

EXERCISE 13 (A)**Question 1:**

Calculate the co-ordinates of the point P which divides the line segment joining:

- (i) A (1, 3) and B (5, 9) in the ratio 1 : 2
(ii) A (-4, 6) and B(3, -5) in the ratio 3 : 2

Solution 1:

- (i) Let the co-ordinates of the point P be (x, y)

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times 5 + 2 \times 1}{1 + 2} = \frac{7}{3}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1 \times 9 + 2 \times 3}{1 + 2} = \frac{15}{3} = 5$$

Thus, the co-ordinates of point P are $\left(\frac{7}{3}, 5\right)$

- (ii) Let the co-ordinates of the point P be (x, y).

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{3 \times 3 + 2 \times (-4)}{3 + 2} = \frac{1}{5}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{3 \times (-5) + 2 \times 6}{3 + 2} = \frac{-3}{5}$$

Thus, the co-ordinates of point P are $\left(\frac{1}{5}, \frac{-3}{5}\right)$

Question 2:

In what ratio is the line joining (2, -3) and (5, 6) divided by the x-axis.

Solution 2:

Let the line joining points A (2, -3) and B (5, 6) be divided by point P (x, 0) in the ratio k: 1.

$$y = \frac{ky_2 + y_1}{k + 1}$$

$$0 = \frac{k \times 6 + 1 \times (-3)}{k + 1}$$

$$0 = 6k - 3$$

$$k = \frac{1}{2}$$

Thus, the required ratio is 1: 2.

Question 3:

In what ratio is the line joining (2, -4) and (-3, 6) divided by the y – axis.

Solution 3:

Let the line joining points A (2, -4) and B (-3, 6) be divided by point P (0, y) in the ratio k: 1.

$$x = \frac{Kx_2 + x_1}{k + 1}$$

$$0 = \frac{k \times (-3) + 1 \times 2}{k + 1}$$

$$0 = -3k + 2$$

$$k = \frac{2}{3}$$

Thus, the required ratio is 2: 3.

Question 4:

In what ratio does the point (1, a) divide the join of (-1, 4) and (4, -1)?

Also, find the value of a.

Solution 4:

$$\frac{K}{A(-1, 4)} \quad \frac{1}{P(1, a)} \quad B(4, -1)$$

Let the point P (1, a) divides the line segment AB in the ratio k: 1.

Using section formula, we have:

$$1 = \frac{4K - 1}{K + 1}$$

$$\Rightarrow K + 1 = 4K - 1$$

$$\Rightarrow 2 = 3K$$

$$\Rightarrow K = \frac{2}{3} \quad \dots\dots(1)$$

$$\Rightarrow a = \frac{-k + 4}{k + 1}$$

Hence, the required is 2:3 and the value of a is 2.

Question 5:

In what ratio does the point (a, 6) divide the join of (-4, 3) and (2, 8)?

Also, find the value of a.

Solution 5:

Let the point P (a, 6) divides the line segment joining A (-4, 3) and B (2, 8) in the ratio k: 1.

Using section formula, we have:

$$6 = \frac{8K + 3}{K + 1}$$

$$\Rightarrow 6K + 6 = 8K + 3$$

$$\Rightarrow 3 = 2k$$

$$\Rightarrow k = \frac{3}{2} \quad \dots\dots(1)$$

$$\Rightarrow a = \frac{2k - 4}{k + 1}$$

$$\Rightarrow a = \frac{2 \times \frac{3}{2} - 4}{\frac{3}{2} + 1} \quad (\text{from(1)})$$

$$\Rightarrow a = -\frac{2}{5}$$

Hence, the required ratio is 3:2 and the value of a is $-\frac{2}{5}$

Question 6:

In what ratio is the join of (4, 3) and (2, -6) divided by the x-axis. Also, find the co-ordinates of the point of intersection.

Solution 6:

Let the point P (x, 0) on x-axis divides the line segment joining A (4, 3) and B (2, -6) in the ratio k: 1.

Using section formula, we have:

$$0 = \frac{-6k + 3}{k + 1}$$

$$0 = -6k + 3$$

$$k = \frac{1}{2}$$

Thus, the required ratio is 1: 2.

Also, we have:

$$x = \frac{2k + 4}{k + 1}$$

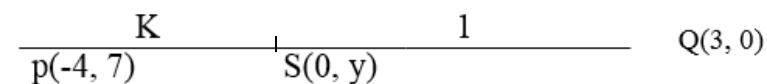
$$\begin{aligned}
 &= \frac{2 \times \frac{1}{2} + 4}{\frac{1}{2} + 1} \\
 &= \frac{10}{3}
 \end{aligned}$$

Thus, the required co-ordinates of the point of intersection are $\left(\frac{10}{3}, 0\right)$

Question 7:

Find the ratio in which the join of $(-4, 7)$ and $(3, 0)$ is divided by the y-axis. Also, find the co-ordinates of the point of intersection.

Solution 7:



$$0 = \frac{3k - 4}{k + 1}$$

$$3k = 4$$

$$k = \frac{4}{3} \quad \dots\dots(1)$$

$$y = \frac{0 + 7}{k + 1}$$

$$y = \frac{7}{\frac{4}{3} + 1} \quad (\text{from (1)})$$

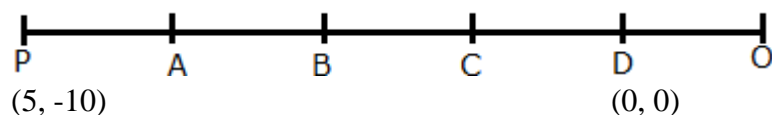
$$y = 3$$

Hence, the required ratio is 4:3 and the required point is $S(0, 3)$

Question 8:

Points A, B, C and D divide the line segment joining the point $(5, -10)$ and the origin in five equal parts. Find the co-ordinates of B and D.

Solution 8:



Point A divides PO in the ratio 1: 4.

Co-ordinates of point A are:

$$\left(\frac{1 \times 0 + 4 \times 5}{1 + 4}, \frac{1 \times 0 + 4 \times (-10)}{1 + 4} \right) = \left(\frac{20}{5}, \frac{-40}{5} \right) = (4, -8)$$

Point B divides PO in the ratio 2: 3.

Co-ordinates of point B are:

$$\left(\frac{2 \times 0 + 3 \times 5}{2 + 3}, \frac{2 \times 0 + 3 \times (-10)}{2 + 3} \right) = \left(\frac{15}{5}, \frac{-30}{5} \right) = (3, -6)$$

Point C divides PO in the ratio 3: 2.

Co-ordinates of point C are:

$$\left(\frac{3 \times 0 + 2 \times 5}{3 + 2}, \frac{3 \times 0 + 2 \times (-10)}{3 + 2} \right) = \left(\frac{10}{5}, \frac{-20}{5} \right) = (2, -4)$$

Point D divides PO in the ratio 4: 1.

Co-ordinates of point D are:

$$\left(\frac{4 \times 0 + 1 \times 5}{4 + 1}, \frac{4 \times 0 + 1 \times (-10)}{4 + 1} \right) = \left(\frac{5}{5}, \frac{-10}{5} \right) = (1, -2)$$

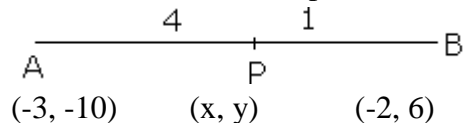
Question 9:

The line joining the points A (-3, -10) and B (-2, 6) is divided by the point P such that $\frac{PB}{AB} = \frac{1}{5}$.

Find the co-ordinates of P.

Solution 9:

Let the co-ordinates of point P are (x, y).



Given: $PB : AB = 1 : 5$

$\therefore PB : PA = 1 : 4$

Coordinates of P are

$$(x, y) = \left(\frac{4 \times (-2) + 1 \times (-3)}{5}, \frac{4 \times 6 + 1 \times (-10)}{5} \right) = \left(\frac{-11}{5}, \frac{14}{5} \right)$$

Question 10:

P is a point on the line joining A(4, 3) and B(-2, 6) such that $5AP = 2BP$. Find the co-ordinates of P.

Solution 10:

$$5AP = 2BP$$

$$\frac{AP}{BP} = \frac{2}{5}$$

The co-ordinates of the point P are

$$\left(\frac{2 \times (-2) + 5 \times 4}{2 + 5}, \frac{2 \times 6 + 5 \times 3}{2 + 5} \right)$$

$$\left(\frac{16}{7}, \frac{27}{7} \right)$$

Question 11:

Calculate the ratio in which the line joining the points (-3, -1) and (5, 7) is divided by the line $x = 2$. Also, find the co-ordinates of the point of intersection.

Solution 11:

The co-ordinates of every point on the line $x = 2$ will be of the type (2, y).

Using section formula, we have:

$$x = \frac{m_1 \times 5 + m_2 \times (-3)}{m_1 + m_2}$$

$$2 = \frac{5m_1 - 3m_2}{m_1 + m_2}$$

$$2m_1 + 2m_2 = 5m_1 - 3m_2$$

$$5m_2 = 3m_1$$

$$\frac{m_1}{m_2} = \frac{5}{3}$$

Thus, the required ratio is 5: 3.

$$y = \frac{m_1 \times 7 + m_2 \times (-1)}{m_1 + m_2}$$

$$y = \frac{5 \times 7 + 3 \times (-1)}{5 + 3}$$

$$y = \frac{35 - 3}{8}$$

$$y = \frac{32}{8} = 4$$

Thus, the required co-ordinates of the point of intersection are (2, 4).

Question 12:

Calculate the ratio in which the line joining A(6, 5) and B(4, -3) is divided by the line $y = 2$.

Solution 12:

The co-ordinates of every point on the line $y = 2$ will be of the type $(x, 2)$.

Using section formula, we have:

$$y = \frac{m_1 \times (-3) + m_2 \times 5}{m_1 + m_2}$$

$$2 = \frac{-3m_1 + 5m_2}{m_1 + m_2}$$

$$2m_1 + 2m_2 = -3m_1 + 5m_2$$

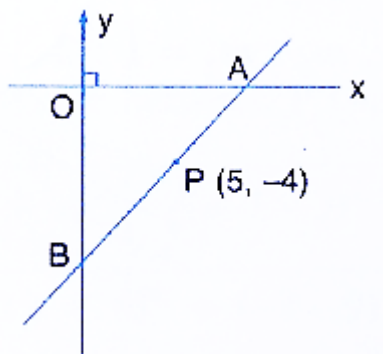
$$5m_1 = 3m_2$$

$$\frac{m_1}{m_2} = \frac{3}{5}$$

Thus, the required ratio is 3 : 5.

Question 13:

The point P (5, -4) divides the line segment AB, as shown in the figure, in the ratio 2 : 5. Find the co-ordinates of points A and B.



Solution 13:

Point A lies on x-axis. So, let the co-ordinates of A be $(x, 0)$.

Point B lies on y-axis. So, let the co-ordinates of B be $(0, y)$.

P divides AB in the ratio 2: 5.

We have:

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$5 = \frac{2 \times 0 + 5 \times x}{2 + 5}$$

$$5 = \frac{5x}{7}$$

$$x = 7$$

Thus, the co-ordinates of point A are (7, 0).

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$-4 = \frac{2 \times y + 5 \times 0}{2 + 5}$$

$$-4 = \frac{2y}{7}$$

$$-2 = \frac{y}{7}$$

$$y = -14$$

Thus, the co-ordinates of point B are (0, -14).

Question 14:

Find the co-ordinates of the points of tri-section of the line joining the points (-3, 0) and (6, 6)

Solution 14:

Let P and Q be the point of trisection of the line segment joining the points A (-3, 0) and B (6, 6).

So, AP = PQ = QB

We have AP: PB = 1 : 2

Co-ordinates of the point P are

$$\left(\frac{1 \times 6 + 2 \times (-3)}{1 + 2}, \frac{1 \times 6 + 2 \times 0}{1 + 2} \right)$$

$$= \left(\frac{6 - 6}{3}, \frac{6}{3} \right)$$

$$= (0, 2)$$

We have AQ : QB = 2 : 1

Co-ordinates of the point Q are

$$\left(\frac{2 \times 6 + 1 \times (-3)}{2 + 1}, \frac{2 \times 6 + 1 \times 0}{2 + 1} \right)$$

$$= \left(\frac{9}{3}, \frac{12}{3} \right)$$

$$= (3, 4)$$

Question 15:

Show that the line segment joining the points (-5, 8) and (10, -4) is trisected by the co-ordinate axes.

Solution 15:

Let P and Q be the point of trisection of the line segment joining the points A (-5, 8) and B (10, -4).

So, AP = PQ = QB

We have AP:PB = 1 : 2

Co-ordinates of the point P are

$$\left(\frac{1 \times 10 + 2 \times (-5)}{1 + 2}, \frac{1 \times (-4) + 2 \times 8}{1 + 2} \right)$$

$$= \left(\frac{10 - 10}{3}, \frac{12}{3} \right)$$

$$= (0, 4)$$

So, point P lies on the y-axis

We have AQ : QB = 2 : 1

Co-ordinates of the point Q are

$$\left(\frac{2 \times 10 + 1 \times (-5)}{2 + 1}, \frac{2 \times (-4) + 1 \times 8}{2 + 1} \right)$$

$$= \left(\frac{20 - 5}{3}, \frac{-8 + 8}{3} \right)$$

$$= (5, 0)$$

So, point Q lies on the x-axis.

Hence, the line segment joining the given points A and B is trisected by the co-ordinate axes.

Question 16:

Show that A (3, -2) is a point of trisection of the line segment joining the points (2, 1) and (5, -8).

Also, find the co-ordinates of the other point of trisection.

Solution 16:

Let A and B be the point of trisection of the line segment joining the points P (2, 1) and Q (5, -8).

So, PA = AB = BQ

We have PA : AQ = 1 : 2

Co-ordinates of the point A are

$$\begin{aligned} & \left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, \frac{1 \times (-8) + 2 \times 1}{1 + 2} \right) \\ &= \left(\frac{9}{3}, \frac{-6}{3} \right) \\ &= (3, -2) \end{aligned}$$

Hence, A (3, -2) is a point of trisection of PQ.

We have PB : BQ = 2 : 1

Co-ordinates of the point B are

$$\begin{aligned} & \left(\frac{2 \times 5 + 1 \times 2}{2 + 1}, \frac{2 \times (-8) + 1 \times 1}{2 + 1} \right) \\ &= \left(\frac{10 + 2}{3}, \frac{-16 + 1}{3} \right) \\ &= (4, -5) \end{aligned}$$

Question 17:

If A = (-4, 3) and B = (8, -6)

(i) Find the length of AB

(ii) In what ratio is the line joining A and B, divided by the x-axis?

Solution 17:

(i) A (-4,3) and B (8, -6)

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 + 4)^2 + (-6 - 3)^2} \\ &= \sqrt{144 + 81} \end{aligned}$$

$$= \sqrt{225}$$

$$= 15 \text{ units}$$

(ii) Let P be the point, which divides AB on the x-axis in the ratio $k : 1$.

Therefore, y-co-ordinate of P = 0.

$$\Rightarrow \frac{-6k + 3}{k + 1} = 0$$

$$\Rightarrow -6k + 3 = 0$$

$$\Rightarrow k = \frac{1}{2}$$

\therefore Required ratio is 1 : 2.

Question 18:

The line segment joining the points M(5, 7) and N(-3, 2) is intersected by the y-axis at point L. Write down the abscissa of L. Hence, find the ratio in which L divides MN.

Also, find the co-ordinates of L.

Solution 18:

Since, point L lies on y-axis, its abscissa is 0.

Let the co-ordinates of point L be (0, y). Let L divides MN in the ratio $k : 1$.

Using section formula, we have:

$$x = \frac{k \times (-3) + 1 \times 5}{k + 1}$$

$$0 = \frac{-3k + 5}{k + 1}$$

$$-3k + 5 = 0$$

$$k = \frac{5}{3}$$

Thus, the required ratio is 5 : 3.

$$\text{Now, } y = \frac{k \times 2 + 1 \times 7}{k + 1}$$

$$= \frac{\frac{5}{3} \times 2 + 7}{\frac{5}{3} + 1}$$

$$= \frac{10 + 21}{5 + 3}$$

$$= \frac{31}{8}$$

$$= \frac{31}{8}$$

Question 19:

A (2, 5), B (-1, 2) and C (5, 8) are the co-ordinates of the vertices of the triangle ABC. Points P and Q lie on AB and AC respectively,

Such that: $AP : PB = AQ : QC = 1 : 2$

(i) Calculate the co-ordinates of P and Q.

(ii) Show that $PQ = \frac{1}{3} BC$

Solution 19:

(i) Co-ordinates of P are

$$\left(\frac{1 \times (-1) + 2 \times 2}{1 + 2}, \frac{1 \times 2 + 2 \times 5}{1 + 2} \right)$$

$$= \left(\frac{3}{3}, \frac{12}{3} \right)$$

$$= (1, 4)$$

Co-ordinates of Q are

$$\left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, \frac{1 \times 8 + 2 \times 5}{1 + 2} \right)$$

$$= \left(\frac{9}{3}, \frac{18}{3} \right)$$

$$= (3, 6)$$

(ii) Using distance formula, we have:

$$BC = \sqrt{(5+1)^2 + (8-2)^2} = \sqrt{36+36} = 6\sqrt{2}$$

$$PQ = \sqrt{(3-1)^2 + (6-4)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Hence, $PQ = \frac{1}{3} BC$.

Question 20:

A (-3, 4), B (3, -1) and C (-2, 4) are the vertices of a triangle ABC. Find the length of line segment AP, where point P lies inside BC, such that $BP : PC = 2 : 3$

Solution 20:

$BP : PC = 2 : 3$

Co-ordinates of P are

$$\left(\frac{2 \times (-2) + 3 \times 3}{2 + 3}, \frac{2 \times 4 + 3 \times (-1)}{2 + 3} \right)$$

$$= \left(\frac{-4+9}{5}, \frac{8-3}{5} \right)$$

$$= (1, 1)$$

Using distance formula, we have:

$$AP = \sqrt{(1+3)^2 + (1-4)^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ units.}$$

Question 21:

The line segment joining A (2, 3) and B (6, -5) is intercepted by x-axis at the point K. Write down the ordinate of the point K. Hence, find the ratio in which K divides AB. Also find the co-ordinates of the point K.

Solution 21:

Since, point K lies on x-axis, its ordinate is 0.

Let the point K (x, 0) divides AB in the ratio k: 1.

We have,

$$y = \frac{k \times (-5) + 1 \times 3}{k + 1}$$

$$0 = \frac{-5k + 3}{k + 1}$$

$$k = \frac{3}{5}$$

Thus, K divides AB in the ratio 3: 5.

Also, we have:

$$x = \frac{k \times 6 + 1 \times 2}{k + 1}$$

$$x = \frac{\frac{3}{5} \times 6 + 2}{\frac{3}{5} + 1}$$

$$x = \frac{18 + 10}{3 + 5}$$

$$x = \frac{28}{8} = \frac{7}{2} = 3\frac{1}{2}$$

Thus, the co-ordinates of the point K are $\left(3\frac{1}{2}, 0 \right)$.

Question 22:

The line segment joining A (4, 7) and B (-6, -2) is intercepted by the y – axis at the point K. write down the abscissa of the point K. hence, find the ratio in which K divides AB. Also, find the co-ordinates of the point K.

Solution 22:

Since, point K lies on y-axis, its abscissa is 0.

Let the point K (0, y) divides AB in the ratio k : 1

We have,

$$x = \frac{k \times (-6) + 1 \times 4}{k + 1}$$

$$0 = \frac{-6k + 4}{k + 1}$$

$$k = \frac{4}{6} = \frac{2}{3}$$

Thus, K divides AB in the ratio 2: 3.

Also, we have:

$$y = \frac{k \times (-2) + 1 \times 7}{k + 1}$$

$$y = \frac{-2k + 7}{k + 1}$$

$$y = \frac{-2 \times \frac{2}{3} + 7}{\frac{2}{3} + 1}$$

$$y = \frac{-4 + 21}{2 + 3}$$

$$y = \frac{17}{5}$$

Thus, the co-ordinates of the point K are $\left(0, \frac{17}{5}\right)$

Question 23:

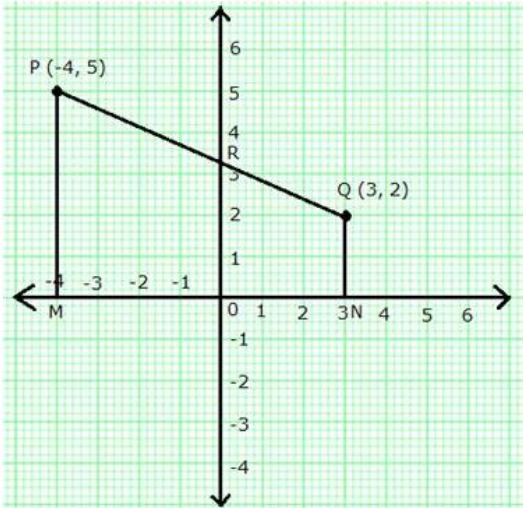
The line joining P(-4, 5) and Q(3, 2) intersects the y-axis at point R. PM and QN are perpendicular from P and Q on the x-axis Find:

(i) the ratio PR : RQ

(ii) the coordinates of R.

(iii) the area of the quadrilateral PMNQ.

Solution 23:



(i) Let point R (0, y) divides PQ in the ratio k: 1.

We have:

$$x = \frac{k \times 3 + 1 \times (-4)}{k + 1}$$

$$0 = \frac{3k - 4}{k + 1}$$

$$0 = 3k - 4$$

$$k = \frac{4}{3}$$

Thus, PR: RQ = 4: 3

(ii) Also, we have:

$$y = \frac{k \times 2 + 1 \times 5}{k + 1}$$

$$y = \frac{2k + 5}{k + 1}$$

$$y = \frac{2 \times \frac{4}{3} + 5}{\frac{4}{3} + 1}$$

$$y = \frac{8 + 15}{4 + 3}$$

$$y = \frac{23}{7}$$

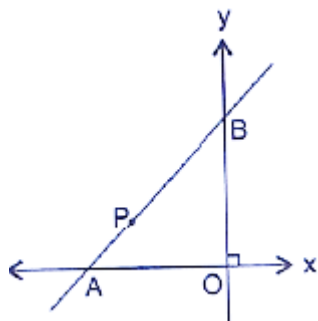
Thus, the co-ordinates of point R are $\left(0, \frac{23}{7}\right)$.

(iii) Area of quadrilateral PMNQ

$$\begin{aligned} &= \frac{1}{2} \times (\text{PM} + \text{QN}) \times \text{MN} \\ &= \frac{1}{2} \times (5 + 2) \times 7 \\ &= \frac{1}{2} \times 7 \times 7 \\ &= 24.5 \text{ sq units} \end{aligned}$$

Question 24:

In the given figure line APB meets the x-axis at point A and y-axis at point B. P is the point (-4, 2) and $AP : PB = 1 : 2$. Find the co-ordinates of A and B.



Solution 24:

Given, A lies on x-axis and B lies on y-axis.

Let the co-ordinates of A and B be $(x, 0)$ and $(0, y)$ respectively.

Given, P is the point $(-4, 2)$ and $AP : PB = 1 : 2$.

Using section formula, we have:

$$-4 = \frac{1 \times 0 + 2 \times x}{1 + 2}$$

$$-4 = \frac{2x}{3}$$

$$x = \frac{-4 \times 3}{2} = -6$$

Also,

$$2 = \frac{1 \times y + 2 \times 0}{1 + 2}$$

$$2 = \frac{y}{3}$$

$$y = 6$$

Thus, the co-ordinates of points A and B are $(-6, 0)$ and $(0, 6)$ respectively.

Question 25:

Given a line segment AB joining the points A (-4, 6) and B (8, -3). Find:

- (i) the ratio in which AB is divided by the y-axis
- (ii) find the coordinates of the point of intersection
- (iii) the length of AB.

Solution 25:

- (i) Let the required ratio be $m_1 : m_2$

Consider $A(-4, 6) = (x_1, y_1)$; $B(8, -3) = (x_2, y_2)$ and let

$P(x, y)$ be the point of intersection of the line segment

And the y-axis

By section formula, we have,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{8m_1 - 4m_2}{m_1 + m_2}, y = \frac{-3m_1 + 6m_2}{m_1 + m_2}$$

The equation of the y-axis is $x = 0$

$$\Rightarrow x = \frac{8m_1 - 4m_2}{m_1 + m_2} = 0$$

$$\Rightarrow 8m_1 - 4m_2 = 0$$

$$\Rightarrow 8m_1 = 4m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{4}{8}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{1}{2}$$

- (ii) from the previous subpart, we have,

$$\frac{m_1}{m_2} = \frac{1}{2}$$

$$\Rightarrow m_1 = k \text{ and } m_2 = 2k, \text{ where } k$$

Is any constant.

Also, we have,

$$x = \frac{8m_1 - 4m_2}{m_1 + m_2}, y = \frac{-3m_1 + 6m_2}{m_1 + m_2}$$

$$\Rightarrow x = \frac{8 \times k - 4 \times 2k}{k + 2k}, y = \frac{-3 \times k + 6 \times 2k}{k + 2k}$$

$$\Rightarrow x = \frac{8k - 8k}{3k}, y = \frac{-3k + 12k}{3k}$$

$$\Rightarrow x = \frac{0}{3k}, y = \frac{9k}{3k}$$

$$\Rightarrow x = 0, y = 3$$

Thus, the point of intersection is p (0, 3)

(iii) The length of AB = distance between two points A and B.

The distance between two given points

A(x₁, y₁) and B (x₂, y₂) is given by,

$$\begin{aligned} \text{Distance AB} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 + 4)^2 + (-3 - 6)^2} \\ &= \sqrt{(12)^2 + (9)^2} \\ &= \sqrt{144 + 81} \\ &= \sqrt{225} \\ &= 15 \text{ units} \end{aligned}$$

EXERCISE. 13 (B)

Question 1:

Find the mid – point of the line segment joining the point:

(i) (-6, 7) and (3, 5) (ii) (5, -3) and (-1, 7)

Solution 1:

(i) A (-6, 7) and B (3, 5)

$$\text{Mid-point of AB} = \left(\frac{-6+3}{2}, \frac{7+5}{2} \right) = \left(\frac{-3}{2}, 6 \right)$$

(ii) A (5, -3) and B (-1, 7)

$$\text{Mid-point of AB} = \left(\frac{5-1}{2}, \frac{-3+7}{2} \right) = (2, 2)$$

Question 2:

Points A and B have co-ordinates (3, 5) and (x, y) respectively. The mid point of AB is (2, 3). Find the values of x and y.

Solution 2:

Mid-point of AB = (2, 3)

$$\therefore \left(\frac{3+x}{2}, \frac{5+y}{2} \right) = (2, 3)$$

$$\Rightarrow \frac{3+x}{2} = 2 \quad \text{and} \quad \frac{5+y}{2} = 3$$

$$\Rightarrow 3+x = 4 \quad \text{and} \quad 5+y = 6$$

$$\Rightarrow x = 1 \quad \text{and} \quad y = 1$$

Question 3:

A (5, 3), B(-1, 1) and C(7, -3) are the vertices of triangle ABC. If L is the mid-point of AB and M is the mid-point of AC, show that $LM = \frac{1}{2} BC$

Solution 3:

Given, L is the mid-point of AB and M is the mid-point of AC.

Co-ordinates of L are

$$\left(\frac{5-1}{2}, \frac{3+1}{2} \right) = (2, 2)$$

Co-ordinates of M are

$$\left(\frac{5+7}{2}, \frac{3-3}{2} \right) = (6, 0)$$

Using distance formula, we have:

$$BC = \sqrt{(7+1)^2 + (-3-1)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$LM = \sqrt{(6-2)^2 + (0-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Hence, } LM = \frac{1}{2} BC$$

Question 4:

Given M is the mid point of AB, find the co-ordinates of:

(i) A; if M = (1, 7) and B = (-5, 10)

(ii) B; if A = (3, -1) and M = (-1, 3)

Solution 4:

(i) Let the co-ordinates of A be (x, y).

$$\therefore (1,7) = \left(\frac{x-5}{2}, \frac{y+10}{2} \right)$$

$$\Rightarrow 1 = \frac{x-5}{2} \quad \text{and} \quad 7 = \frac{y+10}{2}$$

$$\Rightarrow 2 = x-5 \quad \text{and} \quad 14 = y+10$$

$$\Rightarrow x = 7 \quad \text{and} \quad y = 4$$

Hence, the co-ordinates of A are (7, 4).

(ii) Let the co-ordinates of B be (x, y).

$$\therefore (-1,3) = \left(\frac{3+x}{2}, \frac{-1+y}{2} \right)$$

$$\Rightarrow -1 = \frac{3+x}{2} \quad \text{and} \quad 3 = \frac{-1+y}{2}$$

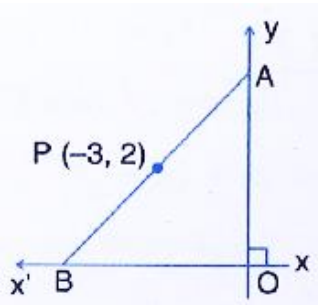
$$\Rightarrow -2 = 3+x \quad \text{and} \quad 6 = -1+y$$

$$\Rightarrow x = -5 \quad \text{and} \quad y = 7$$

Hence, the co-ordinates of B are (-5, 7).

Question 5:

P (-3, 2) is the mid-point of line segment AB as shown in the given figure. Find the co-ordinates of points A and B.



Solution 5:

Point A lies on y-axis, so let its co-ordinates be (0, y).

Point B lies on x-axis, so let its co-ordinates be (x, 0).

P (-3, 2) is the mid-point of line segment AB.

$$\therefore (-3,2) = \left(\frac{0+x}{2}, \frac{y+0}{2} \right)$$

$$\Rightarrow (-3,2) = \left(\frac{x}{2}, \frac{y}{2} \right)$$

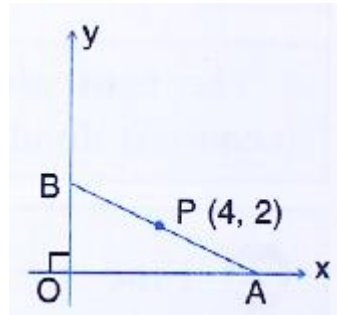
$$\Rightarrow -3 = \frac{x}{2} \quad \text{and} \quad 2 = \frac{y}{2}$$

$$\Rightarrow -6 = x \quad \text{and} \quad 4 = y$$

Thus, the co-ordinates of points A and B are (0, 4) and (-6, 0) respectively.

Question 6:

In the given figure, P (4, 2) is mid-point of line segment AB. Find the co-ordinates of A and B.



Solution 6:

Point A lies on x-axis, so let its co-ordinates be (x, 0).

Point B lies on y-axis, so let its co-ordinates be (0, y).

P (4, 2) is mid-point of line segment AB.

$$\therefore (4, 2) = \left(\frac{x+0}{2}, \frac{0+y}{2} \right)$$

$$\Rightarrow 4 = \frac{x}{2} \quad \text{and} \quad 2 = \frac{y}{2}$$

$$\Rightarrow 8 = x \quad \text{and} \quad 4 = y$$

Hence, the co-ordinates of points A and B are (8, 0) and (0, 4) respectively.

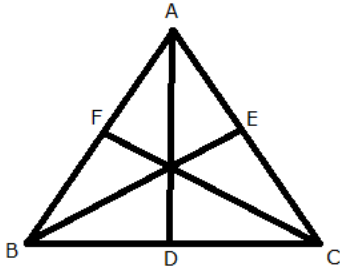
Question 7:

(-5, 2), (3, -6) and (7, 4) are the vertices of a triangle. Find the length of its median through the vertex (3, -6)

Solution 7:

Let A (-5, 2), B (3, -6) and C (7, 4) be the vertices of the given triangle.

Let AD be the median through A, BE be the median through B and CF be the median through C.



We know that median of a triangle bisects the opposite side.

Co-ordinates of point F are

$$\left(\frac{-5+3}{2}, \frac{2-6}{2}\right) = \left(\frac{-2}{2}, \frac{-4}{2}\right) = (-1, -2)$$

Co-ordinates of point D are

$$\left(\frac{3+7}{2}, \frac{-6+4}{2}\right) = \left(\frac{10}{2}, \frac{-2}{2}\right) = (5, -1)$$

Co-ordinates of point E are

$$\left(\frac{-5+7}{2}, \frac{2+4}{2}\right) = \left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3)$$

The median of the triangle through the vertex B(3, -6) is BE

Using distance formula,

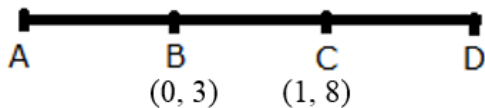
$$BE = \sqrt{(1-3)^2 + (3+6)^2} = \sqrt{4+81} = \sqrt{85} = 9.22$$

Question 8:

Given a line ABCD in which $AB = BC = CD$, $B = (0, 3)$ and $C = (1, 8)$

Find the co-ordinates of A and D.

Solution 8:



Given, $AB = BC = CD$

So, B is the mid-point of AC. Let the co-ordinates of point A be (x, y).

$$\therefore (0, 3) = \left(\frac{x+1}{2}, \frac{y+8}{2}\right)$$

$$\Rightarrow 0 = \frac{x+1}{2} \quad \text{and} \quad 3 = \frac{y+8}{2}$$

$$\Rightarrow 0 = x + 1 \quad \text{and} \quad 6 = y + 8$$

$$\Rightarrow -1 = x \quad \text{and} \quad -2 = y$$

Thus, the co-ordinates of point A are (-1, -2).

Also, C is the mid-point of BD. Let the co-ordinates of point D be (p, q).

$$\therefore (1, 8) = \left(\frac{0+p}{2}, \frac{3+q}{2} \right)$$

$$\Rightarrow 1 = \frac{0+p}{2} \quad \text{and} \quad 8 = \frac{3+q}{2}$$

$$\Rightarrow 2 = 0+p \quad \text{and} \quad 16 = 3+q$$

$$\Rightarrow -2 = p \quad \text{and} \quad 13 = q$$

Thus, the co-ordinates of point D are (2, 13).

Question 9:

One end of the diameter of a circle is (-2, 5). Find the co-ordinates of the other end of it, if the centre of the circle is (2, -1)

Solution 9:

We know that the centre is the mid-point of diameter.

Let the required co-ordinates of the other end of mid-point be (x, y).

$$\therefore (2, -1) = \left(\frac{-2+x}{2}, \frac{5+y}{2} \right)$$

$$\Rightarrow 2 = \frac{-2+x}{2} \quad \text{and} \quad -1 = \frac{5+y}{2}$$

$$\Rightarrow 4 = -2+x \quad \text{and} \quad -2 = 5+y$$

$$\Rightarrow 6 = x \quad \text{and} \quad -7 = y$$

Thus, the required co-ordinates are (6, -7).

Question 10:

A (2, 5), B (1, 0), C (-4, 3) and D (-3, 8) are the vertices of quadrilateral ABCD. Find the co-ordinates of the mid-points of AC and BD. Give a special name to the quadrilateral.

Solution 10:

Co-ordinates of the mid-point of AC are

$$\left(\frac{2-4}{2}, \frac{5+3}{2} \right) = \left(\frac{-2}{2}, \frac{8}{2} \right) = (-1, 4)$$

Co-ordinates of the mid-point of BD are

$$\left(\frac{1-3}{2}, \frac{0+8}{2}\right) = \left(\frac{-2}{2}, \frac{8}{2}\right) = (-1, 4)$$

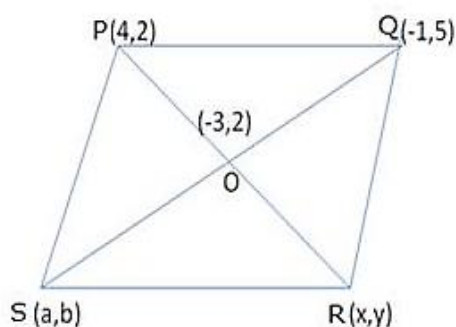
Since, mid-point of AC = mid-point of BD

Hence, ABCD is a parallelogram.

Question 11:

P (4, 2) and Q (-1, 5) are the vertices of parallelogram PQRS and (-3, 2) are the co-ordinates of the point of intersection of its diagonals. Find co-ordinates of R and S.

Solution 11:



Let the coordinates of R and S be (x, y) and (a, b) respectively.

Mid-point of PR is O.

$$\therefore O(-3, 2) = O\left(\frac{4+x}{2}, \frac{2+y}{2}\right)$$

$$-3 = \frac{4+x}{2}, 2 = \frac{2+y}{2}$$

$$-6 = 4+x, 4 = 2+y$$

$$x = -10, y = 2$$

Hence, R = (-10, 2)

Similarly, the mid-point of SQ is O.

$$\therefore O(-3, 2) = O\left(\frac{a-1}{2}, \frac{b+5}{2}\right)$$

$$-3 = \frac{a-1}{2}, 2 = \frac{b+5}{2}$$

$$-6 = a-1, 4 = b+5$$

$$a = -5, b = -1$$

Hence, S = (-5, -1)

Thus, the coordinates of the point R and S are (-10, 2) and (-5, -1).

Question 12:

A (-1, 0), B (1, 3) and D (3, 5) are the vertices of a parallelogram ABCD. Find the co-ordinates of vertex C.

Solution 12:

Let the co-ordinates of vertex C be (x, y).

ABCD is a parallelogram.

∴ Mid-point of AC = Mid-point of BD

$$\left(\frac{-1+x}{2}, \frac{0+y}{2}\right) = \left(\frac{1+3}{2}, \frac{3+5}{2}\right)$$

$$\left(\frac{-1+x}{2}, \frac{y}{2}\right) = (2, 4)$$

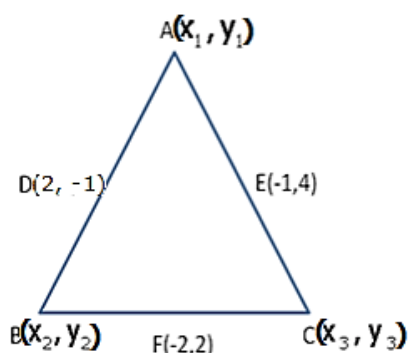
$$\frac{-1+x}{2} = 2 \quad \text{and} \quad \frac{y}{2} = 4$$

$$x = 5 \quad \text{and} \quad y = 8$$

Thus, the co-ordinates of vertex C is (5, 8).

Question 13:

The points (2, -1), (-1, 4) and (-2, 2) are mid points of the sides of a triangle. Find its vertices.

Solution 13:

Let A(x₁, y₁), B(x₂, y₂) and C (x₃, y₃) be the co-ordinates of the vertices of ΔABC .

Midpoint of AB, i.e. D

$$D(2, -1) = D\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$2 = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} = \dots\dots\dots(1)$$

$$x_1 + x_2 = 4 \quad \text{---} \quad (1) \quad y_1 + y_2 = -2 \dots\dots\dots(2)$$

Similarly

$$x_1 + x_3 = -2 \dots \dots \dots (3)$$

$$y_1 + y_3 = 8 \dots \dots \dots (4)$$

$$x_1 + x_3 = -4 \dots \dots \dots (5)$$

$$y_2 + y_3 = 4 \dots \dots \dots (6)$$

Adding (1), (3) and (5), we get,

$$2(x_1 + x_2 + x_3) = -2$$

$$x_1 + x_2 + x_3 = -1$$

$$4 + x_3 = -1 \text{ [from (1)]}$$

$$x_3 = -5$$

From (3)

$$x_1 - 5 = -2$$

$$x_1 = 3$$

From (5)

$$x_2 - 5 = -4$$

$$x_2 = 1$$

Adding (2), (4) and (6), we get,

$$2(y_1 + y_2 + y_3) = 10$$

$$y_1 + y_2 + y_3 = 5$$

$$-2 + y_3 = 5 \text{ [from (2)]}$$

$$y_3 = 7$$

From (4)

$$y_1 + 7 = 8$$

$$y_1 = 1$$

From (6)

$$y_2 + 7 = 4$$

$$y_2 = -3$$

Thus, the co-ordinates of the vertices of ΔABC are (3, 1), (1, -3) and (-5, 7).

Question 14:

Points A (-5, x), B (y, 7) and C (1, -3) are collinear (i.e. lie on the same straight line) such that $AB = BC$. Calculate the values of x and y.

Solution 14:

Given, $AB = BC$, i.e., B is the mid-point of AC.

$$\therefore (y, 7) = \left(\frac{-5+1}{2}, \frac{x-3}{2} \right)$$

$$(y, 7) = \left(-2, \frac{x-3}{2} \right)$$

$$\Rightarrow y = -2 \text{ and } 7 = \frac{x-3}{2}$$

$$\Rightarrow y = -2 \text{ and } x = 17$$

Question 15:

Points P (a, -4), Q (-2, b) and R (0, 2) are collinear. If Q lies between P and R, such that $PR = 2QR$, calculate the values of a and b.

Solution 15:

Given, $PR = 2QR$

Now, Q lies between P and R, so, $PR = PQ + QR$

$$\therefore PQ + QR = 2QR$$

$$\Rightarrow PQ = QR$$

\Rightarrow Q is the mid-point of PR.

$$\therefore (-2, b) = \left(\frac{a+0}{2}, \frac{-4+2}{2} \right)$$

$$(-2, b) = \left(\frac{a}{2}, -1 \right)$$

$$\Rightarrow a = -4 \text{ and } b = -1$$

Question 16:

Calculate the co-ordinates of the centroid of the triangle ABC, if A = (7, -2), B = (0, 1) and C = (-1, 4).

Solution 16:

Co-ordinates of the centroid of triangle ABC are

$$\left(\frac{7+0-1}{3}, \frac{-2+1+4}{3} \right)$$

$$= \left(\frac{6+3}{3}, \frac{3+3}{3} \right)$$

$$= (2, 1)$$

Question 17:

The co-ordinates of the centroid of a triangle PQR are (2, -5). If Q = (-6, 5) and R = (11, 8); calculate the co-ordinates of vertex P.

Solution 17:

Let G be the centroid of DPQR whose coordinates are (2, -5) and let (x, y) be the coordinates of vertex P.

Coordinates of G are,

$$G(2, -5) = G\left(\frac{x-6+11}{3}, \frac{y+5+8}{3}\right)$$

$$2 = \frac{x+5}{3}, \quad -5 = \frac{y+13}{3}$$

$$6 = x + 5, \quad -15 = y + 13$$

$$x = 1, \quad y = -28$$

Coordinates of vertex P are (1, -28)

Question 18:

A (5, x), B (-4, 3) and C (y, -2) are the vertices of the triangle ABC whose centroid is the origin. Calculate the values of x and y.

Solution 18:

Given, centroid of triangle ABC is the origin.

$$\therefore (0, 0) = \left(\frac{5-4+y}{3}, \frac{x+3-2}{3} \right)$$

$$(0, 0) = \left(\frac{1+y}{3}, \frac{x+1}{3} \right)$$

$$0 = \frac{1+y}{3} \quad \text{and} \quad 0 = \frac{x+1}{3}$$

$$y = -1 \quad \text{and} \quad x = -1$$

EXERCISE 13 (C)**Question 1:**

Given a triangle ABC in which A = (4, -4), B = (0, 5) and C = (5, 10). A point P lies on BC such that BP : PC = 3 : 2, Find the length of line segment AP.

Solution 1:

Given, BP: PC = 3: 2

Using section formula, the co-ordinates of point P are

$$\left(\frac{3 \times 5 + 2 \times 0}{3 + 2}, \frac{3 \times 10 + 2 \times 5}{3 + 2} \right)$$

$$= \left(\frac{15}{5}, \frac{40}{5} \right)$$

$$= (3, 8)$$

Using distance formula, we have:

$$AP = \sqrt{(3-4)^2 + (8+4)^2} = \sqrt{1+144} = \sqrt{145} = 12.04$$

Question 2:

A(20, 0) and B(10, -20) are two fixed points Find the co-ordinates of the point P in AB such that : 3PB = AB, Also, find the co-ordinates of some other point Q in AB such that AB = 6 AQ.

Solution 2:

Given, 3PB = AB

$$\Rightarrow \frac{AB}{PB} = \frac{3}{1}$$

$$\Rightarrow \frac{AB - PB}{PB} = \frac{3 - 1}{1}$$

$$\Rightarrow \frac{AP}{PB} = \frac{2}{1}$$

Using section formula,

Coordinates of P are

$$P(x, y) = P \left(\frac{2 \times 10 + 1 \times 20}{2 + 1}, \frac{2 \times (-20) + 1 \times 0}{2 + 1} \right)$$

$$= P \left(\frac{40}{3}, -\frac{40}{3} \right)$$

Given, AB = 6AQ

$$\Rightarrow \frac{AQ}{AB} = \frac{1}{6}$$

$$\Rightarrow \frac{AQ}{AB - AQ} = \frac{1}{6 - 1}$$

$$\Rightarrow \frac{AQ}{QB} = \frac{1}{5}$$

Using section formula,

Coordinates of Q are

$$Q(x, y) = Q\left(\frac{1 \times 10 + 5 \times 20}{1 + 5}, \frac{1 \times (-20) + 5 \times 0}{1 + 5}\right)$$

$$= Q\left(\frac{110}{6}, -\frac{20}{6}\right)$$

$$= Q\left(\frac{55}{3}, -\frac{10}{3}\right)$$

Question 3:

A(-8, 0), B(0, 16) and C(0, 0) are the vertices of a triangle ABC. Point P lies on AB and Q lies on AC such that AP : PB = 3 : 5 and AQ : QC = 3 : 5

Show that : $PQ = \frac{3}{8} BC$

Solution 3:

Given that, point P lies on AB such that AP: PB = 3: 5.

The co-ordinates of point P are

$$\left(\frac{3 \times 0 + 5 \times (-8)}{3 + 5}, \frac{3 \times 16 + 5 \times 0}{3 + 5}\right)$$

$$= \left(\frac{-40}{8}, \frac{48}{8}\right)$$

$$= (-5, 6)$$

Also, given that, point Q lies on AC such that AQ: QC = 3: 5.

The co-ordinates of point Q are

$$\left(\frac{3 \times 0 + 5 \times (-8)}{3 + 5}, \frac{3 \times 0 + 5 \times 0}{3 + 5}\right)$$

$$= \left(\frac{-40}{8}, \frac{0}{8} \right)$$

$$= (-5, 0)$$

Using distance formula,

$$PQ = \sqrt{(-5+5)^2 + (0-6)^2} = \sqrt{0+36} = 6$$

$$BC = \sqrt{(0-0)^2 + (0-16)^2} = \sqrt{0+(16)^2} = 16$$

$$\text{Now, } \frac{3}{8}BC = \frac{3}{8} \times 16 = 6 = PQ$$

Hence, proved

Question 4:

Find the co-ordinates of points of trisection of the line segment joining the point (6, -9) and the origin.

Solution 4:

Let P and Q be the points of trisection of the line segment joining A (6, -9) and B (0, 0).

P divides AB in the ratio 1: 2. Therefore, the co-ordinates of point P are

$$\left(\frac{1 \times 0 + 2 \times 6}{1+2}, \frac{1 \times 0 + 2 \times (-9)}{1+2} \right)$$

$$= \left(\frac{12}{3}, \frac{-18}{3} \right)$$

$$= (4, -6)$$

Q divides AB in the ratio 2: 1. Therefore, the co-ordinates of point Q are

$$\left(\frac{2 \times 0 + 1 \times 6}{2+1}, \frac{2 \times 0 + 1 \times (-9)}{2+1} \right)$$

$$= \left(\frac{6}{3}, \frac{-9}{3} \right)$$

$$= (2, -3)$$

Thus, the required points are (4, -6) and (2, -3).

Question 5:

A line segment joining $A(-1, \frac{5}{3})$ and $B(a, 5)$ is divided in the ratio 1 : 3 at P, the point where the line segment AB intersects the y-axis.

- (i) calculate the value of 'a'
 (ii) Calculate the co-ordinates of 'P'.

Solution 5:

Since, the line segment AB intersects the y-axis at point P, let the co-ordinates of point P be $(0, y)$.

P divides AB in the ratio 1: 3.

$$\therefore (0, y) = \left(\frac{1 \times a + 3 \times (-1)}{1 + 3}, \frac{1 \times 5 + 3 \times \frac{5}{3}}{1 + 3} \right)$$

$$(0, y) = \left(\frac{a - 3}{4}, \frac{10}{4} \right)$$

$$0 = \frac{a - 3}{4} \quad \text{and} \quad y = \frac{10}{4}$$

$$a = 3 \quad \text{and} \quad y = \frac{5}{2} = 2\frac{1}{2}$$

Thus, the value of a is 3 and the co-ordinates of point P are $\left(0, 2\frac{1}{2}\right)$

Question 6:

In what ratio is the line joining $A(0, 3)$ and $B(4, -1)$ divided by the x-axis? Write the co-ordinates of the point where AB intersects the x-axis

Solution 6:

Let the line segment AB intersects the x-axis by point P $(x, 0)$ in the ratio k: 1.

$$\therefore (x, 0) = \left(\frac{k \times 4 + 1 \times 0}{k + 1}, \frac{k \times (-1) + 1 \times 3}{k + 1} \right)$$

$$(x, 0) = \left(\frac{4k}{k + 1}, \frac{-k + 3}{k + 1} \right)$$

$$\Rightarrow 0 = \frac{-k + 3}{k + 1}$$

$$\Rightarrow k = 3$$

Thus, the required ratio in which P divides AB is 3: 1.

Also, we have:

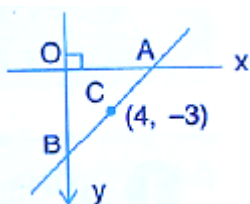
$$x = \frac{4k}{k+1}$$

$$\Rightarrow x = \frac{4 \times 3}{3+1} = \frac{12}{4} = 3$$

Thus, the co-ordinates of point P are (3, 0).

Question 7:

The mid-point of the segment AB, as shown in diagram, is C(4, -3). Write down the co-ordinates of A and B.



Solution 7:

Since, point A lies on x-axis, let the co-ordinates of point A be (x, 0).

Since, point B lies on y-axis, let the co-ordinates of point B be (0, y).

Given, mid-point of AB is C (4, -3).

$$\therefore (4, -3) = \left(\frac{x+0}{2}, \frac{0+y}{2} \right)$$

$$\Rightarrow (4, -3) = \left(\frac{x}{2}, \frac{y}{2} \right)$$

$$\Rightarrow 4 = \frac{x}{2} \quad \text{and} \quad -3 = \frac{y}{2}$$

$$\Rightarrow x = 8 \quad \text{and} \quad y = -6$$

Thus, the co-ordinates of point A are (8, 0) and the co-ordinates of point B are (0, -6).

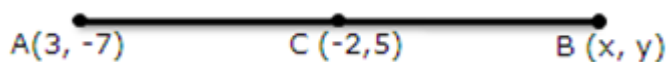
Question 8:

AB is a diameter of a circle with centre C = (-2, 5). If A = (3, -7), find

(i) the length of radius AC

(ii) the coordinates of B.

Solution 8:



$$\begin{aligned}
 \text{(i) Radius AC} &= \sqrt{(3+2)^2 + (-7-5)^2} \\
 &= \sqrt{5^2 + (-12)^2} \\
 &= \sqrt{25+144} \\
 &= \sqrt{169} \\
 &= 13 \text{ units}
 \end{aligned}$$

(ii) Let the co-ordinates of B be (x , y)

Using mid – point formula, we have

$$-2 = \frac{3+x}{2} \quad \text{and} \quad 5 = \frac{-7+y}{2}$$

$$\Rightarrow -4 = 3+x \quad \text{and} \quad 10 = -7+y$$

$$\Rightarrow x = -7 \quad \text{and} \quad y = 17$$

Thus, the coordinates of B are (-7,17)

Question 9:

Find the co-ordinates of the centroid of a triangle ABC whose vertices are:

A(-1, 3), B(1, -1) and C(5, 1)

Solution 9:

Co- ordinates of the centroid of triangle ABC are

$$\begin{aligned}
 &\left(\frac{-1+1+5}{3}, \frac{3-1+1}{3} \right) \\
 &= \left(\frac{5}{3}, 1 \right)
 \end{aligned}$$

Question 10:

The mid point of the line segment joining (4a, 2b - 3) and (-4, 3b) is (2, -2a). Find the values of a and b.

Solution 10:

It is given that the mid-point of the line-segment joining (4a, 2b - 3) and (-4, 3b) is (2, -2a).

$$\therefore (2, -2a) = \left(\frac{4a-4}{2}, \frac{2b-3+3b}{2} \right)$$

$$\Rightarrow 2 = \left(\frac{4a-4}{2} \right)$$

$$\Rightarrow 4a - 4 = 4$$

$$\Rightarrow 4a = 8$$

$$\Rightarrow a = 2$$

Also,

$$-2a = \frac{2b - 3 + 3b}{2}$$

$$\Rightarrow -2 \times 2 = \frac{5b - 3}{2}$$

$$\Rightarrow 5b - 3 = -8$$

$$\Rightarrow 5b = -5$$

$$\Rightarrow b = -1$$

Question 11:

The mid point of the line segment joining $(2a, 4)$ and $(-2, 2b)$ is $(1, 2a + 1)$. Find the values of a and b .

Solution 11:

Mid-point of $(2a, 4)$ and $(-2, 2b)$ is $(1, 2a + 1)$, therefore using mid-point formula, we have:

$$x = \frac{x_1 + x_2}{2} \quad y = \frac{y_1 + y_2}{2}$$

$$1 = \frac{2a - 2}{2} \quad 2a + 1 = \frac{4 + 2b}{2}$$

$$1 = a - 1$$

$$\therefore a = 2 \quad 2a + 1 = 2 + b$$

Putting, $a = 2$ in $2a + 1 = 2 + b$, we get,

$$5 - 2 = b \Rightarrow b = 3$$

Therefore, $a = 2$, $b = 3$.

Question 12:

(i) write down the co-ordinates of the point P that divides the line joining $A(-4, 1)$ and $B(17, 10)$ in the ratio $1 : 2$.

(ii) Calculate the distance OP , where O is the origin.

(iii) In what ratio does the Y -axis divide the line AB ?

Solution 12:

(i) Co-ordinates of point P are

$$\begin{aligned} & \left(\frac{1 \times 17 + 2 \times (-4)}{1+2}, \frac{1 \times 10 + 2 \times 1}{1+2} \right) \\ &= \left(\frac{17-8}{3}, \frac{10+2}{3} \right) \\ &= \left(\frac{9}{3}, \frac{12}{3} \right) \\ &= (3, 4) \end{aligned}$$

$$(ii) \text{ OP} = \sqrt{(0-3)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

(iii) Let AB be divided by the point P(0, y) lying on y-axis in the ratio k : 1

$$\therefore (0, y) = \left(\frac{k \times 17 + 1 \times (-4)}{k+1}, \frac{k \times 10 + 1 \times 1}{k+1} \right)$$

$$\Rightarrow (0, y) = \left(\frac{17k-4}{k+1}, \frac{10k+1}{k+1} \right)$$

$$\Rightarrow 0 = \frac{17k-4}{k+1}$$

$$\Rightarrow 17k-4=0$$

$$\Rightarrow k = \frac{4}{17}$$

Thus, the ratio in which the y-axis divide the line AB is 4: 17.

Question 13:

Prove that the points A(-5, 4); B(-1, -2) and C(5, 2) are the vertices of an isosceles right angled triangle. Find the co-ordinates of D so that ABCD is a square.

Solution 13:

We have:

$$AB = \sqrt{(-1+5)^2 + (-2-4)^2} = \sqrt{16+36} = \sqrt{52}$$

$$BC = \sqrt{(-1+5)^2 + (-2-2)^2} = \sqrt{36+16} = \sqrt{52}$$

$$AC = \sqrt{(5+5)^2 + (2-4)^2} = \sqrt{100+4} = \sqrt{104}$$

$$AB^2 + BC^2 = 52 + 52 = 104$$

$$AC^2 = 104$$

$$\therefore AB = BC \text{ and } AB^2 + BC^2 = AC^2$$

\therefore ABC is an isosceles right-angled triangle.

Let the coordinates of D be (x, y).

If ABCD is a square, then,

Mid-point of AC = Mid-point of BD

$$\left(\frac{-5+5}{2}, \frac{4+2}{2}\right) = \left(\frac{x-1}{2}, \frac{y-2}{2}\right)$$

$$0 = \frac{x-1}{2}, 3 = \frac{y-2}{2}$$

$$x = 1, y = 8$$

Thus, the co-ordinates of point D are (1, 8).

Question 14:

M is the mid-point of the line segment joining the points A(-3, 7) and B(9, -1). Find the co-ordinates of point M. Further, if R(2, 2) divides the line segment joining M and the origin in the ratio p : q, find the ratio p : q

Solution 14:

Given, M is the mid-point of the line segment joining the points A (-3, 7) and B (9, -1).
The co-ordinates of point M are

$$\left(\frac{-3+9}{2}, \frac{7-1}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{6}{2}\right)$$

$$= (3, 3)$$

Also, given that, R (2, 2) divides the line segment joining M and the origin in the ratio p : q.

$$\therefore (2, 2) = \left(\frac{p \times 0 + q \times 3}{p + q}, \frac{p \times 0 + q \times 3}{p + q}\right)$$

$$\Rightarrow \frac{p \times 0 + q \times 3}{p + q} = 2$$

$$\Rightarrow \frac{3q}{p + q} = 2$$

$$\Rightarrow 3q = 2p + 2q$$

$$\Rightarrow 3q - 2q = 2p$$

$$\Rightarrow q = 2p$$

$$\Rightarrow \frac{p}{q} = \frac{1}{2}$$

Thus the ratio p : q is 1 : 2.