

EXERCISE**Question 1:**

Find, which of the following points lie on the line $x - 2y + 5 = 0$

- (i) (1, 3) (ii) (0, 5) (iii) (-5, 0) (iv) (5, 5) (v) (2, -1.5) (vi) (-2, -1.5)

Solution 1:

The given line is $x - 2y + 5 = 0$.

- (i) Substituting $x = 1$ and $y = 3$ in the given equation, we have:

$$\begin{aligned}\text{L.H.S.} &= 1 - 2 \times 3 + 5 \\ &= 1 - 6 + 5 \\ &= 6 - 6 \\ &= 0 \\ &= \text{R.H.S.}\end{aligned}$$

Thus, the point (1, 3) lies on the given line.

- (ii) Substituting $x = 0$ and $y = 5$ in the given equation, we have:

$$\begin{aligned}\text{L.H.S.} &= 0 - 2 \times 5 + 5 \\ &= -10 + 5 \\ &= -5 \neq \text{R.H.S.}\end{aligned}$$

Thus, the point (0, 5) does not lie on the given line.

- (iii) Substituting $x = -5$ and $y = 0$ in the given equation, we have:

$$\begin{aligned}\text{L.H.S.} &= -5 - 2 \times 0 + 5 \\ &= -5 - 0 + 5 \\ &= 5 - 5 \\ &= 0 = \text{R.H.S.}\end{aligned}$$

Thus, the point (-5, 0) lie on the given line.

- (iv) Substituting $x = 5$ and $y = 5$ in the given equation, we have:

$$\begin{aligned}\text{L.H.S.} &= 5 - 2 \times 5 + 5 \\ &= 5 - 10 + 5 \\ &= 10 - 10 \\ &= 0 = \text{R.H.S.}\end{aligned}$$

Thus, the point (5, 5) lies on the given line.

- (v) Substituting $x = 2$ and $y = -1.5$ in the given equation, we have:

$$\begin{aligned}\text{L.H.S.} &= 2 - 2 \times (-1.5) + 5 \\ &= 2 + 3 + 5 \\ &= 10 \neq \text{R.H.S.}\end{aligned}$$

Thus, the point (2, -1.5) does not lie on the given line.

- (vi) Substituting $x = -2$ and $y = -1.5$ in the given equation, we have:

$$\text{L.H.S.} = -2 - 2 \times (-1.5) + 5$$

$$= -2 + 3 + 5$$

$$= 6 \neq \text{R.H.S.}$$

Thus, the point $(-2, -1.5)$ does not lie on the given line.

Question 2:

State, true or false:

(i) the line $\frac{x}{2} + \frac{y}{3} = 0$ passes through the point $(2, 3)$

(ii) the line $\frac{x}{2} + \frac{y}{3} = 0$ passes through the point $(4, -6)$

(iii) the point $(8, 7)$ lies on the line $y - 7 = 0$

(iv) the point $(-3, 0)$ lies on the line $x + 3 = 0$

(v) if the point $(2, a)$ lies on the line $2x - y = 3$, then $a = 5$.

Solution 2:

(i) The given line is $\frac{x}{2} + \frac{y}{3} = 0$

Substituting $x = 2$ and $y = 3$ in the given equation,

$$\text{L.H.S} = \frac{x}{2} + \frac{y}{3} = 1 + 1 = 2 \neq \text{R.H.S}$$

Thus, the given statement is false.

(ii) The given line is $\frac{x}{2} + \frac{y}{3} = 0$

Substituting $x = 4$ and $y = -6$ in the given equation,

$$\text{L.H.S.} = \frac{4}{2} + \frac{-6}{3} = 2 - 2 = 0 = \text{R.H.S}$$

Thus, the given statement is true.

(iii) $\text{L.H.S} = y - 7 = 7 - 7 = 0 = \text{R.H.S.}$

Thus, the point $(8, 7)$ lies on the line $y - 7 = 0$.

The given statement is true.

(iv) $\text{L.H.S.} = x + 3 = -3 + 3 = 0 = \text{R.H.S}$

Thus, the point $(-3, 0)$ lies on the line $x + 3 = 0$.

The given statement is true.

(v) The point $(2, a)$ lies on the line $2x - y = 3$.

$$\therefore 2(2) - a = 3$$

$$4 - a = 3$$

$$a = 4 - 3 = 1$$

Thus, the given statement is false.

Question 3:

The line given by the equation $2x - \frac{y}{3} = 7$ passes through the point $(k, 6)$; calculate the value of k .

Solution 3:

Given, the line given by the equation $2x - \frac{y}{3} = 7$ passes through the point $(k, 6)$.

Substituting $x = k$ and $y = 6$ in the given equation, we have:

$$2x - \frac{6}{3} = 7$$

$$2k - 2 = 7$$

$$2k = 9$$

$$k = \frac{9}{2} = 4.5$$

Question 4:

For what value of k will the point $(3, -k)$ lie on the line $9x + 4y = 3$?

Solution 4:

The given equation of the line is $9x + 4y = 3$.

Put $x = 3$ and $y = -k$, we have:

$$9(3) + 4(-k) = 3$$

$$27 - 4k = 3$$

$$4k = 27 - 3 = 24$$

$$k = 6$$

Question 5:

The line $\frac{3x}{5} - \frac{2y}{3} + 1 = 0$ contains the point $(m, 2m - 1)$; calculate the value of m .

Solution 5:

The equation of the given line is $\frac{3x}{5} - \frac{2y}{3} + 1 = 0$

Putting $x = m$, $y = 2m - 1$, we have:

$$\frac{3m}{5} - \frac{2(2m-1)}{3} + 1 = 0$$

$$\frac{3m}{5} - \frac{4m-2}{3} = -1$$

$$\frac{9m - 20m + 10}{15} = -1$$

$$9m - 20m + 10 = -15$$

$$-11m = -25$$

$$m = \frac{25}{11} = 2\frac{3}{11}$$

Question 6:

Does the line $3x - 5y = 6$ bisect the join of $(5, -2)$ and $(-1, 2)$?

Solution 6:

The given line will bisect the join of A $(5, -2)$ and B $(-1, 2)$, if the co-ordinates of the mid-point of AB satisfy the equation of the line.

The co-ordinates of the mid-point of AB are

$$\left(\frac{5-1}{2}, \frac{-2+2}{2}\right) = (2, 0)$$

Substituting $x = 2$ and $y = 0$ in the given equation, we have:

$$\begin{aligned}\text{L.H.S.} &= 3x - 5y \\ &= 3(2) - 5(0) \\ &= 6 - 0 \\ &= 6 = \text{R.H.S.}\end{aligned}$$

Hence, the line $3x - 5y = 6$ bisect the join of $(5, -2)$ and $(-1, 2)$.

Question 7:

(i) the line $y = 3x - 2$ bisects the join of $(a, 3)$ and $(2, -5)$, Find the value of a .

(ii) the line $x - 6y + 11 = 0$ bisects the join of $(8, -1)$ and $(0, k)$. Find the value of k .

Solution 7:

(i) The given line bisects the join of A $(a, 3)$ and B $(2, -5)$, so the co-ordinates of the mid-point of AB will satisfy the equation of the line.

The co-ordinates of the mid-point of AB are

$$\left(\frac{a+2}{2}, \frac{3-5}{2}\right) = \left(\frac{a+2}{2}, -1\right)$$

Substituting $x = \frac{a+2}{2}$ and $y = -1$ in the given equation, we have:

$$y = 3x - 2$$

$$-1 = 3 \times \frac{a+2}{2} - 2$$

$$3 \times \frac{a+2}{2} = 1$$

$$a+2 = \frac{2}{3}$$

$$a = \frac{2}{3} - 2 = \frac{2-6}{3} = \frac{-4}{3}$$

- (ii) The given line bisects the join of A (8, -1) and B (0, k), so the co-ordinates of the mid-point of AB will satisfy the equation of the line.

The co-ordinates of the mid-point of AB are

$$\left(\frac{8+0}{2}, \frac{-1+k}{2} \right) = \left(4, \frac{-1+k}{2} \right)$$

Substituting $x = 4$ and $y = \frac{-1+k}{2}$ in the given equation, we have:

$$x - 6y + 11 = 0$$

$$4 - 6\left(\frac{-1+k}{2}\right) + 11 = 0$$

$$6\left(\frac{-1+k}{2}\right) = 15$$

$$\frac{-1+k}{2} = \frac{15}{6}$$

$$\frac{-1+k}{2} = \frac{5}{2}$$

$$-1+k = 5$$

$$k = 6$$

Question 8:

- (i) the point (-3, 2) lies on the line $ax + 3y + 6 = 0$, calculate the value of a.
(ii) The line $y = mx + 8$ contains the point (-4, 4), calculate the value of m.

Solution 8:

- (i) Given, the point (-3, 2) lies on the line $ax + 3y + 6 = 0$.

Substituting $x = -3$ and $y = 2$ in the given equation, we have:

$$a(-3) + 3(2) + 6 = 0$$

$$-3a + 12 = 0$$

$$3a = 12$$

$$a = 4$$

- (ii) Given, the line $y = mx + 8$ contains the point (-4, 4).

Substituting $x = -4$ and $y = 4$ in the given equation, we have:

$$4 = -4m + 8$$

$$4m = 4 = m = 1$$

Question 9:

The point P divides the join of (2, 1) and (−3, 6) in the ratio 2 : 3. Does P lies on the line $x - 5y + 15 = 0$?

Solution 9:

Given, the point P divides the join of (2, 1) and (−3, 6) in the ratio 2 : 3.

Co-ordinates of the point P are

$$\left(\frac{2 \times (-3) + 3 \times 2}{2 + 3}, \frac{2 \times 6 + 3 \times 1}{2 + 3} \right)$$

$$= \left(\frac{-6 + 6}{5}, \frac{12 + 3}{5} \right)$$

$$= (0, 3)$$

Substituting $x = 0$ and $y = 3$ in the given equation, we have:

$$\text{L.H.S.} = 0 - 5(3) + 15$$

$$= -15 + 15$$

$$= 0 = \text{R.H.S.}$$

Hence, the point P lies on the line $x - 5y + 15 = 0$.

Question 10:

The line segment joining the points (5, −4) and (2, 2) is divided by the points Q in the ratio 1:2 Does the line $x - 2y = 0$ contain Q?

Solution 10:

Given, the line segment joining the points (5, −4) and (2, 2) is divided by the point Q in the ratio 1: 2.

Co-ordinates of the point Q are

$$\left(\frac{1 \times 2 + 2 \times 5}{1 + 2}, \frac{1 \times 2 + 2 \times (-4)}{1 + 2} \right)$$

$$= \left(\frac{2 + 10}{3}, \frac{2 - 8}{3} \right)$$

$$= (4, -2)$$

Substituting $x = 4$ and $y = -2$ in the given equation, we have:

$$\text{L.H.S.} = x - 2y$$

$$= 4 - 2(-2)$$

$$= 4 + 4$$

$$= 8 \neq \text{R.H.S.}$$

Hence, the given line does not contain point Q.

Question 11:

Find the point of intersection of the lines: $4x + 3y = 1$ and $3x - y + 9 = 0$. If this point lies on the line $(2k - 1)x - 2y = 4$; find the value of k .

Solution 11:

Consider the given equations:

$$4x + 3y = 1 \dots(1)$$

$$3x - y + 9 = 0 \dots(2)$$

Multiplying (2) with 3, we have:

$$9x - 3y = -27 \dots(3)$$

Adding (1) and (3), we get,

$$13x = -26$$

$$x = -2$$

$$\text{From (2), } y = 3x + 9 = -6 + 9 = 3$$

Thus, the point of intersection of the given lines (1) and (2) is $(-2, 3)$.

The point $(-2, 3)$ lies on the line $(2k - 1)x - 2y = 4$.

$$(2k - 1)(-2) - 2(3) = 4$$

$$-4k + 2 - 6 = 4$$

$$-4k = 8$$

$$k = -2$$

Question 12:

Show that the lines $2x + 5y = 1$, $x - 3y = 6$ and $x + 5y + 2 = 0$ are concurrent.

Solution 12:

We know that two or more lines are said to be concurrent if they intersect at a single point.

We first find the point of intersection of the first two lines.

$$2x + 5y = 1 \dots(1)$$

$$x - 3y = 6 \dots(2)$$

Multiplying (2) by 2, we get,

$$2x - 6y = 12 \dots(3)$$

Subtracting (3) from (1), we get,

$$11y = -11$$

$$y = -1$$

$$\text{From (2), } x = 6 + 3y = 6 - 3 = 3$$

So, the point of intersection of the first two lines is $(3, -1)$.

If this point lie on the third line, i.e., $x + 5y + 2 = 0$, then the given lines will be concurrent.

Substituting $x = 3$ and $y = -1$, we have:

$$\text{L.H.S.} = x + 5y + 2$$

$$= 3 + 5(-1) + 2$$

$$= 5 - 5$$

$$= 0 = \text{R.H.S.}$$

Thus, $(3, -1)$ also lie on the third line.

Hence, the given lines are concurrent.

EXERCISE. 14 (B)**Question 1:**

Find the slope of the line whose inclination is:

- (i) 0° (ii) 30° (iii) $72^\circ 30'$ (iv) 46°

Solution 1:

(i) Slope = $\tan 0^\circ = 0$

(ii) Slope = $\tan 30^\circ = \frac{1}{\sqrt{3}}$

(iii) Slope = $\tan 72^\circ 30' = 3.1716$

(iv) Slope = $\tan 46^\circ = 1.0355$

Question 2:

Find the inclination of the line whose slope is:

- (i) 0 (ii) $\sqrt{3}$ (iii) 0.7646 (iv) 1.0875

Solution 2:

(i) Slope = $\tan \theta = 0$
 $\Rightarrow \theta = 0^\circ$

(ii) Slope = $\tan \theta = \sqrt{3}$
 $\Rightarrow \theta = 60^\circ$

(iii) Slope = $\tan \theta = 0.7646$
 $\Rightarrow \theta = 37^\circ 24'$

(iv) Slope = $\tan \theta = 1.0875$
 $\Rightarrow \theta = 47^\circ 24'$

Question 3:

Find the slope of the line passing through the following pairs of points:

- (i) $(-2, -3)$ and $(1, 2)$
(ii) $(-4, 0)$ and origin
(iii) $(a, -b)$ and $(b, -a)$

Solution 3:

We know:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

(i) Slope = $\frac{2 - (-3)}{1 - (-2)} = \frac{5}{3}$

$$(ii) \text{ Slope} = \frac{0 - 0}{0 + 4} = \frac{0}{4} = 0$$

$$(iii) \text{ Slope} = \frac{-a + b}{b - a} = 1$$

Question 4:

Find the slope of the line parallel to AB if:

(i) $A = (-2, 4)$ and $B = (0, 6)$

(ii) $A = (0, -3)$ and $B = (-2, 5)$

Solution 4:

$$(i) \text{ Slope of AB} = \frac{6 - 4}{0 + 2} = \frac{2}{2} = 1$$

Slope of the line parallel to AB = Slope of AB = 1

$$(ii) \text{ Slope of AB} = \frac{5 + 3}{-2 - 0} = \frac{8}{-2} = -4$$

Slope of the line parallel to AB = Slope of AB = -4

Question 5:

Find the slope of the line perpendicular to AB if:

(i) $A = (0, -5)$ and $B = (-2, 4)$

(ii) $A = (3, -2)$ and $B = (-1, 2)$

Solution 5:

$$(i) \text{ Slope of AB} = \frac{4 - (-5)}{-2 - 0} = \frac{9}{-2} = -\frac{9}{2}$$

$$\text{Slope of the line perpendicular to AB} = \frac{-1}{\text{Slope of AB}} = \frac{-1}{-\frac{9}{2}} = \frac{2}{9}$$

$$(ii) \text{ Slope of AB} = \frac{2 - (-2)}{-1 - 3} = \frac{4}{-4} = -1$$

$$\text{Slope of the line perpendicular to AB} = \frac{-1}{\text{Slope of AB}} = \frac{-1}{-1} = 1$$

Question 6:

The line passing through (0, 2) and (-3, -1) is parallel to the line passing through (-1, 5) and (4, a), Find a.

Solution 6:

$$\text{Slope of the line passing through (0, 2) and (-3, -1)} = \frac{-1-2}{-3-0} = \frac{-3}{-3} = 1$$

$$\text{Slope of the line passing through (-1, 5) and (4, a)} = \frac{a-5}{4+1} = \frac{a-5}{5}$$

Since, the lines are parallel.

$$\therefore 1 = \frac{a-5}{5}$$

$$a - 5 = 5$$

$$a = 10$$

Question 7:

The line passing through (-4, -2) and (2, -3) is perpendicular to the line passing through (a, 5) and (2, -1). Find a.

Solution 7:

$$\text{Slope of the line passing through (-4, -2) and (2, -3)} = \frac{-3+2}{2+4} = \frac{-1}{6}$$

$$\text{Slope of the line passing through (a, 5) and (2, -1)} = \frac{-1-5}{2-a} = \frac{-6}{2-a}$$

Since, the lines are perpendicular.

$$\therefore \frac{-1}{6} = \frac{-1}{-6} \cdot \frac{-6}{2-a}$$

$$\frac{-1}{6} = \frac{2-a}{6}$$

$$2 - a = -1$$

$$a = 3$$

Question 8:

Without using the distance formula, show that the points A (4, -2), B (-4, 4) and C (10, 6) are the vertices of a right angled triangle.

Solution 8:

The given points are A (4, -2), B (-4, 4) and C (10, 6).

$$\text{Slope of AB} = \frac{4 + 2}{-4 - 4} = \frac{6}{-8} = \frac{-3}{4}$$

$$\text{Slope of BC} = \frac{6 - 4}{10 + 4} = \frac{2}{14} = \frac{1}{7}$$

$$\text{Slope of AC} = \frac{6 + 2}{10 - 4} = \frac{8}{6} = \frac{4}{3}$$

It can be seen that:

$$\text{Slope of AB} = \frac{-1}{\text{Slope of AC}}$$

Hence, $AB \perp AC$

Thus, the given points are the vertices of a right – angled triangle.

Question 9:

Without using the distance formula, show that the points A (4, 5), B (1, 2), C (4, 3) and D (7, 6) are the vertices of a parallelogram.

Solution 9:

The given points are A (4, 5), B (1, 2), C (4, 3) and D (7, 6).

$$\text{Slope of AB} = \frac{2 - 5}{1 - 4} = \frac{-3}{-3} = 1$$

$$\text{Slope of CD} = \frac{6 - 3}{7 - 4} = \frac{3}{3} = 1$$

Since, slope of AB = slope of CD

Therefore, $AB \parallel CD$

$$\text{Slope of BC} = \frac{3 - 2}{4 - 1} = \frac{1}{3}$$

$$\text{Slope of DA} = \frac{5 - 6}{4 - 7} = \frac{-1}{-3} = \frac{1}{3}$$

Since, slope of BC = slope of DA

Therefore, $BC \parallel DA$

Hence, ABCD is a parallelogram

Question 10:

(-2, 4), B (4, 8), C (10, 7) and D (11, -5) are the vertices of a quadrilateral. Show that the quadrilateral, obtained on joining the mid-points of its sides, is a parallelogram.

Solution 10:

Let the given points be A (-2, 4), B (4, 8), C (10, 7) and D (11, -5).

Let P, Q, R and S be the mid-points of AB, BC, CD and DA respectively.

$$\text{Co-ordinates of P are } \left(\frac{-2+4}{2}, \frac{4+8}{2} \right) = (1, 6)$$

$$\text{Co-ordinates of Q are } \left(\frac{4+10}{2}, \frac{8+7}{2} \right) = \left(7, \frac{15}{2} \right)$$

$$\text{Co-ordinates of R are } \left(\frac{10+11}{2}, \frac{7-5}{2} \right) = \left(\frac{21}{2}, 1 \right)$$

$$\text{Co-ordinates of S are } \left(\frac{11-2}{2}, \frac{-5+4}{2} \right) = \left(\frac{9}{2}, -\frac{1}{2} \right)$$

$$\text{Slope of PQ} = \frac{\frac{15}{2} - 6}{7 - 1} = \frac{\frac{15-12}{2}}{6} = \frac{3}{12} = \frac{1}{4}$$

$$\text{Slope of RS} = \frac{-\frac{1}{2} - 1}{\frac{9}{2} - \frac{21}{2}} = \frac{\frac{-1-2}{2}}{\frac{9-21}{2}} = \frac{-3}{-12} = \frac{1}{4}$$

Since, slope of PQ = Slope of RS, PQ $\parallel$ RS.

$$\text{Slope of QR} = \frac{1 - \frac{15}{2}}{\frac{21}{2} - 7} = \frac{\frac{2-15}{2}}{\frac{21-14}{2}} = \frac{-13}{7}$$

$$\text{Slope of SP} = \frac{6 + \frac{1}{2}}{1 - \frac{9}{2}} = \frac{\frac{12+1}{2}}{\frac{2-9}{2}} = \frac{13}{-7} = \frac{-13}{7}$$

Since, slope of QR = Slope of SP, QR $\parallel$ SP.

Hence, PQRS is a parallelogram.

Question 11:

Show that the points P (a, b + c), Q (b, c + a) and R (c, a + b) are collinear.

Solution 11:

The points P, Q, R will be collinear if slope of PQ and QR is the same.

$$\text{Slope of PQ} = \frac{c + a - b - c}{b - c} = \frac{a - b}{b - a} = -1$$

$$\text{Slope of QR} = \frac{a + b - c - a}{c - b} = \frac{b - c}{c - b} = -1$$

Hence, the points P, Q, and R are collinear.

Question 12:

Find x, if the slope of the line joining (x, 2) and (8, -11) is $-\frac{3}{4}$

Solution 12:

Let A = (x, 2) and B = (8, -11)

$$\text{Slope of AB} = \frac{-11 - 2}{8 - x}$$

$$\frac{-11 - 2}{8 - x} = \frac{-3}{4} \quad (\text{Given})$$

$$\frac{13}{8 - x} = \frac{3}{4}$$

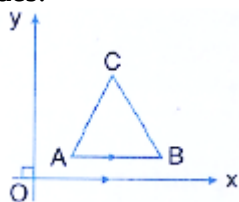
$$52 = 24 - 3x$$

$$3x = 24 - 52 = -28$$

$$x = \frac{-28}{3}$$

Question 13:

The side AB of an equilateral triangle ABC is parallel to the x-axis. Find the slopes of all its sides.



Solution 13:

We know that the slope of any line parallel to x-axis is 0.

Therefore, slope of AB = 0

Since, ABC is an equilateral triangle, $\angle A = 60^\circ$

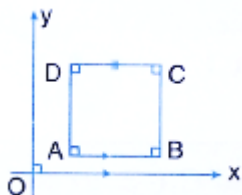
$$\text{Slope of AC} = \tan 60^\circ = \sqrt{3}$$

$$\text{Slope of BC} = -\tan 60^\circ = -\sqrt{3}$$

Question 14:

The side AB of a square ABCD is parallel to x-axis. Find the slopes of all its sides. Also, find:

- (i) the slope of the diagonal AC.
- (ii) the slope of the diagonal BD.

**Solution 14:**

We know that the slope of any line parallel to x-axis is 0.
Therefore, slope of AB = 0

As CD $\parallel$ AB, slope of CD = Slope of AB = 0

As BC $\perp$ AB, slope of BC = $-\frac{1}{\text{slope of AB}} = \frac{-1}{0}$ = not defined

As AD $\perp$ AB, slope of AD = $-\frac{1}{\text{slope of AB}} = \frac{-1}{0}$ = not defined

(i) The diagonal AC makes an angle of 45° with the positive direction of x axis.
 $\therefore$ Slope of AC = $\tan 45^\circ = 1$

(ii) The diagonal BC makes an angle of -45° with the positive direction of x axis.
 $\therefore$ Slope of BC = $\tan (-45^\circ) = -1$

Question 15:

A (5, 4), B (-3, -2) and C (1, -8) are the vertices of a triangle ABC. Find:

- (i) the slope of the altitude of AB.
- (ii) the slope of the median AD and
- (iii) the slope of the line parallel to AC.

Solution 15:

Given, A (5, 4), B (-3, -2) and C (1, -8) are the vertices of a triangle ABC.

(i) Slope of AB = $\frac{-2-4}{-3-5} = \frac{-6}{-8} = \frac{3}{4}$

Slope of the altitude of AB = $\frac{-1}{\text{slope of AB}} = \frac{-1}{\frac{3}{4}} = \frac{-4}{3}$

(ii) Since, D is the mid-point of BC.

Co-ordinates of point D are $\left(\frac{-3+1}{2}, \frac{-2-8}{2}\right) = (-1, -5)$

$$\text{Slope of AD} = \frac{-5-4}{-1-5} = \frac{-9}{-6} = \frac{3}{2}$$

$$\text{(iii) Slope of AC} = \frac{-8-4}{1-5} = \frac{-12}{-4} = 3$$

Slope of line parallel to AC = Slope of AC = 3

Question 16:

The slope of the side BC of a rectangle ABCD is $\frac{2}{3}$ Find:

(i) the slope of the side AB.

(ii) the slope of the side AD.

Solution 16:

(i) Since, BC is perpendicular to AB,

$$\text{Slope of AB} = \frac{-1}{\text{slope of BC}} = \frac{-1}{\frac{2}{3}} = \frac{-3}{2}$$

(ii) Since, AD is parallel to BC,

$$\text{Slope of AD} = \text{Slope of BC} = \frac{2}{3}$$

Question 17:

Find the slope and the inclination of the line AB if:

(i) A = (-3, -2) and B = (1, 2)

(ii) A = (0, $-\sqrt{3}$) and B = (3, 0)

(iii) A = (-1, $2\sqrt{3}$) and B = (-2, $\sqrt{3}$)

Solution 17:

(i) A = (-3, -2) and B = (1, 2)

$$\text{Slope of AB} = \frac{2+2}{1+3} = \frac{4}{4} = 1 = \tan \theta$$

$$\text{Inclination of line AB} = \theta = 45^\circ$$

(ii) A = (0, $\sqrt{3}$) and B = (3, 0)

$$\text{Slope of AB} = \frac{0 + \sqrt{3}}{3 - 0} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \tan \theta$$

$$\text{Inclination of line AB} = \theta = 30^\circ$$

$$\text{(iii) A} = (-1, 2\sqrt{3}) \text{ and B} = (-2, \sqrt{3})$$

$$\text{Slope of AB} = \frac{\sqrt{3} - 2\sqrt{3}}{-2 + 1} = \frac{-\sqrt{3}}{-1} = \sqrt{3} = \tan \theta$$

$$\text{Inclination of line AB} = \theta = 60^\circ$$

Question 18:

The points $(-3, 2)$, $(2, -1)$ and $(a, 4)$ are collinear. Find a .

Solution 18:

Given, points A $(-3, 2)$, B $(2, -1)$ and C $(a, 4)$ are collinear.

$\therefore$ Slope of AB = Slope of BC

$$\frac{-1 - 2}{2 + 3} = \frac{4 + 1}{a - 2}$$

$$\frac{-3}{5} = \frac{5}{a - 2}$$

$$-3a + 6 = 25$$

$$-3a = 25 - 6 = 19$$

$$a = \frac{-19}{3} = -6\frac{1}{3}$$

Question 19:

The points $(K, 3)$, $(2, -4)$ and $(-K + 1, -2)$ are collinear. Find K .

Solution 19:

Given, points A $(K, 3)$, B $(2, -4)$ and C $(-K + 1, -2)$ are collinear.

$\therefore$ Slope of AB = Slope of BC

$$\frac{-4 - 3}{2 - k} = \frac{-2 + 4}{-k + 1 - 2}$$

$$\frac{-7}{2 - k} = \frac{2}{-k - 1}$$

$$7k + 7 = 4 - 2k$$

$$9k = -3$$

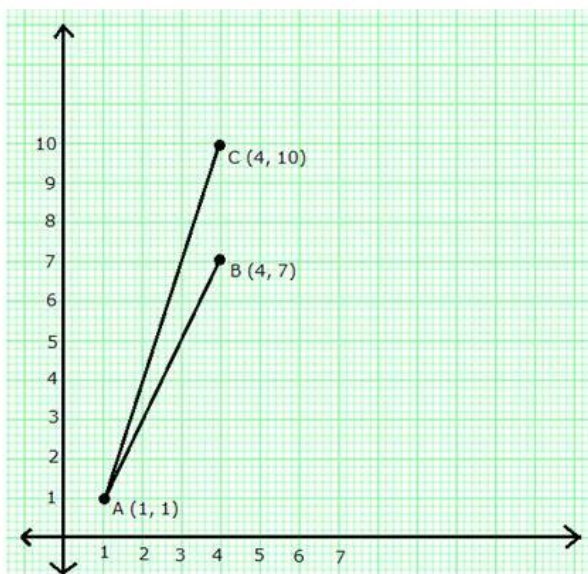
$$k = \frac{-1}{3}$$

Question 20:

Plot the points A (1, 1), B (4, 7) and C(4, 10) on a graph paper. Connect A and B and also A and C.

Which segment appears to have the steeper slope, AB or AC?

Justify your conclusion by calculating the slopes of AB and AC.

Solution 20:

From the graph, clearly, AC has steeper slope.

$$\text{Slope of AB} = \frac{7-1}{4-1} = \frac{6}{3} = 2$$

$$\text{Slope of AC} = \frac{10-1}{4-1} = \frac{9}{3} = 3$$

The line with greater slope is steeper. Hence, AC has steeper slope.

Question 21:

Find the value(s) of k so that PQ will be parallel to RS. Given:

- (i) P (2, 4), Q (3, 6), R (8, 1) and S (10, k)
- (ii) P (3, -1), Q (7, 11), R (-1, -1) and S (1, k)
- (iii) P (5, -1), Q (6, 11), R (6, -4k) and S (7, k²)

Solution 21:

Since, PQ $\parallel$ RS,

Slope of PQ = Slope of RS

$$(i) \text{ Slope of PQ} = \frac{6-4}{3-2} = 2$$

$$\text{Slope of RS} = \frac{k-1}{10-8} = \frac{k-1}{2}$$

$$\therefore 2 = \frac{k-1}{2}$$

$$k-1 = 4$$

$$k = 5$$

$$\text{(ii) Slope of PQ} = \frac{11+1}{7-3} = \frac{12}{4} = 3$$

$$\text{Slope of RS} = \frac{k+1}{1+1} = \frac{k+1}{2}$$

$$\therefore 3 = \frac{k+1}{2}$$

$$k+1 = 6$$

$$k = 5$$

$$\text{(iii) Slope of PQ} = \frac{11+1}{6-5} = \frac{12}{1} = 12$$

$$\text{Slope of RS} = \frac{k^2+4k}{7-6} = k^2+4k$$

$$\therefore 12 = k^2 + 4k$$

$$k^2 + 4k - 12 = 0$$

$$(k+6)(k-2) = 0$$

$$k = -6 \text{ and } 2$$

EXERCISE. 14 (C)

Question 1:

Find the equation of a line whose:

y- intercept = 2 and slope = 3

Solution 1:

Given, y-intercept = c = 2 and slope = m = 3.

Substituting the values of c and m in the equation $y = mx + c$, we get,

$y = 3x + 2$, which is the required equation.

Question 2:

Find the equation of a line whose:

y – intercept = -1 and inclination = 45°

Solution 2:

Given, y-intercept = $c = -1$ and inclination = 45° .

Slope = $m = \tan 45^\circ = 1$

Substituting the values of c and m in the equation $y = mx + c$, we get,
 $y = x - 1$, which is the required equation.

Question 3:

Find the equation of the line whose slope is $-\frac{4}{3}$ and which passes through $(-3, 4)$

Solution 3:

Given, slope = $-\frac{4}{3}$

The equation passes through $(-3, 4) = (x_1, y_1)$

Substituting the values in $y - y_1 = m(x - x_1)$, we get,

$$y - 4 = -\frac{4}{3}(x + 3)$$

$$3y - 12 = -4x - 12$$

$4x + 3y = 0$, which is the required equation.

Question 4:

Find the equation of a line which passes through $(5, 4)$ and makes an angle of 60° with the positive direction of the x-axis.

Solution 4:

Slope of the line = $\tan 60^\circ = \sqrt{3}$

The line passes through the point $(5, 4) = (x_1, y_1)$

Substituting the values in $y - y_1 = m(x - x_1)$, we get,

$$y - 4 = \sqrt{3}(x - 5)$$

$$y - 4 = \sqrt{3}x - 5\sqrt{3}$$

$y = \sqrt{3}x + 4 - 5\sqrt{3}$, which is the required equation.

Question 5:

Find the equation of the line passing through:

- (i) (0, 1) and (1, 2) (ii) (-1, -4) and (3, 0)

Solution 5:

- (i) Let (0, 1) = (x₁, y₁) and (1, 2) = (x₂, y₂)

$$\therefore \text{slope of the line} = \frac{2-1}{1-0} = 1$$

The required equation of the line is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 1(x - 0)$$

$$y - 1 = x$$

$$y = x + 1$$

- (ii) Let (-1, -4) = (x₁, y₁) and (3, 0) = (x₂, y₂)

$$\therefore \text{slope of the line} = \frac{0+4}{3+1} = \frac{4}{4} = 1$$

The required equation of the line is given by:

$$y - y_1 = m(x - x_1)$$

$$y + 4 = 1(x + 1)$$

$$y + 4 = x + 1$$

$$y = x - 3$$

Question 6:

The co-ordinates of two points P and Q are (2, 6) and (-3, 5) respectively Find:

- (i) the gradient of PQ;
(ii) the equation of PQ;
(iii) the co-ordinates of the point where PQ intersects the x-axis.

Solution 6:

Given, co-ordinates of two points P and Q are (2, 6) and (-3, 5) respectively.

$$(i) \text{ Gradient of PQ} = \frac{5-6}{-3-2} = \frac{-1}{-5} = \frac{1}{5}$$

- (ii) The equation of the line PQ is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{1}{5}(x - 2)$$

$$5y - 30 = x - 2$$

$$5y = x + 28$$

- (iii) Let the line PQ intersects the x-axis at point A (x, 0).

Putting y = 0 in the equation of the line PQ, we get,

$$0 = x + 28$$

$$x = -28$$

Thus, the co-ordinates of the point where PQ intersects the x-axis are A (-28, 0).

Question 7:

The co-ordinates of two points A and B are (-3, 4) and (2, -1) Find:

- the equation of AB
- the co-ordinates of the point where the line AB intersects the y-axis.

Solution 7:

- Given, co-ordinates of two points A and B are (-3, 4) and (2, -1).

$$\text{Slope} = \frac{-1-4}{2+3} = \frac{-5}{5} = -1$$

The equation of the line AB is given by:

$$y - y_1 = m(x - x_1)$$

$$y + 1 = -1(x - 2)$$

$$y + 1 = -x + 2$$

$$x + y = 1$$

- Let the line AB intersects the y-axis at point (0, y).

Putting $x = 0$ in the equation of the line, we get,

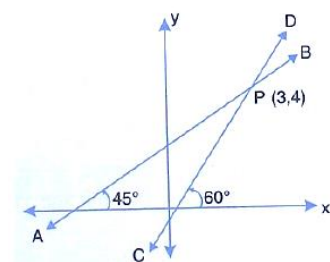
$$0 + y = 1$$

$$y = 1$$

Thus, the co-ordinates of the point where the line AB intersects the y-axis are (0, 1).

Question 8:

The figure given alongside shows two straight lines AB and CD intersecting each other at point P (3, 4). Find the equations of AB and CD.



Solution 8:

$$\text{Slope of line AB} = \tan 45^\circ = 1$$

The line AB passes through P (3, 4). So, the equation of the line AB is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 1(x - 3)$$

$$y - 4 = x - 3$$

$$y = x + 1$$

$$\text{Slope of line CD} = \tan 60^\circ = \sqrt{3}$$

The line CD passes through P (3, 4). So, the equation of the line CD is given by:

$$y - y_1 = m (x - x_1)$$

$$y - 4 = \sqrt{3} (x - 3)$$

$$y - 4 = \sqrt{3} x - 3 \sqrt{3}$$

$$y = \sqrt{3} x + 4 - 3 \sqrt{3}$$

Question 9:

In $\triangle ABC$, A = (3, 5), B = (7, 8) and C = (1, -10). Find the equation of the median through A.

Solution 9:

The vertices of the triangle are given as vertices are A (3, -5), B (1, 2) and C (-7, 4).

$$\text{Slope of AB} = \frac{2+5}{1-3} = \frac{7}{-2} = -\frac{7}{2}$$

The equation of the line AB is given by:

$$y - y_1 = m(x - x_1)$$

$$y + 5 = -\frac{7}{2} (x - 3)$$

$$2y + 10 = -7x + 21$$

$$7x + 2y = 11$$

$$\text{Slope of BC} = \frac{4-2}{-7-1} = \frac{2}{-8} = -\frac{1}{4}$$

The equation of the line BC is given by:

$$y - y_1 = m (x - x_1)$$

$$y - 2 = -\frac{1}{4} (x - 1)$$

$$4y - 8 = -x + 1$$

$$x + 4y = 9$$

$$\text{Slope of AC} = \frac{4+5}{-7-3} = \frac{9}{-10} = -\frac{9}{10}$$

The equation of the line AC is given by:

$$y - y_1 = m (x - x_1)$$

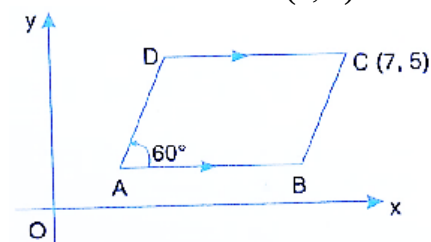
$$y - 4 = -\frac{9}{10} (x + 7)$$

$$10y - 40 = -9x - 63$$

$$9x + 10y + 23 = 0$$

Question 10:

The following figure shows a parallelogram ABCD whose side AB is parallel to the x-axis. $\angle A = 60^\circ$ and vertex $C = (7, 5)$. Find the equations of BC and CD.

**Solution 10:**

Since, ABCD is a parallelogram,

$$\angle A + \angle B = 180^\circ$$

$$\angle B = 180^\circ - 60^\circ = 120^\circ$$

$$\text{Slope of BC} = \tan 120^\circ = \tan (90^\circ + 30^\circ) = \cot 30^\circ = \sqrt{3}$$

Equation of the line BC is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \sqrt{3}(x - 7)$$

$$y - 5 = \sqrt{3}x - 7\sqrt{3}$$

$$y = \sqrt{3}x + 5 - 7\sqrt{3}$$

Since, $CD \parallel AB$ and $AB \parallel x\text{-axis}$, slope of $CD = \text{Slope of } AB = 0$

Equation of the line CD is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 0(x - 7)$$

$$y = 5$$

Question 11:

Find the equation of the straight line passing through origin and the point of intersection of the lines $x + 2y = 7$ and $x - y = 4$.

Solution 11:

The given equations are:

$$x + 2y = 7 \dots(1)$$

$$x - y = 4 \dots(2)$$

Subtracting (2) from (1), we get,

$$3y = 3$$

$$y = 1$$

$$\text{From (2), } x = 4 + y = 4 + 1 = 5$$

The required line passes through (0, 0) and (5, 1).

$$\text{Slope of the line} = \frac{1-0}{5-0} = \frac{1}{5}$$

Required equation of the line is given by:

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 0 = \frac{1}{5}(x - 0)$$

$$\Rightarrow 5y = x$$

$$\Rightarrow x - 5y = 0$$

Question 12:

In triangle ABC, the co-ordinates of vertices A, B and C are (4, 7), (-2, 3) and (0, 1) respectively. Find the equation of median through vertex A.

Also, find the equation of the line through vertex B and parallel to AC.

Solution 12:

Given, the co-ordinates of vertices A, B and C of a triangle ABC are (4, 7), (-2, 3) and (0, 1) respectively.

Let AD be the median through vertex A.

Co-ordinates of the point D are

$$\left(\frac{-2+0}{2}, \frac{3+1}{2} \right)$$

$$(-1, 2)$$

$$\therefore \text{Slope of AD} = \frac{2-7}{-1-4} = \frac{-5}{-5} = 1$$

The equation of the median AD is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 1(x + 1)$$

$$y - 2 = x + 1$$

$$y = x + 3$$

The slope of the line which is parallel to line AC will be equal to the slope of AC.

$$\text{Slope of AC} = \frac{1-7}{0-4} = \frac{-6}{-4} = \frac{3}{2}$$

The equation of the line which is parallel to AC and passes through B is given by:

$$y - 3 = \frac{3}{2}(x + 2)$$

$$2y - 6 = 3x + 6$$

$$2y = 3x + 12$$

Question 13:

A, B and C have co-ordinates (0, 3), (4, 4) and (8, 0) respectively. Find the equation of the line through A and perpendicular to BC.

Solution 13:

$$\text{Slope of BC} = \frac{0-4}{8-4} = \frac{-4}{4} = -1$$

$$\text{Slope of line perpendicular to BC} = \frac{-1}{\text{slope of BC}} = 1$$

The equation of the line through A and perpendicular to BC is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 1(x - 0)$$

$$y - 3 = x$$

$$y = x + 3$$

Question 14:

Find the equation of the perpendicular dropped from the point $(-1, 2)$ onto the line joining the points $(1, 4)$ and $(2, 3)$

Solution 14:

Let $A = (1, 4)$, $B = (2, 3)$, and $C = (-1, 2)$.

$$\text{Slope of AB} = \frac{3-4}{2-1} = -1$$

$$\text{Slope of equation perpendicular to AB} = \frac{-1}{\text{slope of AB}} = 1$$

The equation of the perpendicular drawn through C onto AB is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 1(x + 1)$$

$$y - 2 = x + 1$$

$$y = x + 3$$

Question 15:

Find the equation of the line, whose:

(i) x-intercept = 5 and y-intercept = 3

(ii) x-intercept = -4 and y-intercept = 6

(iii) x-intercept = -8 and y-intercept = -4

Solution 15:

(i) When x-intercept = 5, corresponding point on x-axis is $(5, 0)$

When y-intercept = 3, corresponding point on y-axis is $(0, 3)$.

Let $(x_1, y_1) = (5, 0)$ and $(x_2, y_2) = (0, 3)$

$$\text{Slope} = \frac{3-0}{0-5} = \frac{-3}{5}$$

The required equation is:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-3}{5} (x - 5)$$

$$5y = -3x + 15$$

$$3x + 5y = 15$$

(ii) When x-intercept = -4, corresponding point on x-axis is (-4, 0)

When y-intercept = 6, corresponding point on y-axis is (0, 6).

Let $(x_1, y_1) = (-4, 0)$ and $(x_2, y_2) = (0, 6)$

$$\text{Slope} = \frac{6-0}{0+4} = \frac{6}{4} = \frac{3}{2}$$

The required equation is:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{3}{2} (x + 4)$$

$$2y = 3x + 12$$

(iii) When x-intercept = -8, corresponding point on x-axis is (-8, 0)

When y-intercept = -4, corresponding point on y-axis is (0, -4).

Let $(x_1, y_1) = (-8, 0)$ and $(x_2, y_2) = (0, -4)$

$$\text{Slope} = \frac{-4-0}{0+8} = \frac{-4}{8} = \frac{-1}{2}$$

The required equation is:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-1}{2} (x + 8)$$

$$2y = -x - 8$$

$$x + 2y + 8 = 0$$

Question 16:

Find the equation of the line whose slope is $-\frac{5}{6}$ and x-intercept is 6.

Solution 16:

Since, x-intercept is 6, so the corresponding point on x-axis is (6, 0).

$$\text{Slope} = m = \frac{-5}{6}$$

Required equation of the line is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-5}{6} (x - 6)$$

$$6y = -5x + 30$$

$$5x + 6y = 30$$

Question 17:

Find the equation of the line with x-intercept 5 and a point on it (-3, 2)

Solution 17:

Since, x-intercept is 5, so the corresponding point on x-axis is (5, 0).

The line also passes through (-3, 2).

$$\therefore \text{Slope of the line} = \frac{2 - 0}{-3 - 5} = \frac{2}{-8} = \frac{-1}{4}$$

Required equation of the line is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-1}{4} (x - 5)$$

$$4y = -x + 5$$

$$x + 4y = 5$$

Question 18:

Find the equations of the line through (1, 3) and making an intercept of 5 on the y-axis.

Solution 18:

Since, y-intercept = 5, so the corresponding point on y-axis is (0, 5).

The line passes through (1, 3).

$$\therefore \text{Slope of the line} = \frac{3 - 5}{1 - 0} = \frac{-2}{1} = -2$$

Required equation of the line is given by:

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -2(x - 0)$$

$$y - 5 = -2x$$

$$2x + y = 5$$

Question 19:

Find the equations of the lines passing through point $(-2, 0)$ and equally inclined to the co-ordinate axes.

Solution 19:

Let AB and CD be two equally inclined lines.

For line AB:

$$\text{Slope} = m = \tan 45^\circ = 1$$

$$(x_1, y_1) = (-2, 0)$$

Equation of the line AB is:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 1(x + 2)$$

$$y = x + 2$$

For line CD:

$$\text{Slope} = m = \tan (-45^\circ) = -1$$

$$(x_1, y_1) = (-2, 0)$$

Equation of the line CD is:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -1(x + 2)$$

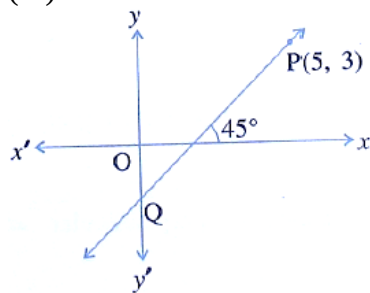
$$y = -x - 2$$

$$x + y + 2 = 0$$

Question 20:

The line through P $(5, 3)$ intersects y-axis at Q.

- (i) write the slope of the line
- (ii) write the equation of the line
- (iii) Find the co-ordinates of Q.

**Solution 20:**

- (i) The equation of the y-axis is $x = 0$

Given that the required line through P $(5, 3)$

Intersects the y-axis at Q and the angle of inclination is 45°

Therefore slope of the line PQ $= \tan 45^\circ = 1$

- (ii) The equation of a line passing through the point

A(x_1, y_1) with slope 'm' is

$$y - y_1 = m(x - x_1)$$

Therefore, the equation of the line passing through the point P (5, 3) with slope 1 is

$$y - 3 = 1 \times (x - 5)$$

$$\Rightarrow y - 3 = x - 5$$

$$\Rightarrow x - y = 2$$

(iii) From subpart (ii), the equation of the line PQ

$$\text{Is } x - y = 2$$

Given that the line intersects with the y – axis, $x = 0$

Thus, substituting $x = 0$ in the equation $x - y = 2$

$$\text{We have, } 0 - y = 2$$

$$\Rightarrow y = -2$$

Thus, the coordinates point of intersection Q

Are $q(0, -2)$

Question 21:

Write down the equation of the line whose gradient is $-\frac{2}{5}$ and which passes through point P, where P divides the line segment joining A(4, -8) and B (12, 0) in the ratio 3 : 1

Solution 21:

Given, P divides the line segment joining A (4, -8) and B (12, 0) in the ratio 3: 1.

Co-ordinates of point P are

$$\left(\frac{3 \times 12 + 1 \times 4}{3 + 1}, \frac{3 \times 0 + 1 \times (-8)}{3 + 1} \right)$$

$$= \left(\frac{36 + 4}{4}, \frac{-8}{4} \right)$$

$$= (10, -2)$$

$$\text{Slope} = m = \frac{-2}{5} \text{ (Given)}$$

Thus, the required equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y + 2 = \frac{-2}{5}(x - 10)$$

$$5y + 10 = -2x + 20$$

$$2x + 5y = 10$$

Question 22:

A (1, 4), B (3, 2) and C (7, 5) are vertices of a triangle ABC. Find:

- (i) the co-ordinates of the centroid of triangle ABC.
 (ii) the equation of a line, through the centroid and parallel to AB.

Solution 22:

- (i) Co-ordinates of the centroid of triangle ABC are

$$\left(\frac{1+3+7}{3}, \frac{4+2+5}{3} \right)$$

$$= \left(\frac{11}{3}, \frac{11}{3} \right)$$

- (ii) Slope of AB = $\frac{2-4}{3-1} = \frac{-2}{2} = -1$

Slope of the line parallel to AB = Slope of AB = -1

Thus, the required equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - \frac{11}{3} = -1 \left(x - \frac{11}{3} \right)$$

$$3y - 11 = -3x + 11$$

$$3x + 3y = 22$$

Question 23

A (7, -1), B (4, 1) and C (-3, 4) are the vertices of a triangle ABC. Find the equation of a line through the vertex B and the point P in AC; such that AP : CP = 2 : 3.

Solution 23:

Given, AP: CP = 2: 3

∴ Co-ordinates of P are

$$\left(\frac{2 \times (-3) + 3 \times 7}{2 + 3}, \frac{2 \times 4 + 3(-1)}{2 + 3} \right)$$

$$= \left(\frac{-6 + 21}{5}, \frac{8 - 3}{5} \right)$$

$$= \left(\frac{15}{5}, \frac{5}{5} \right)$$

$$= (3, 1)$$

$$\text{Slope of BP} = \frac{1-1}{3-4} = 0$$

Required equation of the line passing through points B and P is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 0(x - 3)$$

$$y = 1$$

EXERCISE. 14 (D)

Question 1:

Given $3x + 2y + 4 = 0$

(i) express the equation in the form $y = mx + c$

(ii) Find the slope and y-intercept of the line $3x + 2y + 4 = 0$

Solution 1:

(i) $3x + 2y + 4 = 0$

$$2y = -3x - 4$$

$$y = \frac{-3}{2}x - 2$$

This is of the form $y = mx + c$.

(ii) Slope = $m = \frac{-3}{2}$

$$\text{y-intercept} = c = -2$$

Question 2:

Find the slope and y-intercept of the line:

(i) $y = 4$ (ii) $ax - by = 0$

(iii) $3x - 4y = 5$

Solution 2:

(i) $y = 4$

Comparing this equation with $y = mx + c$, we have:

$$\text{Slope} = m = 0$$

$$\text{y-intercept} = c = 4$$

(ii) $ax - by = 0$

$$\Rightarrow by = ax \Rightarrow y = \frac{a}{b}x$$

Comparing this equation with $y = mx + c$, we have:

$$\text{Slope} = m = \frac{a}{b}$$

$$\text{y-intercept} = c = 0$$

$$(iii) 3x - 4y = 5 \Rightarrow 4y = 3x - 5 \Rightarrow y = \frac{3}{4}x - \frac{5}{4}$$

Comparing this equation with $y = mx + c$, we have:

$$\text{Slope} = m = \frac{3}{4}$$

$$\text{y-intercept} = c = -\frac{5}{4}$$

Question 3:

The equation of a line is $x - y = 4$. Find its slope and y – intercept. Also, find its inclination.

Solution 3:

Given equation of a line is $x - y = 4$

$$\Rightarrow y = x - 4$$

Comparing this equation with $y = mx + c$. We have:

$$\text{Slope} = m = 1$$

$$\text{y-intercept} = c = -4$$

Let the inclination be θ .

$$\text{Slope} = 1 = \tan \theta = \tan 45^\circ$$

$$\therefore \theta = 45^\circ$$

Question 4:

(i) Is the line $3x + 4y + 7 = 0$ perpendicular to the line $28x - 21y + 50 = 0$?

(ii) Is the line $x - 3y = 4$ perpendicular to the line $3x - y = 7$?

(iii) Is the line $3x + 2y = 5$ parallel to the line $x + 2y = 1$?

(iv) Determine x so that slope of the line through (1, 4) and (x, 2) is 2.

Solution 4:

$$(i) 3x + 4y + 7 = 0$$

$$\Rightarrow 4y = -3x - 7$$

$$\Rightarrow y = -\frac{3}{4}x - \frac{7}{4}$$

$$\text{Slope of this line} = \frac{-3}{4}$$

$$28x - 21y + 50 = 0$$

$$\Rightarrow 21y = 28x + 50$$

$$\Rightarrow y = \frac{28}{21}x + \frac{50}{21}$$

$$\Rightarrow y = \frac{4}{3}x + \frac{50}{21}$$

$$\text{Slope of this line} = \frac{4}{3}$$

Since, product of slopes of the two lines = -1, the lines are perpendicular to each other.

(ii) $x - 3y = 4$

$$3y = x - 4$$

$$y = \frac{1}{3}x - \frac{4}{3}$$

$$\text{Slope of this line} = \frac{1}{3}$$

$$3x - y = 7$$

$$y = 3x - 7$$

$$\text{Slope of this line} = 3$$

$$\text{Product of slopes of the two lines} = 1 \neq -1$$

So, the lines are not perpendicular to each other.

(iii) $3x + 2y = 5$

$$2y = -3x + 5$$

$$y = \frac{-3x}{2} + \frac{5}{2}$$

$$\text{Slope of this line} = \frac{-3}{2}$$

$$x + 2y = 1$$

$$2y = -x + 1$$

$$y = \frac{-1x}{2} + \frac{1}{2}$$

$$\text{Slope of this line} = \frac{-1}{2}$$

$$\text{Product of slopes of the two lines} = 3 \neq -1$$

So, the lines are not perpendicular to each other.

(iv) Given, the slope of the line through (1, 4) and (x, 2) is 2.

$$\therefore \frac{2-4}{x-1} = 2$$

$$\frac{-2}{x-1} = 2$$

$$\frac{-1}{x-1} = 1$$

$$-1 = x - 1$$

$$x = 0$$

Question 5:

Find the slope of the line which is parallel to:

(i) $x + 2y + 3 = 0$ (ii) $\frac{x}{2} - \frac{y}{3} - 1 = 0$

Solution 5:

(i) $x + 2y + 3 = 0$

$$2y = -x - 3$$

$$y = \frac{-1}{2}x - \frac{3}{2}$$

$$\text{Slope of this line} = \frac{-1}{2}$$

$$\text{Slope of the line which is parallel to the given line} = \text{Slope of the given line} = \frac{-1}{2}$$

(ii) $\frac{x}{2} - \frac{y}{3} - 1 = 0$

$$\frac{y}{3} = \frac{x}{2} - 1$$

$$y = \frac{3x}{2} - 3$$

$$\text{Slope of this line} = \frac{3}{2}$$

$$\text{Slope of the line which is parallel to the given line} = \text{Slope of the given line} = \frac{3}{2}$$

Question 6:

Find the slope of the line which is perpendicular to:

(i) $x - \frac{y}{2} + 3 = 0$ (ii) $\frac{x}{3} - 2y = 4$

Solution 6:

(i) $x - \frac{y}{2} + 3 = 0$

$$\frac{y}{2} = x + 3$$

$$y = 2x + 6$$

$$\text{Slope of this line} = 2$$

$$\text{Slope of the line which is perpendicular to the given line}$$

$$= \frac{-1}{\text{Slope of the given line}} = \frac{-1}{2}$$

$$(ii) \frac{x}{3} - 2y = 4$$

$$2y = \frac{x}{3} - 4$$

$$y = \frac{x}{6} - 2$$

$$\text{Slope of this line} = \frac{1}{6}$$

$$\text{Slope of the line which is perpendicular to the given line} = \frac{-1}{\text{Slope of this line}} = \frac{-1}{\frac{1}{6}} = -6$$

Question 7:

(i) Lines $2x - by + 5 = 0$ and $ax + 3y = 2$ are parallel to each other. Find the relation connecting a and b .

(ii) Lines $mx + 3y + 7 = 0$ and $5x - ny - 3 = 0$ are perpendicular to each other. Find the relation connecting m and n .

Solution 7:

$$(i) 2x - by + 3 = 0$$

$$by = 2x + 3$$

$$y = \frac{2x}{b} + \frac{3}{b}$$

$$\text{Slope of this line} = \frac{2}{b}$$

$$ax + 3y = 2$$

$$3y = -ax + 2$$

$$y = \frac{-ax}{3} + \frac{2}{3}$$

$$\text{Slope of this line} = \frac{-a}{3}$$

Since, the lines are parallel, so the slopes of the two lines are equal.

$$\therefore \frac{2}{b} = \frac{-a}{3}$$

$$ab = -6$$

$$(ii) mx + 3y + 7 = 0$$

$$3y = -mx - 7$$

$$y = \frac{-mx}{3} - \frac{7}{3}$$

$$\text{Slope of this line} = \frac{-m}{3}$$

$$5x - ny - 3 = 0$$

$$ny = 5x - 3$$

$$y = \frac{5x}{n} - \frac{3}{n}$$

$$\text{Slope of this line} = \frac{5}{n}$$

Since, the lines are perpendicular; the product of their slopes is -1.

$$\therefore \left(\frac{-m}{3} \right) \left(\frac{5}{n} \right) = -1$$

$$5m = 3n$$

Question 8:

Find the value of p if the lines, whose equations are $2x - y + 5 = 0$ and $px + 3y = 4$ are perpendicular to each other.

Solution 8:

$$2x - y + 5 = 0$$

$$y = 2x + 5$$

$$\text{Slope of this line} = 2$$

$$px + 3y = 4$$

$$3y = -px + 4$$

$$y = \frac{-px}{3} + \frac{4}{3}$$

$$\text{Slope of this line} = \frac{-p}{3}$$

Since, the lines are perpendicular to each other, the product of the slopes is -1.

$$\therefore (2) \left(\frac{-p}{3} \right) = -1$$

$$\frac{2p}{3} = 1$$

$$p = \frac{3}{2}$$

Question 9:

The equation of a line AB is $2x - 2y + 3 = 0$

(i) Find the slope of line AB.

(ii) Calculate the angle that the line AB makes with the positive direction of the x-axis.

Solution 9:

(i) $2x - 2y + 3 = 0$

$$2y = 2x + 3$$

$$y = x + \frac{3}{2}$$

Slope of the line AB = 1

(ii) Required angle = θ

$$\text{Slope} = \tan \theta = 1 = \tan 45^\circ$$

$$\theta = 45^\circ$$

Question 10:

The lines represented by $4x + 3y = 9$ and $px - 6y + 3 = 0$ are parallel. Find the value of p.

Solution 10:

$$4x + 3y = 9$$

$$3y = -4x + 9$$

$$y = \frac{-4}{3}x + 3$$

$$\text{Slope of this line} = \frac{-4}{3}$$

$$px - 6y + 3 = 0$$

$$6y = px + 3$$

$$y = \frac{px}{6} + \frac{1}{2}$$

$$\text{Slope of this line} = \frac{p}{6}$$

Since, the lines are parallel, their slopes will be equal.

$$\therefore \frac{-4}{3} = \frac{p}{6}$$

$$-4 = \frac{p}{2}$$

$$p = -8$$

Question 11:

If the lines $y = 3x + 7$ and $2y + px = 3$ are perpendicular to each other, find the value of p .

Solution 11:

$$y = 3x + 7$$

Slope of this line = 3

$$2y + px = 3$$

$$2y = -px + 3$$

$$y = -\frac{px}{2} + \frac{3}{2}$$

$$\text{Slope of this line} = -\frac{p}{2}$$

Since, the lines are perpendicular to each other, the product of their slopes is -1.

$$\therefore (3)\left(-\frac{p}{2}\right) = -1$$

$$\frac{3p}{2} = 1$$

$$p = \frac{2}{3}$$

Question 12:

The line through A (-2, 3) and B (4, b) is perpendicular to the line $2x - 4y = 5$. Find the value of b .

Solution 12:

The slope of the line passing through two given points A(x_1 , y_1) and B (x_2 , y_2) is

$$\text{Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line passing through two

Given points A(-2, 3) and B (4, b) is

$$\text{Slope of AB} = \frac{b - 3}{4 - (-2)} = \frac{b - 3}{4 + 2} = \frac{b - 3}{6}$$

Equation of the given line is $2x - 4y = 5$

$$\Rightarrow \text{Equation is } 4y = 2x - 5$$

$$\Rightarrow \text{Equation is } y = \frac{1}{4}(2x - 5)$$

$$\Rightarrow \text{Equation is } y = \frac{x}{2} - \frac{5}{4}$$

Comparing this equation with the general equation,

$$Y = mx + c, \text{ we have } m = \frac{1}{2}$$

Since the given line and AB are perpendicular to each other, the product of their slopes is -1

$$\therefore \left(\frac{b-3}{6} \right) \times \frac{1}{2} = -1$$

$$\Rightarrow b - 3 = -12$$

$$\Rightarrow b = 3 - 12$$

$$\Rightarrow b = -9$$

Question 13:

Find the equation of the line passing through $(-5, 7)$ and parallel to:

- (i) x-axis (ii) y - axis

Solution 13:

- (i) The slope of the line parallel to x-axis is 0.

$$(x_1, y_1) = (-5, 7)$$

Required equation of the line is

$$y - y_1 = m (x - x_1)$$

$$y - 7 = 0(x + 5)$$

$$y = 7$$

- (ii) The slope of the line parallel to y-axis is not defined.

That is slope of the line is $\tan 90^\circ$ and hence the given line is parallel to y-axis.

$$(x_1, y_1) = (-5, 7)$$

Required equation of the line is

$$x - x_1 = 0$$

$$\Rightarrow x + 5 = 0$$

Question 14:

- (i) Find the equation of the line passing through $(5, -3)$ and parallel to $x - 3y = 4$.

- (ii) Find the equation of the line parallel to the line $3x + 2y = 8$ and passing through the point $(0, 1)$.

Solution 14:

- (i) $x - 3y = 4$

$$\Rightarrow 3y = x - 4$$

$$\Rightarrow y = \frac{1}{3}x - \frac{4}{3}$$

$$\text{Slope of this line} = \frac{1}{3}$$

Slope of a line parallel to this line = $\frac{1}{3}$

Required equation of the line passing through (5, -3) is

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \frac{1}{3}(x - 5)$$

$$3y + 9 = x - 5$$

$$x - 3y - 14 = 0$$

(ii) $2y = -3x + 8$

Or $y = -\frac{3}{2}x + \frac{8}{2}$

Slope of given line $= -\frac{3}{2}$

Since the required line is parallel to given straight line.

$\therefore$ Slope of required line (m) = $-\frac{3}{2}$

Now the equation of the required line is given by:

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 1 = -\frac{3}{2}(x - 0)$$

$$\Rightarrow 2y - 2 = -3x$$

$$\Rightarrow 3x + 2y = 2$$

Question 15:

Find the equation of the line passing through (-2, 1) and perpendicular to $4x + 5y = 6$.

Solution 15:

$$4x + 5y = 6$$

$$5y = -4x + 6$$

$$y = \frac{-4x}{5} + \frac{6}{5}$$

Slope of this line $= \frac{-4}{5}$

The required line is perpendicular to the line $4x + 5y = 6$.

$$\therefore \text{Slope of the required line} = \frac{-1}{\text{slope of the given line}} = \frac{-1}{\frac{-4}{5}} = \frac{5}{4}$$

The required equation of the line is given by

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{5}{4} (x + 2)$$

$$4y - 4 = 5x + 10$$

$$5x - 4y + 14 = 0$$

Question 16:

Find the equation of the perpendicular bisector of the line segment obtained on joining the points (6, -3) and (0, 3)

Solution 16:

Let A = (6, -3) and B = (0, 3).

We know the perpendicular bisector of a line is perpendicular to the line and it bisects the line, that is, it passes through the mid-point of the line.

Co-ordinates of the mid-point of AB are

$$\left(\frac{6+0}{2}, \frac{-3+3}{2} \right) = (3, 0)$$

Thus, the required line passes through (3, 0).

$$\text{Slope of AB} = \frac{3+3}{0-6} = \frac{6}{-6} = -1$$

$$\therefore \text{Slope of the required line} = \frac{-1}{\text{slope of AB}} = 1$$

Thus, the equation of the required line is given by:

$$y - y_1 = m (x - x_1)$$

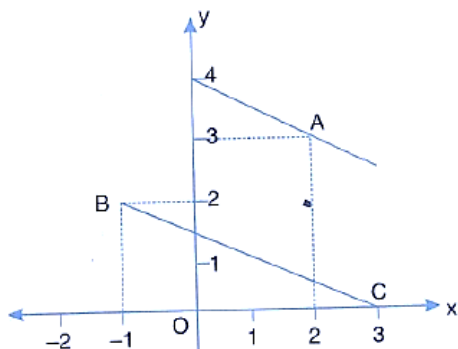
$$y - 0 = 1(x - 3)$$

$$y = x - 3$$

Question 17:

In the following diagram, write down:

- the co-ordinates of the points A, B and C.
- the equation of the line through A and parallel to BC.



Solution 17:

(i) The co-ordinates of points A, B and C are (2, 3), (-1, 2) and (3, 0) respectively.

$$(ii) \text{ Slope of BC} = \frac{0-2}{3+1} = \frac{-2}{4} = \frac{-1}{2}$$

$$\text{Slope of a line parallel to BC} = \text{Slope of BC} = \frac{-1}{2}$$

Required equation of a line passing through A and parallel to BC is given by

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{-1}{2}(x - 2)$$

$$2y - 6 = -x + 2$$

$$x + 2y = 8$$

Question 18:

B (-5, 6) and D (1, 4) are the vertices of rhombus ABCD. Find the equations of diagonals BD and AC.

Solution 18:

We know that in a rhombus, diagonals bisect each other at right angle.

Let O be the point of intersection of the diagonals AC and BD.

Co-ordinates of O are

$$\left(\frac{-5+1}{2}, \frac{6+4}{2} \right) = (-2, 5)$$

$$\text{Slope of BD} = \frac{4-6}{1+5} = \frac{-2}{6} = \frac{-1}{3}$$

For line BD:

$$\text{Slope} = m = \frac{-1}{3}, (x_1, y_1) = (-5, 6)$$

Equation of the line BD is

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{-1}{3}(x + 5)$$

$$3y - 18 = -x - 5$$

$$x + 3y = 13$$

For line AC:

$$\text{Slope} = m = \frac{-1}{\text{slope of BD}} = 3, (x_1, y_1) = (-2, 5)$$

Equation of the line AC is

$$\begin{aligned}
 y - y_1 &= m(x - x_1) \\
 y - 5 &= 3(x + 2) \\
 y - 5 &= 3x + 6 \\
 y &= 3x + 11
 \end{aligned}$$

Question 19:

A (7, -2) and C = (-1, -6) are the vertices of square ABCD. Find the equations of diagonals AC and BD.

Solution 19:

We know that in a square, diagonals bisect each other at right angle.

Let O be the point of intersection of the diagonals AC and BD.

Co-ordinates of O are

$$\left(\frac{7-1}{2}, \frac{-2-6}{2} \right) = (3, -4)$$

$$\text{Slope of AC} = \frac{-6+2}{-1-7} = \frac{-4}{-8} = \frac{1}{2}$$

For line AC:

$$\text{Slope} = m = \frac{1}{2}, (x_1, y_1) = (7, -2)$$

Equation of the line AC is

$$y - y_1 = m(x - x_1)$$

$$y + 2 = \frac{1}{2}(x - 7)$$

$$2y + 4 = x - 7$$

$$2y = x - 11$$

For line BD:

$$\text{Slope} = m = \frac{-1}{\text{slope of AC}} = \frac{-1}{\frac{1}{2}} = -2,$$

$$(x_1, y_1) = (3, -4)$$

Equation of the line BD is

$$y - y_1 = m(x - x_1)$$

$$y + 4 = -2(x - 3)$$

$$y + 4 = -2x + 6$$

$$2x + y = 2$$

Question 20:

A (1, -5), B (2, 2) and C (-2, 4) are the vertices of triangle ABC, Find the equation of:

- (i) the median of the triangle through A.
- (ii) the altitude of the triangle through B.
- (iii) the line through C and parallel to AB.

Solution 20:

(i) We know the median through A will pass through the mid-point of BC. Let AD be the median through A.

Co-ordinates of the mid-point of BC, i.e., D are

$$\left(\frac{2-2}{2}, \frac{2+4}{2} \right) = (0, 3)$$

$$\text{Slope of AD} = \frac{3+5}{0-1} = -8$$

Equation of the median AD is

$$y - 3 = -8(x - 0)$$

$$8x + y = 3$$

(ii) Let BE be the altitude of the triangle through B.

$$\text{Slope of AC} = \frac{4+5}{-2-1} = \frac{9}{-3} = -3$$

$$\therefore \text{Slope of BE} = \frac{-1}{\text{slope of AC}} = \frac{1}{3}$$

Equation of altitude BE is

$$y - 2 = \frac{1}{3}(x - 2)$$

$$3y - 6 = x - 2$$

$$3y = x + 4$$

$$\text{(iii) Slope of AB} = \frac{2+5}{2-1} = 7$$

Slope of the line parallel to AB = Slope of AB = 7

So, the equation of the line passing through C and parallel to AB is

$$y - 4 = 7(x + 2)$$

$$y - 4 = 7x + 14$$

$$y = 7x + 18$$

Question 21:

(i) Write down the equation of the line AB, through (3, 2) and perpendicular to the line

$$2y = 3x + 5$$

(ii) AB meets the x-axis at A and the y-axis at B. Write down the co-ordinates of A and B. Calculate the area of triangle OAB, where O is the origin.

Solution 21:

(i) $2y = 3x + 5$

$$\Rightarrow y = \frac{3x}{2} + \frac{5}{2}$$

Slope of this line $= \frac{3}{2}$

Slope of the line AB $= \frac{-1}{\frac{3}{2}} = \frac{-2}{3}$

$(x_1, y_1) = (3, 2)$

The required equation of the line AB is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-2}{3}(x - 3)$$

$$3y - 6 = -2x + 6$$

$$2x + 3y = 12$$

(ii) For the point A (the point on x-axis), the value of $y = 0$.

$$2x + 3y = 12 \Rightarrow 2x = 12 \Rightarrow x = 6$$

Co-ordinates of point A are (6, 0).

For the point B (the point on y-axis), the value of $x = 0$.

$$\therefore 2x + 3y = 12$$

$$\Rightarrow 3y = 12$$

$$\Rightarrow y = 4$$

Co-ordinates of point B are (0, 4).

$$\text{Area of } \triangle OAB = \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times 6 \times 4$$

$$= 12 \text{ sq units}$$

Question 22:

The line $4x - 3y + 12 = 0$ meets x-axis at A. Write the co-ordinates of A. determine the equation of the line through A and perpendicular to $4x - 3y + 12 = 0$

Solution 22:

For the point A (the point on x-axis), the value of $y = 0$.

$$4x - 3y + 12 = 0$$

$$\Rightarrow 4x = -12$$

$$\Rightarrow x = -3$$

Co-ordinates of point A are (-3, 0).

Here, $(x_1, y_1) = (-3, 0)$

The given line is $4x - 3y + 12 = 0$

$$3y = 4x + 12$$

$$y = \frac{4}{3}x + 4$$

$$\text{Slope of this line} = \frac{4}{3}$$

$$\therefore \text{Slope of a line perpendicular to the given line} = \frac{-1}{\frac{4}{3}} = \frac{-3}{4}$$

Required equation of the line passing through A is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-3}{4}(x + 3)$$

$$4y = -3x - 9$$

$$3x + 4y + 9 = 0$$

Question 23:

The point P is the foot of perpendicular from A (-5, 7) to the line whose equation is $2x - 3y + 18 = 0$. Determine:

(i) the equation of the line AP

(ii) the co-ordinates of P.

Solution 23:

(i) The given equation is

$$2x - 3y + 18 = 0$$

$$3y = 2x + 18$$

$$y = \frac{2}{3}x + 6$$

$$\text{Slope of this line} = \frac{2}{3}$$

$$\text{Slope of a line perpendicular to this line} = \frac{-1}{\frac{2}{3}} = \frac{-3}{2}$$

$$(x_1, y_1) = (-5, 7)$$

The required equation of the line AP is given by

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{-3}{2} (x + 5)$$

$$2y - 14 = -3x - 15$$

$$3x + 2y + 1 = 0$$

(ii) P is the foot of perpendicular from point A.

So P is the point of intersection of the lines $2x - 3y + 18 = 0$ and $3x + 2y + 1 = 0$.

$$2x - 3y + 18 = 0$$

$$\Rightarrow 4x - 6y + 36 = 0$$

$$3x + 2y + 1 = 0$$

$$\Rightarrow 9x + 6y + 3 = 0$$

Adding the two equations, we get,

$$13x + 39 = 0$$

$$x = -3$$

$$\therefore 3y = 2x + 18 = -6 + 18 = 12$$

$$y = 4$$

Thus, the co-ordinates of the point P are $(-3, 4)$.

Question 24:

The points A, B and C are $(4, 0)$, $(2, 2)$ and $(0, 6)$ respectively. Find the equations of AB and BC.

If AB cuts the y-axis at P and BC cuts the x-axis at Q, Find the co-ordinates of P and Q.

Solution 24:

For the line AB:

$$\text{Slope of AB} = m = \frac{2-0}{2-4} = \frac{2}{-2} = -1$$

$$(x_1, y_1) = (4, 0)$$

Equation of the line AB is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -1(x - 4)$$

$$y = -x + 4$$

$$x + y = 4 \dots(1)$$

For the line BC:

$$\text{Slope of BC} = m = \frac{6-2}{0-2} = \frac{4}{-2} = -2$$

$$(x_1, y_1) = (2, 2)$$

Equation of the line BC is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -2(x - 2)$$

$$y - 2 = -2x + 4$$

$$2x + y = 6 \dots(2)$$

Given that AB cuts the y-axis at P. So, the abscissa of point P is 0.

Putting $x = 0$ in (1), we get,

$$y = 4$$

Thus, the co-ordinates of point P are (0, 4).

Given that BC cuts the x-axis at Q. So, the ordinate of point Q is 0.

Putting $y = 0$ in (2), we get,

$$2x = 6$$

$$\Rightarrow x = 3$$

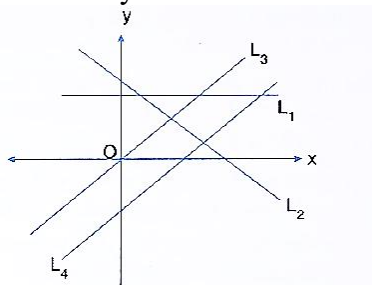
Thus, the co-ordinates of point Q are (3, 0).

Question 25:

Match the equations A, B, C and D with the lines L_1 , L_2 , L_3 and L_4 , whose graphs are roughly drawn in the given diagram.

$$A \equiv y = 2x; \quad B \equiv Y - 2x + 2 = 0$$

$$C \equiv 3x + 2y = 6 \quad D \equiv Y = 2$$



Solution 25:

Putting $x = 0$ and $y = 0$ in the equation $y = 2x$, we have:

$$\text{LHS} = 0 \text{ and RHS} = 0$$

Thus, the line $y = 2x$ passes through the origin.

Hence, $A = L_3$

Putting $x = 0$ in $y - 2x + 2 = 0$, we get, $y = -2$

Putting $y = 0$ in $y - 2x + 2 = 0$, we get, $x = 1$

So, x-intercept = 1 and y-intercept = -2

So, x-intercept is positive and y-intercept is negative.

Hence, $B = L_4$

Putting $x = 0$ in $3x + 2y = 6$, we get, $y = 3$

Putting $y = 0$ in $3x + 2y = 6$, we get, $x = 2$

So, both x-intercept and y-intercept are positive.

Hence, $C = L_2$

The slope of the line $y = 2$ is 0.

So, the line $y = 2$ is parallel to x-axis.

Hence, $D = L_1$

EXERCISE. 14 (E)**Question 1:**

Point P divides the line segment joining the points A (8, 0) and B (16, -8) in the ratio 3:5 Find its co-ordinates of point P.

Also, find the equation of the line through P and parallel to $3x + 5y = 7$

Solution 1:

Using section formula, the co-ordinates of the point P are

$$\left(\frac{3 \times 16 + 5 \times 8}{3 + 5}, \frac{3 \times (-8) + 5 \times 0}{3 + 5} \right)$$

$$= (11, -3) = (x_1, y_1)$$

$$3x + 5y = 7$$

$$\Rightarrow y = \frac{-3}{5}x + \frac{7}{5}$$

$$\text{Slope of this line} = \frac{-3}{5}$$

As the required line is parallel to the line $3x + 5y = 7$,

$$\text{Slope of the required line} = \text{Slope of the given line} = \frac{-3}{5}$$

Thus, the equation of the required line is

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \frac{-3}{5}(x - 11)$$

$$5y + 15 = -3x + 33$$

$$3x + 5y = 18$$

Question 2:

The line segment joining the points A (3, -4) and B (-2, 1) is divided in the ratio 1 : 3 at point P in it. Find the co-ordinates of P.

Also, find the equation of the line through P and perpendicular to the line $5x - 3y = 4$.

Solution 2:

Using section formula, the co-ordinates of the point P are

$$\left(\frac{1 \times (-2) + 3 \times 3}{1 + 3}, \frac{1 \times 1 + 3 \times (-4)}{1 + 3} \right)$$

$$= \left(\frac{7}{4}, \frac{-11}{4} \right) = (x_1, y_1)$$

The equation of the given line is

$$5x - 3y + 4 = 0$$

$$\Rightarrow y = \frac{5x}{3} + \frac{4}{3}$$

$$\text{Slope of this line} = \frac{5}{3}$$

Since, the required line is perpendicular to the given line,

$$\text{Slope of the required line} = \frac{-1}{\frac{5}{3}} = \frac{-3}{5}$$

Thus, the equation of the required line is

$$y - y_1 = m(x - x_1)$$

$$y + \frac{11}{4} = \frac{-3}{5} \left(x - \frac{7}{4} \right)$$

$$\frac{4y + 11}{4} = \frac{-3}{5} \left(\frac{4x - 7}{4} \right)$$

$$20y + 55 = -12x + 21$$

$$12x + 20y + 34 = 0$$

$$6x + 10y + 17 = 0$$

Question 3:

A line $5x + 3y + 15 = 0$ meets y – axis at point P. Find the co-ordinates of points P. Find the equation of a line through P and perpendicular to $x - 3y + 4 = 0$.

Solution 3:

Point P lies on y -axis, so putting $x = 0$ in the equation $5x + 3y + 15 = 0$, we get, $y = -5$

Thus, the co-ordinates of the point P are $(0, -5)$.

$$x - 3y + 4 = 0 \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$$

$$\text{Slope of this line} = \frac{1}{3}$$

The required equation is perpendicular to given equation $x - 3y + 4 = 0$.

$$\therefore \text{Slope of the required line} = \frac{-1}{\frac{1}{3}} = -3$$

$$(x_1, y_1) = (0, -5)$$

Thus, the required equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y + 5 = -3(x - 0)$$
$$3x + y + 5 = 0$$

Question 4:

Find the value of K for which the lines $kx - 5y + 4 = 0$ and $5x - 2y + 5 = 0$ are perpendicular to each other.

Solution 4:

$$kx - 5y + 4 = 0$$

$$\Rightarrow 5y = kx + 4$$

$$\Rightarrow y = \frac{k}{5}x + \frac{4}{5}$$

$$\text{Slope of this line} = m_1 = \frac{k}{5}$$

$$5x - 2y + 5 = 0$$

$$\Rightarrow 2y = 5x + 5$$

$$\Rightarrow y = \frac{5}{2}x + \frac{5}{2}$$

$$\text{Slope of this line} = m_2 = \frac{5}{2}$$

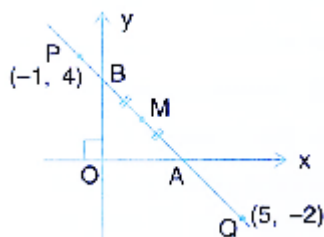
Since, the lines are perpendicular, $m_1 \times m_2 = -1$

$$\Rightarrow \frac{k}{5} \times \frac{5}{2} = -1$$

$$\Rightarrow k = -2$$

Question 5:

A straight line passes through the points P (-1, 4) and Q (5, -2). It intersects the co-ordinate axes at points A and B. M is the mid-point of the segment AB. Find:



- (i) The equation of the line
- (ii) The co-ordinates of A and B.
- (iii) The co-ordinates of M.

Solution 5:

$$(i) \text{ Slope of PQ} = \frac{-2-4}{5+1} = \frac{-6}{6} = -1$$

Equation of the line PQ is given by

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -1(x + 1)$$

$$y - 4 = -x - 1$$

$$x + y = 3$$

(ii) For point A (on x-axis), $y = 0$.

Putting $y = 0$ in the equation of PQ, we get,

$$x = 3$$

Thus, the co-ordinates of point A are (3, 0).

For point B (on y-axis), $x = 0$.

Putting $x = 0$ in the equation of PQ, we get,

$$y = 3$$

Thus, the co-ordinates of point B are (0, 3).

(iii) M is the mid-point of AB.

So, the co-ordinates of point M are

$$\left(\frac{3+0}{2}, \frac{0+3}{2} \right) = \left(\frac{3}{2}, \frac{3}{2} \right)$$

Question 6:

(1, 5) and (-3, -1) are the co-ordinates of vertices A and C respectively of rhombus ABCD. Find the equations of the diagonals AC and BD.

Solution 6:

A = (1, 5) and C = (-3, -1)

We know that in a rhombus, diagonals bisect each other at right angle.

Let O be the point of intersection of the diagonals AC and BD.

Co-ordinates of O are

$$\left(\frac{1-3}{2}, \frac{5-1}{2} \right) = (-1, 2)$$

$$\text{Slope of AC} = \frac{-1-5}{-3-1} = \frac{-6}{-4} = \frac{3}{2}$$

For line AC:

$$\text{Slope} = m = \frac{3}{2}, (x_1, y_1) = (1, 5)$$

Equation of the line AC is

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{3}{2} (x - 1)$$

$$2y - 10 = 3x - 3$$

$$3x - 2y + 7 = 0$$

For line BD:

$$\text{Slope} = m = \frac{-1}{\text{slope of AC}} = \frac{-2}{3},$$

$$(x_1, y_1) = (-1, 2)$$

Equation of the line BD is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-2}{3} (x + 1)$$

$$3y - 6 = -2x - 2$$

$$2x + 3y = 4$$

Question 7:

Show that A(3, 2), B (6, -2) and C (2, -5) can be the vertices of a square.

(i) Find the co-ordinates of its fourth vertex D, if ABCD is a square

(ii) Without using the co-ordinates of vertex D, find the equation of side AD of the square and also the equation of diagonal BD.

Solution 7:

Using distance formula, we have:

$$AB = \sqrt{(6-3)^2 + (-2-2)^2} = \sqrt{9+16} = 5$$

$$BC = \sqrt{(2-6)^2 + (-5+2)^2} = \sqrt{16+9} = 5$$

Thus, AC = BC

$$\text{Also, Slope of AB} = \frac{-2-2}{6-3} = \frac{-4}{3}$$

$$\text{Slope of BC} = \frac{-5+2}{2-6} = \frac{-3}{-4} = \frac{3}{4}$$

$$\text{Slope of AB} \times \text{Slope of BC} = -1$$

Thus, AB $\perp$ BC

Hence, A, B, C can be the vertices of a square.....

$$(i) \text{ Slope of AB} = \frac{-2-2}{6-3} \text{ Slope of CD}$$

Equation of the line CD is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y + 5 = \frac{-4}{3}(x - 2)$$

$$\Rightarrow 3y + 15 = -4x + 8$$

$$\Rightarrow 4x + 3y = -7 \dots\dots\dots(1)$$

$$\text{Slope of BC} = \left(\frac{-5+2}{2-6} \right) = \frac{-3}{-4} = \frac{3}{4} \text{ Slope of AD}$$

Equation of the line AD is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 2 = \frac{3}{4}(x - 3)$$

$$\Rightarrow 4y - 8 = 3x - 9$$

$$\Rightarrow 3x - 4y = 1 \dots\dots\dots(2)$$

Now, D is the point of intersection of CD and AD.

$$(1) \Rightarrow 16x + 12y = -28$$

$$(2) \Rightarrow 9x - 12y = 3$$

Adding the above two equations we get,

$$25x = -25$$

$$\Rightarrow x = -1$$

$$\text{so, } 4y = 3x - 1 = -3 - 1 = -4$$

$$\Rightarrow y = -1$$

Thus, the co-ordinates of point D are $(-1, -1)$.

(ii) The equation of line AD is found in part (i)

It is $3x - 4y = 1$ Or $4y = 3x - 1$

$$\text{Slope of BD} = \frac{-1+2}{-1-6} = \frac{1}{-7} = \frac{-1}{7}$$

The equation of diagonal BD is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y + 1 = \frac{-1}{7}(x + 1)$$

$$\Rightarrow 7y + 7 = -x - 1$$

$$\Rightarrow x + 7y + 8 = 0$$

Question 8:

A line through origin meets the line

$x = 3y + 2$ at right angles at point X. Find the co-ordinates of X.

Solution 8:

The given line is

$$x = 3y + 2 \dots(1)$$

$$3y = x - 2$$

$$y = \frac{1}{3}x - \frac{2}{3}$$

Slope of this line is $\frac{1}{3}$

The required line intersects the given line at right angle.

$$\therefore \text{Slope of the required line} = \frac{-1}{\frac{1}{3}} = -3$$

The required line passes through $(0, 0) = (x_1, y_1)$

The equation of the required line is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -3(x - 0)$$

$$3x + y = 0 \dots(2)$$

Point X is the intersection of the lines (1) and (2).

Using (1) in (2), we get,

$$9y + 6 + y = 0$$

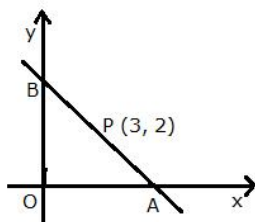
$$y = \frac{-6}{10} = \frac{-3}{5}$$

$$\therefore x = 3y + 2 = \frac{-9}{5} + 2 = \frac{1}{5}$$

Thus, the co-ordinates of the point X are $\left(\frac{1}{5}, \frac{-3}{5}\right)$

Question 9:

A straight line passes through the point $(3, 2)$ and the portion of this line, intercepted between the positive axes, is bisected at this point. Find the equation of the line.

Solution 9:

Let the line intersect the x-axis at point A $(x, 0)$ and y-axis at point B $(0, y)$.

Since, P is the mid-point of AB, we have:

$$\left(\frac{x+0}{2}, \frac{0+y}{2} \right) = (3, 2)$$

$$\left(\frac{x}{2}, \frac{y}{2} \right) = (3, 2)$$

$$x = 6, y = 4$$

Thus, A = (6, 0) and B = (0, 4)

$$\text{Slope of line AB} = \frac{4-0}{0-6} = \frac{4}{-6} = \frac{-2}{3}$$

Let $(x_1, y_1) = (6, 0)$

The required equation of the line AB is given by

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-2}{3} (x - 6)$$

$$3y = -2x + 12$$

$$2x + 3y = 12$$

Question 10:

Find the equation of the line passing through the point of intersection of $7x + 6y = 71$ and $5x - 8y = -23$; and perpendicular to the line $4x - 2y = 1$

Solution 10:

$$7x + 6y = 71 \Rightarrow 28x + 24 = 284 \dots(1)$$

$$5x - 8y = -23 \Rightarrow 15x - 24y = -69 \dots(2)$$

Adding (1) and (2), we get,

$$43x = 215$$

$$x = 5$$

$$\text{From (2), } 8y = 5x + 23 = 25 + 23 = 48 \Rightarrow y = 6$$

Thus, the required line passes through the point (5, 6).

$$4x - 2y = 1$$

$$2y = 4x - 1$$

$$y = 2x - \frac{1}{2}$$

Slope of this line = 2

$$\text{Slope of the required line} = \frac{-1}{2}$$

The required equation of the line is

$$y - y_1 = m(x_1, x_2)$$

$$y - 6 = \frac{-1}{2} (x - 5)$$

$$2y - 12 = -x + 5$$
$$x + 2y = 17$$

Question 11:

Find the equation of the line which is perpendicular to the line $\frac{x}{a} - \frac{y}{b} = 1$ at the point where this line meets y-axis.

Solution 11:

The given line is

$$\frac{x}{a} - \frac{y}{b} = 1$$

$$\Rightarrow \frac{y}{b} = \frac{x}{a} - 1$$

$$\Rightarrow y = \frac{b}{a}x - b$$

$$\text{Slope of this line} = \frac{b}{a}$$

$$\text{Slope of the required line} = \frac{-1}{\frac{b}{a}} = \frac{-a}{b}$$

Let the required line passes through the point P (0, y).

Putting x = 0 in the equation $\frac{x}{a} - \frac{y}{b} = 1$, we get,

$$0 - \frac{y}{b} = 1$$

$$\Rightarrow y = -b$$

Thus, P = (0, -b) = (x₁, y₁)

The equation of the required line is

$$y - y_1 = m(x - x_1)$$

$$y + b = \frac{-a}{b} (x - 0)$$

$$by + b^2 = -ax$$

$$ax + by + b^2 = 0$$

Question 12:

O (0, 0), A (3, 5) and B (−5, −3) are the vertices of triangle OAB. Find:

- (i) the equation of median of triangle OAB through vertex O.
- (ii) the equation of altitude of triangle OAB through vertex B.

Solution 12:

(i) Let the median through O meets AB at D. So, D is the mid-point of AB.

Co-ordinates of point D are

$$\left(\frac{3-5}{2}, \frac{5-3}{2} \right) = (-1, 1)$$

$$\text{Slope of OD} = \frac{1-0}{-1-0} = -1$$

$$(x_1, y_1) = (0, 0)$$

The equation of the median OD is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -1(x - 0)$$

$$x + y = 0$$

(ii) The altitude through vertex B is perpendicular to OA.

$$\text{Slope of OA} = \frac{5-0}{3-0} = \frac{5}{3}$$

$$= \frac{-1}{\frac{5}{3}} = \frac{-3}{5}$$

Slope of the required altitude

The equation of the required altitude through B is

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \frac{-3}{5}(x + 5)$$

$$5y + 15 = -3x - 15$$

$$3x + 5y + 30 = 0$$

Question 13:

Determine whether the line through points (−2, 3) and (4, 1) is perpendicular to the line $3x = y + 1$.

Does line $3x = y + 1$ bisect the line segment joining the two given points?

Solution 13:

Let A = (−2, 3) and B = (4, 1)

$$\text{Slope of AB} = m_1 = \frac{1-3}{4+2} = \frac{-2}{6} = \frac{-1}{3}$$

Equation of line AB is

$$y - y_1 = m_1(x - x_1)$$

$$y - 3 = \frac{-1}{3} (x + 2)$$

$$3y - 9 = -x - 2$$

$$x + 3y = 7 \dots(1)$$

Slope of the given line $3x = y + 1$ is $3 = m_2$.

$$\therefore m_1 \times m_2 = -1$$

Hence, the line through points A and B is perpendicular to the given line.

Given line is $3x = y + 1 \dots(2)$

Solving (1) and (2), we get,

$$x = 1 \text{ and } y = 2$$

So, the two lines intersect at point $P = (1, 2)$.

The co-ordinates of the mid-point of AB are

$$\left(\frac{-2+4}{2}, \frac{3+1}{2} \right) = (1, 2) = P$$

Hence, the line $3x = y + 1$ bisects the line segment joining the points A and B.

Question 14:

Given a straight line $x \cos 30^\circ + y \sin 30^\circ = 2$. Determine the equation of the other line which is parallel to it and passes through $(4, 3)$.

Solution 14:

$$x \cos 30^\circ + y \sin 30^\circ = 2$$

$$\Rightarrow x \frac{\sqrt{3}}{2} + y + \frac{1}{2} = 2$$

$$\Rightarrow \sqrt{3}x + y = 4$$

$$\Rightarrow y = -\sqrt{3}x + 4$$

Slope of this line = $-\sqrt{3}$

Slope of a line which is parallel to this given line = $-\sqrt{3}$

Let $(4, 3) = (x_1, y_1)$

Thus, the equation of the required line is given by:

$$y - y_1 = m_1 (x - x_1)$$

$$y - 3 = -\sqrt{3} (x - 4)$$

$$\sqrt{3}x + y = 4\sqrt{3} + 3$$

Question 15:

Find the value of k such that the line

$(k - 2)x + (k + 3)y - 5 = 0$ is:

(i) perpendicular to the line $2x - y + 7 = 0$

(ii) parallel to it.

Solution 15:

$$(k - 2)x + (k + 3)y - 5 = 0 \dots(1)$$

$$(k + 3)y = -(k - 2)x + 5$$

$$y = \left(\frac{2 - k}{k + 3} \right)x + \frac{5}{k + 3}$$

$$\text{Slope of this line} = m_1 = \frac{2 - k}{k + 3}$$

(i) $2x - y + 7 = 0$

$$y = 2x + 7 = 0$$

$$\text{Slope of this line} = m_2 = 2$$

Line (1) is perpendicular to $2x - y + 7 = 0$

$$\therefore m_1 m_2 = -1$$

$$\Rightarrow \left(\frac{2 - k}{k + 3} \right)(2) = -1$$

$$\Rightarrow 4 - 2k = -k - 3$$

$$\Rightarrow k = 7$$

(ii) Line (1) is parallel to $2x - y + 7 = 0$

$$\therefore m_1 = m_2$$

$$\Rightarrow \frac{2 - k}{k + 3} = 2$$

$$\Rightarrow 2 - k = 2k + 6$$

$$\Rightarrow 3k = -4$$

$$\Rightarrow k = -\frac{4}{3}$$

Question 16:

The vertices of a triangle ABC are A (0, 5), B (-1, -2) and C (11, 7). Write down the equation of BC Find:

(i) the equation of line through A and perpendicular to BC.

(ii) the co-ordinates of the point P, where the perpendicular through A, as obtained in (i), meets BC.

Solution 16:

$$\text{Slope of BC} = \frac{7+2}{11+1} = \frac{9}{12} = \frac{3}{4}$$

Equation of the line BC is given by

$$y - y_1 = m_1 (x - x_1)$$

$$y + 2 = \frac{3}{4} (x + 1)$$

$$4y + 8 = 3x + 3$$

$$3x - 4y = 5 \dots (1)$$

$$(i) \text{ Slope of line perpendicular to BC} = \frac{-1}{\frac{3}{4}} = \frac{-4}{3}$$

Required equation of the line through A (0, 5) and perpendicular to BC is

$$y - y_1 = m_1 (x - x_1)$$

$$y - 5 = \frac{-4}{3} (x - 0)$$

$$3y - 15 = -4x$$

$$4x + 3y = 15 \dots (2)$$

(ii) The required point will be the point of intersection of lines (1) and (2).

$$(1) \Rightarrow 9x - 12y = 15$$

$$(2) \Rightarrow 16x + 12y = 60$$

Adding the above two equations, we get,

$$25x = 75$$

$$x = 3$$

$$\text{So, } 4y = 3x - 5 = 9 - 5 = 4$$

$$y = 1$$

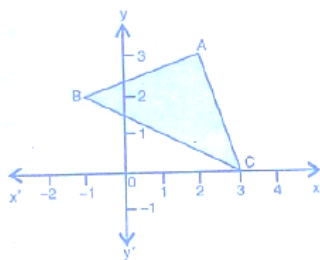
Thus, the co-ordinates of the required point is (3, 1).

Question 17:

From the given figure, find:

(i) the co-ordinates of A, B and C.

(ii) the equation of the line through A and parallel to BC.

**Solution 17:**

(i) $A = (2, 3)$, $B = (-1, 2)$, $C = (3, 0)$

(ii) Slope of $BC = \frac{0-2}{3+1} = -\frac{2}{4} = -\frac{1}{2}$

Slope of required line which is parallel to $BC = \text{Slope of } BC = -\frac{1}{2}$

$(x_1, y_1) = (2, 3)$

The required equation of the line through A and parallel to BC is given by:

$$y - y_1 = m_1(x - x_1)$$

$$y - 3 = -\frac{1}{2}(x - 2)$$

$$2y - 6 = -x + 2$$

$$x + 2y = 8$$

Question 18:

$P(3, 4)$, $Q(7, -2)$ and $R(-2, -1)$ are the vertices of triangle PQR . Write down the equation of the median of the triangle through R .

Solution 18:

The median (say RX) through R will bisect the line PQ .

The co-ordinates of point X are

$$\left(\frac{3+7}{2}, \frac{4-2}{2}\right) = (5, 1)$$

$$\text{Slope of } RX = \frac{1+1}{5+2} = \frac{2}{7} = m$$

$(x_1, y_1) = (-2, -1)$

The required equation of the median RX is given by:

$$y - y_1 = m_1(x - x_1)$$

$$y + 1 = \frac{2}{7}(x + 2)$$

$$7y + 7 = 2x + 4$$

$$7y = 2x - 3$$

Question 19:

A(8, -6), B(-4, 2) and C(0, -10) are vertices of a triangle ABC. If P is the mid-point of AB and Q is the mid-point of AC, use co-ordinate geometry to show that PQ is parallel to BC. Give a special name to quadrilateral PBCQ.

Solution 19:

P is the mid-point of AB. So, the co-ordinate of point P are

$$\left(\frac{8-4}{2}, \frac{-6+2}{2} \right) = (2, -2)$$

Q is the mid-point of AC. So, the co-ordinate of point Q are

$$\left(\frac{8+0}{2}, \frac{-6-10}{2} \right) = (4, -8)$$

$$\text{Slope of PQ} = \frac{-8+2}{4-2} = \frac{-6}{2} = -3$$

$$\text{Slope of BC} = \frac{-10-2}{0+4} = \frac{-12}{4} = -3$$

Since, slope of PQ = Slope of BC,

$\therefore$ PQ $\parallel$ BC

Also, we have:

$$\text{Slope of PB} = \frac{-2-2}{2+4} = \frac{-2}{3}$$

$$\text{Slope of QC} = \frac{-8+10}{4-10} = \frac{1}{2}$$

Thus, PB is not parallel to QC.

Hence, PBCQ is a trapezium.

Question 20:

A line AB meets the x-axis at point A and y-axis at point B. The point P(-4, -2) divides the line segment AB internally such that AP : PB = 1 : 2, Find:

(i) the co-ordinates of A and B

(ii) equation of line through P and perpendicular to AB.

Solution 20:

(i) Let the co-ordinates of point A (lying on x-axis) be (x, 0) and the co-ordinates of point B (lying y-axis) be (0, y).

Given, P = (-4, -2) and AP : PB = 1 : 2

Using section formula, we have:

$$(-4, -2) = \left(\frac{1 \times 0 + 2 \times x}{1+2}, \frac{1 \times y + 2 \times 0}{1+2} \right)$$

$$(-4, -2) = \left(\frac{2x}{3}, \frac{y}{3} \right)$$

$$\Rightarrow -4 = \frac{2x}{3} \quad -2 = \frac{y}{3}$$

$$\Rightarrow x = -6 \quad y = -6$$

Thus, the co-ordinates of A and B are $(-6, 0)$ and $(0, -6)$.

$$(ii) \text{ Slope of AB} = \frac{-6-0}{0+6} = \frac{-6}{6} = -1$$

$$\text{Slope of the required line perpendicular to AB} = \frac{-1}{-1} = 1$$

$$(x_1, y_1) = (-4, -2)$$

Required equation of the line passing through P and perpendicular to AB is given by

$$y - y_1 = m(x - x_1)$$

$$y + 2 = 1(x + 4)$$

$$y + 2 = x + 4$$

$$y = x + 2$$

Question 21:

A line intersects x-axis at point $(-2, 0)$ and cuts off an intercept of 3 units from the positive side of y-axis. Find the equation of the line.

Solution 21:

The required line intersects x-axis at point A $(-2, 0)$.

Also, y-intercept = 3

So, the line also passes through B $(0, 3)$.

$$\text{Slope of line AB} = \frac{3-0}{0+2} = \frac{3}{2} \quad m$$

$$(x_1, y_1) = (-2, 0)$$

Required equation of the line AB is given by

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{3}{2}(x + 2)$$

$$2y = 3x + 6$$

Question 22:

Find the equation of a line passing through the point (2, 3) and having the x-intercept of 4 units.

Solution 22:

The required line passes through A (2, 3).

Also, x-intercept = 4

So, the required line passes through B (4, 0).

$$\text{Slope of AB} = \frac{0-3}{4-2} = \frac{-3}{2} = m$$

$$(x_1, y_1) = (4, 0)$$

Required equation of the line AB is given by

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-3}{2} (x - 4)$$

$$2y = -3x + 12$$

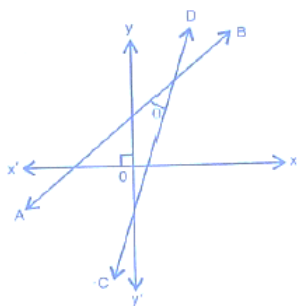
$$3x + 2y = 12$$

Question 23:

The given figure (not drawn to scale) shows two straight lines AB and CD. If equation of the line AB is:

$Y = x + 1$ and equation of line CD is:

$y = \sqrt{3}x - 1$. Write down the inclination of lines AB and CD; also, find the angle θ between AB and CD.

**Solution 23:**

Equation of the line AB is $y = x + 1$

Slope of AB = 1

Inclination of line AB = 45° (Since, $\tan 45^\circ = 1$)

$$\Rightarrow \angle RPQ = 45^\circ$$

Equation of line CD is $y = \sqrt{3}x - 1$

Slope of CD = $\sqrt{3}$

Inclination of line CD = 60° (Since, $\tan 60^\circ = \sqrt{3}$)

$$\Rightarrow \angle DQX = 60^\circ$$

$$\therefore \angle DQP = 180^\circ - 60^\circ = 120^\circ$$

Using angle sum property in ΔPQR ,

$$\theta = 180^\circ - 45^\circ - 120^\circ = 15^\circ$$

Question 24:

Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P. where P divides the line segment joining A(-2, 6) and B(3, -4) in the ratio 2 : 3

Solution 24:

Given, P divides the line segment joining A (-2, 6) and B (3, -4) in the ratio 2: 3.

Co-ordinates of point P are

$$\left(\frac{2 \times 3 + 3 \times (-2)}{2 + 3}, \frac{2 \times (-4) + 3 \times 6}{2 + 3} \right)$$

$$= \left(\frac{6 - 6}{5}, \frac{-8 + 18}{5} \right)$$

$$= (0, 2) = (x_1, y_1)$$

$$\text{Slope of the required line} = m = \frac{3}{2}$$

The required equation of the line is given by

$$y - y_1 = m (x - x_1)$$

$$y - 2 = \frac{3}{2} (x - 0)$$

$$2y - 4 = 3x$$

$$2y = 3x + 4$$

Question 25:

The ordinate of a point lying on the line joining the points (6, 4) and (7, -5) is -23. Find the co-ordinates of that point.

Solution 25:

Let A = (6, 4) and B = (7, -5)

$$\text{Slope of the line AB} = \frac{-5 - 4}{7 - 6} = -9$$

$$(x_1, y_1) = (6, 4)$$

The equation of the line AB is given by

$$y - y_1 = m (x - x_1)$$

$$y - 4 = -9(x - 6)$$

$$y - 4 = -9x + 54$$

$$9x + y = 58 \dots(1)$$

Now, given that the ordinate of the required point is -23.

Putting $y = -23$ in (1), we get,

$$9x - 23 = 58$$

$$9x = 81$$

$$x = 9$$

Thus, the co-ordinates of the required point is (9, -23).

Question 26 :

Point A and B have Co-ordinates (7, -3) and (1, 9) respectively. Find:

(i) the slope of AB

(ii) the equation of perpendicular bisector of the line segment AB.

(iii) the value of 'p' of (-2, p) lies on it.

Solution 26:

Given points are A (7, -3) and B(1, 9).

$$(i) \text{ Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 + 3}{1 - 7} = \frac{12}{-6} = -2$$

$$(ii) \text{ Slope of perpendicular bisector} = \frac{-1}{-2} = \frac{1}{2}$$

$$\text{Mid-point of AB} = \left(\frac{7+1}{2}, \frac{-3+9}{2} \right) = (4, 3)$$

Equation of perpendicular bisector is:

$$y - 3 = \frac{1}{2}(x - 4)$$

$$2y - 6 = x - 4$$

$$x - 2y + 2 = 0$$

(iii) Point (-2, p) lies on $x - 2y + 2 = 0$.

$$\therefore -2 - 2p + 2 = 0$$

$$\Rightarrow 2p = 0$$

$$\Rightarrow p = 0$$

Question 27:

A and B are two points on the x-axis and y-axis respectively. P (2, -3) is the mid point of AB
Find the

(i) co-ordinates of A and B

(ii) slope of line AB

(iii) equation of line AB

Solution 27:

(i) Let the co-ordinates be A (x, 0) and B (0, y).

Mid-point of A and B is given by $\left(\frac{x+0}{2}, \frac{y+0}{2}\right) = \left(\frac{x}{2}, \frac{y}{2}\right)$

$$\Rightarrow (2, -3) = \left(\frac{x}{2}, \frac{y}{2}\right)$$

$$\Rightarrow \frac{x}{2} = 2 \quad \text{and} \quad \frac{y}{2} = -3$$

$$\Rightarrow x = 4 \quad \text{and} \quad y = -6$$

$$\therefore A = (4, 0) \quad \text{and} \quad B = (0, -6)$$

$$(ii) \text{ Slope of line AB, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{0 - 4} = \frac{3}{2} = 1\frac{1}{2}$$

(iii) Equation of line AB, using A (4, 0)

$$y - 0 = \frac{3}{2}(x - 4)$$

$$2y = 3x - 12$$

Question 28:

The equation of a line is $3x + 4y - 7 = 0$. Find:

(i) the slope of the line

(ii) the equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$

Solution 28:

$$3x + 4y - 7 = 0 \dots (1)$$

$$4y = -3x + 7$$

$$y = \frac{-3}{4}x + \frac{7}{4}$$

$$(i) \text{ Slope of the line } = m = \frac{-3}{4}$$

$$(ii) \text{ Slope of the line perpendicular to the given line } = \frac{-1}{\frac{-3}{4}} = \frac{4}{3}$$

Solving the equations $x - y + 2 = 0$ and $3x + y - 10 = 0$, we get $x = 2$ and $y = 4$.
So, the point of intersection of the two given lines is (2, 4).

Given that a line with slope $\frac{4}{3}$ passes through point (2, 4).

Thus, the required equation of the line is

$$y - 4 = \frac{4}{3} (x - 2)$$

$$3y - 12 = 4x - 8$$

$$4x - 3y + 4 = 0$$

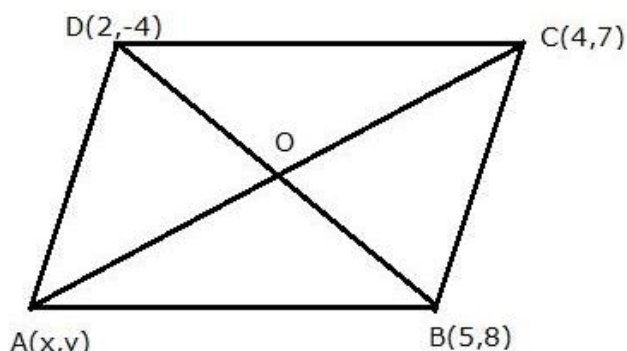
Question 29:

ABCD is a parallelogram where A(x, y), B (5, 8), C (4, 7) and D (2, -4). Find:

(i) co-ordinates of A.

(ii) equation of diagonal BD

Solution 29:



In parallelogram ABCD, A (x, y), B(5, 8), C(4, 7) and D(2, -4).

The diagonals of the parallelogram bisect each other.

O is the point of intersection of AC and BD

Since O is the midpoint of BD, its coordinates will be

$$\left(\frac{2+5}{2}, \frac{-4+8}{2} \right) \text{ or } \left(\frac{7}{2}, \frac{4}{2} \right) \text{ or } \left(\frac{7}{2}, 2 \right)$$

(i) Since O is the midpoint of AC also,

$$\frac{x+4}{2} = \frac{7}{2}$$

$$\Rightarrow x + 4 = 7$$

$$\Rightarrow x = 7 - 4 = 3$$

$$\frac{y+7}{2} = 2$$

$$\Rightarrow y + 7 = 14$$

$$\text{and } \Rightarrow y = 14 - 7 = 7$$

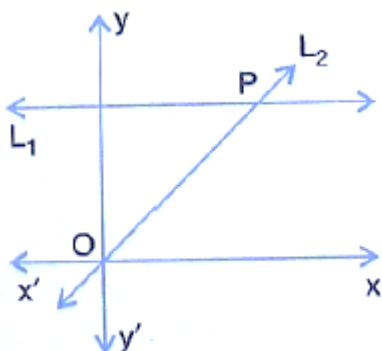
Thus, Coordinates of A are (3, 7)

$$\begin{aligned}
 \text{(ii)} \quad & \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \\
 \Rightarrow & y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)} \times (x - x_1) \\
 \Rightarrow & y + 4 = \frac{8 + 4}{5 - 2} \times (x - 2) \\
 \Rightarrow & y + 4 = \frac{12}{3} \times (x - 2) \\
 \Rightarrow & y + 4 = 4(x - 2) \\
 \Rightarrow & y + 4 = 4x - 8 \\
 \Rightarrow & 4x - y = 12
 \end{aligned}$$

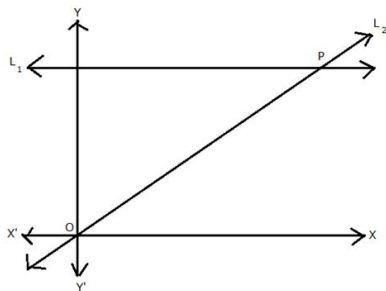
Question 30:

Given equation of line L_1 is $y = 4$.

- (i) Write the slope of line L_2 if L_2 is the bisector of angle O.
- (ii) write the co-ordinates of point P.
- (iii) Find the equation of L_2

**Solution 30:**

- (i) Equation of line L_1 is $y = 4$
 $\therefore L_2$ is the bisector of $\angle O$



$$\therefore \angle POX = 45^\circ$$

$$\text{Slope} = \tan 45^\circ = 1$$

Let coordinates of P be (x, y)

$\therefore$ P lies on L_1

(ii)

$$\therefore \text{Slope of } L_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$1 = \frac{4 - 0}{x - 0} \Rightarrow 1 = \frac{4}{x}$$

$$\Rightarrow x = 4$$

$\therefore$ Coordinates of P are (4, 4)

(iii) Equation of L_2 is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 4 = 1(x - 4)$$

$$\Rightarrow y - 4 = x - 4$$

$$\Rightarrow x = y$$