

EXERCISE 15(A)**Question 1:**

State, true or false:

- (i) Two similar polygons are necessarily congruent.
- (ii) Two congruent polygons are necessarily similar.
- (iii) all equiangular triangles are similar
- (iv) all isosceles triangles are similar.
- (v) Two isosceles – right triangles are similar
- (vi) Two isosceles triangles are similar, if an angle of one is congruent to the corresponding angle of the other.
- (vii) The diagonals of a trapezium, divide each other into proportional segments.

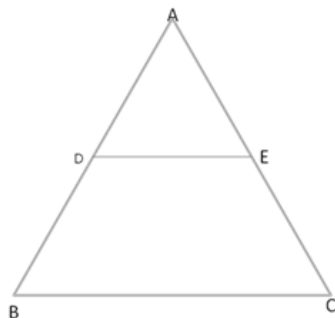
Solution 1:

- (i) False
- (ii) True
- (iii) True
- (iv) False
- (v) True
- (vi) True
- (vii) True

Question 2:

In triangle ABC, DE is parallel to BC; where D and E are the points on AB and AC respectively. Prove: $\triangle ADE \sim \triangle ABC$.

Also, find the length of DE, if $AD = 12$ cm, $BD = 24$ cm $BC = 8$ cm.

Solution 2:

In $\triangle ADE$ and $\triangle ABC$, DE is parallel to BC , so corresponding angles are equal.

$$\angle ADE = \angle ABC$$

$$\angle AED = \angle ACB$$

Hence, $\triangle ADE \sim \triangle ABC$ (By AA similarity criterion)

$$\therefore \frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{12}{12 + 24} = \frac{DE}{8}$$

$$DE = \frac{12}{36} \times 8 = \frac{8}{3} = 2\frac{2}{3}$$

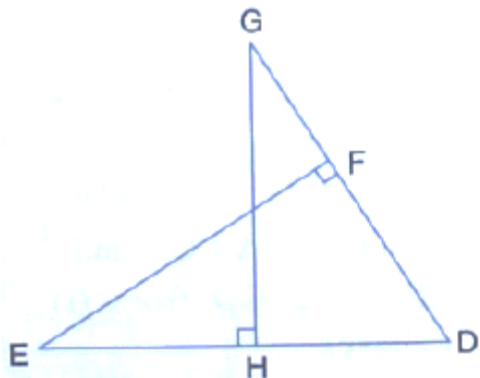
$$\text{Hence, } DE = 2\frac{2}{3} \text{ cm}$$

Question 3:

Given: $\angle GHE = \angle DFE = 90^\circ$,

$DH = 8$, $DF = 12$,

$DG = 3x - 1$ and $DE = 4x + 2$.



Find: the lengths of segments DG and DE

Solution 3:

In $\triangle DHG$ and $\triangle DFE$,

$$\angle GHD = \angle DFE = 90^\circ$$

$$\angle D = \angle D \quad (\text{Common})$$

$\therefore \triangle DHG \sim \triangle DFE$

$$\Rightarrow \frac{DH}{DF} = \frac{DG}{DE}$$

$$\Rightarrow \frac{8}{12} = \frac{3x-1}{4x+2}$$

$$\Rightarrow 32x + 16 = 36x - 12$$

$$\Rightarrow 28 = 4x$$

$$\Rightarrow x = 7$$

$$\therefore DG = 3 \times 7 - 1 = 20$$

$$DE = 4 \times 7 + 2 = 30$$

Question 4:

D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that: $CA^2 = CB \times CD$

Solution 4:

In $\triangle ADC$ and $\triangle BAC$,

$$\angle ADC = \angle BAC \quad (\text{Given})$$

$$\angle ACD = \angle ACB \quad (\text{Common})$$

$$\therefore \triangle ADC \sim \triangle BAC$$

$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

$$\text{Hence, } CA^2 = CB \times CD$$

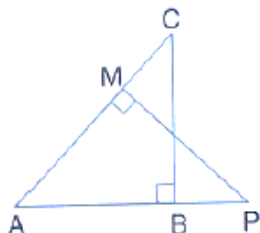
Question 5:

In the given figure, $\triangle ABC$ and $\triangle AMP$ are right angled at B and M respectively.

Given $AC = 10$ cm, $AP = 15$ cm and $PM = 12$ cm.

(i) Prove $\triangle ABC \sim \triangle AMP$

(ii) Find AB and BC



Solution 5:

(i) In $\triangle ABC$ and $\triangle AMP$,

$$\angle BAC = \angle PAM \quad [\text{Common}]$$

$$\angle ABC = \angle PMA \text{ [Each} = 90^\circ]$$

$$\triangle ABC \sim \triangle AMP \text{ [AA Similarity]}$$

(ii)

$$AM = \sqrt{AP^2 - PM^2} = \sqrt{15^2 - 12^2} = 11$$

Since $\triangle ABC \sim \triangle AMP$,

$$\frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$$

$$\Rightarrow \frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$$

$$\Rightarrow \frac{AB}{11} = \frac{BC}{12} = \frac{10}{15}$$

From this we can write,

$$\frac{AB}{11} = \frac{10}{15}$$

$$\Rightarrow AB = \frac{10 \times 11}{15} = 7.33$$

$$\frac{BC}{12} = \frac{10}{15}$$

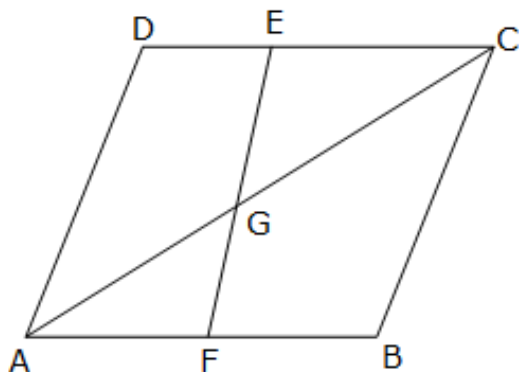
$$\Rightarrow BC = 8\text{cm}$$

Question 6:

E and F are the points in sides DC and AB respectively of parallelogram ABCD. If diagonal AC and segment EF intersect at G; prove that:

$$AG \times EG = FG \times CG$$

Solution 6:



In $\triangle EGC$ and $\triangle FGA$

$\angle ECG = \angle FAG$ (Alternate angles as $AB \parallel CD$)

$\angle EGC = \angle FGA$ (Vertically opposite angles)

$\triangle EGC \sim \triangle FGA$ (By AA – similarity)

$$\therefore \frac{EG}{FG} = \frac{CG}{AG}$$

$$AG \times EG = FG \times CG$$

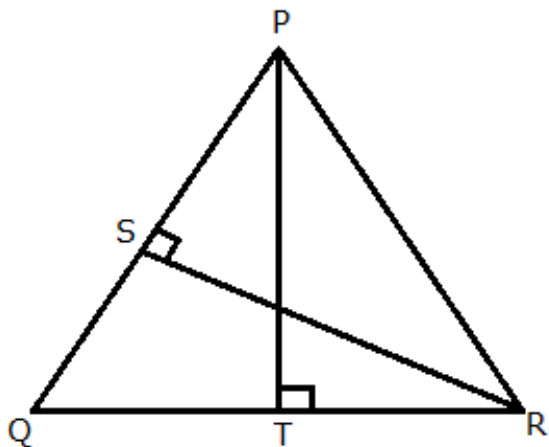
Question 7:

Given: RS and PT are altitudes of $\triangle PQR$. Prove that:

(i) $\triangle PQT \sim \triangle QRS$

(ii) $PQ \times QS = RQ \times QT$

Solution 7:



(i)

In $\triangle PQT$ and $\triangle QRS$,

$\angle PTQ = \angle RSQ = 90^\circ$ (Given)

$\angle PQT = \angle RQS$ (Common)

$\triangle PQT \sim \triangle RQS$ (By AA similarity)

(ii)

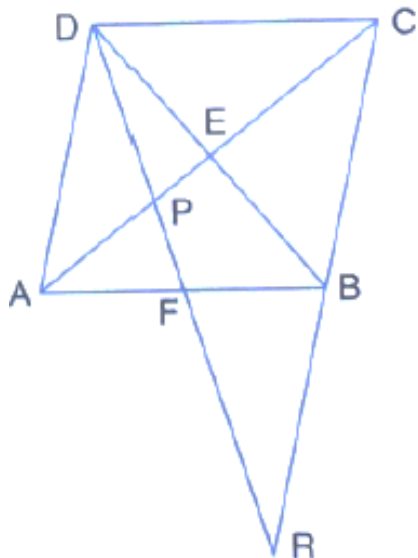
Since, triangle PQT and RQS are similar

$$\therefore \frac{PQ}{RQ} = \frac{QT}{QS}$$

$$\Rightarrow PQ \times QS = RQ \times QT$$

Question 8:

Given: ABCD is a rhombus, DRP and CBR are straight lines.



Prove that:

$$DP \times CR = DC \times PR$$

Solution 8:

In $\triangle DPA$ and $\triangle RPC$,

$$\angle DPA = \angle RPC \quad (\text{Vertically opposite angles})$$

$$\angle PAD = \angle PCR \quad (\text{Alternate angles})$$

$$\triangle DPA \sim \triangle RPC$$

$$\therefore \frac{DP}{PR} = \frac{AD}{CR}$$

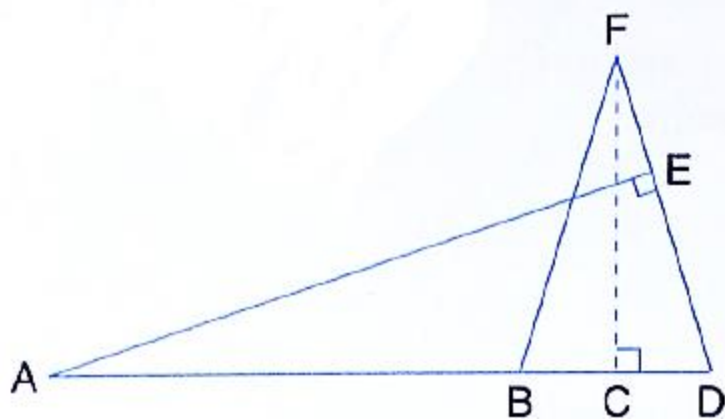
$$\frac{DP}{PR} = \frac{DC}{CR} \quad (AD = DC, \text{ as } ABCD \text{ is rhombus})$$

Hence, $DP \times CR = DC \times PR$

Question 9:

Given: $FB = FD$, $AE \perp FD$ and $FC \perp AD$

Prove: $\frac{FB}{AD} = \frac{BC}{ED}$

**Solution 9:**

Given, $FB = FD$

$$\therefore \angle FDB = \angle FBD \quad \dots\dots\dots(1)$$

In $\triangle AED = \triangle FCB$,

$$\angle AED = \angle FCB = 90^\circ$$

$$\angle ADE = \angle FBC \quad [\text{using (1)}]$$

$$\triangle AED \sim \triangle FCB \quad [\text{By AA similarity}]$$

$$\therefore \frac{AD}{FB} = \frac{ED}{BC}$$

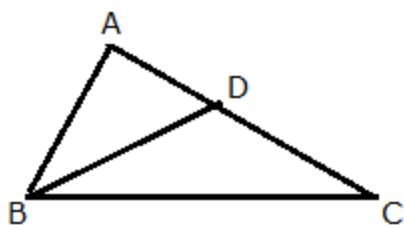
$$\frac{FB}{AD} = \frac{BC}{ED}$$

Question 10:

In $\triangle ABC$, $\angle B = 2\angle C$ and the bisector of angle B meets CA at point D. Prove that:

(i) $\triangle ABC$ and $\triangle ABD$ are similar,

(ii) $DC:AD = BC:AB$

Solution 10:

(i) Since, BD is the bisector of angle B,

$$\angle ABD = \angle DBC$$

Also, given $\angle B = 2\angle C$

$$\therefore \angle ABD = \angle DBC = \angle ACB \dots (1)$$

In $\triangle ABC$ and $\triangle ABD$,

$$\angle BAC = \angle DAB \quad (\text{Common})$$

$$\angle ACB = \angle ABD \quad (\text{Using (1)})$$

$$\therefore \triangle ABC \sim \triangle ADB \quad (\text{By AA similarity})$$

(ii) Since, triangles ABC and ADB are similar,

$$\therefore \frac{BC}{BD} = \frac{AB}{AD}$$

$$\frac{BC}{AB} = \frac{BD}{AD}$$

$$\frac{BC}{AB} = \frac{DC}{AD} \quad (\angle DBC = \angle DCB \Rightarrow DC = BD)$$

$$BC : AB = DC : AD$$

Question 11:

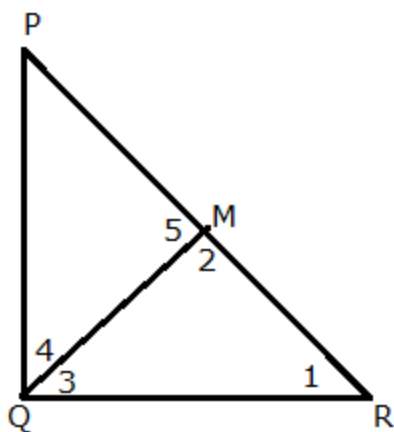
In $\triangle PQR$, $\angle Q = 90^\circ$ and QM is perpendicular to PR . Prove that:

$$(i) PQ^2 = PM \times PR$$

$$(ii) QR^2 = PM \times MR$$

$$(iii) PQ^2 + QR^2 = PR^2$$

Solution 11:



(i) In $\triangle PQM$ and $\triangle PQR$,

$$\angle PMQ = \angle PQR = 90^\circ$$

$$\angle QPM = \angle RPQ \quad (\text{Common})$$

$$\therefore \triangle PQM \sim \triangle PRQ \quad (\text{By AA Similarity})$$

$$\therefore \frac{PQ}{PR} = \frac{PM}{PQ}$$

$$\Rightarrow PQ^2 = PM \times PR$$

(ii) In $\triangle QMR$ and $\triangle PQR$,

$$\angle QMR = \angle PQR = 90^\circ$$

$$\angle QRM = \angle QRP \text{ (Common)}$$

$\therefore \triangle QRM \sim \triangle PQR$ (By AA similarity)

$$\therefore \frac{QR}{PR} = \frac{MR}{QR}$$

$$\Rightarrow QR^2 = PR \times MR$$

(iii) Adding the relations obtained in (i) and (ii), we get,

$$PQ^2 + QR^2 = PM \times PR + PR \times MR$$

$$= PR(PM + MR)$$

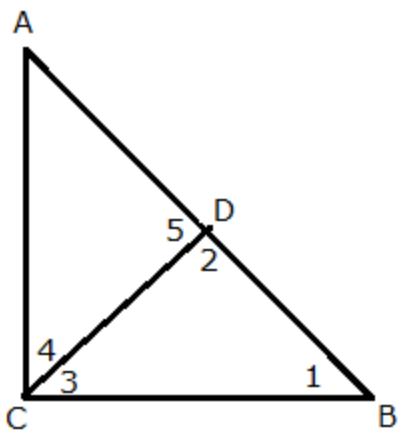
$$= PR \times PR$$

$$= PR^2$$

Question 12:

In $\triangle ABC$, right – angled at C, $CD \perp AB$. Prove: $CD^2 = AD \times DB$

Solution 12:



In $\triangle CDB$,

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

$$\angle 1 + \angle 3 = 90^\circ \text{ (1) (Since, } \angle 2 = 90^\circ \text{)}$$

$$\angle 3 + \angle 4 = 90^\circ \text{ (2) (Since, } \angle ACB = 90^\circ \text{)}$$

From (1) and (2),

$$\angle 1 + \angle 3 = \angle 3 + \angle 4$$

$$\angle 1 = \angle 4$$

$$\text{Also, } \angle 2 = \angle 5 = 90^\circ$$

$\therefore \triangle BDC \sim \triangle CDA$ (By AA similarity)

$$\Rightarrow \frac{DB}{CD} = \frac{CD}{AD}$$

$$\Rightarrow CD^2 = AD \times DB$$

Question 13:

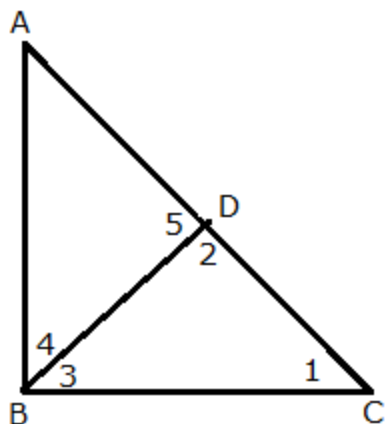
In $\triangle ABC$, $\angle B = 90^\circ$ and $BD \perp AC$.

(i) If $CD = 10$ cm and $BD = 8$ cm; find AD .

(ii) If $AC = 18$ cm and $AD = 6$ cm; find BD .

(iii) If $AC = 9$ cm and $AB = 7$ cm; find AD .

Solution 13:



(i) In $\triangle CDB$,

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

$$\angle 1 + \angle 3 = 90^\circ \dots (1) \text{ (Since, } \angle 2 = 90^\circ \text{)}$$

$$\angle 3 + \angle 4 = 90^\circ \dots (2) \text{ (Since, } \angle ABC = 90^\circ \text{)}$$

From (1) and (2),

$$\angle 1 + \angle 3 = \angle 3 + \angle 4$$

$$\angle 1 = \angle 4$$

$$\text{Also, } \angle 2 = \angle 5 = 90^\circ$$

$\therefore \triangle CDB \sim \triangle BDA$ (By AA similarity)

$$\Rightarrow \frac{CD}{BD} = \frac{BD}{AD}$$

$$\Rightarrow BD^2 = AD \times CD$$

$$\Rightarrow (8)^2 = AD \times 10$$

$$\Rightarrow AD = 6.4$$

Hence, $AD = 6.4$ cm

(ii) Also, by similarity, we have:

$$\frac{BD}{DA} = \frac{CD}{BD}$$

$$BD^2 = 6 \times (18 - 6)$$

$$BD^2 = 72$$

Hence, $BD = 8.5$ cm

(iii)

Clearly, $\triangle ADB \sim \triangle ABC$

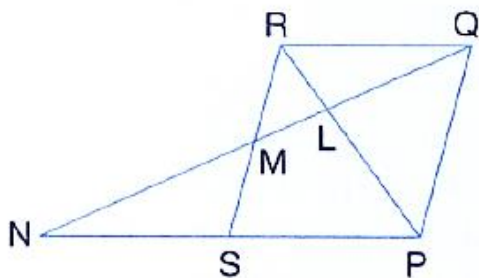
$$\therefore \frac{AD}{AB} = \frac{AB}{AC}$$

$$AD = \frac{7 \times 7}{9} = \frac{49}{9} = 5\frac{4}{9}$$

Hence, $AD = 5\frac{4}{9}$ cm

Question 14:

In the figure, PQRS is a parallelogram with $PQ = 16$ cm and $QR = 10$ cm, L is a point on PR such that $RL:LP = 2:3$. QL produced meets RS at M and PS produced at N.



Find the lengths of PN and RM.

Solution 14:

In $\triangle RLQ$ and $\triangle PLN$,

$\angle RLQ = \angle PLN$ (Vertically opposite angles)

$\angle L R Q = \angle L P N$ (Alternate angles)

$\triangle RLQ \sim \triangle PLN$ (AA Similarity)

$$\therefore \frac{RL}{LP} = \frac{RQ}{PN}$$

$$\frac{2}{3} = \frac{10}{PN}$$

$$PN = 15 \text{ cm}$$

In $\triangle RLM$ and $\triangle PLQ$

$\angle RLM = \angle PLQ$ (Vertically opposite angles)

$\angle LRM = \angle LPQ$ (Alternate angles)

$\triangle RLM \sim \triangle PLQ$ (AA Similarity)

$$\therefore \frac{RM}{PQ} = \frac{RL}{LP}$$

$$\frac{RM}{16} = \frac{2}{3}$$

$$RM = \frac{32}{3} = 10\frac{2}{3} \text{ cm}$$

Question 15:

In quadrilateral ABCD, diagonals AC and BD intersect at point E such that

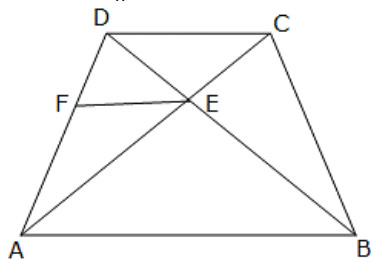
AE : EC = BE : ED

Show that ABCD is a parallelogram.

Solution 15:

Given, AE : EC = BE : ED

Draw EF $\parallel$ AB



In $\triangle ABD$, EF $\parallel$ AB

Using Basic Proportionality theorem,

$$\frac{DF}{FA} = \frac{DE}{EB}$$

But, given $\frac{DE}{EB} = \frac{CE}{EA}$

$$\therefore \frac{DF}{FA} = \frac{CE}{EA}$$

Thus, in $\triangle DCA$, E and F are points on CA and DA respectively such that $\frac{DF}{FA} = \frac{CE}{EA}$

Thus, by converse of Basic proportionality theorem, $FE \parallel DC$.

But, $FE \parallel AB$.

Hence, $AB \parallel DC$.

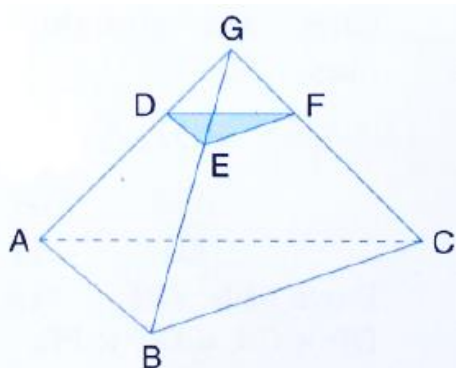
Thus, ABCD is a trapezium.

Question 16:

Given: $AB \parallel DE$ and $BC \parallel EF$. Prove that:

$$(i) \frac{AD}{DG} = \frac{CF}{FG}$$

$$(ii) \triangle DFG \sim \triangle ACG$$



Solution 16:

(i) In $\triangle AGB$, $DE \parallel AB$, by Basic proportionality theorem,

$$\frac{GD}{DA} = \frac{GE}{EB} \dots\dots\dots(1)$$

In $\triangle GBC$, $EF \parallel BC$, by Basic proportionality theorem,

$$\frac{GE}{EB} = \frac{GF}{FC} \dots\dots\dots(2)$$

From (1) and (2), we get,

$$\begin{aligned} \frac{GD}{DA} &= \frac{GF}{FC} \\ \frac{AD}{DG} &= \frac{CF}{FG} \end{aligned}$$

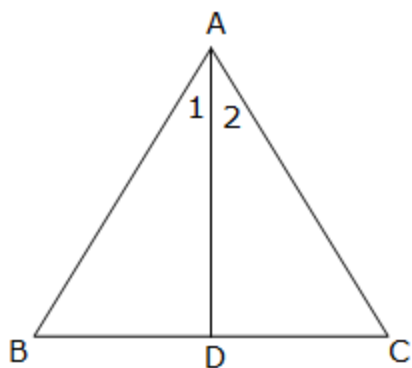
(ii)

From (i), we have:

$$\frac{AD}{DG} = \frac{CF}{FG}$$

$$\angle DGF = \angle AGC \text{ (Common)}$$

$$\therefore \triangle DFG \sim \triangle ACG \text{ (SAS similarity)}$$

Question 17:In triangle ABC, AD is perpendicular to side BC and $AD^2 = BD \times DC$.Show that angle BAC = 90° **Solution 17:**Given $AD^2 = BD \times DC$

$$\frac{AD}{DC} = \frac{BD}{AD}$$

$$\angle ADB = \angle ADC = 90^\circ$$

$$\therefore \triangle DBA \sim \triangle DAC \text{ (SAS similarity)}$$

So, these two triangles will be equiangular.

$$\therefore \angle 1 = \angle C \text{ and } \angle 2 = \angle B$$

$$\angle 1 + \angle 2 = \angle B + \angle C$$

$$\angle A = \angle B + \angle C$$

By angle sum property,

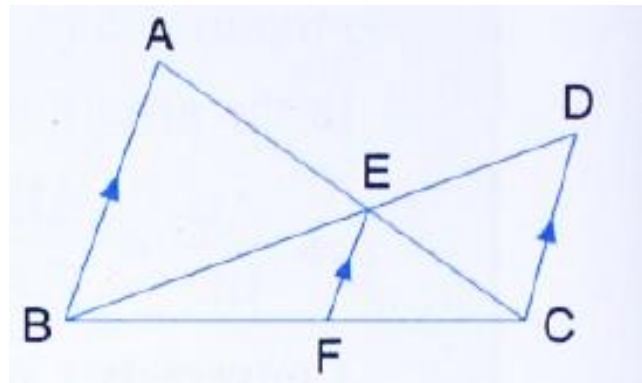
$$\angle A + \angle B + \angle C = 180^\circ$$

$$2\angle A = 180^\circ$$

$$\angle A = \angle BAC = 90^\circ$$

Question 18:

In the given figure, $AB \parallel EF \parallel DC$; $AB = 67.5$ cm, $DC = 40.5$ cm and $AE = 52.5$ cm.



- (i) Name the three pairs of similar triangles.
 (ii) Find the lengths of EC and EF.

Solution 18:

- (i) The three pairs of similar triangles are:

$\triangle BEF$ and $\triangle BDC$

$\triangle CEF$ and $\triangle CAB$

$\triangle ABE$ and $\triangle CDE$

- (ii) Since, $\triangle ABE$ and $\triangle CDE$ are similar,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{67.5}{40.5} = \frac{52.5}{CE}$$

$$CE = 31.5 \text{ cm}$$

Since, $\triangle CEF$ and $\triangle CAB$ are similar,

$$\frac{CE}{CA} = \frac{EF}{AB}$$

$$\frac{31.5}{52.5 + 31.5} = \frac{EF}{67.5}$$

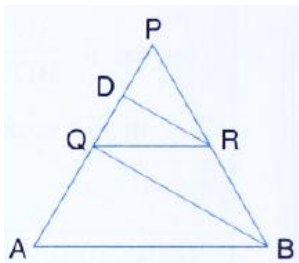
$$\frac{31.5}{84} = \frac{EF}{67.5}$$

$$EF = \frac{2126.25}{84}$$

$$EF = \frac{405}{16} = 25\frac{5}{16} \text{ cm}$$

Question 19:

In the given figure, QR is parallel to AB and DR is parallel to AB and DR is parallel to QB. Prove that:



$$PQ^2 = PD \times PA$$

Solution 19:

Given, QR is parallel to AB. Using Basic proportionality theorem,

$$\Rightarrow \frac{PQ}{PA} = \frac{PR}{PB} \dots\dots\dots(1)$$

Also, DR is parallel to QB. Using Basic proportionality theorem,

$$\Rightarrow \frac{PD}{PQ} = \frac{PR}{PB} \dots\dots\dots(2)$$

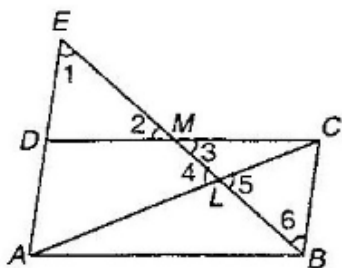
From (1) and (2), we get,

$$\frac{PQ}{PA} = \frac{PD}{PQ}$$

$$PQ^2 = PD \times PA$$

Question 20:

Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting diagonal AC in L and AD produced in E. Prove that: $EL = 2 BL$.

Solution 20:

$$\angle 1 = \angle 6 \text{ (Alternate interior angles)}$$

$$\angle 2 = \angle 3 \text{ (Vertically opposite angles)}$$

DM = MC (M is the mid-point of CD)

$\therefore \triangle DEM \cong \triangle CBM$ (AAS congruence criterion)

So, DE = BC (Corresponding parts of congruent triangles)

Also, AD = BC (Opposite sides of a parallelogram)

$\Rightarrow AE = AD + DE = 2BC$

Now, $\angle 1 = \angle 6$ and $\angle 4 = \angle 5$

$\therefore \triangle ELA \sim \triangle ABC$ (AA similarity)

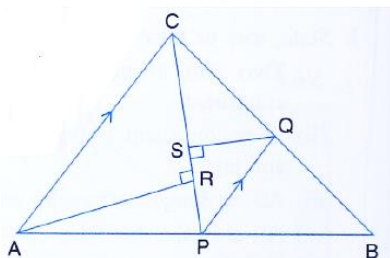
$$\Rightarrow \frac{EL}{BL} = \frac{EA}{BC}$$

$$\Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} = 2$$

$$\Rightarrow E = 2BL$$

Question 21:

In the figure, given below, P is a point on AB such that $AP : PB = 4 : 3$. PQ is parallel to AC.



(i) Calculate the ratio $PQ : AC$, giving reason for your answer.

(ii) In triangle ARC, $\angle ARC = 90^\circ$. Given $QS = 6\text{cm}$, calculate the length of AR.

Solution 21:

(i) Given, $AP : PB = 4 : 3$.

Since, $PQ \parallel AC$. Using Basic Proportionality theorem,

$$\frac{AP}{PB} = \frac{CQ}{QB}$$

$$\Rightarrow \frac{CQ}{QB} = \frac{4}{3}$$

$$\Rightarrow \frac{BQ}{BC} = \frac{3}{7} \dots\dots\dots(1)$$

Now, $\angle PQB = \angle ACB$ (Corresponding angles)

$\angle QPB = \angle CAB$ (Corresponding angles)

$\therefore \triangle PBQ \sim \triangle ABC$ (AA similarity)

$$\Rightarrow \frac{PQ}{AC} = \frac{BQ}{BC}$$

$$\Rightarrow \frac{PQ}{AC} = \frac{3}{7} \quad [\text{using (1)}]$$

$$(ii) \angle ARC = \angle QSP = 90^\circ$$

$$\angle ACR = \angle SPQ \text{ (Alternate angles)}$$

$$\therefore \triangle ARC \sim \triangle QSP \quad (\text{AA similarity})$$

$$\Rightarrow \frac{AR}{QS} = \frac{AC}{PQ}$$

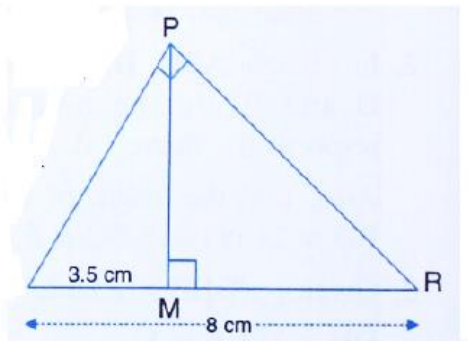
$$\Rightarrow \frac{AR}{QS} = \frac{7}{3}$$

$$\Rightarrow AR = \frac{7 \times 6}{3} = 14 \text{ cm}$$

Question 22:

In the right-angled triangle QPR, PM is an altitude.

Given that QR = 8 cm and MQ = 3.5 cm, calculate the value of PR.



Solution 22:

We have

$$\angle QPR = \angle PMR = 90^\circ$$

$$\angle PRQ = \angle PRM \text{ (common)}$$

$$\triangle PQR \sim \triangle MPR \quad (\text{AA similarity})$$

$$\therefore \frac{QR}{PR} = \frac{PR}{MR}$$

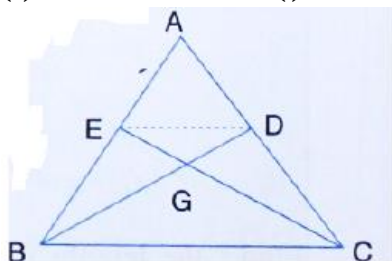
$$PR^2 = 8 \times 4.5 = 36$$

$$PR = 6 \text{ cm}$$

Question 23:

In the figure, given below, the medians BD and CE of a triangle ABC meet at G. Prove that:

- (i) $\triangle EGD \sim \triangle CGB$ and
 (ii) $BG = 2 GD$ from (i) above.

**Solution 23:**

(i) Since, BD and CE are medians.

$$AD = DC$$

$$AE = BE$$

Hence, by converse of Basic Proportionality theorem,

$$ED \parallel BC$$

In $\triangle EGD$ and $\triangle CGB$,

$$\angle DEG = \angle GCB \text{ (alternate angles)}$$

$$\angle EGD = \angle BGC \text{ (Vertically opposite angles)}$$

$$\triangle EGD \sim \triangle CGB \text{ (AA similarity)}$$

(ii) since, $\triangle EGD \sim \triangle CGB$

$$\frac{GD}{GB} = \frac{ED}{BC} \dots\dots\dots (1)$$

In $\triangle AED$ and $\triangle ABC$,

$$\angle AED = \angle ABC \text{ (Corresponding angles)}$$

$$\angle EAD = \angle BAC \text{ (Common)}$$

$$\triangle AED \sim \triangle ABC \text{ (AA similarity)}$$

$$\therefore \frac{ED}{BC} = \frac{AE}{AB} = \frac{1}{2} \text{ (since, E is the mid - point of AB)}$$

$$\Rightarrow \frac{ED}{BC} = \frac{1}{2}$$

From (1),

$$\frac{GD}{GB} = \frac{1}{2}$$

$$GB = 2GD$$

EXERCISE. 15(B)**Question 1:**

- (i) The ratio between the corresponding sides of two similar triangles is 2 is to 5. Find the ratio between the areas of these triangles.
- (ii) Area of two similar triangles are 98 sq.cm and 128 sq.cm. Find the ratio between the lengths of their corresponding sides.

Solution 1:

We know that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

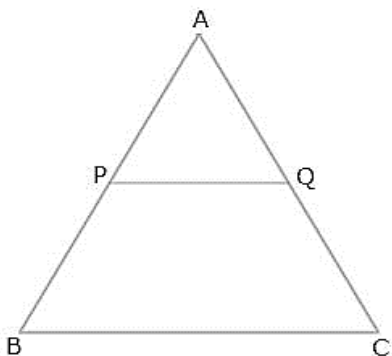
(i) Required ratio = $\frac{2^2}{5^2} = \frac{4}{25}$

(ii) Required ratio = $\sqrt{\frac{98}{128}} = \sqrt{\frac{49}{64}} = \frac{7}{8}$

Question 2:

A line PQ is drawn parallel to the base BC of $\triangle ABC$ which meets sides AB and AC at points P and Q respectively. If $AP = \frac{1}{3} PB$; find the value of:

- (i) $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle APQ}$
- (ii) $\frac{\text{Area of } \triangle APQ}{\text{Area of trapezium PBCQ}}$

Solution 2:

(i) $AP = \frac{1}{3} PB \Rightarrow \frac{AP}{PB} = \frac{1}{3}$

In $\triangle APQ$ and $\triangle ABC$,

As $PQ \parallel BC$, corresponding angles are equal

$$\angle APQ = \angle ABC$$

$$\angle AQP = \angle ACB$$

$$\triangle APQ \sim \triangle ABC$$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle APQ} = \frac{AB^2}{AP^2}$$

$$= \frac{4^2}{1^2} = 16 : 1$$

$$\left(\frac{AP}{PB} = \frac{1}{3} \Rightarrow \frac{AB}{AP} = \frac{4}{1} \right)$$

$$\begin{aligned} & \frac{\text{Area of } \triangle APQ}{\text{Area of trapezium PBCQ}} \\ &= \frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ABC - \text{Area of } \triangle APQ} \\ &= \frac{1}{16 - 1} = 1 : 5 \end{aligned}$$

Question 3:

The perimeter of two similar triangles are 30 cm and 24 cm. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

Solution 3:

Let $\triangle ABC \sim \triangle DEF$

$$\begin{aligned} \text{Then, } \frac{AB}{DE} &= \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB + BC + AC}{DE + EF + DF} \\ &= \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} \end{aligned}$$

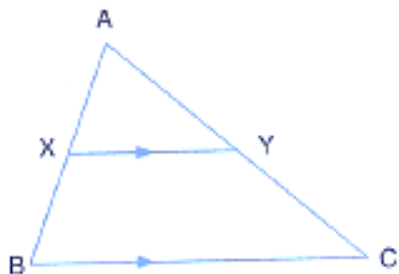
$$\Rightarrow \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB}{DE}$$

$$\Rightarrow \frac{30}{24} = \frac{12}{DE}$$

$$\Rightarrow DE = 9.6 \text{ cm}$$

Question 4:

In the given figure, $AX : XB = 3 : 5$



Find:

- (i) the length of BC, if the length of XY is 18 cm.
- (ii) the ratio between the areas of trapezium XBCY and triangle ABC.

Solution 4:

Given, $\frac{AX}{XB} = \frac{3}{5} \Rightarrow \frac{AX}{AB} = \frac{3}{8}$ (1)

(i)

In $\triangle AXY$ and $\triangle ABC$,

As $XY \parallel BC$, Corresponding angles are equal

$$\angle AXY = \angle ABC$$

$$\angle AYX = \angle ACB$$

$$\triangle AXY \sim \triangle ABC$$

$$\Rightarrow \frac{AX}{AB} = \frac{XY}{BC}$$

$$\Rightarrow \frac{3}{8} = \frac{18}{BC}$$

$$\Rightarrow BC = 48 \text{ cm}$$

(ii)

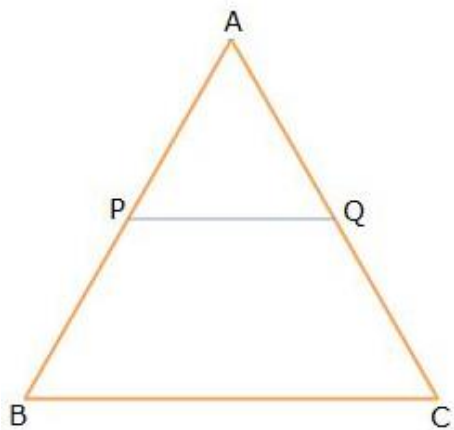
$$\frac{\text{Area of } \triangle AXY}{\text{Area of } \triangle ABC} = \frac{AX^2}{AB^2} = \frac{9}{64}$$

$$\frac{\text{Area of } \triangle ABC - \text{Area of } \triangle AXY}{\text{Area of } \triangle ABC} = \frac{64 - 9}{64} = \frac{55}{64}$$

$$\frac{\text{Area of trapezium XBCY}}{\text{Area of } \triangle ABC} = \frac{55}{64}$$

Question 5:

ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that $PQ \parallel BC$ and divides triangle ABC into two parts equal in area. Find the value of ratio BP : AB.

Solution 5:

From the given information, we have:

$$\text{ar}(\triangle APQ) = \frac{1}{2} \text{ar}(\triangle ABC)$$

$$\Rightarrow \frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)} = \frac{1}{2}$$

$$\Rightarrow \frac{AP^2}{AB^2} = \frac{1}{2}$$

$$\Rightarrow \frac{AP}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{AB - BP}{AB} = \frac{1}{\sqrt{2}}$$

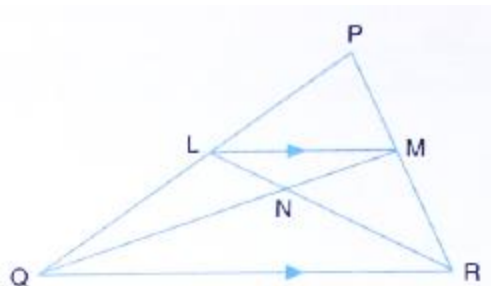
$$\Rightarrow 1 - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}} = \frac{2 - \sqrt{2}}{2}$$

Question 6:

In the given triangle PQR, LM is parallel to QR and $PM : MR = 3 : 4$



Calculate the value of ratio:

(i) $\frac{PL}{PQ}$ and then $\frac{LM}{QR}$

(ii) $\frac{\text{Area of } \triangle LMN}{\text{Area of } \triangle MNR}$ (iii) $\frac{\text{Area of } \triangle LQM}{\text{Area of } \triangle LQN}$

Solution 6:

(i)

In $\triangle PLM$ and $\triangle PQR$,

As $LM \parallel QR$, Corresponding angles are equal

$$\angle PLM = \angle PQR$$

$$\angle PML = \angle PRQ$$

$$\triangle PLM \sim \triangle PQR$$

$$\Rightarrow \frac{3}{7} = \frac{LM}{QR} \left(\because \frac{PM}{MR} = \frac{3}{4} \Rightarrow \frac{PM}{PR} = \frac{3}{7} \right)$$

Also, by using basic proportionality theorem, we have:

$$\frac{PL}{LQ} = \frac{PM}{MR} = \frac{3}{4}$$

$$\Rightarrow \frac{LQ}{PL} = \frac{4}{3}$$

$$\Rightarrow 1 + \frac{LQ}{PL} = 1 + \frac{4}{3}$$

$$\Rightarrow \frac{PL + LQ}{PL} = \frac{3 + 4}{3}$$

$$\Rightarrow \frac{PQ}{PL} = \frac{7}{3}$$

$$\Rightarrow \frac{PL}{PQ} = \frac{3}{7}$$

(ii) Since $\triangle LMN$ and $\triangle MNR$ have common vertex at M and their bases LN and NR are along the same straight line

$$\therefore \frac{\text{Area of } \triangle LMN}{\text{Area of } \triangle MNR} = \frac{LN}{NR}$$

Now, in $\triangle LNM$ and $\triangle RNQ$

$\angle NLM = \angle NRQ$ (Alternate angles)

$\angle LMN = \angle NQR$ (Alternate angles)

$\triangle LMN \sim \triangle RNQ$ (AA Similarity)

$$\therefore \frac{MN}{QN} = \frac{LN}{NR} = \frac{LM}{QR} = \frac{3}{7}$$

$$\therefore \frac{\text{Area of } \triangle LMN}{\text{Area of } \triangle MNR} = \frac{LN}{NR} = \frac{3}{7}$$

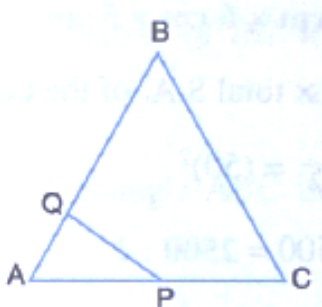
(iii) Since $\triangle LQM$ and $\triangle LQN$ have common vertex at L and their bases QM and QN are along the same straight line

$$\frac{\text{Area of } \triangle LQM}{\text{Area of } \triangle MNR} = \frac{QM}{QN} = \frac{10}{7}$$

$$\left(\because \frac{MN}{QN} = \frac{3}{7} \Rightarrow \frac{QM}{QN} = \frac{10}{7} \right)$$

Question 7:

The given diagram shows two isosceles triangles which are similar also. In the given diagram, PQ and BC are not parallel; PC = 4, AQ = 3, QB = 12, BC = 15 and AP = PQ



Calculate:

- (i) the length of AP,
- (ii) the ratio of the areas of triangle APQ and triangle ABC.

Solution 7:

(i)

Given, $\triangle AQP \sim \triangle ACB$

$$\Rightarrow \frac{AQ}{AC} = \frac{AP}{AB}$$

$$\Rightarrow \frac{3}{4 + AP} = \frac{AP}{3 + 12}$$

$$\Rightarrow AP^2 + 4AP - 45 = 0$$

$$\Rightarrow (AP + 9)(AP - 5) = 0$$

$$\Rightarrow AP = 5 \text{ units (as length cannot be negative)}$$

(ii)

Since, $\triangle AQP \sim \triangle ACB$

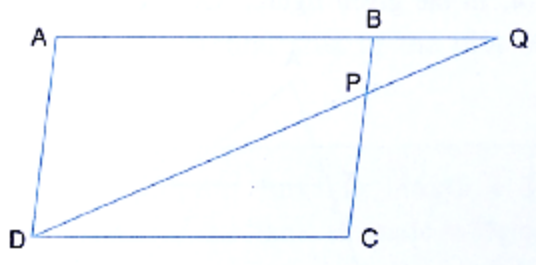
$$\therefore \frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ACB)} = \frac{PQ^2}{BC^2}$$

$$\Rightarrow \frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)} = \frac{AP^2}{BC^2} \quad (PQ = AP)$$

$$\Rightarrow \frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)} = \left(\frac{5}{15}\right)^2 = \frac{1}{9}$$

Question 8:

In the figure, given below, ABCD is a parallelogram. P is a point on BC such that $BP : PC = 1 : 2$. DP produced meets AB produces at Q. Given the area of triangle CPQ = 20 cm^2 .



Calculate:

- (i) area of triangle CDP,
- (ii) area of parallelogram ABCD.

Solution 8:(i) In $\triangle BPQ$ and $\triangle CPD$ $\angle BPQ = \angle CPD$ (Vertically opposite angles)

$\angle BQP = \angle PDC$ (Alternate angles)

$\triangle BPQ \sim \triangle CPD$ (AA similarity)

$$\therefore \frac{BP}{PC} = \frac{PQ}{PD} = \frac{BQ}{CD} = \frac{1}{2} \left(\because \frac{BP}{PC} = \frac{1}{2} \right)$$

$$\text{Also, } \frac{\text{ar}(\triangle BPQ)}{\text{ar}(\triangle CPD)} = \left(\frac{BP}{PC} \right)^2$$

$$\Rightarrow \frac{10}{\text{ar}(\triangle CPD)} = \frac{1}{4} \quad [\text{ar}(\triangle BPQ) = \frac{1}{2} \times \text{ar}(\triangle CPQ), \text{ar}(\triangle CPQ) = 20]$$

$$\Rightarrow \text{ar}(\triangle CPD) = 40 \text{ cm}^2$$

(ii) In $\triangle BAP$ and $\triangle AQD$

As $BP \parallel AD$, corresponding angles are equal

$\angle QBP = \angle QAD$

$\angle BQP = \angle AQD$ (Common)

$\triangle BQP \sim \triangle AQD$ (AA similarity)

$$\therefore \frac{AQ}{BQ} = \frac{QD}{QP} = \frac{AD}{BP} = 3 \left(\because \frac{BP}{PC} = \frac{PQ}{PD} = \frac{1}{2} \Rightarrow \frac{PQ}{QD} = \frac{1}{3} \right)$$

$$\text{Also, } \frac{\text{ar}(\triangle AQD)}{\text{ar}(\triangle BQP)} = \left(\frac{AQ}{BQ} \right)^2$$

$$\Rightarrow \frac{\text{ar}(\triangle AQD)}{10} = 9$$

$$\Rightarrow \text{ar}(\triangle AQD) = 90 \text{ cm}^2$$

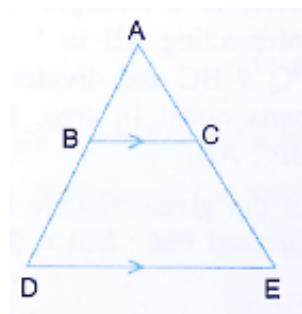
$$\therefore \text{ar}(\triangle ADP) = \text{ar}(\triangle AQD) - \text{ar}(\triangle BQP) = 90 \text{ cm}^2 - 10 \text{ cm}^2 = 80 \text{ cm}^2$$

$$\text{ar}(\triangle ABC) = \text{ar}(\triangle ADP) + \text{ar}(\triangle BQP) = 80 \text{ cm}^2 + 10 \text{ cm}^2 = 90 \text{ cm}^2$$

Question 9:

In the given figure, BC is parallel to DE . Area of triangle $ABC = 25 \text{ cm}^2$, Area of trapezium $BCED = 24 \text{ cm}^2$ and $DE = 14 \text{ cm}$. Calculate the length of BC .

Also, find the area of triangle BCD .



Solution 9:

In $\triangle ABC$ and $\triangle ADE$,

As $BC \parallel DE$, corresponding angles are equal

$$\angle ABC = \angle ADE$$

$$\angle ACB = \angle AED$$

$$\triangle ABC \sim \triangle ADE$$

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle ADE)} = \frac{BC^2}{DE^2}$$

$$\frac{25}{49} = \frac{BC^2}{14^2} \quad (\text{ar}(\triangle ADE) = \text{ar}(\triangle ABC) + \text{ar}(\text{trapezium } BCED))$$

$$BC^2 = 100$$

$$BC = 10 \text{ cm}$$

In trapezium $BCED$,

$$\text{Area} = \frac{1}{2} (\text{Sum of parallel sides}) \times h$$

Given : Area of trapezium $BCED = 24 \text{ cm}^2$, $BC = 10 \text{ cm}$, $DE = 14 \text{ cm}$

$$\therefore 24 = \frac{1}{2} (10 + 14) \times h$$

$$\Rightarrow h = \frac{48}{(10 + 14)}$$

$$\Rightarrow h = \frac{48}{24}$$

$$\Rightarrow h = 2$$

$$\text{Area of } \triangle BCD = \frac{1}{2} \times \text{base} \times \text{height}$$

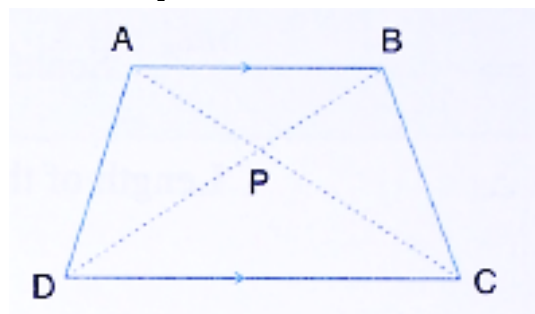
$$= \frac{1}{2} \times BC \times h$$

$$= \frac{1}{2} \times 10 \times 2$$

$$\therefore \text{Area of } \triangle BCD = 10 \text{ cm}^2$$

Question 10:

The given figure shows a trapezium in which AB is parallel to DC and diagonals AC and BD intersect at point P. If AP : CP = 3 : 5,



Find:

- (i) $\triangle APB : \triangle CPB$ (ii) $\triangle DPC : \triangle APB$
 (iii) $\triangle ADP : \triangle APB$ (iv) $\triangle APB : \triangle ADB$

Solution 10:

(i) Since $\triangle APB$ and $\triangle CPB$ have common vertex at B and their bases AP and PC are along the same straight line

$$\therefore \frac{\text{ar}(\triangle APB)}{\text{ar}(\triangle CPB)} = \frac{AP}{PC} = \frac{3}{5}$$

(ii) Since $\triangle DPC$ and $\triangle BPA$ are similar

$$\therefore \frac{\text{ar}(\triangle DPC)}{\text{ar}(\triangle BPA)} = \left(\frac{PC}{AP}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

(iii) Since $\triangle ADP$ and $\triangle APB$ have common vertex at A and their bases DP and PB are along the same straight line

$$\therefore \frac{\text{ar}(\triangle ADP)}{\text{ar}(\triangle APB)} = \frac{DP}{PB} = \frac{5}{3}$$

$$\left(\triangle APB \sim \triangle CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5} \right)$$

(iv) Since $\triangle APB$ and $\triangle ADB$ have common vertex at A and their bases BP and BD are along the same straight line.

$$\therefore \frac{\text{ar}(\triangle APB)}{\text{ar}(\triangle ADB)} = \frac{BP}{BD} = \frac{3}{8}$$

$$\left(\triangle APB \sim \triangle CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5} \Rightarrow \frac{BP}{BD} = \frac{3}{8} \right)$$

Question 11:

On a map, drawn to a scale of 1 : 250000, a triangular plot PQR of land has the following measurements : PQ = 3cm, QR = 4 cm and angles PQR = 90°

Calculate:

- (i) the actual lengths of PQ and PR in kilometer.
- (ii) the actual area of the plot in sq . km.

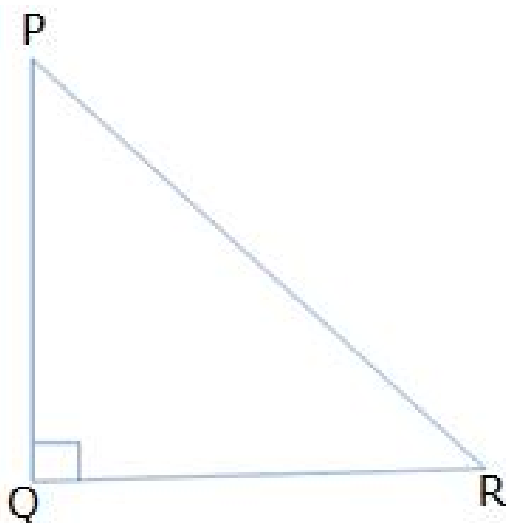
Solution 11:

Scale:-1 : 250000

∴ 1 cm represents 250000cm

$$= \frac{250000}{1000 \times 100} = 2.5 \text{ km}$$

∴ 1 cm represents 2.5 km



(i)

Actual length of PQ = $3 \times 2.5 = 7.5$ km

Actual length of QR = $4 \times 2.5 = 10$ km

Actual length of PR = $\sqrt{(7.5)^2 + (10)^2}$ km = 12.5km

(ii)

$$\text{Area of } \triangle PQR = \frac{1}{2} \times PQ \times QR = \frac{1}{2} (3)(4) \text{ cm}^2 = 6 \text{ cm}^2$$

1cm represents 2.5 km

1 cm² represents 2.5×2.5 km²

The area of plot = $2.5 \times 2.5 \times 6$ km² = 37.5 km²

Question 12:

A model of a ship is made to a scale of 1 : 200.

- (i) The length of the model is 4 m; calculate the length of the ship.
- (ii) The area of the deck of the ship is 160000 m²; find the area of the deck of the model.
- (iii) The volume of the model is 200 litres; calculate the volume of the ship in m³.

Solution 12:

$$\text{Scale factor} = k = \frac{1}{200}$$

(i) Length of model = $k \times$ actual length of the ship

$$\Rightarrow \text{Actual length of the ship} = 4 \times 200 = 800 \text{ m}$$

(ii) Area of the deck of the model = $k^2 \times$ area of the deck of the ship

$$= \left(\frac{1}{200} \right)^2 \times 160000 \text{ m}^2 = 4 \text{ m}^2$$

(iii) Volume of the model = $k^3 \times$ volume of the ship

Volume of the ship

$$= \frac{1}{k^3} \times 200 \text{ liters}$$

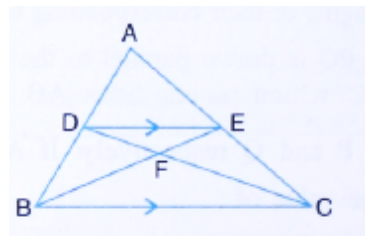
$$= (200)^3 \times 200 \text{ liters}$$

$$= 1600000000 \text{ liters}$$

$$= 1600000 \text{ m}^3$$

Question 13:

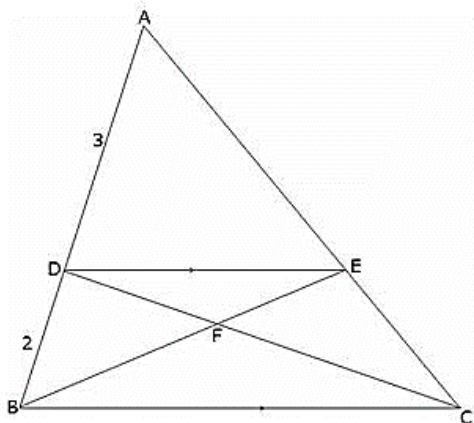
In the given figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$



(i) Determine the ratios $\frac{AD}{AB}$ and $\frac{DE}{BC}$

(ii) Prove that $\triangle DEF$ is similar to $\triangle CBF$. Hence, find $\frac{EF}{FB}$.

(iii) What is the ratio of the areas of $\triangle DEF$ and $\triangle BFC$.

Solution 13:

(i) Given, $DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{2}$

In $\triangle ADE$ and $\triangle ABC$,

$\angle A = \angle A$ (Corresponding Angles)

$\angle ADE = \angle ABC$ (Corresponding Angles)

$\therefore \triangle ADE \sim \triangle ABC$ (By AA- similarity)

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \dots\dots\dots (1)$$

$$\text{Now } \frac{AD}{AB} = \frac{AD}{AD + DB} = \frac{3}{3 + 2} = \frac{3}{5}$$

$$\text{Using (1), we get } \frac{AD}{AE} = \frac{3}{5} = \frac{DE}{BC} \dots\dots\dots (2)$$

(ii) In $\triangle DEF$ and $\triangle CBF$,

$\angle FDE = \angle FCB$ (Alternate Angle)

$\angle DFE = \angle BFC$ (Vertically Opposite Angle)

$\therefore \triangle DEF \sim \triangle CBF$ (By AA- similarity)

$$\frac{EF}{FB} = \frac{DE}{BC} = \frac{3}{5} \text{ Using (2)}$$

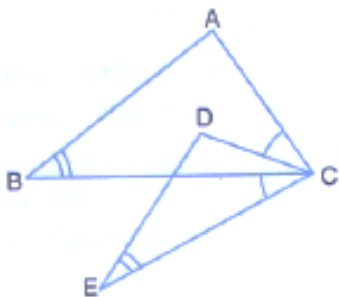
$$\frac{EF}{FB} = \frac{3}{5}$$

(iii) Since the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides, therefore.

$$\frac{\text{Area of } \triangle DFE}{\text{Area of } \triangle CBF} = \frac{EF^2}{FB^2} = \frac{3^2}{5^2} = \frac{9}{25}$$

Question 14:

In the given figure, $\angle B = \angle E$, $\angle ACD = \angle BCE$, $AB = 10.4\text{ cm}$ and $DE = 7.8\text{ cm}$. Find the ratio between areas of the $\triangle ABC$ and $\triangle DEC$

**Solution 14:**

Given, $\angle ACD = \angle BCE$

$$\angle ACD + \angle BCD = \angle BCE + \angle BCD$$

$$\angle ACB = \angle DCE$$

Also, given $\angle B = \angle E$

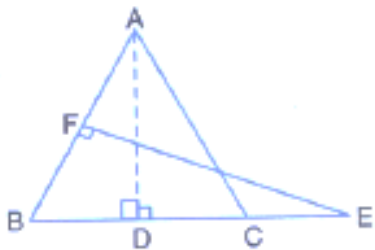
$$\therefore \triangle ABC \sim \triangle DEC$$

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEC)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{10.4}{7.8}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Question 15:

Triangle ABC is an isosceles triangle in which $AB = AC = 13\text{ cm}$ and $BC = 10\text{ cm}$. AD is perpendicular to BC . If $CE = 8\text{ cm}$ and $EF \perp AB$, find:

- (i) $\frac{\text{area of } \triangle ADC}{\text{area of } \triangle FEB}$ (ii) $\frac{\text{area of } \triangle AFE}{\text{area of } \triangle ABC}$

**Solution 15:**

(i) $AB = AC$ (Given)

$$\therefore \angle FBE = \angle ACD$$

$$\angle BFE = \angle ADC$$

$$\triangle EFB \sim \triangle ADC \quad (\text{AA similarity})$$

$$\begin{aligned}
 \therefore \frac{\text{ar}(\triangle ADC)}{\text{ar}(\triangle EFB)} &= \left(\frac{AC}{BE}\right)^2 \\
 &= \left(\frac{AC}{BC + CE}\right)^2 \\
 &= \left(\frac{13}{18}\right)^2 = \frac{169}{324} \dots\dots\dots(1)
 \end{aligned}$$

(ii) Similarly, it can be proved that $\triangle ADB \sim \triangle EFB$

$$\begin{aligned}
 \therefore \frac{\text{ar}(\triangle ADB)}{\text{ar}(\triangle EFB)} &= \left(\frac{AB}{BE}\right)^2 \\
 &= \left(\frac{13}{18}\right)^2 \\
 &= \frac{169}{324} \dots\dots\dots(2)
 \end{aligned}$$

From (1) and (2),

$$\begin{aligned}
 \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle EFB)} &= \frac{169 + 169}{324} = \frac{338}{324} = \frac{169}{162} \\
 \therefore \text{ar}(\triangle EFB) : \text{ar}(\triangle ABC) &= 162 : 169
 \end{aligned}$$

Question 16:

An aeroplane is 30 m long and its model is 15 cm long. If the total outer surface area of the model is 150 cm^2 , find the cost of painting the outer surface of the aeroplane at the rate of Rs. 120 per sq. m. Given that 50 sq. m of the surface of the aeroplane is left for windows.

Solution 16:

15cm represents = 30 m

1cm represents $\frac{30}{15} = 2\text{m}$

1 cm^2 represents $2\text{m} \times 2\text{m} = 4 \text{ m}^2$

Surface area of the model = 150 cm^2

Actual surface area of aeroplane = $150 \times 2 \times 2 \text{ m}^2 = 600 \text{ m}^2$

50 m^2 is left out for windows

Area to be painted = $600 - 50 = 550 \text{ m}^2$

Cost of painting per m^2 = Rs. 120

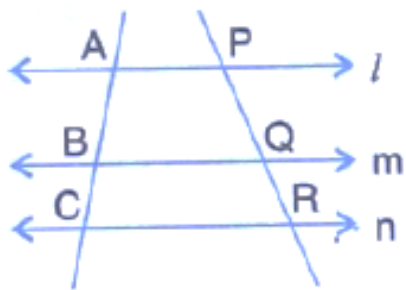
Cost of painting $550 \text{ m}^2 = 120 \times 550 = \text{Rs. } 66000$

EXERCISE. 15 (C)**Question 1:**

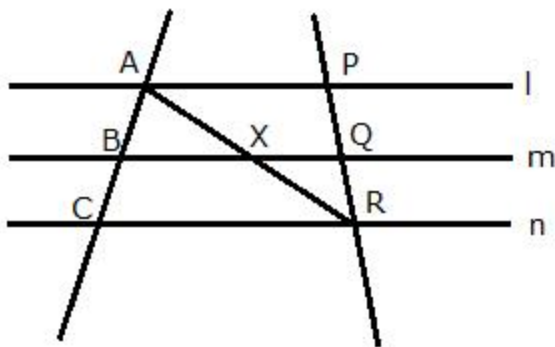
In the given figure, lines l , m and n are such that $l \parallel m \parallel n$.

Prove that:

$$\frac{AB}{BC} = \frac{PQ}{QR}$$

**Solution 1:**

Join AR.



In $\triangle ACR$, $BX \parallel CR$. By Basic Proportionality theorem,

$$\frac{AB}{BC} = \frac{AX}{XR} \quad \dots\dots\dots(1)$$

In $\triangle APR$, $XQ \parallel AP$. By Basic Proportionality theorem,

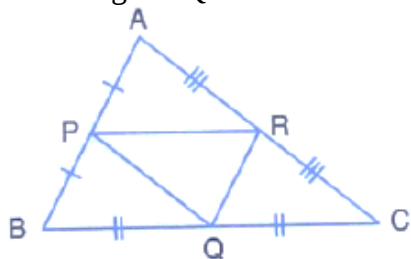
$$\frac{PQ}{QR} = \frac{AX}{XR} \quad \dots\dots\dots(2)$$

From (1) and (2), we get,

$$\frac{AB}{BC} = \frac{PQ}{QR}$$

Question 2:

In the given triangle P, Q and R are the mid points of sides AB, BC and AC respectively. Prove that triangle PQR is similar to triangle ABC.

**Solution 2:**

In $\triangle ABC$, $PR \parallel BC$. By Basic proportionality theorem,

$$\frac{AP}{PB} = \frac{AR}{RC}$$

Also, in $\triangle PAR$ and $\triangle ABC$,

$$\angle PAR = \angle BAC \quad (\text{common})$$

$$\angle APR = \angle ABC \quad (\text{Corresponding angles})$$

$$\triangle PAR \sim \triangle BAC \quad (\text{AA similarity})$$

$$\frac{PR}{BC} = \frac{AP}{AB}$$

$$\frac{PR}{BC} = \frac{1}{2} \quad (\text{As P is the mid-point of AB})$$

$$\frac{PR}{BC} = \frac{1}{2} BC$$

$$\text{Similarly, } PQ = \frac{1}{2} AC$$

$$RQ = \frac{1}{2} AB$$

$$\text{Thus, } \frac{PR}{BC} = \frac{PQ}{AC} = \frac{RQ}{AB}$$

$$\Rightarrow \triangle QRP \sim \triangle ABC \quad (\text{SSS similarity})$$

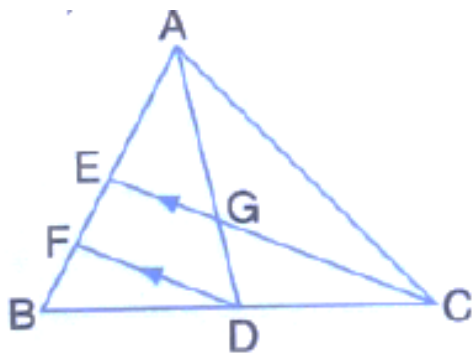
Question 3:

In the following figure, AD and CE are medians of $\triangle ABC$. DF is drawn parallel to CE.

Prove that :

(i) $EF = FB$,

(ii) $AG : GD = 2 : 1$

**Solution 3:**

(i)

In $\triangle BFD$ and $\triangle BEC$, $\angle BFD = \angle BEC$ (Corresponding angles) $\angle FBD = \angle EBC$ (Common) $\triangle BFD \sim \triangle BEC$ (AA Similarity)

$$\therefore \frac{BF}{BE} = \frac{BD}{BC}$$

$$\frac{BF}{BE} = \frac{1}{2} \quad (\text{As } D \text{ is the mid - point of } BC)$$

$$BE = 2BF$$

$$BF = FE = 2BF$$

Hence, $EF = FB$ (ii) In $\triangle AFD$, $EG \parallel FD$. Using Basic Proportionality theorem,

$$\frac{AE}{EF} = \frac{AG}{GD} \dots\dots\dots (1)$$

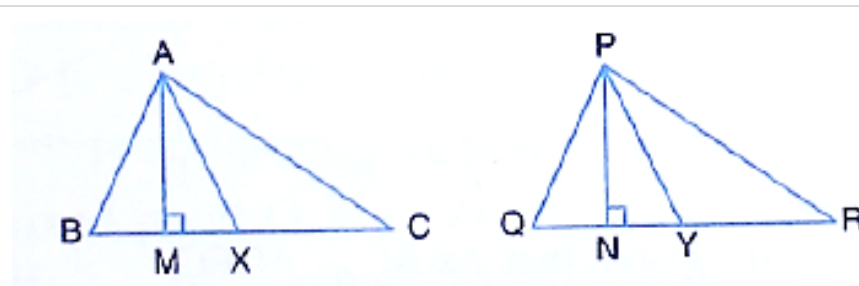
Now, $AE = EB$ (as E is the mid-point of AB) $AE = 2EF$ (Since, $EF = FB$, by (i))

From (1),

$$\frac{AG}{GD} = \frac{2}{1}$$

Hence, $AG : GD = 2 : 1$ **Question 4:**

In the given figure, triangle ABC is similar to triangle PQR . AM and PN are altitudes whereas AX and PY are medians.



Prove that : $\frac{AM}{PN} = \frac{AX}{PY}$

Solution 4:

Since $\triangle ABC \sim \triangle PQR$

So, their respective sides will be in proportion

$$\text{Or, } \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

Also, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

In $\triangle ABM$ and $\triangle PQN$,

$\angle ABM = \angle PQN$ (Since, ABC and PQR are similar)

$\angle AMB = \angle PNQ = 90^\circ$

$\triangle ABM \sim \triangle PQN$ (AA similarity)

$$\therefore \frac{AM}{PN} = \frac{AB}{PQ} \dots\dots\dots (1)$$

Since, AX and PY are medians so they will divide their opposite sides.

$$\text{Or, } BX = \frac{BC}{2} \text{ and } QY = \frac{QR}{2}$$

Therefore, we have:

$$\frac{AB}{PQ} = \frac{BX}{QY}$$

$$\angle B = \angle Q$$

So, we had observed that two respective sides are in same proportion in both triangles and also angle included between them is respectively equal.

Hence, $\triangle ABX \sim \triangle PQY$ (by SAS similarity rule)

$$\text{So, } \frac{AB}{PQ} = \frac{AX}{PY} \dots\dots\dots (2)$$

From (1) and (2),

$$\frac{AM}{PN} = \frac{AX}{PY}$$

Question 5:

The two similar triangles are equal in area. Prove that the triangles are congruent.

Solution 5:

Let us assume two similar triangles as $\triangle ABC \sim \triangle PQR$

$$\text{Now } \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Since $\text{area}(\triangle ABC) = \text{area}(\triangle PQR)$

Therefore, $AB = PQ$

$BC = QR$

$AC = PR$

So, respective sides of two similar triangles

Are also of same length

So, $\triangle ABC \cong \triangle PQR$ (by SSS rule)

Question 6:

The ratio between the altitudes of two similar triangles is 3 : 5; write the ratio between their:

(i) medians (ii) perimeters (iii) areas

Solution 6:

The ratio between the altitudes of two similar triangles is same as the ratio between their sides.

(i) The ratio between the medians of two similar triangles is same as the ratio between their sides.

$\therefore$ Required ratio = 3 : 5

(ii) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.

$\therefore$ Required ratio = 3 : 5

(iii) The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides.

$\therefore$ Required ratio = $(3)^2 : (5)^2 = 9 : 25$

Question 7:

The ratio between the areas of two similar triangles is 16 : 25, Find the ratio between their:

(i) perimeters (ii) altitudes (iii) medians

Solution 7:

The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides.

So, the ratio between the sides of the two triangles = 4 : 5

(i) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.

∴ Required ratio = 4 : 5

(ii) The ratio between the altitudes of two similar triangles is same as the ratio between their sides.

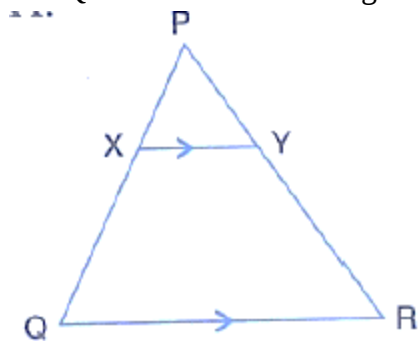
∴ Required ratio = 4 : 5

(iii) The ratio between the medians of two similar triangles is same as the ratio between their sides.

∴ Required ratio = 4 : 5

Question 8:

The following figure shows a triangle PQR in which XY is parallel to QR. If $PX : XQ = 1 : 3$ and $QR = 9$ cm. find the length of XY.



Further, if the area of $\triangle PXY = x$ cm²; find, in terms of x the area of:

(i) triangle PQR (ii) trapezium XQRY

Solution 8:

In $\triangle PXY$ and $\triangle PQR$, XY is parallel to QR, so corresponding angles are equal.

$$\angle PXY = \angle PQR$$

$$\angle PYX = \angle PRQ$$

Hence, $\triangle PXY \sim \triangle PQR$ (By AA similarity criterion)

$$\frac{PX}{PQ} = \frac{XY}{QR}$$

$$\Rightarrow \frac{1}{4} = \frac{XY}{9} \quad (PX : XQ = 1 : 3 \Rightarrow PX : PQ = 1 : 4)$$

$$\Rightarrow \frac{1}{4} = \frac{XY}{9}$$

$$\Rightarrow XY = 2.25 \text{ cm}$$

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\frac{\text{Ar}(\triangle PXY)}{\text{Ar}(\triangle PQR)} = \left(\frac{PX}{PQ}\right)^2$$

$$\frac{x}{\text{Ar}(\triangle PQR)} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

$$\text{Ar}(\triangle PQR) = 16x \text{ cm}^2$$

(ii) $\text{Ar}(\text{trapezium } XQRY) = \text{Ar}(\triangle PQR) - \text{Ar}(\triangle PXY)$

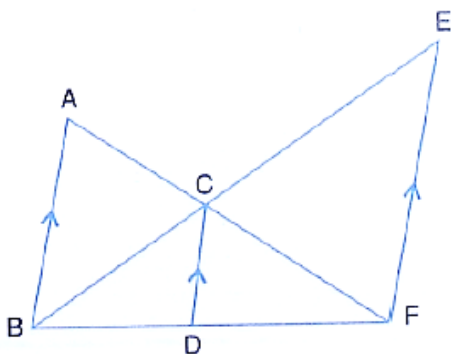
$$= (16x - x) \text{ cm}^2$$

$$= 15x \text{ cm}^2$$

Question 9:

In the following figure, AB, CD and EF are parallel lines. AB = 6cm, CD = y cm, EF = 10 cm, AC = 4 cm and CF = x cm.

Calculate x and y



Solution 9:

In $\triangle FDC$ and $\triangle FBA$,

$\angle FDC = \angle FDA$ (Corresponding angles)

$\angle DFC = \angle BFA$ (Common)

$\triangle FDC \sim \triangle FBA$ (AA similarity)

$$\frac{CD}{AB} = \frac{FC}{FA}$$

$$\frac{y}{6} = \frac{x}{x+4} \dots\dots\dots (1)$$

In $\triangle FCE$ and $\triangle ACB$,

$\angle FCE = \angle ACB$ (vertically opposite angles)

$\angle CFE = \angle CAB$ (Alternate angles)

$\triangle FCE \sim \triangle ACB$ (AA similarity)

$$\frac{FC}{AC} = \frac{EF}{AB}$$

$$\frac{x}{4} = \frac{10}{6} \Rightarrow x = \frac{20}{3} = 6\frac{2}{3} \text{ cm}$$

From (1):

$$y = \frac{6 \times \frac{20}{3}}{\frac{20}{3} + 4} = 3.75$$

Question 10:

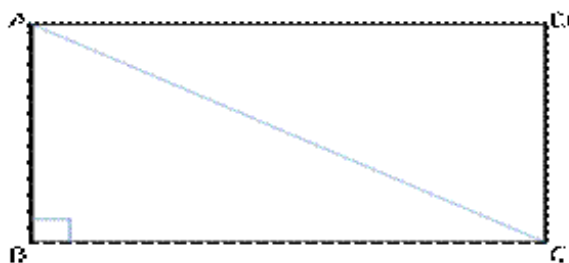
On a map, drawn to a scale of 1 : 20000, a rectangular plot of land ABCD has AB = 24cm and BC = 32 cm. Calculate:

- the diagonal distance of the plot in kilometer
- the area of the plot in sq.km

Solution 10:

Scale :- 1 : 20000

1 cm represents 20000 cm = $\frac{20000}{1000 \times 100} = 0.2 \text{ km}$



(i)

$$AC^2 = AB^2 + BC^2$$

$$= 24^2 + 32^2$$

$$= 576 + 1024 = 1600$$

$$AC = 40 \text{ cm}$$

Actual length of diagonal = $40 \times 0.2 \text{ km} = 8 \text{ km}$

(ii)

1 cm represents 0.2 km

1 cm² represents 0.2×0.2 km²

The area of the rectangle ABCD = AB × BC

$$= 24 \times 32 = 768 \text{ cm}^2$$

$$\text{Actual area of the plot} = 0.2 \times 0.2 \times 768 \text{ km}^2 = 30.72 \text{ km}^2$$

Question 11:

The dimensions of the model of a multistoreyed building are 1 m by 60 cm by 1.20 m. if the scale factor is 1 : 50, find the actual dimensions of the building.

Also find:

(i) the floor area of a room of the building, if the floor area of the corresponding room in the model is 50 sq. cm

(ii) the space (volume) inside a room of the model, if the space inside the corresponding room of the building is 90 m³.

Solution 11:

The dimensions of the building are calculated as below.

$$\text{Length} = 1 \times 50 \text{ m} = 50 \text{ m}$$

$$\text{Breadth} = 0.60 \times 50 \text{ m} = 30 \text{ m}$$

$$\text{Height} = 1.20 \times 50 \text{ m} = 60 \text{ m}$$

Thus, the actual dimensions of the building are 50 m × 30 m × 60 m.

(i)

Floor area of the room of the building =

$$50 \times \left(\frac{50}{1}\right)^2 = 125000 \text{ cm}^2 = \frac{125000}{100 \times 100} = 12.5 \text{ m}^2$$

(ii)

Volume of the model of the building

$$\begin{aligned} &= 90 \left(\frac{1}{50}\right)^3 = 90 \times \left(\frac{1}{50}\right) \times \left(\frac{1}{50}\right) \times \left(\frac{1}{50}\right) = 90 \times \left(\frac{100 \times 100 \times 100}{50 \times 50 \times 50}\right) \text{ cm}^3 \\ &= 720 \text{ cm}^3 \end{aligned}$$

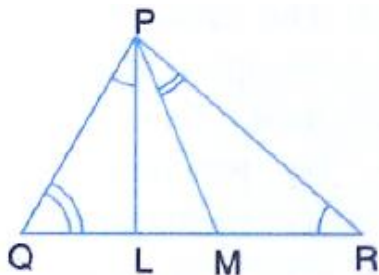
Question 12:

In a triangle PQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$. Prove that:

(i) ΔPQL and ΔRMP

(ii) $QL \times RM = PL \times PM$

(iii) $PQ^2 = QR \times QL$

**Solution 12:**

In $\triangle PQL$ and $\triangle RMP$

$\angle LPQ = \angle QRP$ (Given)

$\angle RQP = \angle RPM$ (Given)

$\triangle PQL \sim \triangle RMP$ (AA similarity)

(ii)

As $\triangle PQL \sim \triangle RMP$ (proved above)

$$\frac{PQ}{RP} = \frac{QL}{PM} = \frac{PL}{RM}$$

$$\Rightarrow QL \times RM = PL \times PM$$

(iii)

$\angle LPQ = \angle QRP$ (Given)

$\angle Q = \angle Q$ (Common)

$\triangle PQL \sim \triangle RQP$ (AA similarity)

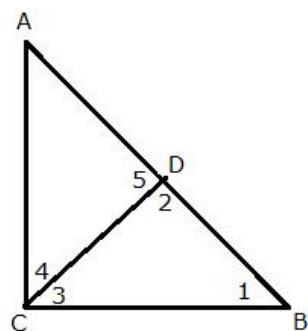
$$= \frac{PQ}{RQ} = \frac{QL}{QP} = \frac{PL}{PR}$$

$$\Rightarrow PQ^2 = QR \times QL$$

Question 13:

In $\triangle ABC$, $\angle ACB = 90^\circ$ and $CD \perp AB$.

Prove that : $\frac{BC^2}{AC^2} = \frac{BD}{AD}$

Solution 13:

In $\triangle CDB$,

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

$$\angle 1 + \angle 3 = 90^\circ \dots (1) \text{ (Since, } \angle 2 = 90^\circ)$$

$$\angle 3 + \angle 4 = 90^\circ \dots (2) \text{ (Since, } \angle ACB = 90^\circ)$$

From (1) and (2),

$$\angle 1 + \angle 3 = \angle 3 + \angle 4$$

$$\angle 1 = \angle 4$$

$$\text{Also, } \angle ADC = \angle ACB = 90^\circ$$

$\therefore \triangle ACD \sim \triangle ABC$ (AA similarity)

$$\therefore \frac{AC}{AB} = \frac{AD}{AC}$$

$$AC^2 = AB \times AD \dots\dots\dots (1)$$

$$\text{Now } \angle BDC = \angle ACB = 90^\circ$$

$$\angle CBD = \angle ABC \text{ (common)}$$

$\triangle BCD \sim \triangle BAC$ (AA similarity)

$$\therefore \frac{BC}{BA} = \frac{BD}{BC} \dots\dots\dots (2)$$

$$BC^2 = BA \times BD$$

From (1) and (2), we get,

$$\frac{BC^2}{AC^2} = \frac{BA \times BD}{AB \times AD} = \frac{BD}{AD}$$

Question 14:

A triangle ABC with $AB = 3$ cm, $BC = 6$ cm and $AC = 4$ cm is enlarged to $\triangle DEF$ such that the longest side of $\triangle DEF = 9$ cm. Find the scale factor and hence, the lengths of the other sides of $\triangle DEF$.

Solution 14:

Triangle ABC is enlarged to DEF . So, the two triangles will be similar.

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Longest side in $\triangle ABC = BC = 6$ cm

Corresponding longest side in $\triangle DEF = EF = 9$ cm

$$\text{Scale factor} = \frac{EF}{BC} = \frac{9}{6} = \frac{3}{2} = 1.5$$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{2}{3}$$

$$DE = \frac{3}{2} AB = \frac{9}{2} = 4.5 \text{ cm}$$

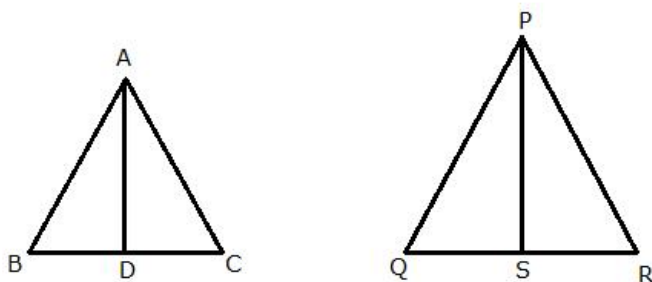
$$DF = \frac{3}{2} AC = \frac{12}{2} = 6 \text{ cm}$$

Question 15:

Two isosceles triangles have equal vertical angles. Show that the triangles are similar.

If the ratio between the areas of these two triangles is 16 : 25, find the ratio between their corresponding altitudes.

Solution 15:



Let ABC and PQR be two isosceles triangles.

$$\text{Then, } \frac{AB}{AC} = \frac{1}{1} \text{ and } \frac{PQ}{PR} = \frac{1}{1}$$

Also, $\angle A = \angle P$ (Given)

$\therefore \triangle ABC \sim \triangle PQR$ (SAS similarity)

Let AD and PS be the altitude in the respective triangles.

We know that the ratio of areas of two similar triangles is equal to the square of their corresponding altitudes.

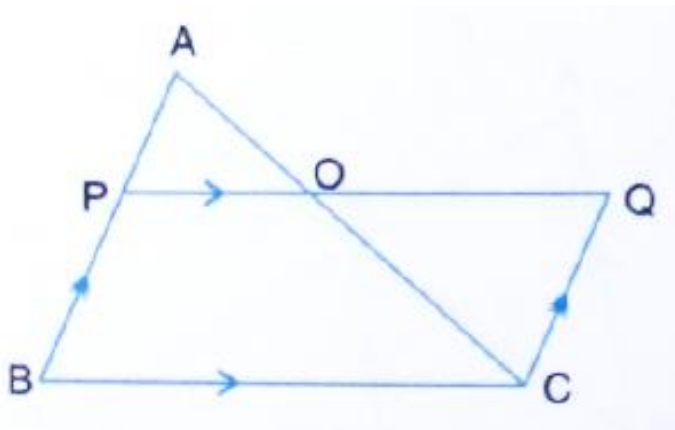
$$\frac{\text{Ar}(\triangle ABC)}{\text{Ar}(\triangle PQR)} = \left(\frac{AD}{PS} \right)^2$$

$$\frac{16}{25} = \left(\frac{AD}{PS} \right)^2$$

$$\frac{AD}{PS} = \frac{4}{5}$$

Question 16:

In $\triangle ABC$, $AP : PB = 2 : 3$. PO is parallel to BC and is extended to Q so that CQ is parallel to BA .



Find:

- (i) area $\triangle APO$: area $\triangle ABC$.
- (ii) area $\triangle APO$: area $\triangle CQO$.

Solution 16:

In triangle ABC , $PO \parallel BC$. Using Basic proportionality theorem,

$$\frac{AP}{PB} = \frac{AO}{OC}$$

$$\Rightarrow \frac{AO}{OC} = \frac{2}{3} \dots\dots\dots(1)$$

(i) $\angle PAO = \angle BAC$ (common)

$\angle APO = \angle ABC$ (Corresponding angles)

$\triangle APO \sim \triangle ABC$ (AA similarity)

$$\therefore \frac{\text{Ar}(\triangle APO)}{\text{Ar}(\triangle ABC)} = \left(\frac{AO}{AC}\right)^2 = \left(\frac{2}{2+3}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

(ii)

$\angle POA = \angle COQ$ (vertically opposite angles)

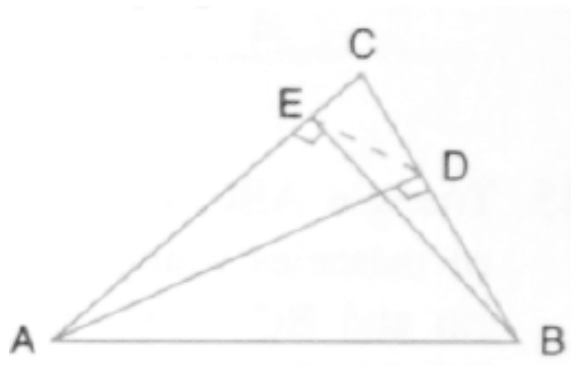
$\angle PAO = \angle QCO$ (alternate angles)

$\triangle AOP \sim \triangle COQ$ (AA similarity)

$$\therefore \frac{\text{Ar}(\triangle AOP)}{\text{Ar}(\triangle COQ)} = \left(\frac{AO}{CO}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Question 17:

The following figure shows a triangle ABC in which AD and BE are perpendiculars to BC and AC respectively.



Show that;

- (i) $\triangle ADC \sim \triangle BEC$
- (ii) $CA \times CE = CB \times CD$
- (iii) $\triangle ABC \sim \triangle DEC$
- (iv) $CD \times AB = CA \times DE$

Solution 17:

$$\angle ADC = \angle BEC = 90^\circ$$

$$\angle ACD = \angle BCE \text{ (Common)}$$

$$\triangle ADC \sim \triangle BEC \text{ (AA similarity)}$$

(ii) From part (i),

$$\frac{AC}{BC} = \frac{CD}{EC} \dots\dots\dots (1)$$

$$\Rightarrow CA \times CE = CB \times CD$$

(iii) In $\triangle ABC$ and $\triangle DEC$,

From (1),

$$\frac{AC}{BC} = \frac{CD}{EC} \Rightarrow \frac{AC}{CD} = \frac{BC}{EC}$$

$$\angle DCE = \angle BCA \text{ (Common)}$$

$$\triangle ABC \sim \triangle DEC \text{ (SAS similarity)}$$

(iv) From part (iii),

$$\frac{AC}{DC} = \frac{AB}{DE}$$

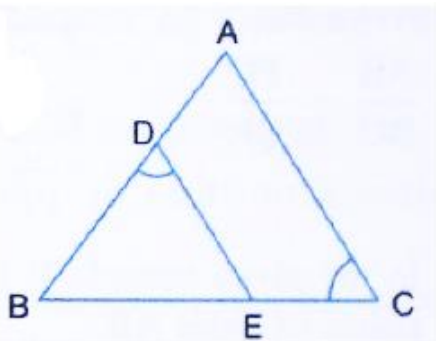
$$\Rightarrow CD \times AB = CA \times DE$$

Question 18:

In the give figure, ABC is a triangle with $\angle EDB = \angle ACB$. Prove that $\triangle ABC \sim \triangle EBD$.

If $BE = 6$ cm, $EC = 4$ cm, $BD = 5$ cm and area of $\triangle BED = 9$ cm². Calculate the:

- (i) length of AB
- (ii) area of $\triangle ABC$

**Solution 18:**

In $\triangle ABC$ and $\triangle EBD$,

$\angle ACB = \angle EDB$ (given)

$\angle ABC = \angle EBD$ (common)

$\triangle ABC \sim \triangle EBD$ (by AA – similarity)

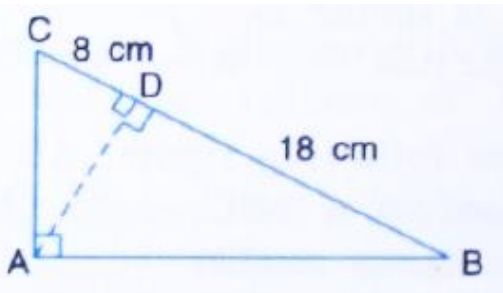
(i) we have, $\frac{AB}{BE} = \frac{BC}{BD} \Rightarrow AB = \frac{6 \times 10}{5} = 12$ cm

(ii) $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle BED} = \left(\frac{AB}{BE}\right)^2$

$$\begin{aligned} \Rightarrow \text{Area of } \triangle ABC &= \left(\frac{12}{6}\right)^2 \times 9 \text{ cm}^2 \\ &= 4 \times 9 \text{ cm}^2 = 36 \text{ cm}^2 \end{aligned}$$

Question 19:

In the given figure, ABC is a right angled triangle with $\angle BAC = 90^\circ$.



- (i) Prove $\triangle ADB \sim \triangle CDA$.
(ii) If $BD = 18$ cm and $CD = 8$ cm, find AD .
(iii) Find the ratio of the area of $\triangle ADB$ is to area of $\triangle CDA$.

Solution 19:

(i) let $\angle CAD = x$

$$\Rightarrow m \angle dab = 90^\circ - x$$

$$\Rightarrow m \angle DBA = 180^\circ - (90^\circ + 90^\circ - x) = x$$

$$\Rightarrow \angle CDA = \angle DBA \dots\dots\dots (1)$$

In $\triangle ADB$ and $\triangle CDA$,

$$\angle ADB = \angle CDA \dots\dots \text{[each } 90^\circ]$$

$$\angle ABD = \angle CAD \dots\dots \text{[From (1)]}$$

$$\therefore \triangle ADB \sim \triangle CDA \dots\dots\dots \text{[By A.A]}$$

(ii) Since the corresponding sides of similar triangles are proportional, we have.

$$\frac{BD}{AD} = \frac{AD}{CD}$$

$$\Rightarrow \frac{18}{AD} = \frac{AD}{8}$$

$$\Rightarrow AD^2 = 18 \times 8 = 144$$

$$\Rightarrow AD = 12 \text{ cm}$$

(iii) The ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

$$\Rightarrow \frac{\text{Ar}(\triangle ADB)}{\text{Ar}(\triangle CDA)} = \frac{AD^2}{CD^2} = \frac{12^2}{8^2} = \frac{144}{64} = \frac{9}{4} = 9:4$$