EXERCISE. 9 (A)

Question 1:

State, whether the following statements are true or false. If false, give a reason.

(i) If A and B are two matrices of orders 3 × 2 and 2 × 3 respectively; then their sum A + B is possible.

(ii) The matrices $A_{2 \times 3}$ and $B_{2 \times 3}$ are conformable for subtraction.

(iii) Transpose of a 2 × 1 matrix is a 2 × 1 matrix

(iv) Transpose of a square matrix is a square matrix

(v) A column matrix has many columns and only one row.

Solution 1:

(i) False

The sum A + B is possible when the order of both the matrices A and B are same.

(ii) True

(iii) False

Transpose of a 2 × 1 matrix is a 1 × 2 matrix.

(iv) True

(v) False

A column matrix has only one column and many rows.

Question 2:

Given : $\begin{bmatrix} x & y+2 \\ 3 & z-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$; find x, y and z.

Solution 2:

If two matrices are equal, then their corresponding elements are also equal. Therefore, we have:

$x = 3$,

$y + 2 = 1 \Rightarrow y = -1$

$z - 1 = 2 \Rightarrow z = 3$

Question 3:

Solve for a, b and c; if:

(i) $\begin{bmatrix} -4 & a+5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} b+4 & 2 \\ 3 & c-1 \end{bmatrix}$

(ii) $\begin{bmatrix} a & a-b \\ b+c & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$

Solution 3:

If two matrices are equal, then their corresponding elements are also equal.

(i)

$a + 5 = 2 \Rightarrow a = -3$

$-4 = b + 4 \Rightarrow b = -8$

$2 = c - 1 \Rightarrow c = 3$

(ii) $a = 3$

$a - b = -1$

$\Rightarrow b = a + 1 = 4$

$b + c = 2$

$\Rightarrow c = 2 - b = 2 - 4 = -2$

Question 4:

If $A = \begin{bmatrix} 8 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \end{bmatrix}$; find:

(i) A + B (ii) B – A

Solution 4:

(i) $A + B = \begin{bmatrix} 8 & -3 \end{bmatrix} + \begin{bmatrix} 4 & -5 \end{bmatrix} = \begin{bmatrix} 8+4 & -3-5 \end{bmatrix} = \begin{bmatrix} 12 & -8 \end{bmatrix}$

(ii) $B - A = \begin{bmatrix} 4 & -5 \end{bmatrix} - \begin{bmatrix} 8 & -3 \end{bmatrix}$

$= \begin{bmatrix} 4-8 & -5+3 \end{bmatrix}$

$= \begin{bmatrix} -4 & -2 \end{bmatrix}$

Question 5:

If $A = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $C = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$, find:

(i) B + C (ii) A – C

(iii) A + B – C (iv) A – B +C

Solution 5:

(i) $B + C = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 1+6 \\ 4-2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$

(ii) $A - C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 2-6 \\ 5+2 \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \end{bmatrix}$

(iii) $A + B - C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

$= \begin{bmatrix} 2+1-6 \\ 5+4+2 \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \end{bmatrix}$

(iv) $A - B + C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

$= \begin{bmatrix} 2-1+6 \\ 5-4-2 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$

Question 6:

Wherever possible, write each of the following as a single matrix.

(i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 1 & -7 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 3 \\ 6 & -1 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 0 & 1 & 2 \\ 4 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$

Solution 6:

(i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 1 & -7 \end{bmatrix} = \begin{bmatrix} 1-1 & 2-2 \\ 3+1 & 4-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 4 & -3 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 3 \\ 6 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2-0 & 3-2 & 4-3 \\ 5-6 & 6+1 & 7-0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 7 & 7 \end{bmatrix}$

(iii) Addition is not possible, because both matrices are not of same order.

Question 7:

Find, x and y from the following equations:

(i) $\begin{bmatrix} 5 & 2 \\ -1 & y-1 \end{bmatrix} - \begin{bmatrix} 1 & x-1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} -8 & x \end{bmatrix} + \begin{bmatrix} y & -2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \end{bmatrix}$

Solution 7:

(i) $\begin{bmatrix} 5 & 2 \\ -1 & y-1 \end{bmatrix} - \begin{bmatrix} 1 & x-1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5-1 & 2-x-1 \\ -1-2 & y-1+3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4 & 3-x \\ -3 & y+2 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$

Equating the corresponding elements, we get,

$3 - x = 7$ and $y + 2 = 2$

Thus, we get, $x = -4$ and $y = 0$.

(ii)

$\begin{bmatrix} -8 & x \end{bmatrix} + \begin{bmatrix} y & -2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -8+y & x-2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \end{bmatrix}$

Equating the corresponding elements, we get,

$-8 + y = -3$ and $x - 2 = 2$

Thus, we get, $x = 4$ and $y = 5$.

Question 8:

Given: $M = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$, find its transpose matrix M^t. If possible, find:

(i) $M + M^t$ (ii) $M^t - M$

Solution 8:

$M = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$

$M^t = \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix}$

(i) $M + M^t = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 5+5 & -3-2 \\ -2-3 & 4+4 \end{bmatrix} = \begin{bmatrix} 10 & -5 \\ -5 & 8 \end{bmatrix}$

(i) $M^t - M = \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 5 \ -5 & -2 \ +3 \\ -3 \ +2 & 4 \ -4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Question 9:

Write the additive inverse of matrices A, B and C:

Where $A = \begin{bmatrix} 6 & -5 \end{bmatrix}$; $B = \begin{bmatrix} -2 & 0 \\ 4 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -7 \\ 4 \end{bmatrix}$

Solution 9:

We know additive inverse of a matrix is its negative.

Additive inverse of A = $-A = -\begin{bmatrix} 6 & -5 \end{bmatrix} = \begin{bmatrix} -6 & 5 \end{bmatrix}$

Additive inverse of B = $-B = -\begin{bmatrix} -2 & 0 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -4 & 1 \end{bmatrix}$

Additive inverse of C = $-C = -\begin{bmatrix} -7 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$

Question 10:

Given $A = \begin{bmatrix} 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 4 \end{bmatrix}$; Find the matrix X in each of the following:

(i) X + B = C – A

(ii) A – X = B + C

Solution 10:

(i) X + B = C – A

$$X + \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -3 \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 4 + & 3 \end{bmatrix} = \begin{bmatrix} -3 & 7 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} -3-0 & 7-2 \end{bmatrix} = \begin{bmatrix} -3 & 5 \end{bmatrix}$$

(ii) A – X = B + C

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - X = \begin{bmatrix} 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - X = \begin{bmatrix} 0-1 & 2+4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - X = \begin{bmatrix} -1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 6 \end{bmatrix} = X$$

$$X = \begin{bmatrix} 2+1 & -3-6 \end{bmatrix} = \begin{bmatrix} 3 & -9 \end{bmatrix}$$

Question 11:

Given $A = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix}$; find the matrix X in each of the following:

(i) A + X = B (ii) A – X = B (iii) X – B = A

Solution 11:

(i) A + X = B

X = B – A

$$X = \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3+1 & -3-0 \\ -2-2 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -4 & 4 \end{bmatrix}$$

(ii) A – X = B

X = A – B

$$X = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -1-3 & 0+3 \\ 2+2 & -4-0 \end{bmatrix} = \begin{bmatrix} -4 & -3 \\ 4 & -4 \end{bmatrix}$$

(iii) X – B = A

X = A + B

$$X = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -1+3 & 0-3 \\ 2-2 & -4+0 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 0 & -4 \end{bmatrix}$$

EXERCISE. 9 (B)

Question 1:

Evaluate:

(i) $3\begin{bmatrix} 5 & -2 \end{bmatrix}$

(ii) $7\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

(iii) $2\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix}$

(iv) $6\begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} -8 \\ 1 \end{bmatrix}$

Solution 1:

(i) $3\begin{bmatrix} 5 & -2 \end{bmatrix} = \begin{bmatrix} 15 & -6 \end{bmatrix}$

(ii) $7\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 14 \\ 0 & 7 \end{bmatrix}$

(iii) $2\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 4 & -6 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} -2+3 & 0+3 \\ 4+5 & -6+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix}$

(iv) $6\begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 18 \\ -12 \end{bmatrix} - \begin{bmatrix} -16 \\ 2 \end{bmatrix} = \begin{bmatrix} 18+16 \\ -12-2 \end{bmatrix} = \begin{bmatrix} 34 \\ -14 \end{bmatrix}$

Question 2:

Find x and y if:

(i) $3\begin{bmatrix} 4 & x \end{bmatrix} + 2\begin{bmatrix} y & -3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$

(ii) $x\begin{bmatrix} -1 \\ 2 \end{bmatrix} - 4\begin{bmatrix} -2 \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$

Solution 2:

(i) $3\begin{bmatrix} 4 & x \end{bmatrix} + 2\begin{bmatrix} y & -3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$

$\begin{bmatrix} 12 & 3x \end{bmatrix} + \begin{bmatrix} 2y & -6 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$

$\begin{bmatrix} 12+2y & 3x-6 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$

Comparing the corresponding elements, we get,

$12 + 2y = 10$ and $3x - 6 = 0$

Simplifying, we get, $y = -1$ and $x = 2$.

(ii) $x\begin{bmatrix} -1 \\ 2 \end{bmatrix} - 4\begin{bmatrix} -2 \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$

$\begin{bmatrix} -x \\ 2x \end{bmatrix} - \begin{bmatrix} -8 \\ 4y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$

$\begin{bmatrix} -x+8 \\ 2x-4y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$

Comparing corresponding the elements, we get,

$-x + 8 = 7$ and $2x - 4y = -8$

Simplifying, we get,

$x = 1$ and $y = \frac{5}{2} = 2.5$

Question 3:

Given $A = 2\begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix} + \begin{vmatrix} -3 & -1 \\ 0 & 0 \end{vmatrix}$; find

(i) 2A – 3B + C (ii) A + 2C – B

Solution 3:

(i) 2A − 3B + C

$$= 2\begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix} + \begin{vmatrix} -3 & -1 \\ 0 & 0 \end{vmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 3 \\ 15 & 6 \end{bmatrix} + \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3-3 & 2-3-1 \\ 6-15+0 & 0-6+0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ -9 & -6 \end{bmatrix}$$

(ii) A + 2C – B

$$= \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} + 2\begin{vmatrix} -3 & -1 \\ 0 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} + \begin{bmatrix} -6 & -2 \\ 0 & 0 \end{bmatrix} - \begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 2-6-1 & 1-2-1 \\ 3+0-5 & 0+0-2 \end{vmatrix}$$

$$= \begin{bmatrix} -5 & -2 \\ -2 & -2 \end{bmatrix}$$

Question 4:

If $\begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + 3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}$; find A.

Solution 4:

$$\begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + 3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}$$

$$3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix}$$

$$3A = \begin{bmatrix} -2-4 & -2+2 \\ 1-4 & -3-0 \end{bmatrix}$$

$$3A = \begin{bmatrix} -6 & 0 \\ -3 & -3 \end{bmatrix}$$

$$A = \frac{1}{3}\begin{bmatrix} -6 & 0 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix}$$

Question 5:

Given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$

(i) find the matrix 2A+B

(ii) find a matrix C such that:

$$C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Solution 5:

$$\text{(i) } 2\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 2-4 & 8-1 \\ 4-3 & 6-2 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 1 & 4 \end{bmatrix}$$

$$\text{(ii) } C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 0+4 & 0+1 \\ 0+3 & 0+2 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

Question 6:

If $2\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 3 \\ y & 2 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$; find the values of x, y and z.

Solution 6:

$$2\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 3 \\ y & 2 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2x \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 9 \\ 3y & 6 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 2x+9 \\ 3y & 8 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$2x + 9 = -7 \Rightarrow 2x = -16 \Rightarrow x = -8$

$3y = 15 \Rightarrow y = 5$

$z = 9$

Question 7:

Given $A = \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$ and A^t is its transpose matrix Find:

(i) $2A + 3A^t$

(ii) $2A^t - 3A$

(iii) $\frac{1}{2}A - \frac{1}{3}A^t$

(iv) $A^t - \frac{1}{3}A$

Solution 7:

$$A = \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$$

$$A^t = \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix}$$

(i) $2A + 3A^t$

$$= 2\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix} + 3\begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 12 \\ 0 & -18 \end{bmatrix} + \begin{bmatrix} -9 & 0 \\ 18 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 12 \\ 18 & -45 \end{bmatrix}$$

(ii) $2A^t - 3A$

$$= 2\begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix} - 3\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 0 \\ 12 & -18 \end{bmatrix} - \begin{bmatrix} -9 & 18 \\ 0 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -18 \\ 12 & 9 \end{bmatrix}$$

(iii) $\frac{1}{2}A - \frac{1}{3}A^t$

$$= \frac{1}{2}\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-3}{2} & 3 \\ 0 & \frac{-9}{2} \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{2} & 3 \\ -2 & \frac{-3}{2} \end{bmatrix}$$

(iv) $A^t - \frac{1}{3}A$

$$= \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ 6 & -6 \end{bmatrix}$$

Question 8:

Given $A = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

(i) $X + 2A = B$

(ii) $3X + B + 2A = 0$

(iii) $3A - 2X = X - 2B$

Solution 8:

(i) $X + 2A = B$

$X = B - 2A$

$$X=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}-2\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}$$

$$X=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}-\begin{bmatrix}2 & 2\\ -4 & 0\end{bmatrix}$$

$$X=\begin{bmatrix}0 & -3\\ 5 & 1\end{bmatrix}$$

(ii) $3X + B + 2A = O$

$3X = -2A - B$

$$3x=-2\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}-\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$$

$$3x=\begin{bmatrix}-2 & -2\\ 4 & 0\end{bmatrix}-\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$$

$$3x=\begin{bmatrix}-4 & -1\\ 3 & -1\end{bmatrix}$$

$$x=\begin{bmatrix}\frac{-4}{3} & \frac{-1}{3}\\ 1 & \frac{-1}{3}\end{bmatrix}$$

(iii) $3A - 2X = X - 2B$

$3A + 2B = X + 2X$

$3X = 3A + 2B$

$$3x=3\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}+2\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$$

$$3x=\begin{bmatrix}3 & 3\\ -6 & 0\end{bmatrix}+\begin{bmatrix}4 & -2\\ 2 & 2\end{bmatrix}$$

$$3x=\begin{bmatrix}7 & 1\\ -4 & 2\end{bmatrix}$$

$$x=\begin{bmatrix}\frac{7}{3} & \frac{1}{3}\\ \frac{-4}{3} & \frac{2}{3}\end{bmatrix}$$

Question 9:

If $M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, show that;

$3M + 5N = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Solution 9:

$3M + 5N$

$= 3\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Question 10:

If I is the unit matrix of order 2 × 2; find the matrix M, such that;

(i) $M - 2I = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix}$

(ii) $5M + 3I = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix}$

Solution 10:

(i) $M - 2I = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix}$

$M = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} + 2I$

$M = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$M = \begin{bmatrix} -3 & 0 \\ 12 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$M = \begin{bmatrix} -1 & 0 \\ 12 & 5 \end{bmatrix}$

(ii) $5M + 3I = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix}$

$5M = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix} - 3I$

$5M = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$5M = \begin{bmatrix} 8 & -20 \\ 0 & -12 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$5M = \begin{bmatrix} 5 & -20 \\ 0 & -15 \end{bmatrix}$

$M = \frac{1}{5}\begin{bmatrix} 5 & -20 \\ 0 & -15 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 0 & -3 \end{bmatrix}$

Question 11:

If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, Find the matrix M.

Solution 11:

$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$

$\Rightarrow 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$

$\Rightarrow M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$

EXERCISE. 9 (C)

Question 1:

Evaluate; if possible

(i) $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

(iv) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 \end{bmatrix}$

Solution 1:

(i) $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6+0 \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -2+2 & 3-8 \end{bmatrix} = \begin{bmatrix} 0 & -5 \end{bmatrix}$

(iii) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -6+12 \\ -3-3 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$

(iv) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 \end{bmatrix}$

The number of columns in the first matrix is not equal to the number of rows in the second matrix. Thus, the product is not possible.

Question 2:

If $A = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$ and I is a unit matrix of order 2×2, find;

(i) AB (ii) BA (iii) AI (iv) IB (v) A^2 (vi) B^2

Solution 2:

(i) $AB = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$

$$=\begin{bmatrix} 0+6 & 0+4 \\ 5-6 & -5-4 \end{bmatrix}$$

$$=\begin{bmatrix} 6 & 4 \\ -1 & -9 \end{bmatrix}$$

$$\text{(ii)}BA=\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} 0-5 & 2+2 \\ 0+10 & 6-4 \end{bmatrix}$$

$$=\begin{bmatrix} -5 & 4 \\ 10 & 2 \end{bmatrix}$$

$$\text{(iii) } AI=\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 0+0 & 0+2 \\ 5-0 & 0-2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}=A$$

$$\text{(iv) } IB=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 3 & 21 \end{bmatrix}$$

$$=\begin{bmatrix} 1+0 & -1+0 \\ 0+3 & 0+2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}=B$$

$$\text{(v)}A^2=\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} 0+10 & 0-4 \\ 0-10 & 10+4 \end{bmatrix}$$

$$=\begin{bmatrix} 10 & -4 \\ -10 & 14 \end{bmatrix}$$

$$\text{(vi)}B^2=\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 1-3 & -1-2 \\ 3+6 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix}$$

$$B^2A = \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0-15 & -4+6 \\ 0+5 & 18-2 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 2 \\ 5 & 16 \end{bmatrix}$$

Question 3:

If $M = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$; find M^2, M^3 and M^5.

Solution 3:

$$M = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4+1 & 2-2 \\ 2-2 & 1+4 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$M^3 = MM^2 = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 10+0 & 0+5 \\ 5-0 & 0-10 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 5 & -10 \end{bmatrix}$$

$$M^5 = M^2.M^3 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 10 & 5 \\ 5 & -10 \end{bmatrix} = \begin{bmatrix} 50+0 & 25+0 \\ 0+25 & 0-50 \end{bmatrix} = \begin{bmatrix} 50 & 25 \\ 25 & -50 \end{bmatrix}$$

Question 4:

Find X and y, if:

(i) $\begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$

(ii) $\begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

Solution 4:

(i) $\begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$

$$\begin{bmatrix} 20+3x \\ 5x-2 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$5x - 2 = 8 \Rightarrow x = 2$

$20 + 3x = y \Rightarrow y = 20 + 6 = 26$

(ii) $\begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

$$\begin{bmatrix} x+0 & x+0 \\ -3+0 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x & x \\ -3 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$x = 2$

$-3 + y = -2 \Rightarrow y = 1$

Question 5:

If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, find; (i) (AB) C (ii)) A(BC) Is A(BC) = (AB)C?

Solution 5:

(i) $AB = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1+12 & 2+9 \\ 2+16 & 4+12 \end{bmatrix} = \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix}$

$(AB)C = \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 52+11 & 39+22 \\ 72+16 & 54+32 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$

(ii) $BC = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+2 & 3+4 \\ 16+3 & 12+6 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix}$

$A(BC) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix} = \begin{bmatrix} 6+57 & 7+54 \\ 12+76 & 14+72 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$

Question 6:

Given $A = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}$, find if possible:

(i) AB (ii) BA (iii) A^2

Solution 6:

(i) $AB = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}$

$= \begin{bmatrix} 0-4-30 & 0+8-36 \\ 0-0+5 & 3+0+6 \end{bmatrix}$

$= \begin{bmatrix} -34 & -28 \\ 5 & 9 \end{bmatrix}$

(ii) $BA = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix} \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 0+3 & 0+0 & 0-1 \\ 0+6 & -4+0 & -6-2 \\ 0-18 & -20-0 & -30+6 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 & -1 \\ 6 & -4 & -8 \\ -18 & -20 & -24 \end{bmatrix}$

(iii) Product AA ($=A^2$) is not possible as the number of columns of matrix A is not equal to its number of rows.

Question 7:

If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$;

(i) Write down the product matrix AB.

(ii) would it be possible to form the product matrix BA? If so, compute BA; if not give a reason why it is not possible.

Solution 7:

(i) $AB = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2-6+1 & 1-4+1 \\ 4+3+3 & 2+2+3 \end{bmatrix}$

$= \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$

(ii) Product BA is possible

$BA = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2+2 & -4+1 & 2+3 \\ 3+4 & -6+2 & 3+6 \\ 1+2 & -2+1 & 1+3 \end{bmatrix}$

$= \begin{bmatrix} 4 & -3 & 5 \\ 7 & -4 & 9 \\ 3 & -1 & 4 \end{bmatrix}$

Question 8:

If $M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and I is a unit matrix of the same order as that of M; show that:

$M^2 = 2M + 3I$

Solution 8:

$M^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

$2M + 3I = 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

Hence, $M^2 = 2M + 3I$.

Question 9:

If $A = \begin{bmatrix} a & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -b \\ 1 & 0 \end{bmatrix}$, $M = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $BA = M^2$, find the values of a and b.

Solution 9:

$$BA = \begin{bmatrix} 0 & -b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0-2b \\ a+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2b \\ a & 0 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & -1-1 \\ 1+1 & -1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

Given, $BA = M^2$

$$\begin{bmatrix} 0 & -2b \\ a & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a = 2$

$-2b = -2 \Longrightarrow b = 1$

Question 10:

Given $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, find:

(i) $A - B$ (ii) A^2 (iii) AB (iv) A^2-AB+2B

Solution 10:

(i) $A - B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

(ii) $A^2 = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 16+2 & 4+3 \\ 8+6 & 2+9 \end{bmatrix}$

$= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix}$

(iii) $AB = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4-2 & 0+1 \\ 2-6 & 0+3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$

(iv) $A^2 - AB + 2B$

$= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 16 & 6 \\ 18 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 18 & 6 \\ 14 & 10 \end{bmatrix}$

Question 11:

If $A = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$; find:

(i) $(A + B)^2$ (ii) $A^2 + B^2$ (iii) Is $(A + B)^2 = A^2 + B^2$?

Solution 11:

(i) $A+B=\begin{bmatrix}1 & 4\\ 1 & -3\end{bmatrix}+\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}=\begin{bmatrix}2 & 6\\ 0 & -4\end{bmatrix}$

$(A+B)^2=\begin{bmatrix}2 & 6\\ 0 & -4\end{bmatrix}\begin{bmatrix}2 & 6\\ 0 & -4\end{bmatrix}$

$=\begin{bmatrix}4+0 & 12-24\\ 0+0 & 0+16\end{bmatrix}$

$=\begin{bmatrix}4 & -12\\ 0 & 16\end{bmatrix}$

(ii) $A^2=\begin{bmatrix}1 & 4\\ 1 & -3\end{bmatrix}\begin{bmatrix}1 & 4\\ 1 & -3\end{bmatrix}$

$=\begin{bmatrix}1+4 & 4-12\\ 1-3 & 4+9\end{bmatrix}$

$\begin{bmatrix}5 & -8\\ -2 & 13\end{bmatrix}$

$B^2=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}$

$=\begin{bmatrix}1-2 & 2-2\\ -1+1 & -2+1\end{bmatrix}$

$=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

$A^2+B^2=\begin{bmatrix}5 & -8\\ -2 & 13\end{bmatrix}+\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

$=\begin{bmatrix}4 & -8\\ -2 & 12\end{bmatrix}$

(iii) Clearly, $(A+B)^2 \neq A^2+B^2$

Question 12:

Find the matrix A, If $B=\begin{bmatrix}2 & 1\\ 0 & 1\end{bmatrix}$ and $B^2=B+\frac{1}{2}A$

Solution 12:

$B^2=B+\frac{1}{2}A$

$$\frac{1}{2}A = B^2 - B$$

$$A = 2\left(B^2 - B\right)$$

$$B^2 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$B^2 - B = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$$

$$\therefore A = 2\left(B^2 - B\right)$$

$$= 2\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}$$

Question 13:

If $A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$ and $A^2 = I$; find a and b.

Solution 13:

$$A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}\begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$$

$$= \begin{bmatrix} 1+a & -1+b \\ -a+ab & a+b^2 \end{bmatrix}$$

It is given that $A^2 = I$.

$$\therefore \ = \begin{bmatrix} 1+a & -1+b \\ -a+ab & a+b^2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements, we get,
$1 + a = 1$
Therefore, $a = 0$
$-1 + b = 0$
Therefore, $b = 1$

Question 14:

If $A = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$; then show that:

(i) A(B + C) = AB + AC
(ii) (B – A)C = BC – AC

Solution 14:

(i) $B + C = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 4 & 3 \end{bmatrix}$

$A(B + C) = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 6+4 & 14+3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 17 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4+4 & 6+1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 0 & 0 \end{bmatrix}$

$AC = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix}$

$AB + AC = \begin{bmatrix} 8 & 7 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 17 \\ 0 & 0 \end{bmatrix}$

Hence, $A(B + C) = AB + AC$

(ii)$B - A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 4 & 1 \end{bmatrix}$

$(B - A)C = \begin{bmatrix} 0 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0+4 \\ 4+0 & 16+2 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 18 \end{bmatrix}$

$BC = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+6 \\ 4+0 & 16+2 \end{bmatrix} = \begin{bmatrix} 2 & 14 \\ 4 & 18 \end{bmatrix}$

$$AC=\begin{bmatrix}2&1\\0&0\end{bmatrix}\begin{bmatrix}1&4\\0&2\end{bmatrix}=\begin{bmatrix}2+0&8+2\\0&0\end{bmatrix}=\begin{bmatrix}2&10\\0&0\end{bmatrix}$$

$$BC-AC=\begin{bmatrix}2&14\\4&18\end{bmatrix}-\begin{bmatrix}2&10\\0&0\end{bmatrix}=\begin{bmatrix}0&4\\4&18\end{bmatrix}$$

Hence, $(B-A)C=BC-AC$

Question 15:

If $A=\begin{bmatrix}1&4\\2&1\end{bmatrix}$, $B=\begin{bmatrix}-3&2\\4&0\end{bmatrix}$, $C=\begin{bmatrix}1&0\\0&2\end{bmatrix}$, simplify: A^2+BC

Solution 15:

$$A^2=\begin{bmatrix}1&4\\2&1\end{bmatrix}\begin{bmatrix}1&4\\2&1\end{bmatrix}=\begin{bmatrix}1+8&4+4\\2+2&8+1\end{bmatrix}=\begin{bmatrix}9&8\\4&9\end{bmatrix}$$

$$BC=\begin{bmatrix}-3&2\\4&0\end{bmatrix}\begin{bmatrix}1&0\\0&2\end{bmatrix}=\begin{bmatrix}-3+0&0+4\\4+0&0+0\end{bmatrix}=\begin{bmatrix}-3&4\\4&0\end{bmatrix}$$

$$A^2+BC=\begin{bmatrix}9&8\\4&9\end{bmatrix}+\begin{bmatrix}-3&4\\4&0\end{bmatrix}=\begin{bmatrix}6&12\\8&9\end{bmatrix}$$

Question 16:

Solve for x and y:

(i) $\begin{bmatrix}2&5\\5&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-7\\14\end{bmatrix}$

(ii) $\begin{bmatrix}3&-1\\2&-1\end{bmatrix}\begin{bmatrix}-2\\4\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$

(iii) $\begin{bmatrix}x+y&x-4\end{bmatrix}\begin{bmatrix}-1&-2\\2&2\end{bmatrix}=\begin{bmatrix}-7&-11\end{bmatrix}$

Solution 16:

(i) $\begin{bmatrix}2&5\\5&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-7\\14\end{bmatrix}$

$$\begin{bmatrix}2x+5y\\5x+2y\end{bmatrix}=\begin{bmatrix}-7\\14\end{bmatrix}$$

Comparing the corresponding elements, we get,
$2x + 5y = -7$ ...(1)
$5x + 2y = 14$ ...(2)
Multiplying (1) with 2 and (2) with 5, we get,
$4x + 10y = -14$ ...(3)
$25x + 10y = 70$ ...(4)
Subtracting (3) from (4), we get,
$21x = 84 \Rightarrow x = 4$
From (2), $2y = 14 - 5x = 14 - 20 = -6 \Rightarrow y = -3$

(ii) $\begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} -6 & -4 \\ -4 & -4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} -10 \\ -8 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

Comparing the corresponding elements, we get,
$x = -10$ and $y = -8$

(iii) $\begin{bmatrix} x+y & x-4 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$

$\begin{bmatrix} -x-y+2x-8 & -2x-2y+2x-8 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$

$\begin{bmatrix} -y+x-8 & -2y-8 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$

Comparing the corresponding elements, we get,
$-2y - 8 = -11 \Rightarrow -2y = -3 \Rightarrow y = \frac{3}{2}$
$-y + x - 8 = -7$

$\Rightarrow -\frac{3}{2} + x - 8 = -7$

$\Rightarrow x = 1 + \frac{3}{2} = \frac{5}{2}$

Question 17:

In each case given below, find:
(a) the order of matrix M.
(b) the matrix M.

(i) $M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times M = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$

Solution 17:

We know, the product of two matrices is defined only when the number of columns of first matrix is equal to the number of rows of the second matrix.

(i) Let the order of matrix M be a × b.

$$M_{a\times b} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 2 \end{bmatrix}_{1\times 2}$$

Clearly, the order of matrix M is 1 × 2.

$$\text{Let } M = \begin{bmatrix} a & b \end{bmatrix}$$

$$M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a+0 & a+2b \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a = 1$ and $a + 2b = 2 \Longrightarrow 2b = 2 - 1 = 1 \Longrightarrow b = \frac{1}{2}$

$$\therefore M = \begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix}$$

(ii) Let the order of matrix M be a x b.

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}_{2\times 2} \times M_{a\times b} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}_{2\times 1}$$

Clearly, the order of matrix M is 2 x 1.

$$\text{Let } M = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times M = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} a+4b \\ 2a+b \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a + 4b = 13$ ....(1)

$2a + b = 5$ ....(2)

Multiplying (2) by 4, we get,
$8a + 4b = 20$ ....(3)
Subtracting (1) from (3), we get,
$7a = 7 \Longrightarrow a = 1$
From (2), we get,
$b = 5 - 2a = 5 - 2 = 3$

$$\therefore M = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Question 18:

If $A = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$; find the value of x, given that: $A^2 = B$

Solution 18:

$$A^2 = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 2x+x \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix}$$

Given, $A^2 = B$

$$\begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$$

Comparing the two matrices, we get,
$3x = 36 \Rightarrow x = 12$

Question 19:

Find the positive integers p and q such that:

$$\begin{bmatrix} p & q \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$$

Solution 19:

$$\begin{bmatrix} p & q \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$$

$$\begin{bmatrix} p^2 + q^2 \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$$

$$\therefore p^2 + q^2 = 25$$

Since, p and q are positive integers, and $(3)^2 + (4)^2 = 9 + 16 = 25$.
Hence, p = 3 and q = 4 or p = 4 and q = 3

Question 20:

If A and B are any two 2 × 2 matrices such that AB = BA = B and B is not a zero matrix, what can you say about the matrix A?

Solution 20:

AB = BA = B

We know that AI = IA = I, where I is the identity matrix.

Hence, B is the identity matrix.

Question 21:

Given $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ and that AB = A + B; find the values of a, b and c.

Solution 21:

$$AB = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3a+0 & 3b+0 \\ 0+0 & 0+4c \end{bmatrix} = \begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3+a & b \\ 0 & 4+c \end{bmatrix}$$

Given, AB = A+B

$$\therefore \begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix} = \begin{bmatrix} 3+a & b \\ 0 & 4+c \end{bmatrix}$$

Comparing the corresponding elements, we get,

$3a = 3 + a$

$\Longrightarrow 2a = 3$

$\Longrightarrow a = \frac{3}{2}$

$3b = b \Longrightarrow b = 0$

$4c = 4 + c \Longrightarrow 3c = 4 \Longrightarrow c = \frac{4}{3}$

Question 22:

If $P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, then compute:

(i) $P^2 - Q^2$ (ii) $(P + Q)(P - Q)$ Is $(P + Q)(P - Q) = P^2 - Q^2$ true for matrix algebra?

Solution 22:

$$\text{(i) } P^2 = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1+4 & 2-2 \\ 2-2 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$Q^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ 2+2 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$P^2 - Q^2 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -4 & 4 \end{bmatrix}$$

$$P + Q = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 0 \end{bmatrix}$$

$$P - Q = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & -2 \end{bmatrix}$$

$$(P+Q)(P-Q) = \begin{bmatrix} 2 & 2 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0+0 & 4-4 \\ 0+0 & 8-0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 8 \end{bmatrix}$$

Clearly, it can be said that:

$(P + Q)(P - Q) = P^2 - Q^2$ not true for matrix algebra.

Question 23:

Given the matrices:

$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$. Find:

(i) ABC (ii) ACB state whether ABC = ACB.

Solution 23:

$$\text{(i) } AB = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 6-1 & 8-2 \\ 12-2 & 16-4 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix}$$

$$ABC = \begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -15+0 & 5-12 \\ -30+0 & 10-24 \end{bmatrix} = \begin{bmatrix} -15 & -7 \\ -30 & -14 \end{bmatrix}$$

$$\text{(ii) } AC = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -6+0 & 2-2 \\ -12+0 & 4-4 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix}$$

$$ACB = \begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -18-0 & -24-0 \\ -36-0 & -48-0 \end{bmatrix} = \begin{bmatrix} -18 & -24 \\ -36 & -48 \end{bmatrix}$$

Hence, ABC = ACB.

Question 24:

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}$; find each of the following and state if they are equal:

(i) CA + B (ii) A + CB

Solution 24:

$$\text{(i) } CA = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2-9 & -4-12 \\ 0+3 & 0+4 \end{bmatrix} = \begin{bmatrix} -11 & -16 \\ 3 & 4 \end{bmatrix}$$

$$CA + B = \begin{bmatrix} -11 & -16 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -15 \\ 4 & 5 \end{bmatrix}$$

$$\text{(ii) } CB = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -12-3 & -2-3 \\ 0+1 & 0+1 \end{bmatrix} = \begin{bmatrix} -15 & -5 \\ 1 & 1 \end{bmatrix}$$

$$A + CB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -14 & -3 \\ 4 & 5 \end{bmatrix}$$

Thus, $CA + B \neq A + CB$

Question 25:

If $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$; find the matrix X such that AX = B.

Solution 25:

Let the order of the matrix X be $a \times b$

AX=B

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}_{2\times 2} \times X_{a\times b} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}_{2\times 1}$$

Clearly, the order of matrix X is 2 x 1.

$$\text{Let } X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$$

$$\begin{bmatrix} 2x+y \\ x+3y \end{bmatrix} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$$

Comparing the two matrices, we get,

$2x + y = 3$ … (1)

x + 3y = − 11 … (2)
Multiplying (1) with 3, we get,
6x + 3y = 9 … (3)
Subtracting (2) from (3), we get,
5x = 20
x = 4
From (1), we have:
y = 3 − 2x = 3 − 8 = −5

$$\therefore X = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

Question 26:

If $A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$, find (A – 2I) (A – 3I)

Solution 26:

$$A - 2I = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}$$

$$A - 3I = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$$(A - 2I)(A - 3I) = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4-4 \\ 1-1 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

Question 27:

If $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$, Find:

(i) $A^t \cdot A$ (ii) $A \cdot A^t$

Where A^t is the transpose of matrix A.

Solution 27:

$$A=\begin{bmatrix}2 & 1 & -1\\ 0 & 1 & -2\end{bmatrix}$$

$$A^t=\begin{bmatrix}2 & 0\\ 1 & 1\\ -1 & -2\end{bmatrix}$$

$$\text{(i)}A^t.A=\begin{bmatrix}2 & 0\\ 1 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}2 & 1 & -1\\ 0 & 1 & -2\end{bmatrix}$$

$$=\begin{bmatrix}4+0 & 2+0 & -2-0\\ 2+0 & 1+1 & -1-2\\ -2-0 & -1-2 & 1+4\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & -2\\ 2 & 2 & -3\\ -2 & -3 & 5\end{bmatrix}$$

$$\text{(ii)}\,A.A^t=\begin{bmatrix}2 & 1 & -1\\ 0 & 1 & -2\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & 1\\ -1 & -2\end{bmatrix}$$

$$=\begin{bmatrix}4+1+1 & 0+1+2\\ 0+1+2 & 0+1+4\end{bmatrix}$$

$$=\begin{bmatrix}6 & 3\\ 3 & 5\end{bmatrix}$$

Question 28:

If $M=\begin{bmatrix}4 & 1\\ -1 & 2\end{bmatrix}$, show that : $6M - M^2 = 9I$; where I is a 2×2 unit matrix.

Solution 28:

$$M^2=\begin{bmatrix}4 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}4 & 1\\ -1 & 2\end{bmatrix}=\begin{bmatrix}16-1 & 4+2\\ -4-2 & -1+4\end{bmatrix}=\begin{bmatrix}15 & 6\\ -6 & 3\end{bmatrix}$$

$$6M-M^2=6\begin{bmatrix}4 & 1\\ -1 & 2\end{bmatrix}-\begin{bmatrix}15 & 6\\ -6 & 3\end{bmatrix}$$

$$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9I$$

Hence Proved.

Question 29:

If $P = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$ and $Q = \begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix}$; find x and y such that PQ = null matrix.

Solution 29:

$$PQ = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}\begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix} = \begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix}$$

PQ = Null matrix

$$\therefore \begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$2x + 12 = 0$

Therefore $x = -6$

$6 + 6y = 0$

Therefore $y = -1$

Question 30:

Evaluate:

$$\begin{bmatrix} 2\cos 60^\circ & -2\sin 30^\circ \\ -\tan 45^\circ & \cos 0^\circ \end{bmatrix}\begin{bmatrix} \cot 45^\circ & \operatorname{cosec} 30^\circ \\ \sec 60^\circ & \sin 90^\circ \end{bmatrix}$$

Solution 30:

$$\begin{bmatrix} 2\cos 60^\circ & -2\sin 30^\circ \\ -\tan 45^\circ & \cos 0^\circ \end{bmatrix}\begin{bmatrix} \cot 45^\circ & \operatorname{cosec} 30^\circ \\ \sec 60^\circ & \sin 90^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 2\times\frac{1}{2} & -2\times\frac{1}{2} \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 2-1 \\ -1+2 & -2+1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

Question 31:

State with reason whether the following are true or false. A, B and C are matrices of order 2 × 2.

(i) A + B = B + A

(ii) A – B = B – A

(iii) (B . C). A = B . (C . A)

(iv) (A + B) . C = A . C + B . C

(v) A. (B – C) = A . B – A . C

(vi) (A – B) . C = A . C – B . C

(vii) $A^2 - B^2 = (A + B)(A - B)$

(viii) $(A - B)^2 = A^2 - 2A . B + B^2$

Solution 31:

(i) True.

Addition of matrices is commutative.

(ii) False.

Subtraction of matrices is commutative.

(iii) True.

Multiplication of matrices is associative.

(iv) True.

Multiplication of matrices is distributive over addition.

(v) True.

Multiplication of matrices is distributive over subtraction.

(vi) True.

Multiplication of matrices is distributive over subtraction.

(vii) False.

Laws of algebra for factorization and expansion are not applicable to matrices.

(viii) False.

Laws of algebra for factorization and expansion are not applicable to matrices.

EXERCISE 9 (D)

Question 1:

Find x and y, if :-

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$$

Solution 1:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$$

$$\begin{bmatrix} 6x & -2 \\ -2x & +4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-10 \\ -2x+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

Comparing the corresponding elements, we get,

$6x - 10 = 8$

$\Longrightarrow 6x = 18$

$\Longrightarrow x = 3$

$-2x + 14 = 4y$

$\Longrightarrow 4y = -6 + 14 = 8$

$\Longrightarrow y = 2$

Question 2:

Find x and y, if:

$$\begin{bmatrix} 3x & 8 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3\begin{bmatrix} 2 & -7 \end{bmatrix} = 5\begin{bmatrix} 3 & 2y \end{bmatrix}$$

Solution 2:

$$\begin{bmatrix} 3x & 8 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3\begin{bmatrix} 2 & -7 \end{bmatrix} = 5\begin{bmatrix} 3 & 2y \end{bmatrix}$$

$$\begin{bmatrix} 3x+24 & 12x+56 \end{bmatrix} - \begin{bmatrix} 6 & -21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

$$\begin{bmatrix} 3x+24-6 & 12x+56+21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

$$\begin{bmatrix} 3x+18 & 12x+77 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

Comparing the corresponding elements, we get,

$3x + 18 = 15$

$3x = -3$
$x = -1$
$12x + 77 = 10y$
$10y = -12 + 77 = 65$
$y = 6.5$

Question 3:

If $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$ and $\begin{bmatrix} -x & y \end{bmatrix}\begin{bmatrix} 2x \\ y \end{bmatrix} = \begin{bmatrix} -2 \end{bmatrix}$; find x and y, if:

(i) x , y **ε** w (whole numbers)

(ii) x , y **ε** z (integers)

Solution 3:

$$\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$$

$x^2 + y^2 = 25$

and

$-2x^2 + y^2 = -2$

(i) x, y Î W (whole numbers)

It can be observed that the above two equations are satisfied when $x = 3$ and $y = 4$.

(ii) x, y Î Z (integers)

It can be observed that the above two equations are satisfied when $x = \pm 3$ and $y = \pm 4$.

Question 4:

Given $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}. X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ Write:

(i) the order of the matrix X

(ii) the matrix X.

Solution 4:

(i) let the order of matrix X be $a \times b$

$$\therefore \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}_{2\times 2} \times X_{a\times b} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}_{2\times 1}$$

$\Rightarrow a = 2$ and $b = 1$

$\therefore$ The order of the matrix $X = a \times b = 2 \times 1$

(ii) Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x+y \\ -3x+4y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$\Rightarrow$ $2x + y = 7$ and $-3x + 4y = 6$

On solving the above simultaneous equations

In x and y, we have, $x = 2$ and $y = 3$

$\therefore$ The matrix $X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Question 5:

Evaluate: $\begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2}\cos 0^\circ & \sin 0^\circ \end{bmatrix} \begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix}$

Solution 5:

$$\begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2}\cos 0^\circ & \sin 0^\circ \end{bmatrix} \begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix}$$

$$= \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \sqrt{2} & 0 \end{vmatrix} \begin{vmatrix} \frac{1}{\sqrt{2}} & 0 \\ 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{2}+\frac{1}{2} & 0+\frac{1}{2} \\ 1+0 & 0+0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0.5 \\ 1 & 0 \end{vmatrix}$$

Question 6:

If $A = \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$ and $3A \times M = 2B$; find matrix M.

Solution 6:

Let the order of matrix M be $a \times b$.

$3A \times M = 2B$

$$3\begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}_{2\times 2} \times M_{a\times b} = 2\begin{bmatrix} -5 \\ 6 \end{bmatrix}_{2\times 1}$$

Clearly, the order of matrix M is 2 × 1

let $M = \begin{bmatrix} x \\ y \end{bmatrix}$

then,

$$3\begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 \\ 12 & -9 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 0-3y \\ 12x-9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} -3y \\ 12x-9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$-3y = -10$

$\Longrightarrow y = \frac{10}{3}$

$\Rightarrow 12x - 9y = 12$

$\Rightarrow 12x - 30 = 12$

$\Rightarrow x = \frac{7}{2}$

$$\therefore\ M = \begin{bmatrix} \frac{7}{2} \\ \frac{10}{3} \end{bmatrix}$$

Question 7:

If $\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$, find the values of a, b and c.

Solution 7:

$$\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a+1 & 2+b \\ 7 & -1-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a + 1 = 5 \Rightarrow a = 4$

$2 + b = 0 \Rightarrow b = -2$

$-1 - c = 3 \Rightarrow c = -4$

Question 8:

If $a = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$; find :

(i) A(BA) (ii) (AB)B

Solution 8:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

(i)

$$BA = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$A(BA) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$$

(ii)

$$AB = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 1+4 \\ 4+1 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$(AB)B = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8+5 & 4+10 \\ 10+4 & 5+8 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

Question 9:

Find x and y, if : $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

Solution 9:

$$\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 2x+3x \\ 2y+4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$5x = 5 \Longrightarrow x = 1$

$6y = 12 \Longrightarrow y = 2$

Question 10:

If matrix $X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$ and $2X - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$; find the matrix 'X' and matrix 'Y'

Solution 10:

$$X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6-8 \\ 4+6 \end{bmatrix}$$

$$= \begin{bmatrix} -14 \\ 10 \end{bmatrix}$$

Given, $2X - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$

$2\begin{bmatrix} -14 \\ 10 \end{bmatrix} - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$

$3Y = 2\begin{bmatrix} -14 \\ 10 \end{bmatrix} - \begin{bmatrix} 10 \\ -8 \end{bmatrix}$

$3Y = \begin{bmatrix} -28 \\ 20 \end{bmatrix} - \begin{bmatrix} 10 \\ -8 \end{bmatrix}$

$3Y = \begin{bmatrix} -38 \\ 28 \end{bmatrix}$

$Y = \frac{1}{3}\begin{bmatrix} -38 \\ 28 \end{bmatrix}$

Question 11:

Given $A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$; find the matrix X such that:

A + X = 2B + C

Solution 11:

Given, A + X = 2B + C

$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = \begin{bmatrix} -5 & 4 \\ 8 & 2 \end{bmatrix}$

$X = \begin{bmatrix} -5 & 4 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$

$X = \begin{bmatrix} -7 & 5 \\ 6 & 2 \end{bmatrix}$

Question 12:

Find the value of x, given that $A^2 = B$,

$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$

Solution 12:

$$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 24+12 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$$

Given, $A^2 = B$

$$\therefore \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$x = 36$

Question 13:

If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ and I is the identity matric of the same order and A^t is the transpose of matrix A, find $A^t.B + BI$

Solution 13:

Transpose of a matrix is the matrix obtained on

Interchanging its rows and columns

Thus, if $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then $A^t = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

Identify matrix of order 2 is $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Two matrices can be multiplied together if and only

If the number of columns in the first matrix is equal

To the number of rows in B square matrices

Since the matrices A^t and B are square matrices

The condition of compatibility for multiplication of matrices is satisfied.

Let us compute A^tB.

$$A^tB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \times \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\Rightarrow A^tB = \begin{bmatrix} 8-1 & -4+3 \\ 20-3 & -10+9 \end{bmatrix}$$

$$\Rightarrow A^tB = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$$

Since I is identity matrix for multiplication

We have BI = B = IB

Two matrices are compatible for addition, only

When they have the same order

Both A^tB and BI are of the same order

Hence matrix addition is possible

Thus,

$$A^tB + BI = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7+4 & -1-2 \\ 17-1 & -1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}$$

Question 14:

Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$. Find the matrix X such that A + 2X = 2B + C.

Solution 14:

$$\text{Given } 2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

On comparing, corresponding elements,

$8 + y = 0 \Rightarrow y = ?8$

$2x + 1 = 5 \Rightarrow 2x = 5 - 1 \Rightarrow 2x = 4 \Rightarrow x = 2$

Hence, $x = 2, y = -8$

Question 15:

Let $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$, Find $A^2 - A + BC$

Solution 15:

$$A^2 = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 16-12 & -8+6 \\ 24-18 & -12+9 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$$

$$BC = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0+2 & 0-2 \\ -2-1 & 3+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

$$A^2 - A + BC = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

Question 16:

Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$, Find $A^2 + AB + B^2$

Solution 16:

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times2 & 1\times0+0\times1 \\ 2\times1+1\times2 & 2\times0+1\times1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$AB = A \times B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 12\times+0\times(-1) & 1\times3+0\times0 \\ 2\times2+1\times(-1) & 2\times3+1\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+3\times(-1) & 2\times3+3\times0 \\ (-1)\times2+0\times(-1) & -1\times3+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$\therefore\ A^2 + AB + B^2 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}$$

Question 17:

If $A = \begin{bmatrix} 3 & a \\ -4 & 8 \end{bmatrix}$, $B = \begin{bmatrix} c & 4 \\ -3 & 0 \end{bmatrix}$ $C = \begin{bmatrix} -1 & 4 \\ 3 & b \end{bmatrix}$ and 3A – 2C = 6B, find the values of a, b and c.

Solution 17:

$$3A - 2C = 6B$$

$$3\begin{bmatrix} 3 & a \\ -4 & 8 \end{bmatrix} - 2\begin{bmatrix} -1 & 4 \\ 3 & b \end{bmatrix} = 6\begin{bmatrix} c & 4 \\ -3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 3a \\ -12 & 24 \end{bmatrix} - \begin{bmatrix} -2 & 8 \\ 6 & 2b \end{bmatrix} = \begin{bmatrix} 6c & 24 \\ -18 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 3a-8 \\ -18 & 24-2b \end{bmatrix} = \begin{bmatrix} 6c & 24 \\ -18 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$3a - 8 = 24 \Longrightarrow 3a = 32 \Longrightarrow a = \frac{32}{3} = 10\frac{2}{3}$

$24 - 2b = 0 \Longrightarrow 2b = 24 \Longrightarrow b = 12$

$11 = 6c \Longrightarrow c = \frac{11}{6} = 1\frac{5}{6}$

Question 18:

Given $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ and $BA = C^2$. Find the values of p and q.

Solution 18:

$$A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}\begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$$BA = C^2 \Rightarrow \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

By comparing,

$-2q = -8 \Rightarrow q = 4$

And $p = 8$

Question 19:

Given $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ and $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Find : $AB + 2C - 4D$

Solution 19:

$$AB = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 18-2 \\ -6+4 \end{bmatrix} = \begin{bmatrix} 16 \\ -2 \end{bmatrix}$$

$$\therefore\ AB + 2C - 4D = \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Question 20:

Evaluate: $\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

Solution 20:

$$\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

Question 21:

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find $A^2 - 5A + 7I$

Solution 21:

Given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

We need to find $A^2 - 5A + 7I$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$