, $A + B = 90^\circ$

$$\begin{aligned}
 \text{(i)} \quad & \cot B + \cos B \\
 &= \cot(90^\circ - A) + \cos(90^\circ - A) \\
 &= \tan A + \sin A \\
 &= \frac{\sin A}{\cos A} + \sin A \\
 &= \frac{\sin A + \sin A \cos A}{\cos A} \\
 &= \frac{\sin A(1 + \cos A)}{\cos A} \\
 &= \sec A \sin A(1 + \cos A) \\
 &= \sec A \sin(90^\circ - B)[1 + \cos(90^\circ - B)] \\
 &= \sec A \cos B(1 + \sin B)
 \end{aligned}$$

(ii)

$$\begin{aligned} & \cot A \cot B - \sin A \cos B - \cos A \sin B \\ &= \cot A \cot(90^\circ - A) - \sin A \cos(90^\circ - A) - \cos A \sin(90^\circ - A) \\ &= \cot A \tan A - \sin A \sin A - \cos A \cos A \\ &= 1 - (\sin^2 A + \cos^2 A) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

(iii)

$$\begin{aligned} & \operatorname{cosec}^2 A + \operatorname{cosec}^2 B \\ &= \operatorname{cosec}^2 A + [\operatorname{cosec}(90^\circ - A)]^2 \\ &= \operatorname{cosec}^2 A + \sec^2 A \\ &= \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} \\ &= \frac{\cos^2 A + \sin^2 A}{\sin^2 A \cos^2 A} \\ &= \frac{1}{\sin^2 A \cos^2 A} \\ &= \operatorname{cosec}^2 A [\sec(90^\circ - B)]^2 \\ &= \operatorname{cosec}^2 A \operatorname{cosec}^2 B \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos B - \cos A}{\cos B + \cos A} \\ &= \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos(90^\circ - A) - \cos(90^\circ - B)}{\cos(90^\circ - A) + \cos(90^\circ - B)} \\ &= \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\sin A - \sin B}{\sin A + \sin B} \\ &= \frac{(\sin A + \sin B)^2 + (\sin A - \sin B)^2}{(\sin A - \sin B)(\sin A + \sin B)} \\ &= \frac{\sin^2 A + \sin^2 B + 2\sin A \sin B + \sin^2 A + \sin^2 B - 2\sin A \sin B}{\sin^2 A - \sin^2 B} \end{aligned}$$

$$\begin{aligned}
&= 2 \frac{\sin^2 A + \sin^2 B}{\sin^2 A - \sin^2 B} \\
&= 2 \frac{\sin^2 A + \sin^2(90^\circ - A)}{\sin^2 A - \sin^2(90^\circ - A)} \\
&= 2 \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A} \\
&= \frac{2}{\sin^2 A - (1 - \sin^2 A)} \\
&= \frac{2}{2\sin^2 A - 1}
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad & \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos B - \cos A}{\cos B + \cos A} \\
&= \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos(90^\circ - A) - \cos(90^\circ - B)}{\cos(90^\circ - A) + \cos(90^\circ - B)} \\
&= \frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\sin A - \sin B}{\sin A + \sin B} \\
&= \frac{(\sin A + \sin B)^2 + (\sin A - \sin B)^2}{(\sin A - \sin B)(\sin A + \sin B)} \\
&= \frac{\sin^2 A + \sin^2 B + 2\sin A \sin B + \sin^2 A + \sin^2 B - 2\sin A \sin B}{\sin^2 A - \sin^2 B} \\
&= 2 \frac{\sin^2 A + \sin^2 B}{\sin^2 A - \sin^2 B} \\
&= 2 \frac{\sin^2 A + \sin^2(90^\circ - A)}{\sin^2 A - \sin^2(90^\circ - A)} \\
&= 2 \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A} \\
&= \frac{2}{\sin^2 A - (1 - \sin^2 A)} \\
&= \frac{2}{2\sin^2 A - 1}
\end{aligned}$$

Question 10.

$$\frac{\cot A - 1}{2 - \sec^2 A} = \frac{\cot A}{1 + \tan A}$$

Solution:

To prove that : $\frac{\cot A - 1}{2 - \sec^2 A} = \frac{\cot A}{1 + \tan A}$

$$\begin{aligned}
\text{L.H.S} &= \frac{\cot A - 1}{2 - \sec^2 A} \\
&= \frac{\frac{1}{\tan A} - 1}{2 - (1 + \tan^2 A)} \\
&= \frac{1 - \tan A}{\tan A (1 - \tan^2 A)} \\
&= \frac{(1 - \tan A)}{\tan A (1 - \tan A)(1 + \tan A)} \\
&= \frac{1}{\tan A (1 + \tan A)} \\
&= \frac{1}{\tan A} \times \frac{1}{(1 + \tan A)} \\
&= \frac{\cot A}{1 + \tan A} \\
&= \text{R.H.S}
\end{aligned}$$

Hence proved.

Question 11.

Prove that:

$$(i) \frac{1}{\sin A - \cos A} - \frac{1}{\sin A + \cos A} = \frac{2 \cos A}{2 \sin^2 A - 1}$$

$$(ii) \frac{\cot^2 A}{\operatorname{cosec} A - 1} - 1 = \operatorname{cosec} A$$

$$(iii) \frac{\cos A}{1 + \sin A} = \sec A - \tan A$$

$$(iv) \cos A(1 + \cot A) + \sin A(1 + \tan A) = \sec A + \operatorname{cosec} A$$

$$(v) (\sin A - \cos A)(1 + \tan A + \cot A) = \frac{\sec A}{\cos^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$$

$$(vi) \sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$$

$$(vii) (\sin A + \cos A)(\sec A + \operatorname{cosec} A) = 2 + \sec A \operatorname{cosec} A$$

$$(viii) (\tan A + \cot A)(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = 1$$

$$(ix) \cot^2 A - \cot^2 B = \frac{\cos^2 A - \cos^2 B}{\sin^2 A \sin^2 B} = \operatorname{cosec}^2 A - \operatorname{cosec}^2 B$$

Solution:

$$\begin{aligned} (i) & \frac{1}{\sin A - \cos A} - \frac{1}{\sin A + \cos A} \\ &= \frac{\sin A + \cos A - \sin A + \cos A}{(\sin A - \cos A)(\sin A + \cos A)} \\ &= \frac{2\cos A}{\sin^2 A - \cos^2 A} \\ &= \frac{2\cos A}{\sin^2 A - (1 - \sin^2 A)} \\ &= \frac{2\cos A}{2\sin^2 A - 1} \end{aligned}$$

$$\begin{aligned} (ii) & \frac{\cot^2 A}{\operatorname{cosec} A - 1} - 1 \\ &= \frac{\cot^2 A - \operatorname{cosec} A + 1}{\operatorname{cosec} A - 1} \\ &= \frac{-\operatorname{cosec} A + \operatorname{cosec}^2 A}{\operatorname{cosec} A - 1} \\ &= \frac{\operatorname{cosec} A (\operatorname{cosec} A - 1)}{\operatorname{cosec} A - 1} \\ &= \operatorname{cosec} A \end{aligned}$$

$$\begin{aligned}
 & \text{(iii)} \frac{\cos A}{1 + \sin A} \\
 &= \frac{\cos A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A} \\
 &= \frac{\cos A(1 - \sin A)}{1 - \sin^2 A} \\
 &= \frac{\cos A(1 - \sin A)}{\cos^2 A} \\
 &= \frac{1 - \sin A}{\cos A} \\
 &= \sec A - \tan A
 \end{aligned}$$

$$\begin{aligned}
 & \text{(iv)} \cos A(1 + \cot A) + \sin A(1 + \tan A) \\
 &= \cos A + \frac{\cos^2 A}{\sin A} + \sin A + \frac{\sin^2 A}{\cos A} \\
 &= \sin A + \frac{\cos^2 A}{\sin A} + \cos A + \frac{\sin^2 A}{\cos A} \\
 &= \left(\frac{\cos^2 A + \sin^2 A}{\sin A} \right) + \left(\frac{\cos^2 A + \sin^2 A}{\cos A} \right) \\
 &= \frac{1}{\sin A} + \frac{1}{\cos A} \\
 &= \operatorname{cosec} A + \sec A
 \end{aligned}$$

$$\begin{aligned}
 & \text{(v)} (\sin A - \cos A)(1 + \tan A + \cot A) \\
 &= \sin A + \frac{\sin^2 A}{\cos A} + \cos A - \cos A - \sin A - \frac{\cos^2 A}{\sin A} \\
 &= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A} \\
 &= \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}
 \end{aligned}$$

$$(vi) LHS = \sqrt{\sec^2 A + \operatorname{cosec}^2 A}$$

$$= \sqrt{\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}}$$

$$= \sqrt{\frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A}}$$

$$= \sqrt{\frac{1}{\sin^2 A \cos^2 A}}$$

$$= \frac{1}{\sin A \cos A}$$

$$RHS = \tan A + \cot A$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{1}{\sin A \cos A}$$

$$LHS = RHS$$

$$(vii) (\sin A + \cos A)(\sec A + \operatorname{cosec} A)$$

$$= \frac{\sin A}{\cos A} + 1 + 1 + \frac{\cos A}{\sin A}$$

$$= 2 + \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$$

$$= 2 + \frac{1}{\sin A \cos A}$$

$$= 2 + \sec A \operatorname{cosec} A$$

$$\begin{aligned}
 & \text{(viii)} (\tan A + \cot A)(\operatorname{cosec} A - \sin A)(\sec A - \cos A) \\
 &= \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right) \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \\
 &= \left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \right) \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \\
 &= \left(\frac{1}{\sin A \cos A} \right) \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right) \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 & \text{(ix)} \cot^2 A - \cot^2 B \\
 &= \frac{\cos^2 A}{\sin^2 A} - \frac{\cos^2 B}{\sin^2 B} \\
 &= \frac{\cos^2 A \sin^2 B - \cos^2 B \sin^2 A}{\sin^2 A \sin^2 B} \\
 &= \frac{\cos^2 A (1 - \cos^2 B) - \cos^2 B (1 - \cos^2 A)}{\sin^2 A \sin^2 B} \\
 &= \frac{\cos^2 A - \cos^2 A \cos^2 B - \cos^2 B + \cos^2 B \cos^2 A}{\sin^2 A \sin^2 B} \\
 &= \frac{\cos^2 A - \cos^2 B}{\sin^2 A \sin^2 B} \\
 &= \frac{1 - \sin^2 A - 1 + \sin^2 B}{\sin^2 A \sin^2 B} \\
 &= \frac{-\sin^2 A + \sin^2 B}{\sin^2 A \sin^2 B} \\
 &= \frac{\sin^2 B}{\sin^2 A \sin^2 B} - \frac{\sin^2 A}{\sin^2 A \sin^2 B} \\
 &= \frac{1}{\sin^2 A} - \frac{1}{\sin^2 B} \\
 &= \operatorname{cosec}^2 A - \operatorname{cosec}^2 B
 \end{aligned}$$

Question 12.

If $4\cos^2 A - 3 = 0$ and $0^\circ \leq A \leq 90^\circ$, then prove that:

(i) $\sin 3A = 3 \sin A - 4 \sin^3 A$

(ii) $\cos 3A = 4 \cos^3 A - 3 \cos A$

Solution:

$$4 \cos^2 A - 3 = 0$$

$$\cos A = \frac{\sqrt{3}}{2}$$

$$\text{We know } \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\text{So, } A = 30^\circ$$

(i)

$$\text{LHS} = \sin 3A = \sin 90^\circ = 1$$

$$\begin{aligned}\text{RHS} &= 3 \sin A - 4 \sin^3 A \\ &= 3 \sin 30^\circ - 4 \sin^3 30^\circ \\ &= 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3 \\ &= \frac{3}{2} - \frac{1}{2} \\ &= 1\end{aligned}$$

$$\text{LHS} = \text{RHS}$$

(ii)

$$\text{LHS} = \cos 3A = \cos 90^\circ = 0$$

$$\begin{aligned}\text{RHS} &= 4 \cos^3 A - 3 \cos A \\ &= 4 \cos^3 30^\circ - 3 \cos 30^\circ \\ &= 4 \left(\frac{\sqrt{3}}{2}\right)^3 - 3 \left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0\end{aligned}$$

$$\text{LHS} = \text{RHS}$$

Question 13.

Find A, if $0^\circ \leq A \leq 90^\circ$ and:

(i) $2 \cos^2 A - 1 = 0$

(ii) $\sin 3A - 1 = 0$

(iii) $4 \sin^2 A - 3 = 0$

(iv) $\cos^2 A - \cos A = 0$

(v) $2 \cos^2 A + \cos A - 1 = 0$

Solution:

$$(i) 2\cos^2 A - 1 = 0$$

$$\Rightarrow \cos^2 A = \frac{1}{2}$$

$$\Rightarrow \cos A = \frac{1}{\sqrt{2}}$$

$$\text{We know } \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Hence, $A = 45^\circ$

$$(ii) \sin 3A - 1 = 0$$

$$\Rightarrow \sin 3A = 1$$

$$\text{We know } \sin 90^\circ = 1$$

$$\therefore 3A = 90^\circ$$

Hence, $A = 30^\circ$

$$(iii) 4\sin^2 A - 3 = 0$$

$$\Rightarrow \sin^2 A = \frac{3}{4}$$

$$\Rightarrow \sin A = \frac{\sqrt{3}}{2}$$

$$\text{We know } \sin 60^\circ = \frac{\sqrt{3}}{2}$$

Hence, $A = 60^\circ$

$$(iv) \cos^2 A - \cos A = 0$$

$$\Rightarrow \cos A(\cos A - 1) = 0$$

$$\Rightarrow \cos A = 0 \text{ or } \cos A = 1$$

$$\text{We know } \cos 90^\circ = 0 \text{ and } \cos 0^\circ = 1$$

Hence, $A = 90^\circ$ or 0°

$$\begin{aligned}
 \text{(v)} \quad & 2\cos^2 A + \cos A - 1 = 0 \\
 \Rightarrow & 2\cos^2 A + 2\cos A - \cos A - 1 = 0 \\
 \Rightarrow & 2\cos A(\cos A + 1) - 1(\cos A + 1) = 0 \\
 \Rightarrow & (2\cos A - 1)(\cos A + 1) = 0 \\
 \Rightarrow & \cos A = \frac{1}{2} \text{ or } \cos A = -1
 \end{aligned}$$

$$\text{We know } \cos 60^\circ = \frac{1}{2}$$

We also know that for no value of $A(0^\circ \leq A \leq 90^\circ)$, $\cos A = -1$.

Hence, $A = 60^\circ$

Question 14.

If $0^\circ < A < 90^\circ$; find A , if:

$$\text{(i)} \quad \frac{\cos A}{1 - \sin A} + \frac{\cos A}{1 + \sin A} = 4$$

$$\text{(ii)} \quad \frac{\sin A}{\sec A - 1} + \frac{\sin A}{\sec A + 1} = 2$$

Solution:

$$\begin{aligned}
 \text{(i)} \quad & \frac{\cos A}{1 - \sin A} + \frac{\cos A}{1 + \sin A} = 4 \\
 \Rightarrow & \frac{\cos A + \cos A \sin A + \cos A - \sin A \cos A}{(1 - \sin A)(1 + \sin A)} = 4 \\
 \Rightarrow & \frac{2\cos A}{1 - \sin^2 A} = 4 \\
 \Rightarrow & \frac{2\cos A}{\cos^2 A} = 4 \\
 \Rightarrow & \frac{1}{\cos A} = 2 \\
 \Rightarrow & \cos A = \frac{1}{2}
 \end{aligned}$$

$$\text{We know } \cos 60^\circ = \frac{1}{2}$$

Hence, $A = 60^\circ$

$$\begin{aligned}
 \text{(ii)} \quad & \frac{\sin A}{\sec A - 1} + \frac{\sin A}{\sec A + 1} = 2 \\
 \Rightarrow & \frac{\sin A \sec A + \sin A + \sec A \sin A - \sin A}{(\sec A - 1)(\sec A + 1)} = 2 \\
 \Rightarrow & \frac{2 \sin A \sec A}{\sec^2 A - 1} = 2 \\
 \Rightarrow & \frac{\sin A \sec A}{\tan^2 A} = 1 \\
 \Rightarrow & \frac{\cos A}{\sin A} = 1 \\
 \Rightarrow & \cot A = 1
 \end{aligned}$$

We know $\cot 45^\circ = 1$

Hence, $A = 45^\circ$

$$\begin{aligned}
 \text{(i)} \quad & \frac{\cos A}{1 - \sin A} + \frac{\cos A}{1 + \sin A} = 4 \\
 \Rightarrow & \frac{\cos A + \cos A \sin A + \cos A - \sin A \cos A}{(1 - \sin A)(1 + \sin A)} = 4 \\
 \Rightarrow & \frac{2 \cos A}{1 - \sin^2 A} = 4 \\
 \Rightarrow & \frac{2 \cos A}{\cos^2 A} = 4 \\
 \Rightarrow & \frac{1}{\cos A} = 2 \\
 \Rightarrow & \cos A = \frac{1}{2}
 \end{aligned}$$

We know $\cos 60^\circ = \frac{1}{2}$

Hence, $A = 60^\circ$

$$\begin{aligned}
 \text{(ii)} \quad & \frac{\sin A}{\sec A - 1} + \frac{\sin A}{\sec A + 1} = 2 \\
 \Rightarrow & \frac{\sin A \sec A + \sin A + \sec A \sin A - \sin A}{(\sec A - 1)(\sec A + 1)} = 2 \\
 \Rightarrow & \frac{2 \sin A \sec A}{\sec^2 A - 1} = 2
 \end{aligned}$$

$$\Rightarrow \frac{\sin A \sec A}{\tan^2 A} = 1$$

$$\Rightarrow \frac{\cos A}{\sin A} = 1$$

$$\Rightarrow \cot A = 1$$

We know $\cot 45^\circ = 1$

Hence, $A = 45^\circ$

Question 15.

Prove that:

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A) \sec^2 A = \tan A$$

Solution:

L.H.S.,

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A) \sec^2 A$$

$$= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \sec^2 A$$

$$= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \sec^2 A$$

$$= \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right) \sec^2 A$$

$$= \frac{\sin A}{\cos A} = \tan A = \text{R.H.S.}$$

Question 16.

Prove the identity $(\sin \theta + \cos \theta)(\tan \theta + \cot \theta) = \sec \theta + \operatorname{cosec} \theta$.

Solution:

$$\begin{aligned}\text{L.H.S.} &= (\sin \theta + \cos \theta)(\tan \theta + \cot \theta) \\&= (\sin \theta + \cos \theta) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) \\&= (\sin \theta + \cos \theta) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right) \\&= \frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta} \\&= \frac{\sin \theta}{\cos \theta \sin \theta} + \frac{\cos \theta}{\cos \theta \sin \theta} \\&= \frac{1}{\cos \theta} + \frac{1}{\sin \theta} \\&= \sec \theta + \operatorname{cosec} \theta \\&= \text{R.H.S.}\end{aligned}$$

Question 17.

Evaluate without using trigonometric tables,

$$\sin^2 28^\circ + \sin^2 62^\circ + \tan^2 38^\circ - \cot^2 52^\circ + \frac{1}{4} \sec^2 30^\circ$$

Solution:

$$\begin{aligned}&\sin^2 28^\circ + \sin^2 62^\circ + \tan^2 38^\circ - \cot^2 52^\circ + \frac{1}{4} \sec^2 30^\circ \\&= \sin^2 28^\circ + [\sin(90 - 28)^\circ]^2 + \tan^2 38^\circ - [\cot(90 - 38)^\circ]^2 + \frac{1}{4} \sec^2 30^\circ \\&= \sin^2 28^\circ + \cos^2 28^\circ + \tan^2 38^\circ - \tan^2 38^\circ + \frac{1}{4} \sec^2 30^\circ \\&= 1 + 0 + \frac{1}{4} \times \left(\frac{2}{\sqrt{3}} \right)^2 \\&= 1 + \frac{1}{3} \\&= \frac{3+1}{3} \\&= \frac{4}{3}\end{aligned}$$