EXERCISE- 7 (A)

Question 1:

If a : b = 5 : 3, find : $\frac{5a-3b}{5a+3b}$

Solution 1:

$$a:b=5:3$$

$$\Rightarrow \frac{a}{b}=\frac{5}{3}$$

$$\frac{5a-3b}{5a+3b}=\frac{5\left(\frac{a}{b}\right)-30}{5\left(\frac{a}{b}\right)+3} \quad \text{(dividing each term by b)}$$

$$=\frac{5\left(\frac{5}{3}\right)-3}{5\left(\frac{5}{3}\right)+3}$$

$$=\frac{\frac{25}{3}-3}{\frac{25}{3}+3}$$

$$=\frac{25-9}{25+9}$$

$$=\frac{16}{34}=\frac{8}{17}$$

Question 2:

If x : y = 4 : 7, find the value of (3x + 2y) : (5x + y)

Solution 2:

$$x:y=4:7$$

$$\Rightarrow \frac{x}{y}=\frac{4}{7}$$

$$\frac{3x+2y}{5x+y}=$$

$$\frac{3\left(\frac{x}{y}\right)+2}{5\left(\frac{x}{y}\right)+1} \quad \text{(Dividing each term by y)}$$

$$=\frac{3\left(\frac{4}{7}\right)+2}{5\left(\frac{4}{7}\right)+1}$$

$$=\frac{\frac{12}{7}+2}{\frac{20}{7}+1}$$

$$=\frac{12+14}{20+7}$$

$$=\frac{26}{27}$$

Question 3:

If a : b = 3 : 8, find the value of $\frac{4a+3b}{6a-b}$

Solution 3:

$$a:b=3:8$$

$$\Rightarrow \frac{a}{b}=\frac{3}{8}$$

$$\frac{4a+3b}{6a-b}=\frac{4\left(\frac{a}{b}\right)+3}{6\left(\frac{a}{b}\right)-1} \quad \text{(Divinding each term by b)}$$

$$=\frac{4\left(\frac{3}{8}\right)+3}{6\left(\frac{3}{8}\right)-1}$$

$$=\frac{\frac{3}{2}+3}{\frac{9}{4}-1}$$

$= \dfrac{\frac{9}{2}}{\frac{5}{4}}$

$= \dfrac{18}{5}$

Question 4:

If (a – b) : (a + b) = 1 : 11, find the ratio (5a + 4b + 15) : (5a - 4b + 3)

Solution 4:

$\dfrac{a-b}{a+b} = \dfrac{1}{11}$

$11a - 11b = a + b$

$10a = 12b$

So, let $a = 6k$ and $b = 5k$

$$\frac{5a+4b+15}{5a-4b+3} = \frac{5(6k)+4(5k)+15}{5(6k)-4(5k)+3}$$

$$= \frac{30k+20k+15}{30k-20k+3}$$

$$= \frac{50k+15}{10k+3}$$

$$= \frac{5(10k+3)}{10k+3}$$

$$= 5$$

Hence, $(5a+4b+15):(5a-4b+3) = 5:1$

Question 5:

If $\dfrac{y-x}{x} = \dfrac{3}{8}$,find the value of $\frac{y}{x}$

Solution 5:

$$\frac{y-x}{x}=\frac{3}{8}$$

$$\Rightarrow \frac{\frac{y}{x}-\frac{x}{x}}{\frac{x}{x}}=\frac{3}{8}$$

$$\Rightarrow \frac{\frac{y}{x}-1}{1}=\frac{3}{8}$$

$$\Rightarrow \frac{y}{x}=\frac{3}{8}+1=\frac{11}{8}$$

Question 6:

If $\frac{m+n}{m+3n}=\frac{2}{3}$,find : $\frac{2n^2}{3m^2+mn}$

Solution 6:

$$\frac{m+n}{m+3n}=\frac{2}{3}$$

$$\Rightarrow 3m+3n=2m+6n$$

$$\Rightarrow m=3n$$

$$\Rightarrow \frac{m}{n}=\frac{3}{1}$$

$$\frac{2n^2}{3m^2+mn}=\frac{2}{3\left(\frac{m}{n}\right)^2+\left(\frac{m}{n}\right)} \left(\text{Dividing each term by } n^2\right)$$

$$=\frac{2}{3\left(\frac{3}{1}\right)^2+\left(\frac{3}{1}\right)}$$

$$=\frac{2}{27+3}=\frac{1}{15}$$

Question 7:

Find $\frac{x}{y}$; when $x^2 + 6y^2 = 5xy$.

Solution 7:

$x^2 + 6y^2 = 5xy$

dividing both sides by y^2, we get,

$\frac{x^2}{y^2} + \frac{6y^2}{y^2} = \frac{5xy}{y^2}$

$\left(\frac{x}{y}\right)^2 + 6 = 5\left(\frac{x}{y}\right)$

$\left(\frac{x}{y}\right)^2 - 5\left(\frac{x}{y}\right) + 6 = 0$

Let $\frac{x}{y} = a$

$\therefore a^2 - 5a + 6 = 0$

$\Rightarrow (a-2)(a-3) = 0$

$\Rightarrow a = 2, 3$

Hence, $\frac{x}{y} = 2, 3$

Question 8:

If the ratio between 8 and 11 is the same as the ratio of 2x – y to x + 2y, find the value of $\frac{7x}{9y}$

Solution 8:

$\frac{2x - y}{x + 2y} = \frac{8}{11}$

$22x - 11y = 8x + 16y$

$14x = 27y$

Given, $\frac{x}{y} = \frac{27}{14}$

$\therefore \frac{7x}{9y} = \frac{7 \times 27}{9 \times 14} = \frac{3}{2}$

Question 9:

Two numbers are in the ratio 2 : 3. If 5 is added to each number, the ratio becomes 5 : 7. Find the numbers.

Solution 9:

Let the two numbers be 2x and 3x.

According to the given information,

$$\frac{2x+5}{3x+5} = \frac{5}{7}$$

$$14x + 35 = 15x + 25$$

$$x = 10$$

Thus, the numbers are 2 × 10 = 20 and 3 × 10 = 30.

Question 10:

Two positive numbers are in the ratio 3 : 5 and the difference between their squares is 400. Find the numbers.

Solution 10:

Let the two numbers be 3x and 5x.

According to the given information

$(5x)^2 - (3x)^2 = 400$

$25x^2 - 9x^2 = 400$

$16x^2 = 400$

$x^2 = 25$

$x = 5$

Thus, the numbers are 3 × 5 = 15 and 5 × 5 = 25.

Question 11:

What quantity must be subtracted from each term of the ratio 9 : 17 to make it equal to 1 : 3?

Solution 11:

Let x be subtracted from each term of the ratio 9: 17.

$$\frac{9-x}{17-x} = \frac{1}{3}$$

$$27 - 3x = 17 - x$$

$$10 = 2x$$

$$x = 5$$

Thus, the required number which should be subtracted is 5.

Question 12:

The monthly pocket money of ravi and sanjeev are in the ratio 5 : 7. Their expenditure are in the ratio 3 : 5. If each saves Rs. 80 every month, find their monthly pocket money.

Solution 12:

Given that the pocket money of Ravi and Sanjeev
Are in the ratio 5:7
Thus, the pocket money of ravi is 5k and that of
Sanjeev is 7k
Also given that the expenditure of ravi and Sanjeev
Are in the ratio 3:5
Thus, the expenditure of ravi is 3m and that of
Sanjeev is 5m
And each of them saves Rs. 80
$\Rightarrow 5k - 3m = 80$ ….. (1)
$7k - 5m = 80$ …….. (2)
Solving equations (1) and (2), We have,
K = 40, m = 40
Hence the monthly pocket money of Ravi is Rs. 200
And that of Sanjeev is Rs. 280

Question 13:

The work done by (x – 2) men in (4x + 1) days and the work done by (4x + 1) men in (2x – 3) days are in the ratio 3 : 8, Find the value of x.

Solution 13:

Assuming that all the men do the same amount of work in one day and one day work of each man = 1 units, we have,
Amount of work done by (x − 2) men in (4x + 1) days
= Amount of work done by (x − 2)(4x + 1) men in one day
= (x − 2)(4x + 1) units of work
Similarly,
Amount of work done by (4x + 1) men in (2x − 3) days
= (4x + 1)(2x − 3) units of work
According to the given information,

$$\frac{(x-2)(4x+1)}{(4x+1)(2x-3)} = \frac{3}{8}$$

$$\frac{x-2}{2x-3} = \frac{3}{8}$$

$8x - 16 = 6x - 9$

$2x = 7$

$x = \frac{7}{2} = 3.5$

Question 14:

The bus fare between two cities is increased in the ratio 7 : 9, find the increase in the fare if:

(i) the original fare is Rs. 245

(ii) the increased fare is Rs. 207

Solution 14:

According to the given information,

Increased (new) bus fare = $\frac{9}{7}$ × original bus fare

(i) We have:

Increased (new) bus fare = $\frac{9}{7}$ × Rs. 245 = Rs. 315

∴ Increase in fare = Rs. 315 – Rs. 245 = Rs. 70

(ii) We have:

Rs 207 = $\frac{9}{7}$ × original bus fare

Original bus fare =Rs.207 × $\frac{7}{9}$ = Rs. 161

∴ Increase in fare = Rs. 207 – Rs. 161 = Rs. 46

Question 15:

By increasing the cost of entry ticket to a fair in the ratio 10 : 13, the number of visitors to the fair has decreased in the ratio 6 : 5. In what ratio has the total collection increased or decreased?

Solution 15:

Let the cost of the entry ticket initially and at present be 10 x and 13x respectively.

Let the number of visitors initially and at present be 6y and 5y respectively.

Initially, total collection = 10x × 6y = 60 xy

At present, total collection = 13x × 5y = 65 xy

Ratio of total collection = 60 xy : 65 xy = 12 : 13

Thus, the total collection has increased in the ratio 12 : 13.

Question 16:

In a basket, the ratio between the number of oranges and the number of apples is 7 : 13. If 8 oranges and 11 apples are eaten, the ratio between the number of oranges and the number of apples becomes 1 : 2 find the original number of oranges and the original number of apples in the basket.

Solution 16:

Let the original number of oranges and apples be 7x and 13x.

According to the given information,

$$\frac{7x-8}{13x-11}=\frac{1}{2}$$

$$14x-16=13x-11$$

$$x=5$$

Thus, the original number of oranges and apples are 7 × 5 = 35 and 13 × 5 = 65 respectively.

Question 17:

The ratio between the number of boys and the number of girls in a class is 4:3 . If there were 20 more boys and 12 less girls, the ratio would have been 2:1. Find the total number of students in the class.

Solution 17:

Let the number of boys and girls in the class be 4x and 3x respectively.

According to the given information,

$$\frac{4x+20}{3x-12}=\frac{2}{1}$$

$$4x+20=6x-24$$

$$44=2x$$

$$x=22$$

Therefore,

Number of boys = 4 × 22 = 88

Number of girls = 3 × 22 = 66

∴ Number of students = 88 + 66 = 154

Question 18:

(a) If A: B = 3 : 4 and B : C = 6 : 7, find:

(i) A : B : C (ii) A : C

(b) If A : B = 2 : 5 and A : C = 3 : 4, find : A : B : C.

Solution 18:

(A)

(i)

$$\frac{A}{B}=\frac{3}{4}=\frac{3}{4}\times\frac{3}{4}=\frac{9}{12}$$

$$\frac{B}{C}=\frac{6}{7}=\frac{6}{7}\times\frac{2}{2}=\frac{12}{14}$$

$$A:B:C=9:12:14$$

(ii)

$$\frac{A}{B}=\frac{3}{4}$$

$$\frac{B}{C}=\frac{6}{7}$$

$$\therefore\ \frac{A}{C}=\frac{\frac{A}{C}}{\frac{C}{B}}=\frac{\frac{3}{4}}{\frac{7}{6}}=\frac{3}{4}\times\frac{6}{7}=\frac{9}{14}$$

$$\therefore A:C=9:14$$

(B) (i) To compare 3 ratios, the consequent of the first Ratio and the antecedent of the 2nd ratio must Be made equal.

Given that A : B = 2 : 5 and A : C = 3 : 4

Interchanging the first ratio, we have,

B : A = 5 : 2 and A : C = 3 : 4

L.C.M of 2 and 3 is 6

$\Rightarrow$ B : A= 5 × 3 : 2 × 3 and A : C = 3 × 2 : 4 × 2

$\Rightarrow$ B : A = 15 : 6 and A : C = 6 : 8

$\Rightarrow$ B : A : C = 15 : 6 :8

$\Rightarrow$ A : B : C = 6 : 15 : 8

Question 19:

If 3A = 4B = 6C; find : A : B: C

Solution 19:

3A = 4B = 6C

3A = 4B $\Rightarrow \frac{A}{B}=\frac{4}{3}$

$4B = 6C \Rightarrow \frac{B}{C} = \frac{6}{4} = \frac{3}{2}$

Hence, A: B: C = 4: 3: 2

Question 20:

Find the compound ratio of :

(i) 3 : 5 and 8 : 15

(ii) 2 : 3, 9 : 14 and 14 : 27

(iii) 2a : 3b, mn : x^2 and x : n

(iv) $\sqrt{2}$: 1, 3 : $\sqrt{5}$ and $\sqrt{20}$: 9

Solution 20:

(i) Required compound ratio = 3 × 8: 5 × 15

$= \frac{3 \times 8}{5 \times 15}$

$= \frac{8}{25} = 8 : 25$

(ii) Required compound ratio = 2 × 9 × 14: 3 × 14 × 27

$= \frac{2 \times 9 \times 14}{3 \times 14 \times 27}$

$= \frac{2}{9} = 2 : 9$

(iii) Required compound ratio = 2a × mn × x: 3b × x^2 × n

$= \frac{2a \times mn \times x}{3b \times x^2 \times n}$

$\frac{2am}{3bx} = 2am : 3bx$

(iv) Required compound ratio = $\sqrt{2} \times 3 \times \sqrt{20} : 1 \times \sqrt{5} \times 9$

$= \frac{\sqrt{2} \times 3 \times \sqrt{20}}{1 \times \sqrt{5} \times 9}$

$= \frac{\sqrt{2} \times \sqrt{4}}{3}$

$= \frac{2\sqrt{2}}{3} = 2\sqrt{2} : 3$

Question 21:

Find duplicate ratio of:

(i) 3: 4 (ii) $3\sqrt{3}:2\sqrt{5}$

Solution 21:

(i) Duplicate ratio of 3: 4 = 3^2: 4^2 = 9: 16

(ii) Duplicate ratio of $3\sqrt{3}:2\sqrt{5}=\left(3\sqrt{3}\right)^2:\left(2\sqrt{5}\right)^2=27:20$

Question 22:

Find triplicate ratio of:

(i) 1 : 3 (ii) $\frac{m}{2}:\frac{n}{3}$

Solution 22:

(i) Triplicate ratio of 1: 3 = 1^3: 3^3 = 1: 27

(ii) Triplicate ratio of

$$\frac{m}{2}:\frac{n}{3}$$

$$=\left(\frac{m}{2}\right)^3:\left(\frac{n}{3}\right)^3=\frac{m^3}{8}:\frac{n^3}{27}=\frac{\frac{m^3}{8}}{\frac{n^3}{27}}=27m^3:8n^3$$

Question 23:

Find sub – duplicate ratio of:

(i) 9:16 (ii) $(x-y)^4:(x+y)^6$

Solution 23:

(i) Sub-duplicate ratio of 9 : 16 = $\sqrt{9}:\sqrt{16}=3:4$

(ii) Sub-duplicate ratio of $(x-y)^4$: $(x+y)^6$

$$=\sqrt{(x-y)^4}:\sqrt{(x+y)^6}=(x-y)^2:(x+y)^3$$

Question 24:

Find sub – triplicate ratio of:

(i) 9 : 16 (ii) $x^3 : 125y^3$

Solution 24:

(i) Sub-triplicate ratio of 64 : 27 = $\sqrt[3]{64} : \sqrt[3]{27} = 4 : 3$

(ii) Sub-triplicate ratio of $x^3 : 125y^3$ = $\sqrt[3]{x^3} : \sqrt[3]{125y^3} = x : 5y$

Question 25:

Find the reciprocal ratio of:

(i) 5 : 8 (ii) $\frac{x}{3} : \frac{y}{7}$

Solution 25:

(i) Reciprocal ratio of 5: 8 = $\frac{1}{5} : \frac{1}{8} = 8 : 5$

(ii) Reciprocal ratio of $\frac{x}{3} : \frac{y}{7} = \frac{1}{\frac{x}{3}} : \frac{1}{\frac{y}{7}} = \frac{3}{x} : \frac{7}{y} = \frac{\frac{3}{x}}{\frac{7}{y}} = \frac{3y}{7x} = 3y : 7x$

Question 26:

If 3x + 4 : x + 5 is the duplicate ratio of 8 : 15, find x.

Solution 26:

$$\frac{3x+4}{x+5} = \frac{(8)^2}{(15)^2}$$

$$\Rightarrow \frac{3x+4}{x+5} = \frac{64}{225}$$

$$\Rightarrow 675x + 900 = 64x + 320$$

$$\Rightarrow 611x = -580$$

$$\Rightarrow x = -\frac{580}{611}$$

Question 27:

If m : n is the duplicate ratio of m + x : n + x; show that x^2 = mn.

Solution 27:

$$\frac{m}{n}=\frac{(m+x)^2}{(n+x)^2}$$

$$\frac{m}{n}=\frac{m^2+x^2+2mx}{n^2+x^2+2nx}$$

$$mn^2+mx^2+2mnx=m^2n+nx^2+2mnx$$

$$x^2(m-n)=mn(m-n)$$

$$x^2=mn$$

Question 28:

If 4x + 4 : 9x – 10 is the triplicate ratio of 4 : 5, find x.

Solution 28:

$$\frac{4x+4}{9x-10}=\frac{(4)^3}{(5)^3}$$

$$\frac{4x+4}{9x-10}=\frac{64}{125}$$

$$500x+500=576x-640$$

$$576x-500x=500+640$$

$$76x=1140$$

$$x=\frac{1140}{76}=15$$

Question 29:

Find the ratio compounded of the reciprocal ratio of 15 : 28, the sub – duplicate ratio of 36 : 49 and the triplicate ratio of 5 : 4

Solution 29:

Reciprocal ratio of 15 : 28 = 28 : 15

Sub-duplicate ratio of 36 : 49 = $\sqrt{36}:\sqrt{49}=6:7$

Triplicate ratio of 5: 4 = $5^3 : 4^3$ = 125 : 64

Required compounded ratio

$$=\frac{28\times6\times125}{15\times7\times64}=\frac{25}{8}=25:8$$

Question 30:

If $\frac{a+b}{am+bn}=\frac{b+c}{mb+nc}=\frac{c+a}{mc+na}$, prove that each of these ratios is equal to $\frac{2}{m+n}$ provided a + b + c ≠ 0

Solution 30:

$$\frac{a+b}{am+bn}=\frac{b+c}{mb+nc}=\frac{c+a}{mc+na}=\frac{\text{sum of antecedents}}{\text{sum of consequents}}$$

$$=\frac{a+b+b+c+c+a}{am+bn+mb+nc+mc+na}$$

$$=\frac{2(a+b+c)}{m(a+b+c)+n(a+b+c)}$$

$$=\frac{2}{m+n}$$

EXERCISE 7 (B)

Question 1:

Find the fourth proportional to:

(i) 1.5, 4.5 and 3.5 (ii) 3a, $6a^2$ and $2ab^2$

Solution 1:

(i) Let the fourth proportional to 1.5, 4.5 and 3.5 be x.

$\Rightarrow 1.5 : 4.5 = 3.5 : x$

$\Rightarrow 1.5 \times x = 3.5 \times 4.5$

$\Rightarrow x = 10.5$

(i) Let the fourth proportional to 3a, $6a^2$ and $2ab^2$ be x.

$\Rightarrow 3a : 6a^2 = 2ab^2 : x$

$\Rightarrow 3a \times x = 2ab^2 \times 6a^2$

$\Rightarrow 3a \times x = 12a^3b^2$

$\Rightarrow x = 4a^2b^2$

Question 2:

Find the third proportional to:

(i) $2\frac{2}{3}$ and 4 (ii) a – b and $a^2 - b^2$

Solution 2:

(i) Let the third proportional to $2\frac{2}{3}$ and 4 be x.

$\Rightarrow 2\frac{2}{3}$, 4, x are in continued proportion.

$\Rightarrow 2\frac{2}{3} : 4 = 4 : x$

$$\Rightarrow \frac{\frac{8}{3}}{4} = \frac{4}{x}$$

$$\Rightarrow x = 16 \times \frac{3}{8} = 6$$

(ii) Let the third proportional to $a - b$ and $a^2 - b^2$ be x.

$\Rightarrow a - b, a^2 - b^2$, x are in continued proportion.

$\Rightarrow a - b : a^2 - b^2 = a^2 - b^2 : x$

$$\Rightarrow \frac{a-b}{a^2-b^2} = \frac{a^2-b^2}{x}$$

$$\Rightarrow x = \frac{\left(a^2-b^2\right)^2}{a-b}$$

$$\Rightarrow x = \frac{(a+b)(a-b)\left(a^2-b^2\right)}{a-b}$$

$$\Rightarrow x = (a+b)\left(a^2-b^2\right)$$

Question 3:

Find the mean proportional between:

(i) 17.5 and 0.007

(ii) $6 + 3\sqrt{3}$ and $8 - 4\sqrt{3}$

(iii) $a - b$ and $a^3 - a^2b$.

Solution 3:

(i) Let the mean proportional between 17.5 and 0.007 be x.

$\Rightarrow$ 17.5, x and 0.007 are in continued proportion.

$\Rightarrow 17.5 : x = x : 0.007$

$\Rightarrow x \times x = 17.5 \times 0.007$

$\Rightarrow x^2 = 0.1225$

$\Rightarrow x = 0.35$

(ii) Let the mean proportional between $6 + 3\sqrt{3}$ and $8 - 4\sqrt{3}$ be x.

$6 + 3\sqrt{3}$, x and $8 - 4\sqrt{3}$ are in continued proportion.

$6 + 3\sqrt{3} : x = x : 8 - 4\sqrt{3}$

$\Rightarrow x \times x = (6 + 3\sqrt{3})(8 - 4\sqrt{3})$

$\Rightarrow x^2 = 48 + 24\sqrt{3} - 24\sqrt{3} - 36$

$\Rightarrow x^2 = 12$

$\Rightarrow x = 2\sqrt{3}$

(iii) Let the mean proportional between $a - b$ and $a^3 - a^2b$ be x.

$\Rightarrow a - b, x, a^3 - a^2b$ are in continued proportion.

$\Rightarrow a - b : x = x : a^3 - a^2b$

$\Rightarrow x \quad x = (a - b)(a^3 - a^2b)$

$\Rightarrow x^2 = (a - b)\, a^2(a - b) = [a(a - b)]^2$

$\Rightarrow x = a(a - b)$

Question 4:

If x + 5 is the mean proportion between x + 2 and x + 9; find the value of x.

Solution 4:

Given, x + 5 is the mean proportional between x + 2 and x + 9.

$\Rightarrow (x + 2), (x + 5)$ and $(x + 9)$ are in continued proportion.

$\Rightarrow (x + 2) : (x + 5) = (x + 5) : (x + 9)$

$\Rightarrow (x + 5)^2 = (x + 2)(x + 9)$

$\Rightarrow x^2 + 25 + 10x = x^2 + 2x + 9x + 18$

$\Rightarrow 25 - 18 = 11x - 10x$

$\Rightarrow x = 7$

Question 5:

What least number must be added to each of the numbers 16, 7, 79 and 43 so that the resulting numbers are in proportion?

Solution 5:

Let the number added be x.

$\therefore (16 + x) : (7 + x) :: (79 + x)\,(43 + x)$

$$\frac{16+x}{7+x} = \frac{79+x}{43+x}$$

$$(16+x)(43+x) = (79+x)(7+x)$$

$$688 + 16x + 43x + x^2 = 553 + 79x + 7x + x^2$$

$$688 - 553 = 86x - 59x$$

$135 = 27x$

$x = 5$

Thus, the required number which must be added is 5.

Question 6:

What least number must be added to each of the numbers 6, 15, 20 and 43 to make them proportional.

Solution 6:

Let the number added be x.

∴ (6 + x) : (15 + x) :: (20 + x) (43 + x)

$$\frac{6+x}{15+x} = \frac{20+x}{43+x}$$

$$(6+x)(43+x) = (20+x)(15+x)$$

$$258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2$$

$$49x - 35x = 300 - 258$$

$$14x = 42$$

$$x = 3$$

Thus, the required number which should be added is 3.

Question 7:

What number must be added to each of the number 16, 26 and 40 so that the resulting numbers may be in continued proportion?

Solution 7:

Let the number added be x.

∴ (16 + x) : (26 + x) :: (26 + x) (40 + x)

$$\frac{16+x}{26+x} = \frac{26+x}{40+x}$$

$$(162+x)(40+x) = (26+x^2)$$

$$640 + 16x + 40x + x^2 = 676 + 52x + x^2$$

$$56x - 52x = 676 - 640$$

$$4x = 36$$

$$x = 9$$

Thus, the required number which should be added is 9.

Question 8:

What least number must be subtracted from each of the numbers 7, 17 and 47 so that the remainders are in continued proportion?

Solution 8:

Let the number subtracted be x.

$\therefore (7 - x) : (17 - x) :: (17 - x)\ (47 - x)$

$$\frac{7-x}{17-x} = \frac{17-x}{47-x}$$

$$(7-x)(46-x) = (17-x)^2$$

$$329 - 47x - 7x + x^2 = 289 - 34x + x^2$$

$$329 - 289 = -34x + 54x$$

$$20x = 40$$

$$x = 2$$

Thus, the required number which should be subtracted is 2.

Question 9:

If y is the mean proportional between x and z; show that xy + yz is the mean proportional between $x^2 + y^2$ and $y^2 + z^2$.

Solution 9:

Since y is the mean proportion between x and z

Therefore, $y^2 = xz$

Now, we have to prove that xy + yz is the mean proportional between $x^2 + y^2$ and $y^2 + z^2$, i.e.,

$$(xy + yz)^2 = (x^2 + y^2)(y^2 + z^2)$$

$$LHS = (xy + yz)^2$$

$$= [y(x + z)]^2$$

$$= y^2 (x + z)^2$$

$$= xz(x + z)^2$$

$$RHS = (x^2 + y^2)(y^2 + z^2)$$

$$= (x^2 + xz)(xz + z^2)$$

$$= x(x + z)z(x + z)$$

$$= xz(x + z)^2$$

LHS = RHS

Hence, proved.

Question 10:

If q is the mean proportional between p and r, show that:

Pqr $(p + q + r)^3 = (pq + qr + pr)^3$.

Solution 10:

Given, q is the mean proportional between p and r.

$\Rightarrow q^2 = pr$

$L.H.S = pqr(p+q+r)^3$

$= qq^2(p+q+r)^3$

$= q^3(p+q+r)^3 \quad [\because q^2 = pr]$

$= [q(p+q+r)]^3$

$= (pq+q^2+qr)^3$

$= (pq+pr+qr)^3 \quad [\because q^2 = pr]$

$= R.H.S$

Question 11:

If three quantities are in continued proportion; show that the ratio of the first to the third is the duplicate ratio of the first to the second.

If the three quantities be x, y and z; then x : y = y : z and to prove that $x : z = x^2 : y^2$.

Solution 11:

Let x, y and z be the three quantities which are in continued proportion.

Then, $x : y :: y : z \Rightarrow y^2 = xz$ .... (1)

Now, we have to prove that

$x : z = x^2 : y^2$

That is we need to prove that

$xy^2 = x^2z$

LHS $= xy^2 = x(xz) = x^2z =$ RHS [Using (1)]

Hence, proved.

Question 12:

If y is the mean proportional between x and z, prove that $\frac{x^2 - y^2 + z^2}{x^{-2} - y^{-2} + z^{-2}} = y^4$.

Solution 12:

Given, y is the mean proportional between x and z.

$\Longrightarrow y^2 = xz$

$$\text{LHS} = \frac{x^2 - y^2 + z^2}{x^{-2} - y^{-2} + z^{-2}}$$

$$= \frac{x^2 - y^2 + z^2}{\frac{1}{x^2} - \frac{1}{y^2} + \frac{1}{z^2}}$$

$$= \frac{x^2 - xz + z^2}{\frac{1}{x^2} - \frac{1}{xz} + \frac{1}{z^2}}$$

$$= \frac{x^2 - xz + z^2}{\frac{z^2 - xz + x^2}{x^2z^2}}$$

$$= x^2z^2$$

$$= (xz)^2$$

$$= (y^2)^2 \qquad (\because Y^2 = XZ)$$

$$= y^4$$

$$= \text{RHS}$$

Question 13:

Given four quantities a, b, c and d are in proportion. Show that:

$(a - c)\, b^2 : (b - d)\, cd$

$= (a^2 - b^2 - ab) : (c^2 - d^2 - cd)$

Given: $\frac{a}{b} = \frac{c}{d} = k$ (let)

$\Longrightarrow$ a = bk and c = dk

Now, find the values of L.H.S and R.H.S of the required result by substituting a = bk and c = dk; and show L.H.S = R.H.S

Solution 13:

$$\text{Let} \frac{a}{b} = \frac{c}{d} = k$$

$$\Rightarrow a = bk \text{ and } c = dk$$

$$\text{LHS} = \frac{(a-c)b^2}{(b-d)cd}$$
$$= \frac{(bk-dk)b^2}{(b-d)dkd}$$
$$= \frac{k(b-d)b^2}{(b-d)d^2k}$$
$$= \frac{b^2}{d^2}$$
$$\text{RHS} = \frac{(a^2-b^2-ab)}{(c^2-d^2-cd)}$$
$$= \frac{(b^2k^2-b^2-bkb)}{(d^2k^2-d^2-dkd)}$$
$$= \frac{b^2(k^2-1-k)}{d^2(k^2-1-k)}$$
$$= \frac{b^2}{d^2}$$
$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence proved.

Question 14:

Find two numbers such that the mean proportional between them is 12 and the third proportional to them is 96.

Solution 14:

Let a and b be the two numbers, whose mean proportional is 12.

$$\therefore ab = 12^2 \Rightarrow ab = 144 \Rightarrow b = \frac{144}{a} \text{.................(i)}$$

Now, third proportional is 96

$$\therefore a : b :: b : 96$$
$$\Rightarrow b^2 = 96a$$
$$\Rightarrow \left(\frac{144}{a}\right)^2 = 96a$$

$$\Rightarrow \frac{(144)^2}{a^2} = 96a$$

$$\Rightarrow a^3 = \frac{144 \times 144}{96}$$

$$\Rightarrow a^3 = 216$$

$$\Rightarrow a = 6$$

$$b = \frac{144}{6} = 24$$

Therefore, the numbers are 6 and 24.

Question 15:

Find the third proportional to $\frac{x}{y} + \frac{y}{x}$ and $\sqrt{x^2 + y^2}$

Solution 15:

Let the required third proportional be p.

$\Rightarrow \frac{x}{y} + \frac{y}{x}, \sqrt{x^2 + y^2}$, p are in continued proportion.

$$\Rightarrow \frac{x}{y} + \frac{y}{x} : \sqrt{x^2 + y^2} = \sqrt{x^2 + y^2} : p$$

$$\Rightarrow p\left(\frac{x}{y} + \frac{y}{x}\right) = \left(\sqrt{x^2 + y^2}\right)^2$$

$$\Rightarrow p\left(\frac{x^2 + y^2}{xy}\right) = x^2 + y^2$$

$$\Rightarrow p = xy$$

Question 16:

If p : q = r : s; then show that: mp + np : q = mr + ns : s.

$$\frac{p}{q} = \frac{r}{s} \Rightarrow \frac{mp}{q} = \frac{mr}{s}$$

$\Rightarrow \frac{mp}{q} + n = \frac{mr}{s} + n$ and so

Solution 16:

$$\frac{p}{q} = \frac{r}{s}$$

$$\Rightarrow \frac{mp}{q} = \frac{mr}{s}$$

$$\Rightarrow \frac{mp}{q} + n = \frac{mr}{s} + n$$

$$\Rightarrow \frac{mp + nq}{q} = \frac{mr + ns}{s}$$

Hence, mp + nq : q = mr + ns : s.

Question 17:

If p + r = mq and $\frac{1}{q} + \frac{1}{s} = \frac{m}{r}$; then prove that p : q = r : s.

Solution 17:

$$\frac{1}{q} + \frac{1}{s} = \frac{m}{r}$$

$$\frac{s + q}{qs} = \frac{m}{r}$$

$$\frac{s + q}{s} = \frac{mq}{r}$$

$$\frac{s + q}{s} = \frac{p + r}{r} \quad (\because p + r = mq)$$

$$1 + \frac{q}{s} = \frac{p}{r} + 1$$

$$\frac{q}{s} = \frac{p}{r}$$

$$\frac{p}{q} = \frac{r}{s}$$

Hence, proved

Question 18:

If $\frac{a}{b}=\frac{c}{d}$, prove that each of the given ratio is equal to:

(i) $\frac{5a+4c}{5b+4d}$

(ii) $\frac{13a-8c}{13b-8d}$

(iii) $\sqrt{\frac{3a^2-10c^2}{3b^2-10d^2}}$

(iv) $\left(\frac{8a^3+15c^3}{8b^3+15d^3}\right)^{\frac{1}{3}}$

Solution 18:

$\text{Let}\frac{a}{b}=\frac{c}{d}=k$

$\text{Then}, a=bk \text{ and } c=dk$

(i) $\frac{5a+4c}{5b+4d}=\frac{5(bk)+4(dk)}{5b+4b}=\frac{k(5b+4d)}{5b+4d}=k=\text{each given ratio}$

(ii) $\frac{13a-8c}{13b-8d}=\frac{13(bk)-8(dk)}{13b-8d}=\frac{k(13b-8d)}{13b-8d}=k=\text{each given ratio}$

(iii) $\sqrt{\frac{3a^2-10c^2}{3b^2-10d^2}}=\sqrt{\frac{3(bk)^2-10(dk)^2}{3b^2-10d^2}}=\sqrt{\frac{k^2(3b^2-10d^2)}{3b^2-10d^2}}=k=\text{each given ratio}$

(iv) $\left(\frac{8a^3+15c^3}{8b^3+15d^3}\right)^{\frac{1}{3}}=\left[\frac{8(bk)^3+15(dk)^3}{8b^3+15d^3}\right]^{\frac{1}{3}}=\left[\frac{k^3(8b)^3+15d^3}{8b^3+15d^3}\right]^{\frac{1}{3}}=k=\text{each given ratio}$

Question 19:

If a , b, c and d are in proportion, prove that:

(i) $\frac{13a+17b}{13c+17d}=\sqrt{\frac{2ma^2-3nb^2}{2mc^2-3nd^2}}$

(ii) $\sqrt{\frac{4a^2+9b^2}{4c^2+9d^2}}=\left(\frac{xa^3-5yb^3}{xc^3-5yd^3}\right)^{\frac{1}{3}}$

Solution 19:

$$\text{Let}\ \frac{a}{b}=\frac{c}{d}=k(\text{say})$$

Then, $a = bk$ and $c = dk$

$$\text{(i)L.H.S}=\frac{13a+17b}{13c+17d}=\frac{13(bk)+17b}{13(bk)+17b}=\frac{b(13k+17)}{b(13k+17)}=\frac{b}{d}$$

$$\text{R.H.S}=\sqrt{\frac{2ma^2-3nb^2}{2mc^2-3nd^2}}=\sqrt{\frac{2m(bk)^2-3nb^2}{2m(dk)^2-3nd^2}}=\sqrt{\frac{b^2(2mk^2-3n)}{d^2(2mk^2-3n)}}=\frac{b}{d}$$

Hence, L.H.S = R.H.S

$$\text{(ii)L.H.S}=\sqrt{\frac{4a^2+9b^2}{4c^2+9d^2}}=\sqrt{\frac{4(bk)^2+9b^2}{4(dk)^2+9d^2}}=\sqrt{\frac{b^2(4k^2+9)}{d^2(4k^2+9)}}=\frac{b}{d}$$

$$\text{R.H.S}=\left(\frac{xa^3-5yb^3}{xc^3-5yd^3}\right)^{\frac{1}{3}}=\left[\frac{x(bk)^3-5yb^3}{x(dk)^3-5yd^3}\right]^{\frac{1}{3}}$$

$$=\left[\frac{b^3(xk^3-5y)}{d^3(xk^3-5y)}\right]^{\frac{1}{3}}$$

$$=\left[\frac{b^3}{d^3}\right]^{\frac{1}{3}}=\frac{b}{d}$$

Hence, L.H.S = R.H.S

Question 20:

If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, prove that $\frac{2x^3-3y^3+4z^3}{2a^3-3b^3+4c^3}=\left(\frac{2x-3y+4z}{2a-3b+4c}\right)^3$

Solution 20:

$$\text{Let}\ \frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$$

Then, x = ak, y = bk and z = ck

$$\text{L.H.S}=\frac{2x^3-3y^3+4z^3}{2a^3-3b^3+4c^3}$$

$$=\frac{2(ak)^3-3(bk)^3+4(ck)^3}{2a^3-3b^3+4c^3}$$

$$=\frac{2a^3k^3-3b^3k^3+4c^3k^3}{2a^3-3b^3+4c^3}$$
$$=\frac{k^3\left(2a^3-3b^3+4c^3\right)}{2a^3-3b^3+4c^3}$$
$$=k^3$$
$$\text{R.H.S}=\left(\frac{2x-3y+4z}{2a-3b+4c}\right)^3$$
$$=\left(\frac{2ak-3bk+4ck}{2a-3b+4c}\right)^3$$
$$=\left[\frac{k\left(2a-3b+4c\right)}{2a-3b+4c}\right]^3$$
$$=K^3$$

Hence, L.H.S = R.H.S

EXERCISE .7 (c)

Question 1:

If a : b = c : d, prove that

(i) 5a + 7b : 5a – 7b = 5c + 7d : 5c – 7d.

(ii) (9a + 13b) (9c – 13d) = (9c + 13d) (9a – 13b).

(iii) xa + yb : xc + yd = b : d

Solution 1:

$$\text{(i) Given, }\frac{a}{b}=\frac{c}{d}$$
$$\Rightarrow \frac{5a}{7b}=\frac{5c}{7d}\quad\left(\text{Multiplying each side by }\frac{5}{7}\right)$$
$$\Rightarrow \frac{5a+7b}{5a-7b}=\frac{5c+7d}{5c-7d}\quad\left(\text{By componend and dividendo}\right)$$
$$\text{(ii) Given, }\frac{a}{b}=\frac{c}{d}$$
$$\Rightarrow \frac{9a}{13b}=\frac{9c}{13d}\quad\left(\text{Multiplying each side by }\frac{9}{13}\right)$$
$$\Rightarrow \frac{9a+13b}{13a-13b}=\frac{9c+13d}{9c-13d}\quad\left(\text{By componend and dividendo}\right)$$
$$\Rightarrow (9a+13b)(9c-13d)=(9c+13d)(9a-13b)$$

(iii) Given, $\frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{xa}{yb}=\frac{xc}{yd}$ $\left(\text{Multiplying each side by } \frac{x}{y}\right)$

$\Rightarrow \frac{xa+yb}{yb}=\frac{xc+yd}{yd}$ (By componend)

$\Rightarrow \frac{xa+yb}{xc+yd}=\frac{yb}{yd}$

$\Rightarrow \frac{xa+yb}{xc+yd}=\frac{b}{d}$

Question 2:

If a : b = c : d, prove that $(6a+7d)(3c-4d)=(6c+7d)(3a-4b)$

Solution 2:

Given, $\frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{6a}{7b}=\frac{6c}{7d}$ $\left(\text{Multiplying each side by } \frac{6}{7}\right)$

$\Rightarrow \frac{6a+7b}{7b}=\frac{6c+7d}{7d}$ (By componend)

$\Rightarrow \frac{6a+7b}{6c+7d}=\frac{7b}{7d}=\frac{b}{d}$

$\Rightarrow \text{Also}, \frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{3a}{4b}=\frac{3c}{4d}$ $\left(\text{Multiplying each side by } \frac{3}{4}\right)$

$\Rightarrow \frac{3a-4b}{4b}=\frac{3c-4d}{4d}$ (By dividendo)

$\Rightarrow \frac{3a-4b}{3c-4d}=\frac{4b}{4d}=\frac{b}{d}$ .... (2)

From (1) and = (2)

$\frac{6a+7b}{6c+7d}=\frac{3a-4b}{3c-4d}$

$(6a+7d)(3c-4d)=(6c+7d)(3a-4b)$

Question 3:

Given, prove that:

$$\frac{3a-5b}{3a+5b}=\frac{3c-5d}{3c+5d}$$

Solution 3:

$$\Rightarrow \frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow \frac{3a}{5b}=\frac{3c}{5d}\left(\text{Multiplying each side by } \frac{3}{5}\right)$$

$$\Rightarrow \frac{3a+5b}{3a-5b}=\frac{3c+5d}{3c-5d} \quad (\text{By componendo and dividendo})$$

$$\Rightarrow \frac{3a-5b}{3a+5b}=\frac{3c-5d}{3c+5d} \quad (\text{By alternendo})$$

Question 4:

If $\frac{5x-6y}{5u-6v}=\frac{5x-6y}{5u-6v}$ then prove that x : y = u : v.

Solution 4:

$$\frac{5x-6y}{5u-6v}=\frac{5x-6y}{5u-6v} \quad (\text{By alternendo})$$

$$\frac{5x+6y}{5x-6y}=\frac{5u+6v}{5u-6v}$$

$$\frac{5x+6y+5x-6y}{5x+6y-5x+6y}=\frac{5u+6v+5u-6v}{5u+6v-5u+6v} \quad (\text{By componendo and dividendo})$$

$$\frac{10x}{12y}=\frac{10u}{12v}$$

$$\frac{x}{y}=\frac{u}{v}$$

Question 5:

If (7a + 8b) (7c – 8d) = (7a – 8b) (7c + 8d); Prove that a : b = c : d.

Solution 5:

$$\text{Given, } \frac{7a+8b}{7a-8b}=\frac{7c+8d}{7c-8d}$$

Applying componendo and dividendo,

$$\frac{7a+8b+7a-8b}{7a+8b-7a+8b}=\frac{7c+8d+7c-8d}{7c+8d-7c+8d}$$

$$\Rightarrow \frac{14a}{16b}=\frac{14c}{16d}$$

$$\Rightarrow \frac{a}{b}=\frac{c}{d}$$

Hence, a : b = c : d.

Question 6:

(i) If $x=\frac{6ab}{a+b}$, find the value of: $\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}$

(ii) If a = $\frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$, find the value of : $\frac{a+2\sqrt{2}}{a-2\sqrt{2}}+\frac{a+2\sqrt{3}}{a-2\sqrt{3}}$

Solution 6:

$$(i)\, x=\frac{6ab}{a+b}$$

$$\Rightarrow \frac{x}{3a}=\frac{2b}{a+b}$$

Applying componendo and dividendo,

$$\frac{x+3a}{x-3a}=\frac{2b+a+b}{2b-a-b}$$

$$\frac{x+3a}{x-3a}=\frac{3b+a}{b-a}$$

$$\text{Again, } x=\frac{6ab}{a+b}$$

$$\Rightarrow \frac{x}{3b}=\frac{2a}{a+b}$$

Applying componendo and dividendo,

$$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$$

$$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{-3b-a+3a+b}{a-b}$$

$$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{2a-2b}{a-b}=2$$

$$(ii)\ a = \frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$$

$$\frac{a}{2\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}}$$

Applying componendo and dividendo,

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{2}+\sqrt{3}}{2\sqrt{3}-\sqrt{2}+\sqrt{3}}$$

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \quad ....(1)$$

Applying componendo and dividendo,

$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{2}-\sqrt{3}}$$

$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \quad ....(2)$$

From (1) and (2),

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{2}+\sqrt{3}-3\sqrt{3}-\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{2\sqrt{2}-2\sqrt{3}}{\sqrt{2}-\sqrt{3}} = 2$$

Question 7:

If (a + b + c + d) (a – b – c + d) = (a + b - c - d) (a – b + c - d)

Prove that a: b = c : d

Solution 7:

$$\text{Given,} \frac{a+b+c+d}{a+b-c-d} = \frac{a-b+c-d}{a-b-c+d}$$

Applying componendo and dividendo,

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)} = \frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{2(a+b)}{2(c+d)} = \frac{2(a-b)}{2(c-d)}$$

$$\frac{a+b}{c+d}=\frac{a-b}{c-d}$$
$$\frac{a+b}{a-d}=\frac{c+d}{c-d}$$
Applying componendo and dividendo,
$$\frac{a+b+a-b}{a+b-a+b}=\frac{c+d+c-d}{c+d-c+d}$$
$$\frac{2a}{2b}=\frac{2c}{2d}$$
$$\frac{a}{b}=\frac{c}{d}$$

Question 8:

If $\frac{a-2b-3c+4d}{a+2b-3c-4d}=\frac{a-2b+3c-4d}{a+2b+3c+4d}$, Show that 2ad = 3bc.

Solution 8:

$$\frac{a-2b-3c+4d}{a+2b-3c-4d}=\frac{a-2b+3c-4d}{a+2b+3c+4d}$$
Applying componendo and dividendo,
$$\frac{(a-2b-3c+4d)+(a+2b-3c-4d)}{(a-2b-3c+4d)-(a+2b-3c-4d)}$$
$$=\frac{(a-2b+3c-4d)+(a+2b+3c+4d)}{(a-2b+3c-4d)-(a+2b+3c+4d)}$$
$$\frac{2(a-3c)}{2(-2b+4d)}=\frac{2(a+3c)}{2(-2b-4d)}$$
$$\frac{a-3c}{a+3c}=\frac{-2b+4d}{-2b-4d}$$
Applying componendo and dividendo,
$$\frac{a-3c+a+3c}{a-3c-a-3c}=\frac{-2b+4d-2b-4d}{-2b+4d+2b+4d}$$
$$\frac{2a}{-6c}=\frac{-4b}{8d}$$
$$\frac{a}{-3c}=\frac{-b}{2d}$$
$$2ad=3bc$$

Question 9:

If $\left(a^2+b^2\right)\left(x^2+y^2\right)=\left(ax+by\right)^2$, Prove that: $\frac{a}{x}=\frac{b}{y}$,

Solution 9:

Given, $\left(a^2+b^2\right)\left(x^2+y^2\right)=\left(ax+by\right)^2$

$a^2x^2+a^2y^2+b^2x^2+b^2y^2=a^2x^2+b^2y^2+2abxy$

$a^2y^2+b^2x^2-2abxy=0$

$\left(ay-bx\right)^2=0$

$ay-bx=0$

$ay=bx$

$\frac{a}{x}=\frac{b}{y}$

Question 10:

If a, b and c are in continued proportion prove that:

(i) $\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a}{c}$

(ii) $\frac{a^2+b^2+c^2}{\left(a+b+c\right)^2}=\frac{a-b+c}{a+b+c}$

Solution 10:

Given, a, b and c are in continued proportion.

$\Rightarrow \frac{a}{b}=\frac{b}{c}=k\left(say\right)$

$\Rightarrow a=bk, b=ck$

$\Rightarrow a=\left(ck\right)k=ck^2, b=ck$

(i) $L..H.S=\frac{a^2+ab+b^2}{b^2+bc+c^2}$

$\frac{\left(ck^2\right)^2+\left(ck^2\right)\left(ck\right)+\left(ck\right)^2}{\left(ck\right)^2+\left(ck\right)c+c^2}$

$=\frac{c^2k^4+c^2k^3+c^2k^2}{c^2k^2+c^2k+c^2}$

$=\frac{c^2k^2\left(k^2+k+1\right)}{c^2\left(k^2+k+1\right)}$

$=k^2$

$\text{R.H.S} = \frac{a}{c} = \frac{ck^2}{c} = k^2$

$\therefore \text{L.H.S} = \text{R.H.S}$

(ii) $\text{L..H.S} = \frac{a^2+b^2+c^2}{(a+b+c)^2}$

$= \frac{(ck^2)^2 + (ck^2) + c^2}{(ck^2+ck+c)^2}$

$= \frac{c^2k^4 + c^2k^2 + c^2}{c^2(k^2+k+1)^2}$

$= \frac{c^2(k^4+k^2+1)}{c^2(k^2+k+1)^2}$

$= \frac{k^4+k^2+1}{(k^2+k+1)^2}$

$\text{R.H.S} = \frac{a-b+c}{a+b+c}$

$= \frac{ck^2-ck+c}{ck^2+ck+c}$

$= \frac{k^2-k+1}{k^2+k+1}$

$= \frac{(k^2-k+1)(k^2+k+1)}{(k^2+k+1)^2}$

$= \frac{k^4+k^3+k^2-k^3-k^2-k+k^2+k+1}{(k^2+k+1)^2}$

$= \frac{k^4+k^2+1}{(k^2+k+1)^2}$

$\therefore \text{L..H.S} = \text{R.H.S}$

Question 11:

Using properties of proportion, solve for x:

(i) $\frac{\sqrt{x+5}+\sqrt{x-16}}{\sqrt{x+5}-\sqrt{x-16}} = \frac{7}{3}$

(ii) $\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}=\frac{4x-1}{2}$

(iii) $\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}}=5$

Solution 11:

(i) $\frac{\sqrt{x+5}+\sqrt{x-16}}{\sqrt{x+5}-\sqrt{x-16}}=\frac{7}{3}$

Applying componendo and dividendo,

$$\frac{\sqrt{x+5}+\sqrt{x-16}+\sqrt{x+5}-\sqrt{x-16}}{\sqrt{x+5}+\sqrt{x-16}-\sqrt{x+5}+\sqrt{x-16}}=\frac{7+3}{7-3}$$

$$\frac{2\sqrt{x+5}}{2\sqrt{x-16}}=\frac{10}{4}$$

$$\frac{\sqrt{x+5}}{\sqrt{x-16}}=\frac{5}{2}$$

Squaring both sides,

$$\frac{x+5}{x-16}=\frac{25}{4}$$

$$4x+20=25x-400$$

$$21x=420$$

$$x=\frac{420}{21}=20$$

(ii) $\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}=\frac{4x-1}{2}$

Applying componendo and dividendo,

$$\frac{\sqrt{x+1}+\sqrt{x-1}+\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}-\sqrt{x+1}+\sqrt{x-1}}=\frac{4x-1+2}{4x-1-2}$$

$$\frac{2\sqrt{x+1}}{2\sqrt{x-1}}=\frac{4x+1}{4x-3}$$

Squaring both sides,

$$\frac{x+1}{x-1}=\frac{16x^2+1+8x}{16x^2+9-24x}$$

Applying componendo and dividendo,

$$\frac{x+1+x-1}{x+1-x+1}=\frac{16x^2+1+8x+16x^2+9-24x}{16x^2+1+8x-16x^2-9+24x}$$

$$\frac{2x}{2}=\frac{32x^2+10-16x}{32x-8}$$

$$16x^2-4x=16x^2+5-8x$$

$$4x=5$$

$$x=\frac{5}{4}$$

(iii) $\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}}=5$

Applying componendo and dividendo,

$$\frac{3x+\sqrt{9x^2-5}+3x-\sqrt{9x^2-5}}{3x+\sqrt{9x^2-5}-3x+\sqrt{9x^2-5}}=\frac{5+1}{5-1}$$

$$\frac{6x}{2\sqrt{9x^2-5}}=\frac{6}{4}$$

$$\frac{x}{\sqrt{9x^2-5}}=\frac{1}{2}$$

Squaring both sides,

$$\frac{x^2}{9x^2-5}=\frac{1}{4}$$

$$4x^2=9x^2-5$$

$$5x^2=5$$

$$x^2=1$$

$$x=1$$

Question 12:

If x = $\frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$, prove that: $3bx^2 - 2ax + 3b = 0$

Solution 12:

Since, $\frac{x}{1}=\frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$

Applying componendo and dividendo, we get,

$$\frac{x+1}{x-1}=\frac{\sqrt{a+3b}+\sqrt{a-3b}+\sqrt{a+3b}-\sqrt{a-3b}}{\sqrt{a+3b}+\sqrt{a-3b}-\sqrt{a+3b}+\sqrt{a-3b}}$$

$$\frac{x+1}{x-1}=\frac{2\sqrt{a-3b}}{-2\sqrt{a-3b}}$$

Squaring both sides,

$$\frac{x^2+2x+1}{x^2-2x+1}=\frac{a+3b}{a-3b}$$

Again applying componendo and dividendo,

$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1}=\frac{a+3b+a-3b}{a+3b-a+3b}$$

$$\frac{2(x^2+1)}{2(2x)}=\frac{2(a)}{2(3b)}$$

$$3b(x^2+1)=2ax$$

$3bx^2 + 3b = 2ax$

$3bx^2 - 2ax + 3b = 0.$

Question 13:

Using the properties of proportion, solve for x, given $\frac{x^4+1}{2x^2}=\frac{17}{8}$.

Solution 13:

$$\frac{x}{y}=\frac{\sqrt{a+b}+\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}$$

Again applying componendo and dividendo,

$$\frac{x+y}{x-y}=\frac{\sqrt{a+b}+\sqrt{a-b}+\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}-\sqrt{a+b}+\sqrt{a-b}}$$

$$\frac{x+y}{x-y}=\frac{2\sqrt{a+b}}{2\sqrt{a-b}}$$

$$\frac{x+y}{x-y}=\frac{\sqrt{a+b}}{\sqrt{a-b}}$$

Squaring both sides,

$$\frac{x^2+y^2+2xy}{x^2+y^2-2xy}=\frac{a+b}{a-b}$$

Again applying componendo and dividendo,

$$\frac{x^2+y^2+2xy+x^2+y^2-2xy}{x^2+y^2+2xy-x^2-y^2+2xy}=\frac{a+b+a-b}{a+b-a+b}$$

$$\frac{2\left(x^2+y^2\right)}{4xy}=\frac{2a}{2b}$$

$$\frac{x^2+y^2}{2xy}=\frac{a}{b}$$

$$bx^2+by^2=2axy$$

$$bx^2-2axy+by^2=0$$

Question 14:

If $x=\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$, express n in terms of x and m.

Solution 14:

$$x=\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

applying componendo and dividendo,

$$\frac{x+1}{x-1}=\frac{\sqrt{m+n}+\sqrt{m-n}+\sqrt{m+n}-\sqrt{m-n}}{c-\sqrt{m-n-\sqrt{m+n}+\sqrt{m-n}}}$$

$$\frac{x+1}{x-1}=\frac{2\sqrt{m+n}}{2\sqrt{m-n}}$$

$$\frac{x+1}{x-1}=\frac{\sqrt{m+n}}{\sqrt{m-n}}$$

Squaring both sides,

$$\frac{x^2+2x+1}{x^2-2x+1}=\frac{m+n}{m-n}$$

applying componendo and dividendo,

$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1}=\frac{m+n+m-n}{m+n-m+n}$$

$$\frac{2x^2+2}{4x}=\frac{2m}{2n}$$

$$\frac{x^2+1}{2x}=\frac{m}{n}$$

$$\frac{x^2+1}{2mx}=\frac{1}{n}$$
$$n=\frac{2mx}{x^2+1}$$

Question 15:

If $\frac{x^3+3xy^2}{3x^2y+y^3}=\frac{m^3+3mn^2}{3m^2n+n^3}$ Show that: nx = my.

Solution 15:

$$\frac{x^3+3xy^2}{3x^2y+y^3}=\frac{m^3+3mn^2}{3m^2n+n^3}$$

applying componendo and dividendo,

$$\frac{x^3+3xy^2}{3x^2y+y^3}=\frac{m^3+3mn^2}{3m^2n+n^3}$$
$$\frac{x^3+3xy^2+3x^2y+y^3}{x^3+3xy^2-3x^2y-y^3}=\frac{m^3+3mn^2+3m^2n+n^3}{m^3+3mn^2-3m^2n-n^3}$$
$$\frac{(x+y)^3}{(x-y)^3}=\frac{(m+n)^3}{(m-n)^3}$$
$$\frac{x+y}{x-y}=\frac{m+n}{m-n}$$

applying componendo and dividendo,

$$\frac{x+y+x-y}{x+y-x+y}=\frac{m+n+m-n}{m+n-m+n}$$
$$\frac{2x}{2y}=\frac{2m}{2n}$$
$$\frac{x}{y}=\frac{m}{n}$$
$$nx=my$$

EXERCISE.7 (D)

Question 1:

If a : b = 3 : 5, find (10a + 3b) : (5a + 2b)

Solution 1:

$$\text{Given,} \frac{a}{b} = \frac{3}{5}$$

$$\frac{10a + 3b}{5a + 2b}$$

$$= \frac{10\left(\frac{a}{b}\right) + 3}{5\left(\frac{a}{b}\right) + 2} b$$

$$= \frac{10\left(\frac{3}{5}\right) + 3}{5\left(\frac{3}{5}\right) + 2}$$

$$= \frac{6 + 3}{3 + 2}$$

$$= \frac{9}{5}$$

Question 2:

If 5x + 6y : 8x + 5y = 8 : 9, find x : y

Solution 2:

$$\frac{5x + 6y}{8x + 5y} = \frac{8}{9}$$

$$45x + 54y = 64x + 40y$$

$$64x - 45x = 54y - 40y$$

$$19x = 14y$$

$$\frac{x}{y} = \frac{14}{19}$$

Question 3:

If (3x − 4y): (2x − 3y) = (5x − 6y): (4x − 5y), find x : y.

Solution 3:

(3x − 4y): (2x − 3y) = (5x − 6y): (4x − 5y)

$$\frac{3x-4y}{2x-3y}=\frac{5x-6y}{4x-5y}$$

applying componendo and dividendo,

$$\frac{3x-4y+2x-3y}{3x-4y-2x+3y}=\frac{5x-6y+4x-5y}{5x-6y-4x+5y}$$

$$\frac{5x-7y}{x-y}=\frac{9x-11y}{x-y}$$

$$5x-7y=9x-11y$$

$$11y-7y=9x-5x$$

$$4y=4x$$

$$\frac{x}{y}=\frac{1}{1}$$

$$x:1=1:1$$

Question 4:

Find the:

(i) duplicate ratio of $2\sqrt{2}:3\sqrt{5}$

(ii) triplicate ratio of 2a : 3b,

(iii) sub-duplicate ratio of $9x^2a^4 : 25y^6b^2$

(iv) Sub-triplicate ratio of 216: 343

(v) Reciprocal ratio of 3:5

(vi) ratio compounded of the duplicate ratio of 5:6, the reciprocal ratio of 25 : 42 and the sub-duplicate ratio of 36 : 49.

Solution 4:

(i) Duplicate ratio of $2\sqrt{2}:3\sqrt{5}=\left(2\sqrt{2}\right)^2:\left(3\sqrt{5}\right)^2=8:45$

(ii) Triplicate ratio of 2a: 3b = $(2a)^3$: $(3b)^3$ = $8a^3 : 27b^3$

(iii) Sub-duplicate ratio of $9x^2a^4 : 25y^6b^2$ = $\sqrt{9x^2a^4}:\sqrt{25y^6b^2}=3xa^2:5y^3b$

(iv) Sub-triplicate ratio of 216: 343 = $\sqrt[3]{216}:\sqrt[3]{343}=6:7$

(v) Reciprocal ratio of 3: 5 = 5: 3

(vi) Duplicate ratio of 5: 6 = 25: 36

Reciprocal ratio of 25: 42 = 42: 25

Sub-duplicate ratio of 36: 49 = 6: 7

Required compound ratio = $\frac{25\times42\times6}{36\times25\times7}=1:1$

Question 5:

Find the value os x, if:

(i) (2x + 3) : (5x – 38) is the duplicate ratio of $\sqrt{5}:\sqrt{6}$

(ii) (2x + 1) : (3x + 13) is the sub-duplicate ratio of 9 : 25.

(iii) (3x – 7) : (4x + 3) is the sub-triplicate ratio of 8: 27.

Solution 5:

(i) (2x + 3): (5x - 38) is the duplicate ratio of $\sqrt{5}:\sqrt{6}$

Duplicate ratio of $\sqrt{5}:\sqrt{6}=5:6$

$$\frac{2x+3}{5x-38}=\frac{5}{6}$$

$$12x+18=25x-190$$

$$25x-12x=190+18$$

$$13x=208$$

$$x=\frac{208}{13}=16$$

(ii) (2x + 1): (3x + 13) is the sub-duplicate ratio of 9: 25

Sub-duplicate ratio of 9: 25 = 3: 5

$$\frac{2x+1}{3x+13}=\frac{3}{5}$$

$$10x+5=9x+39$$

$$10x-9x=39-5$$

$$x=34$$

(iii) (3x - 7): (4x + 3) is the sub-triplicate ratio of 8: 27

Sub-triplicate ratio of 8: 27 = 2: 3

$$\frac{3x-7}{4x+3}=\frac{2}{3}$$

$$9x-21=8x+6$$

$$9x-8x=6+21$$

$$x=27$$

Question 6:

What quantity must be added to each term of the ratio x : y so that it may become equal to c : d?

Solution 6:

Let the required quantity which is to be added be p.

Then, we have:

$$\frac{x+p}{y+p}=\frac{c}{d}$$

$$dx+pd=cy+cp$$

$$pd-cp=cy-dx$$

$$p(d-c)=cy-dx$$

$$p=\frac{cy-dx}{d-c}$$

Question 7:

Two numbers are in the ratio 5:7. If 3 is subtracted from each of them, the ratio between them becomes 2:3. Find the numbers.

Solution 7:

Let the two numbers be 5x and 7x.

From the given information,

$$\frac{5x-3}{7x-3}=\frac{2}{3}$$

$$15x-9=14x-6$$

$$15x-14x=9-6$$

$$x=3$$

Thus, the numbers are 5x = 15 and 7x = 21.

Question 8:

If $15(2x^2 - y^2) = 7xy$, find x : y; if x and y both are positive.

Solution 8:

$$15\left(2x^2-y^2\right)=7xy$$

$$\frac{2x^2-y^2}{xy}=\frac{7}{15}$$

$$\frac{2x}{y}-\frac{y}{x}=\frac{7}{15}$$

$$\text{Let } \frac{x}{y}=a$$

$$\therefore 2a-\frac{1}{a}=\frac{7}{15}$$

$$\frac{2a^2-1}{a}=\frac{7}{15}$$

$$30a^2-15=7a$$

$$30a^2-7a-15=0$$

$$30a^2-25a+18a-15=0$$

$$5a(6a-5)+3(6a-5)=0$$

$$(6a-5)(5a+3)=0$$

$$a=\frac{5}{6},-\frac{3}{5}$$

But, a cannot be negative

$$\therefore a=\frac{5}{6}$$

$$\Rightarrow \frac{x}{y}=\frac{5}{6}$$

$$\Rightarrow x:y=5:6$$

Question 9:

Find the:

(i) fourth proportional to 2xy, x^2 and y^2.

(ii) third proportional to $a^2 – b^2$ and a+b

(iii) mean proportion to (x – y) and $(x^3 – x^2y)$

Solution 9:

(i) Let the fourth proportional to 2xy, x^2 and y^2 be n.

$\Longrightarrow 2xy : x^2 = y^2 : n$

$\Longrightarrow 2xy \times n = x^2 \times y^2$

$$\Longrightarrow n=\frac{x^2y^2}{2xy}=\frac{xy}{2}$$

(ii) Let the third proportional to $a^2 - b^2$ and a + b be n.

$\Longrightarrow a^2 - b^2$, a + b and n are in continued proportion.

$\Longrightarrow a^2 - b^2 : a + b = a + b : n$

$$\Rightarrow n = \frac{(a+b)^2}{a^2 - b^2} = \frac{(a+b)^2}{(a+b)(a-b)} = \frac{a+b}{a-b}$$

(iii) Let the mean proportional to (x − y) and $(x^3 - x^2y)$ be n.

$\Longrightarrow (x - y), n, (x^3 - x^2y)$ are in continued proportion

$\Longrightarrow (x - y) : n = n : (x^3 - x^2y)$

$$\Rightarrow n^2 = (x-y)(x^3 - x^2y)$$

$$\Rightarrow n^2 = x^2(x-y)(x-y)$$

$$\Rightarrow n^2 = x^2(x-y)^2$$

$$\Rightarrow n = x(x-y)$$

Question 10:

Find two numbers such that the mean proportional between them is 14 and third proportional to them is 112.

Solution 10:

Let the required numbers be a and b.

Given, 14 is the mean proportional between a and b.

$\Longrightarrow$ a: 14 = 14: b

$\Longrightarrow$ ab = 196

$$\Rightarrow a = \frac{196}{b} \text{........ } (1)$$

Also, given, third proportional to a and b is 112.

$\Longrightarrow$ a: b = b: 112

$$\Longrightarrow b^2 = 112a \text{..........} (2)$$

Using (1), we have:

$$b^2 = 112 \times \frac{196}{b}$$

$$b^3 = (14)^3 (2)^3$$

$$b = 28$$

From (1),

$$a = \frac{196}{28} = 7$$

Thus, the two numbers are 7 and 28

Question 11:

If x and y be unequal and x : y is the duplicate ratio of x + z and y + z, prove that z is mean proportional between x and y.

Solution 11:

Given, $\frac{x}{y} = \frac{(x+z)^2}{(y+z)^2}$

$x\left(y^2 + z^2 + 2yz\right) = y\left(x^2 + z^2 + 2xz\right)$

$xy^2 + xz^2 + 2xyz = x^2y + yz^2 + 2xyz$

$xy^2 + xz^2 = x^2y + yz^2$

$xy^2 - x^2y = yz^2 - xz^2$

$xy(y - x) = z^2(y - x)$

$xy = z^2$

Hence, z is mean proportional between x and y.

Question 12:

If q is the mean proportional between p and r, prove that: $\frac{p^3 + q^3 + r^3}{p^2q^2r^2} = \frac{1}{p^3} + \frac{1}{q^3} + \frac{1}{r^3}$

Solution 12:

Since, q is the mean proportional between p and r,

$q^2 = pr$

$L.H.S = \frac{p^3 + q^3 + r^3}{p^2q^2r^2}$

$= \frac{p^3 + (pr)q + r^3}{p^2(pr)r^2}$

$= \frac{p^3 + prq + r^3}{p^3r^3}$

$= \frac{1}{r^3} + \frac{q}{p^2r^2} + \frac{1}{p^3}$

$= \frac{1}{r^3} + \frac{q}{\left(q^2\right)^2} + \frac{1}{p^3}$

$$= \frac{1}{r^3} + \frac{1}{q^3} + \frac{1}{p^3}$$

$= R.H.S$

Question 13:

If a, b and c are in continued proportion prove that: a : c = $\left(a^2 + b^2\right) : \left(b^2 + c^2\right)$

Solution 13:

Given, a, b and c are in continued proportion.

$\Longrightarrow$ a: b = b: c

$$\text{Let} \frac{a}{b} = \frac{b}{c} = k\left(\text{say}\right)$$

$\Rightarrow a = bk, b = ck$

$\Rightarrow a = ck^2, b = ck$

$$\text{Now,L.H.S} = \frac{a}{c} = \frac{ck^2}{c} = k^2$$

$$\text{R.H.S} = \frac{a^2 + b^2}{b^2 + c^2}$$

$$= \frac{\left(ck^2\right)^2 + \left(ck\right)^2}{\left(ck\right)^2 + c^2}$$

$$= \frac{c^2k^2 + c^2k^2}{c^2k^2 + c^2}$$

$$= \frac{c^2k^2\left(k^2 + 1\right)}{c^2\left(k^2 + 1\right)}$$

$= k^2$

$\therefore$ L.H.S = R.H.S

Question 14:

If $x = \frac{2ab}{a+b}$, find the value of: $\frac{x+a}{x-a} + \frac{x+b}{x-a}$

Solution 14:

$$x = \frac{2ab}{a+b}$$

$$\frac{x}{a} = \frac{2ab}{a+b}$$

applying componendo and dividendo,

$$\frac{x+a}{x-a} = \frac{2b+a+b}{2b-a-b}$$

$$\frac{x+a}{x-a} = \frac{3b+a}{b-a} \qquad ....(1)$$

$$\text{Also, } x = \frac{2ab}{a+b}$$

applying componendo and dividendo,

$$\frac{x+b}{x-b} = \frac{2a+a+b}{2a-a-b}$$

$$\frac{x+b}{x-b} = \frac{3a+a}{a-b} \qquad ....(2)$$

From (1) and (2)

$$\frac{x+a}{x-a} + \frac{x+b}{x-b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

$$\frac{x+a}{x-a} + \frac{x+b}{x-b} = \frac{-3b-a+3a+b}{a-b}$$

$$\frac{x+a}{x-a} + \frac{x+b}{x-b} = \frac{2a-2b}{a-b} = 2$$

Question 15:

If (4a + 9b) (4c – 9d) = (4a – 9b) (4c + 9d), Prove that: a : b = c : d.

Solution 15:

$$\text{Given, } \frac{4a+9b}{4a-9b} = \frac{4c+9d}{4c-9d}$$

applying componendo and dividendo,

$$\frac{4a+9b+4a-9d}{4a+9b-4a+9b} = \frac{4c+9d+4c-9d}{4c+9d-4c+9d}$$

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{8a}{18b} = \frac{8c}{18d}$$

Question 16:

If $\frac{a}{b} = \frac{c}{d}$, Show that:

(a + b) : (c + d) = $\sqrt{a^2 + b^2} : \sqrt{c^2 + d^2}$

Solution 16:

$$\text{Let} \frac{a}{b} = \frac{c}{d} = k(\text{say})$$

$$\Rightarrow a = bk, c = dk$$

$$\text{L.H.S} = \frac{a+b}{c+d}$$

$$= \frac{bk+b}{dk+d}$$

$$= \frac{b(k+1)}{d(k+1)}$$

$$= \frac{b}{d}$$

$$\text{R.H.S} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$$

$$= \frac{\sqrt{(bk)^2+b^2}}{\sqrt{(dk)^2+b^2}}$$

$$= \frac{\sqrt{b^2(k^2+1)}}{\sqrt{d^2(k^2+1)}}$$

$$= \frac{\sqrt{b^2}}{\sqrt{d^2}}$$

$$= \frac{b}{d}$$

$$\therefore \text{L..H.S} = \text{R.H.S}$$

Question 17:

If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, prove that:

$$\frac{ax-by}{(a+b)(x-y)} + \frac{by-cz}{(b+c)(y-z)} + \frac{cz-ax}{(c+a)(z-x)} = 3$$

Solution 17:

Let $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k(\text{say})$

$\Rightarrow x = ak, y = bk, z = ck$

L.H.S

$$= \frac{ax - by}{(a+b)(x-y)} + \frac{by - cz}{(b+c)(y-z)} + \frac{cz - ax}{(c+a)(z-x)}$$

$$= \frac{a(ak) - b(bk)}{(a+b)(ak-bk)} + \frac{b(bk) - c(ck)}{(b+c)(bk-ck)} + \frac{c(ck) - a(ak)}{(c+a)(ck-ak)}$$

$$= \frac{k(a^2 - b^2)}{k(a+b)(a-b)} + \frac{k(b^2 - c^2)}{k(b+c)(b-c)} + \frac{k(c^2 - a^2)}{k(c+a)(c-a)}$$

$$= \frac{k(a^2 - b^2)}{k(a^2 - b^2)} + \frac{k(b^2 - c^2)}{k(b^2 - c^2)} + \frac{k(c^2 - a^2)}{k(c^2 - a^2)}$$

$= 1 + 1 + 1 = 3 = \text{R.H.S}$

Question 18:

There are 36 members in a student council in a school and the ratio of the number of boys to the number of girls is 3:1. How many more girls should be added to the council so that the ratio of number of boys to the number of girls may be 9:5?

Solution 18:

Ratio of number of boys to the number of girls = 3: 1

Let the number of boys be 3x and number of girls be x.

∴ 3x + x = 36

4x = 36

x = 9∴ Number of boys = 27

Number of girls = 9

Le n number of girls be added to the council.

From given information, we have:

$$\frac{27}{9+n} = \frac{9}{5}$$

$135 = 81 + 9n$

$9n = 54$

$n = 6$

Thus, 6 girls are added to the council.

Question 19:

If $\frac{x}{b-c}=\frac{y}{c-a}=\frac{z}{a-b}$, Prove that $ax+by+cz=0$

Solution 19:

Given, $\frac{x}{b-c}=\frac{y}{c-a}=\frac{z}{a-b}=k(say)$

$x=k(b-c), y=k(c-a), z=k(a-b)$

$ax+by+cz$

$=ak(b-c)+bk(c-a)+ck(a-b)$

$=abk-ack+bck-abk+ack-bck$

$=0$

Question 20:

If 7x – 15y = 4x + y, find the value of x: y. Hence use componendo and dividend to find the Values of:

(i) $\frac{9x+5y}{9x-5y}$ (ii) $\frac{3x^2+2y^2}{3x^2-2y^2}$

Solution 20:

$7x-15y=4x+y$

$7x-4x=y+15y$

$3x=16y$

$\frac{x}{y}=\frac{16}{3}$

(i) $\frac{x}{y}=\frac{16}{3}$

$\Rightarrow \frac{9x}{5y}=\frac{144}{15}$ $\left(\text{Multiplying both sides by } \frac{9}{5}\right)$

$\Rightarrow \frac{9x+5y}{9x-5y}=\frac{144+15}{144-15}$ (applying componendo and dividendo)

$\Rightarrow \frac{9x+5y}{9x-5y}=\frac{159}{129}=\frac{53}{43}$

(ii) $\frac{x}{y}=\frac{16}{3}$

$\Rightarrow \frac{x^2}{y^2}=\frac{259}{9}$

$\Rightarrow \frac{3x^2}{2y^2}=\frac{786}{18}=\frac{128}{3}$ $\left(\text{Multiplying both sides by } \frac{3}{2}\right)$

$$\Rightarrow \frac{3x^2+2y^2}{3x^2-2y^2}=\frac{128+3}{128-3} \text{ (applying componendo and dividendo)}$$

$$\Rightarrow \frac{3x^2+2y^2}{3x^2-2y^2}=\frac{131}{125}$$

Question 21:

If $\frac{4m+3n}{4m-3n}=\frac{7}{4}$, Use properties of proportion to find:

(i) m:n (ii) $\frac{2m^2-11n^2}{2m^2+2y^2}$

Solution 21:

(i) Given, $\frac{4m+3n}{4m-3n}=\frac{7}{4}$

applying componendo and dividendo,

$$\frac{4m+3n+4m-3n}{4m+3n-4m+3n}=\frac{7+4}{7-4}$$

$$\frac{8m}{6n}=\frac{11}{3}$$

$$\frac{m}{n}=\frac{11}{4}$$

(ii) $\frac{m}{n}=\frac{11}{4}$

$$\frac{m^2}{n^2}=\frac{121}{16}$$

$$\frac{2m^2}{11n^2}=\frac{2\times121}{11\times16} \quad \left(\text{Multiplying both sides by } \frac{2}{11}\right)$$

$$\frac{2m^2}{11n^2}=\frac{11}{8}$$

$$\frac{2m^2+11n^2}{2m^2-11n^2}=\frac{11+8}{11-8} \quad \text{(Applying componendo and dividendo)}$$

$$\frac{2m^2+11n^2}{2m^2-11n^2}=\frac{19}{3}$$

$$\frac{2m^2-11n^2}{2m^2+11n^2}=\frac{3}{19} \quad \text{(Applying invertendo)}$$

Question 22:

If x, y, z are in continued proportion, prove that

$$\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$$

Solution 22:

∵ x, y, z are in continued proportion,

$$\therefore \frac{x}{y} = \frac{y}{z} \Rightarrow y^2 = zx \ldots\ldots (1)$$

Therefore,

$$\frac{x+y}{y} = \frac{y+z}{z} \quad \text{(By componendo)}$$

$$\Rightarrow \frac{x+y}{y+z} = \frac{y}{z} \quad \text{(By alternendo)}$$

$$\Rightarrow \frac{(x+y)^2}{(y+z)^2} = \frac{y^2}{z^2} \qquad \text{(Squaring both sides)}$$

$$\Rightarrow \frac{(x+y)^2}{(y+z)^2} = \frac{zx}{z^2} \qquad [\text{from}(1)]$$

$$\Rightarrow \frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$$

Hence Proved.

Question 23:

Given $x = \dfrac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$

Use componendo and dividend to prove that

$b^2 = \frac{2a^2x}{x^2+1}$

Solution 23:

$$x = \frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$$

By componendo and dividendo,

$$\frac{x+1}{x-1}=\frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}+\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}-\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}$$

$$\frac{x+1}{x-1}=\frac{2\sqrt{a^2+b^2}}{2\sqrt{a^2-b^2}}$$

Squaring both sides,

$$\frac{x^2+2x+1}{x^2-2x+1}=\frac{a^2+b^2}{a^2-b^2}$$

By componendo and dividendo,

$$\frac{(x^2+2x+1)+(x^2-2x+1)}{(x^2+2x+1)-(x^2-2x+1)}=\frac{(a^2+b^2)+(a^2-b^2)}{(a^2+b^2)-(a^2-b^2)}$$

$$\Rightarrow \frac{2(x^2+1)}{4x}=\frac{2a^2}{2b^2}$$

$$\Rightarrow \frac{x^2+1}{2x}=\frac{a^2}{b^2}$$

$$\Rightarrow b^2=\frac{2a^2x}{x^2+1}$$

Hence Proved

Question 24:

If $\frac{x^2+y^2}{x^2-y^2}=2\frac{1}{8}$, find:

(i) $\frac{x}{y}$ (ii) $\frac{x^3+y^3}{x^3-y^3}$

Solution 24:

(i) Given, $\frac{x^2+y^2}{x^2-y^2}=2\frac{1}{8}$

$$\frac{x^2+y^2}{x^2-y^2}=\frac{17}{8}$$

Applying componendo and dividendo,

$$\frac{x^2+y^2+x^2-y^2}{x^2+y^2-x^2+y^2}=\frac{17+8}{17-8}$$

$$\frac{2x^2}{2y^2}=\frac{25}{9}$$

$$\frac{x^2}{y^2}=\frac{25}{9}$$

$$\frac{x}{y}=\frac{5}{3}=1\frac{2}{3}$$

(ii) $\frac{x^3+y^3}{x^3-y^3}$

$$=\frac{\left(\frac{x}{y}\right)^3+1}{\left(\frac{x}{y}\right)^3-1}$$

$$=\frac{\left(\frac{5}{3}\right)^3+1}{\left(\frac{5}{3}\right)^3-1}$$

$$=\frac{\frac{125}{27}+1}{\frac{125}{27}-1}$$

$$=\frac{\frac{125+27}{27}}{\frac{125-27}{27}}$$

$$=\frac{125+27}{125-27}$$

$$=\frac{76}{49}=1\frac{27}{49}$$

Question 25:

Using componendo and dividendo, find the value of x:

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}}=9$$

Solution 25:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rations such that $\frac{a}{b}=\frac{c}{d}$,

Then by componendo-dividendo,

We have $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

Given that

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}}=9$$

$$\Rightarrow \frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}}=\frac{9}{1}$$

$$\Rightarrow \frac{\left(\sqrt{3x+4}+\sqrt{3x-5}+\sqrt{3x+4}-\sqrt{3x-5}\right)}{\left(\sqrt{3x+4}+\sqrt{3x-5}-\sqrt{3x+4}-\sqrt{3x-5}\right)}=\frac{9+1}{9-1} \quad [\text{Applying componendo – Dividendo}]$$

$$\Rightarrow \frac{2\sqrt{3x+4}}{2\sqrt{3x-5}}=\frac{10}{8}$$

$$\Rightarrow \frac{\sqrt{3x+4}}{\sqrt{3x-5}}=\frac{5}{4}$$

$$\Rightarrow 4\sqrt{3x+4}=5\sqrt{3x-5}$$

Squaring both the sides of the above equation, we have,

$$16(3x+4)=25(3x-5)$$

$$\Rightarrow 16(3x+4)=25(3x-5)$$

$$\Rightarrow 48x+64=75x-125$$

$$\Rightarrow 27x=189$$

$$\Rightarrow x=\frac{189}{27}$$

$$\Rightarrow x=7$$

Question 26:

If $x=\frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$ using properties of proportion show that:

$X^2 - 2ax + = 0$

Solution 26:

given that, $x=\frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$

By Applying componendo – dividendo,

$$\frac{x+1}{x-1}=\frac{\left(\sqrt{a+1}+\sqrt{a-1}\right)+\left(\sqrt{a+1}+\sqrt{a-1}\right)}{\left(\sqrt{a+1}+\sqrt{a-1}\right)-\left(\sqrt{a+1}-\sqrt{a-1}\right)}$$

$$\Rightarrow \frac{x+1}{x-1}=\frac{2\sqrt{a+1}}{2\sqrt{a-1}}$$

$$\Rightarrow \frac{x+1}{x-1}=\frac{\sqrt{a+1}}{\sqrt{a-1}}$$

Squaring both the sides if the equation, we have,

$$\Rightarrow \left(\frac{x+1}{x-1}\right)^2=\frac{a+1}{a-1}$$

$$\Rightarrow (x+1)^2(a-1)=(x-1)^2(a+1)$$

$$\Rightarrow \left(x^2+2x+1\right)(a-1)=\left(x^2-2x+1\right)(a+1)$$

$$\Rightarrow a\left(x^2+2x+1\right)-\left(x^2+2x+1\right)=a\left(x^2-2x+1\right)+\left(x^2-2x+1\right)$$

$$\Rightarrow 4ax=2x^2+2$$

$$\Rightarrow 2ax=x^2+1$$

$$\Rightarrow x^2-2ax+1=0$$