EXERCISE. 21 (A)

Question 1:

$$\frac{\sec A - 1}{\sec A + 1} = \frac{1 - \cos A}{1 + \cos A}$$

Solution 1:

$$\text{LHS} = \frac{\sec A - 1}{\sec A + 1} = \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1}$$

$$= \frac{1 - \cos A}{1 + \cos A} = \text{RHS}$$

Question 2:

$$\frac{1 + \sin A}{1 - \sin A} = \frac{\text{co sec} A + 1}{\text{co sec} A - 1}$$

Solution 2:

$$\text{LHS} = \frac{1 + \sin A}{1 - \sin A}$$

$$\text{RHS} = \frac{\text{co sec} A + 1}{\text{co sec} A - 1} = \frac{\frac{1}{\sin A} + 1}{\frac{1}{\sin A} - 1}$$

$$= \frac{1 + \sin A}{1 - \sin A}$$

Question 3:

$$\frac{1}{\tan A + \cot A} = \cos A \sin A$$

Solution 3:

$$\frac{1}{\tan A + \cot A} = \sin A \cos A$$

$$\text{LHS} = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\frac{1}{\sin A \cos A}} (\because \sin^2 A + \cos^2 A = 1)$$

$$= \sin A \cos A = \text{RHS}$$

Question 4:

$$\tan A - \cot A = \frac{1 - 2\cos^2 A}{\sin A \cos A}$$

Solution 4:

$$\tan A - \cot A = \frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A - \cos^2 A}{\sin A \cos A}$$

$$= \frac{1 - \cos^2 A - \cos^2 A}{\sin A \cos A} (\because \sin^2 A = 1 - \cos^2 A)$$

$$= \frac{1 - 2\cos^2 A}{\sin A \cos A}$$

Question 5:

$$\sin^4 A - \cos^4 A = 2\sin^2 A - 1$$

Solution 5:

$$\sin^4 A - \cos^4 A$$

$$= \left(\sin^2 A\right)^2 - \left(\cos^2 A\right)^2$$

$$= \left(\sin^2 A + \cos^2 A\right)\left(\sin^2 A - \cos^2 A\right)$$

$$= \sin^2 A - \cos^2 A$$

$$= \sin^2 A - \left(1 - \sin^2 A\right)$$

$$= 2\sin^2 A - 1$$

Question 6:

$(1-\tan A)^2+(1+\tan A)^2=2\sec^2 A$

Solution 6:

$(1-\tan A)^2+(1+\tan A)^2$

$=\left(1+\tan^2 A-2\tan A\right)+\left(1+\tan^2 A+2\tan A\right)$

$=2\left(1+\tan^2 A\right)$

$=2\sec^2 A$

Question 7:

$\text{cosec}^4 A - \text{cosec}^2 A = \cot^4 A + \cot^2 A$

Solution 7:

LHS = $\text{cosec}^4 A - \text{cosec}^2 A$

= $\text{cosec}^2 A (\text{cosec}^2 A - 1)$

RHS = $\cot^4 A + \cot^2 A$

= $\cot^2 A(\cot^2 A + 1)$

= $(\text{cosec}^2 A - 1)\,\text{cosec}^2 A$

Thus, LHS = RHS

Question 8:

$\sec A\left(1-\sin A\right)\left(\sec A+\tan A\right)=1$

Solution 8:

$\text{LHS}=\sec\left(1-\sin A\right)\left(\sec A+\tan A\right)$

$=\frac{1}{\cos A}\left(1-\sin A\right)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$

$=\frac{\left(1-\sin A\right)}{\cos A}\left(\frac{1+\sin A}{\cos A}\right)=\left(\frac{1-\sin^2 A}{\cos^2 A}\right)$

$=\left(\frac{\cos^2 A}{\cos^2 A}\right)=1=\text{RHS}$

Question 9:

$\cos ecA(1+\cos A)(\cos ecA-\cot A)=1$

Solution 9:

$\text{LHS}=\cos ecA(1+\cos A)(\cos ecA-\cot A)$

$=\frac{1}{\sin A}(1+\cos A)\left(\frac{1}{\sin A}-\frac{\cos A}{\sin A}\right)$

$=\frac{(1+\cos A)}{\sin A}\left(\frac{1-\cos A}{\sin A}\right)$

$=\frac{1-\cos^2 A}{\sin^2 A}\left(\frac{\sin^2 A}{\sin^2 A}\right)=1=\text{RHS}$

Question 10:

$\sec^2 A+\cos ec^2 A=\sec^2 A\cos ec^2 A$

Solution 10:

$\text{LHS}=\sec^2 A+\cos ec^2 A$

$=\frac{1}{\cos^2 A}+\frac{1}{\sin^2 A}=\frac{\sin^2 A+\cos^2 A}{\cos^2 A.\sin^2 A}$

$=\frac{1}{\cos^2 A.\sin^2 A}=\sec^2 A\cos ec^2 A=\text{RHS}$

Question 11:

$\frac{(1+\tan^2 A)\cot A}{\cos ec^2 A}=\tan A$

Solution 11:

$\frac{(1+\tan^2 A)\cot A}{\cos ec^2 A}$

$=\frac{\sec^2 A\cot A}{\cos ec^2 A}(\because \sec^2 A=1+\tan^2 A)$

$$= \frac{\frac{1}{\cos^2 A} \times \frac{\cos A}{\sin A}}{\frac{1}{\sin^2 A}} = \frac{\frac{1}{\cos A \sin A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin A}{\cos A} = \tan A$$

Question 12:

$\tan^2 A - \sin^2 A = \tan^2 A . \sin^2 A$

Solution 12:

$\text{LHS} = \tan^2 A - \sin^2 A$

$$= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A}.\sin^2 A = \tan^2 A.\sin^2 A = \text{RHS}$$

Question 13:

$\cot^2 A - \cos^2 A = \cos^2 A.\cot^2 A$

Solution 13:

$\text{LHS} = \cot^2 A - \cos^2 A$

$$= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A = \frac{\cos^2 A\left(1 - \sin^2 A\right)}{\sin^2 A}$$

$$= \cos^2 A \frac{\cos^2 A}{\sin^2 A} = \cos^2 A.\cot^2 A = \text{RHS}$$

Question 14:

(cosecA + sin A) (cosec A – sin A) = $\cot^2 A + \cos^2 A$

Solution 14:

(cosecA + sin A) (cosec A – sin A)

$= \cos ec^2 A - \sin^2 A$

$= \left(1 + \cot^2 A\right) - \left(1 - \cos^2 A\right)$

$= \cot^2 A + \cos^2 A$

Question 15:

$\left(\sec A - \cos A\right)\left(\sec A + \cos A\right) = \sin^2 A + \tan^2 A$

Solution 15:

$\left(\sec A - \cos A\right)\left(\sec A + \cos A\right)$

$= \sec^2 A - \cos^2 A$

$= \left(1 + \tan^2 A\right) - \left(1 - \sin^2 A\right)$

$= \sin^2 A + \tan^2 A$

Question 16:

$\left(\cos A + \sin A\right)^2 + \left(\cos A - \sin A\right)^2 = 2$

Solution 16:

$\text{LHS} = \left(\cos A + \sin A\right)^2 + \left(\cos A - \sin A\right)^2$

$= \cos^2 A + \sin^2 A + 2\cos A.\sin A + \cos^2 A + \sin^2 A - 2\cos A.\sin A$

$= 2\left(\cos^2 A + \sin^2 A\right) = 2 = \text{RHS}$

Question 17:

$\left(\cos ecA - \sin A\right)\left(\sec A - \cos A\right)\left(\tan A + \cot A\right) = 1$

Solution 17:

$\text{LHS} = \left(\cos ecA - \sin A\right)\left(\sec A - \cos A\right)\left(\tan A + \cot A\right)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{1}{\tan A} + \tan A\right)$

$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A.\cos A}\right)$

Question 18:

$$\frac{1}{\sec A + \tan A} = \sec A - \tan A$$

Solution 18:

$$\frac{1}{\sec A + \tan A}$$

$$= \frac{1}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}$$

$$= \frac{\sec A - \tan A}{\sec^2 A - \tan^2 A}$$

$$= \sec A - \tan A$$

Question 19:

$$\cos ecA + \cot A = \frac{1}{\cos ecA - \cot A}$$

Solution 19:

$$\cos ecA + \cot A$$

$$= \frac{\cos ecA + \cot A}{1} \times \frac{\cos ecA - \cot A}{\cos ecA - \cot A}$$

$$= \frac{\cos ec^2 A - \cot^2 A}{\cos ecA - \cot A} = \frac{1 + \cot^2 A - \cot^2 A}{\cos ecA - \cot A}$$

$$= \frac{1}{\cos ecA - \cot A}$$

Question 20:

$$\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2\sec A \tan A + 2\tan^2 A$$

Solution 20:

$$\frac{\sec A - \tan A}{\sec A + \tan A}$$

$$= \frac{\sec A - \tan A}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}$$

$$= \frac{(\sec A - \tan A)^2}{\sec^2 A - \tan^2 A}$$
$$= \frac{\sec^2 A + \tan^2 A - 2\sec A \tan A}{1}$$
$$= 1 + \tan^2 A + \tan^2 A - 2\sec A \tan A$$
$$= 1 - 2\sec A \tan A + 2\tan^2 A$$

Question 21:

$$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

Solution 21:

$$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$
$$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$
$$= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2 + 2$$
$$= 1 + \operatorname{cosec}^2 A + \sec^2 A + 4$$
$$= (1 + \cot^2 A) + (1 + \tan^2 A) + 5$$
$$= 7 + \tan^2 A + \cot^2 A$$

Question 22:

$$\sec^2 A \operatorname{cosec}^2 A = \tan^2 A + \cot^2 A + 2$$

Solution 22:

$$\text{LHS} = \sec^2 A \operatorname{cosec}^2 A = \frac{1}{\cos^2 A . \sin^2 A}$$
$$\text{RHS} = \tan^2 A + \cot^2 A + 2 = \tan^2 A + \cot^2 A + 2\tan A . \cot A$$
$$= (\tan A + \cot A)^2 = \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)^2$$
$$= \left(\frac{\sin^2 A + \cos^2 A}{\sin A . \cos A}\right)^2 = \frac{1}{\cos^2 A . \sin^2 A}$$

Hence, LHS = RHS

Question 23:

$$\frac{1}{1+\cos A}+\frac{1}{1-\cos A}=2\operatorname{cosec}^2 A$$

Solution 23:

$$\frac{1}{1+\cos A}+\frac{1}{1-\cos A}$$

$$=\frac{1-\cos A+1+\cos A}{(1+\cos A)(1-\cos A)}$$

$$=\frac{2}{1-\cos^2 A}$$

$$=\frac{2}{\sin^2 A}$$

$$=2\operatorname{cosec}^2 A$$

Question 24:

$$\frac{1}{1-\sin A}+\frac{1}{1+\sin A}=2\sec^2 A$$

Solution 24:

$$\frac{1}{1-\sin A}+\frac{1}{1+\sin A}$$

$$=\frac{1+\sin A+1-\sin A}{(1-\sin A)(1+\sin A)}$$

$$=\frac{2}{1-\sin^2 A}$$

$$=\frac{2}{\cos^2 A}$$

$$=2\sec^2 A$$

Question 25:

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=2\sec^2 A$$

Solution 25:

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}$$

$$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{\operatorname{cosec}^2 A - 1}$$

$$= \frac{2\operatorname{cosec}^2 A}{\cot^2 A}\left(\because \operatorname{cosec}^2 A - 1 = \cot^2 A\right)$$

$$= \frac{\frac{2}{\sin^2 A}}{\frac{\cos^2 A}{\sin^2 A}} = \frac{2}{\cos^2 A} = 2\sec^2 A$$

Question 26:

$$\frac{\sec A}{\sec A + 1} + \frac{\sec A}{\sec A - 1} = 2\operatorname{cosec}^2 A$$

Solution 26:

$$\frac{\sec A}{\sec A + 1} + \frac{\sec A}{\sec A - 1}$$

$$= \frac{\sec^2 A - \sec A + \sec^2 A + \sec A}{\sec^2 A - 1}$$

$$= \frac{2\sec^2 A}{\tan^2 A}\left(\because \sec^2 A - 1 = \tan^2 A\right)$$

$$= \frac{\frac{2}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A}} = \frac{2}{\sin^2 A} = 2\operatorname{cosec}^2 A$$

Question 27:

$$\frac{1 + \cos A}{1 - \cos A} = \frac{\tan^2 A}{(\sec A - 1)^2}$$

Solution 27:

$$\frac{1 + \cos A}{1 - \cos A}$$

$$= \frac{1 + \frac{1}{\sec A}}{1 - \frac{1}{\sec A}} = \frac{\sec A + 1}{\sec A - 1}$$

$$= \frac{\sec A + 1}{\sec A - 1} \times \frac{\sec A - 1}{\sec A - 1}$$

$$= \frac{\sec^2 A - 1}{(\sec A - 1)^2} = \frac{\tan^2 A}{(\sec A - 1)^2} \left(\because \sec^2 A - 1 = \tan^2 A\right)$$

Question 28:

$$\frac{\cot^2 A}{(\operatorname{cosec} A + 1)^2} = \frac{1 - \sin A}{1 + \sin A}$$

Solution 28:

$$\text{R.H.S} = \frac{1 - \sin A}{1 + \sin A}$$

$$= \frac{1 - \frac{1}{\operatorname{cosec} A}}{1 + \frac{1}{\operatorname{cosec} A}} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \times \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A + 1}$$

$$= \frac{\operatorname{cosec}^2 A - 1}{(\operatorname{cosec} A + 1)^2} = \frac{\cot^2 A}{(\operatorname{cosec} A + 1)^2} \left(\because \operatorname{cosec}^2 A - 1 = \cot^2 A\right)$$

$$= \text{L.H.S}$$

Question 29:

$$\frac{1 + \sin A}{\cos A} + \frac{\cos A}{1 + \sin A} = 2 \sec A$$

Solution 29:

$$\frac{1 + \sin A}{\cos A} + \frac{\cos A}{1 + \sin A}$$

$$= \frac{(1 + \sin A)^2 + \cos^2 A}{\cos A (1 + \sin A)}$$

$$= \frac{1 + \sin^2 A + 2 \sin A + \cos^2 A}{\cos A (1 + \sin A)}$$

$$= \frac{1 + 2 \sin A + 1}{\cos A (1 + \sin A)}$$

$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$

$= 2\sec A$

Question 30:

$\frac{1-\sin A}{1+\sin A} = (\sec A - \tan A)^2$

Solution 30:

$\frac{1-\sin A}{1+\sin A}$

$= \frac{1-\sin A}{1+\sin A} \times \frac{1-\sin A}{1-\sin A}$

$= \frac{(1-\sin A)^2}{1-\sin^2 A}$

$= \frac{(1-\sin A)^2}{\cos^2 A}$

$= \left(\frac{1-\sin A}{\cos A}\right)^2$

$= (\sec A - \tan A)^2$

Question 31:

$(\cot A - \cos ecA)^2 = \frac{1-\cos A}{1+\cos A}$

Solution 31:

$\text{R.H.S} = \frac{1-\cos A}{1+\cos A}$

$= \frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$

$= \frac{(1-\cos A)^2}{1-\cos^2 A}$

$= \frac{(1-\cos A)^2}{\sin^2 A}$

$$=\left(\frac{1-\cos A}{\sin A}\right)^2$$
$$=(\cos ecA-\cot A)^2$$
$$=(\cot A-\cos ecA)^2$$
=L.H.S

Question 32:

$$\frac{\cos ecA-1}{\cos ecA+1}=\left(\frac{\cos A}{1+\sin A}\right)^2$$

Solution 32:

$$\frac{\cos ecA-1}{\cos ecA+1}$$
$$=\frac{\cos ecA-1}{\cos ecA+1}\times\frac{\cos ecA+1}{\cos ecA+1}$$
$$=\frac{\cos ec^2A-1}{(\cos ecA+1)^2}$$
$$=\frac{\cot^2 A}{(\cos ecA+1)^2}$$
$$=\frac{\frac{\cos^2 A}{\sin^2 A}}{\left(\frac{1}{\sin A}+1\right)^2}$$
$$=\left(\frac{\cos A}{1+\sin A}\right)^2$$

Question 33:

$$\tan^2 A-\tan^2 B=\frac{\sin^2 A-\sin B}{\cos^2 A\cos^2 B}$$

Solution 33:

$$\tan^2 A-\tan^2 B$$

$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

$= \frac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B}$

$= \frac{\sin A(1-\sin^2 B) - \sin^2 B\left(1-\sin^2 A\right)}{\cos^2 A \cos^2 B}$

$= \frac{\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B}{\cos^2 A \cos^2 B}$

$= \frac{\sin^2 A - \sin B}{\cos^2 A \cos^2 B}$

Question 34:

$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

Solution 34:

$= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \frac{\sin A(1-2\sin^2 A)}{\cos A\left(2\cos^2 A - 1\right)}$

$= \frac{\sin A\left(\sin^2 A + \cos^2 A - 2\sin^2 A\right)}{\cos A\left(2\cos^2 A - \sin^2 A - \cos^2 A\right)}$

$= \frac{\sin A\left(\cos^2 A - \sin^2 A\right)}{\cos A\left(\cos^2 A - \sin^2 A\right)}$

$= \frac{\sin A}{\cos A}$

$= \tan A$

Question 35:

$\frac{\sin A}{1+\cos A} = \operatorname{cosec} A - \cot A$

Solution 35:

$\frac{\sin A}{1+\cos A}$

$= \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$

$= \frac{\sin A(1-\cos A)}{1-\cos^2 A}$

$= \frac{1-\cos A}{\sin A}$

$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

$= \operatorname{cosec} A - \cot A$

Question 36:

$\frac{\cos A}{1-\sin A} = \sec A + \tan A$

Solution 36:

$\text{L.HS} = \frac{\cos A}{1-\sin A}$

RHS = secA + tanA

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \frac{1+\sin A}{\cos A}$

$= \frac{1+\sin A}{\cos A}\left(\frac{1-\sin A}{1-\sin A}\right) = \left(\frac{1-\sin^2 A}{\cos A(1-\sin A)}\right)$

$= \frac{\cos^2 A}{\cos A(1-\sin A)} = \frac{\cos A}{(1-\sin A)} = \text{LHS}$

Question 37:

$\frac{\sin A \tan A}{1-\cos A} = 1 + \sec A$

Solution 37:

$\frac{\sin A \tan A}{1-\cos A}$

$= \frac{\sin A \tan A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}$

$= \frac{\sin A \tan A(1+\cos A)}{1-\cos^2 A}$

$$= \frac{\sin A \frac{\sin A}{\cos A}(1+\cos A)}{\sin^2 A}$$
$$= \frac{1+\cos A}{\cos A}$$
$$= \frac{1}{\cos A} + \frac{\cos A}{\cos A}$$
$$= \sec A + 1$$

Question 38:

$$(1+\cot A - \cos ecA)(1+\tan A + \sec A) = 2$$

Solution 38:

$$(1+\cot A - \cos ecA)(1+\tan A + \sec A)$$
$$= \left(1+\frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A} - \frac{1}{\cos A}\right)$$
$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$
$$= \frac{(\sin A + \cos A - 1)(\sin A + \cos A + 1)}{\sin A \cos A}$$
$$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cos A}$$
$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A}$$
$$= \frac{1 + 2\sin A \cos A - 1}{\sin A \cos A}$$
$$= \frac{2\sin A \cos A}{\sin A \cos A} = 2$$

Question 39:

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

Solution 39:

$$\sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \sec A + \tan A$$

Question 40:

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \cos ecA - \cot A$$

Solution 40:

$$\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$$

$$= \frac{1-\cos A}{\sin A}$$

$$= \cos ecA - \cot A$$

Question 41:

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}$$

Solution 41:

$$\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$= \frac{\sin A}{1+\cos A}$$

Question 42:

$$\sqrt{\frac{1-\sin A}{1+\sin A}} = \frac{\cos A}{1+\sin A}$$

Solution 42:

$$\sqrt{\frac{1-\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{1-\sin A}{1+\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{1-\sin^2 A}{(1+\sin A)^2}}$$

$$= \sqrt{\frac{\cos^2 A}{(1+\sin A)^2}}$$

$$= \frac{\cos A}{1+\sin A}$$

Question 43:

$$1 - \frac{\cos^2 A}{1+\sin A} = \sin A$$

Solution 43:

$$1 - \frac{\cos^2 A}{1+\sin A}$$

$$= \frac{1 + \sin A - \cos^2 A}{1 + \sin A}$$
$$= \frac{\sin A + \sin^2 A}{1 + \sin A}$$
$$= \frac{\sin A(1 + \sin A)}{1 + \sin A}$$
$$= \sin A$$

Question 44:

$$\frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A} = \frac{2\sin A}{1 - 2\cos^2 A}$$

Solution 44:

$$\frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A}$$
$$= \frac{\sin A - \cos A + \sin A + \cos A}{\sin^2 A - \cos^2 A}$$
$$= \frac{2\sin A}{1 - \cos^2 A - \cos^2 A} = \frac{2\sin A}{1 - 2\cos^2 A}$$

Question 45:

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2\sin^2 A - 1}$$

Solution 45:

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$$
$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A + \cos A)(\sin A - \cos A)}$$
$$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A + \sin^2 A + \cos^2 A - 2\sin\ \cos A}{\sin^2 A - \cos^2 A}$$
$$= \frac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A}$$
$$= \frac{2}{\sin^2 A - \cos^2 A} \quad [\sin^2 A + \cos^2 A = 1]$$

$= \frac{2}{\sin^2 A - \cos^2 A} \quad = \frac{2}{\sin^2 A - \left(1 - \sin^2 A\right)}$

$\Rightarrow \frac{2}{2\sin^2 A - 1}$

Question 46:

$\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A}$

Solution 46:

$\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$

$= \frac{\cot A + \operatorname{cosec} A - \left(\operatorname{cosec}^2 A - \cot^2 A\right)}{\cot A - \operatorname{cosec} A + 1}$ $[\operatorname{cosec}^2 A - \cot^2 A = 1]$

$= \frac{\cot A + \operatorname{cosec} A - \left[\left(\operatorname{cosec} A - \cot A\right)\left(\operatorname{cosec} A + \cot A\right)\right]}{\cot A - \operatorname{cosec} A + 1}$

$= \frac{\cot A + \operatorname{cosec} A\left[1 - \operatorname{cosec} A + \cot A\right]}{\cot A - \operatorname{cosec} A + 1}$

$= \cot A + \operatorname{cosec} A$

$= \frac{\cos A}{\sin A} + \frac{1}{\sin A}$

$= \frac{1 + \cos A}{\sin A}$

Question 47:

$\frac{\sin\theta \tan\theta}{1 - \cos\theta} = 1 + \sec\theta$

Solution 47:

$\frac{\sin\theta \tan\theta}{1 - \cos\theta}$

$= \frac{\sin\theta \tan\theta}{1 - \cos\theta} \times \frac{1 + \cos\theta}{1 + \cos\theta}$

$= \frac{\sin\theta \tan\theta\left(1 + \cos\theta\right)}{1 - \cos^2\theta}$

$$= \frac{\sin\theta \dfrac{\sin\theta}{\cos\theta}(1+\cos\theta)}{\sin^2\theta}$$
$$= \frac{(1+\cos\theta)}{\cos\theta}$$
$$= \frac{1}{\cos\theta} + 1$$
$$= \sec\theta + 1$$

Question 48:

$$\frac{\cos\theta\cot\theta}{1+\sin\theta} = \cos ec\theta - 1$$

Solution 48:

$$\frac{\cos\theta\cot\theta}{1+\sin\theta}$$
$$= \frac{\cos\theta\cot\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$
$$= \frac{\cos\theta\cot\theta(1-\sin\theta)}{1-\sin^2\theta}$$
$$= \frac{\cos\theta \dfrac{\cos\theta}{\sin\theta}(1-\sin\theta)}{\cos^2\theta}$$
$$= \frac{(1-\sin\theta)}{\sin\theta}$$
$$= \frac{1}{\sin\theta} - 1$$
$$= \cos ec\theta - 1$$

EXERCISE. 21 (B)

Question 1:

(i) $\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\sin A+\cos A$

(ii) $\frac{\cos^3 A+\sin^3 A}{\cos^3 A+\sin^3 A}+\frac{\cos^3 A-\sin^3 A}{\cos^3 A-\sin^3 A}=2$

(iii) $\frac{\tan A}{1-\cot A}+\frac{\cot}{1-\tan A}=\sec A \operatorname{cosec} A+1$

(iv) $\left(\tan A+\frac{1}{\cos A}\right)^2+\left(\tan A-\frac{1}{\cos A}\right)^2=2\left(\frac{1+\sin^2 A}{1-\sin^2 A}\right)$

(v) $2\sin^2 A+\cos^4 A=1+\sin^4 A$

(vi) $\frac{\sin A-\sin B}{\cos A+\cos B}+\frac{\cos A-\cos B}{\sin A+\sin B}=0$

(vii) $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$

(viii) $(1+\tan A\tan B)^2+(\tan A-\tan B)^2=\sec^2 A\sec^2 B$

(ix) $\frac{1}{\cos A+\sin A-1}+\frac{1}{\cos A+\sin A+1}=\operatorname{cosec} A+\sec A$

Solution 1:

(i) $\text{LHS}=\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}$

$$=\frac{\cos A}{1-\frac{\sin A}{\cos A}}+\frac{\sin A}{1-\frac{\cos A}{\sin A}}=\frac{\cos A}{\frac{\cos A-\sin A}{\cos A}}+\frac{\sin A}{\frac{\sin A-\cos A}{\sin A}}$$

$$=\frac{\cos^2 A}{\cos A-\sin A}+\frac{\sin^2 A}{\sin A-\cos A}=\frac{\cos^2 A-\sin^2 A}{(\cos A-\sin A)}$$

$=\sin A+\cos A=\text{RHS}$

(ii) $\frac{\cos^3 A+\sin^3 A}{\cos^3 A+\sin^3 A}+\frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}$

$$=\frac{(\cos^3 A+\sin^3 A)(\cos A-\sin A)+(\cos^3 A-\sin^3 A)(\cos A+\sin A)}{\cos^2 A-\sin^2 A}$$

$$=\frac{\cos^4 A-\cos^3 A\sin A+\sin^3 A\cos A-\sin^4 A+\cos^4 A+\cos^3 A\sin A-\sin^3 A\cos A-\sin^4 A}{\cos^2 A-\sin^2 A}$$

$$= \frac{2\left(\cos^4 A - \sin^4 A\right)}{\cos^2 A - \sin A}$$

$$= \frac{2\left(\cos^2 A + \sin^2 A\right)2\left(\cos^2 A - \sin^2 A\right)}{\left(\cos^2 A - \sin^2 A\right)}$$

$$= 2\left(\cos^2 A + \sin^2 A\right)$$

$$= 2\left(\because \cos^2 A + \sin^2 A = 1\right)$$

(iii) $\frac{\tan A}{1 - \cot A} + \frac{\cot}{1 - \tan A}$

$$= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1 - \tan A}$$

$$= \frac{\tan^2 A}{\tan A - 1} + \frac{1}{\tan A\left(1 - \tan A\right)}$$

$$= \frac{\tan^3 A - 1}{\tan A\left(1 - \tan A\right)}$$

$$= \frac{\left(\tan A - 1\right)\left(\tan^2 A + 1 + \tan A\right)}{\tan A\left(\tan A - 1\right)}$$

$$= \frac{\sec^2 A + \tan A}{\tan A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{\sin A}{\cos A}} + 1$$

$$= \frac{1}{\sin A \cos A} + 1$$

$$= \sec A \operatorname{cosec} A + 1$$

(iv) $\left(\tan A + \frac{1}{\cos A}\right)^2 + \left(\tan A - \frac{1}{\cos A}\right)^2$

$$= \left(\frac{\sin A + 1}{\cos A}\right)^2 + \left(\frac{\sin A - 1}{\cos A}\right)^2$$

$$= \frac{\sin^2 A + 1 + 2\sin A + \sin^2 A + 1 - 2\sin A}{\cos^2 A}$$

$= \dfrac{2+2\sin^2 A}{\cos^2 A}$

$= 2\left(\dfrac{1+\sin^2 A}{1-\sin^2 A}\right)$

(v) $2\sin^2 A + \cos^4 A$

$= 2\sin^2 A + \left(1-\sin^2 A\right)^2$

$= 2\sin^2 A + 1 + \sin^4 A - 2\sin^2 A$

$= 1+\sin^4 A$

(vi) $\dfrac{\sin A - \sin B}{\cos A + \cos B} + \dfrac{\cos A - \cos B}{\sin A + \sin B}$

$= \dfrac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$

$= \dfrac{\left(\sin^2 A + \cos^2 A\right) - \left(\sin^2 B + \cos^2 B\right)}{(\cos A + \cos B)(\sin A + \sin B)}$

$= \dfrac{1-1}{(\cos A + \cos B)(\sin A + \sin B)}$

$= 0$

(vii) LHS

$= (\cos ecA - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right)$

$= \left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right)$

$= \left(\dfrac{\cos^2 A}{\sin A}\right)\left(\dfrac{\sin^2 A}{\cos A}\right)$

$= \sin A \cos A$

$\text{RHS} = \dfrac{1}{\tan A + \cot A}$

$= \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A} A}$

$= \dfrac{\sin A \cos A}{\sin^2 A + \cos^2 A}$

$= \sin A \cos A$

$\text{LHS} = \text{RHS}$

(viii) $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2$

$= 1 + \tan^2 A \tan^2 B + 2 \tan A \tan B + \tan^2 A + \tan^2 B - 2 \tan A \tan B$

$= 1 + \tan^2 A + \text{tab}^2 B + \tan^2 A \tan^2 B$

$= \sec^2 A + \tan^2 B(1 + \tan^2 A)$

$= \sec^2 A + \tan^2 B \sec^2 A$

$= \sec^2 A(1 + \tan^2 B)$

$= \sec^2 A \sec^2 B$

(ix) $\dfrac{1}{(\cos A + \sin A) - 1} + \dfrac{1}{(\cos A + \sin A) + 1}$

$= \dfrac{\cos A + \sin A + 1 + \cos A + \sin A - 1}{(\cos A + \sin A)^2 - 1}$

$= \dfrac{2(\cos A + \sin A)}{\cos^2 A + \sin^2 A + 2 \cos A \sin A - 1}$

$= \dfrac{2(\cos A + \sin A)}{1 + 2 \cos A \sin A - 1} = \dfrac{\cos A + \sin A}{\cos A \sin A}$

$= \dfrac{\cos A}{\cos A \sin A} + \dfrac{\sin A}{\cos A \sin A}$

$= \dfrac{1}{\sin A} + \dfrac{1}{\cos A}$

$= \cos ec A + \sec A$

Question 2:

If x cos A + sin A = m and

X sin A – y cos A = n, then prove that: $x^2 + y^2 = m^2 + n^2$

Solution 2:

$m^2 + n^2$

$= (x \cos A + y \sin A)^2 + (x \sin A - y \cos A)^2$

$= x^2 \cos^2 A + y^2 \sin^2 A + 2xy \sin A \cos A$

$\quad + x^2 \sin^2 A + y^2 \cos^2 A - 2xy \sin A \cos A$

$= x^2 (\cos^2 A + \sin^2 A) + y^2 (\cos^2 A + \sin^2 A)$

$= x^2 + y^2$

Hence, $x^2 + y^2 = m^2 + n^2$

Question 3:

If m = a sec A + b tan A and n = a tan A + b sec A, then prove that : $m^2 - n^2 = a^2 - b^2$

Solution 3:

Given,

m = asecA + btanA and n = atanA + bsecA

$m^2 - n^2 = (a\sec A + b\tan A)^2 - (a\tan A + b\sec A)^2$

$= a^2\sec^2 A + b^2\tan^2 A + 2ab\sec A\tan A$

$-(a^2\tan^2 A + b^2\sec^2 A + 2ab\sec A\tan A)$

$= \sec^2 A(a^2 - b^2) + \tan^2 A(b^2 - a^2)$

$= (a^2 - b^2)[\sec^2 A - \tan^2 A]$

$= (a^2 - b^2)[\text{Since } \sec^2 A - \tan^2 A = 1]$

Hence, $m^2 - n^2 = a^2 - b^2$

Question 4:

If x = r sin A cos B, y = r sin A sin B and z = r cos A, then prove that:

$x^2 + y^2 + z^2 = r^2$

Solution 4:

$\text{LHS} = (r\sin A\cos B)^2 + (r\sin A\sin B)^2 + (r\cos A)^2$

$= r^2\sin^2 A\cos^2 B + r^2\sin^2 A\sin^2 B + r^2\cos^2 A$

$= r^2\sin^2 A(\cos^2 B + \sin^2 B) + r^2\cos^2 A$

$= r^2(\sin^2 A + \cos^2 A) = r^2 = \text{RHS}$

Question 5:

If sin A + cos A = m and sec A + cosec A = n, show that: $n(m^2 - 1) = 2m$

Solution 5:

Given:

sin A + cos A = m

and

sec A + cosec A = n

Consider L.H.S = $n\left(m^2-1\right)$

$=\left(\sec A+\operatorname{cos}ecA\right)\left[\left(\sin A+\cos A\right)^2-1\right]$

$=\left(\frac{1}{\cos A}+\frac{1}{\sin A}\right)\left[\sin^2 A+\cos^2 A+2\sin A\cos A-1\right]$

$=\left(\frac{\cos A+\sin A}{\sin A\ \cos A}\right)\left(1+2\ \sin A\cos A-1\right)$

$=\frac{\left(\cos A+\sin A\right)}{\sin A\cos A}\left(2\sin A\cos A\right)$

$=2\left(\sin A+\cos A\right)$

$=2m=$ R.H.S

Question 6:

If x = r cos A cos B, y = r cos A sin B and Z = r sin A, show that:

$x^2+y^2+z^2=r^2$

Solution 6:

$\text{LHS}=\left(r\cos A\cos B\right)^2+\left(r\cos A\sin B\right)^2+\left(r\sin A\right)^2$

$=r^2\cos^2 A\cos^2 B+r^2\cos^2 A\sin^2 B+r^2\sin^2 A$

$=r^2\cos^2 A\left(\cos^2 B+\sin^2 B\right)+r^2\sin^2 A$

$=r^2\left(\cos^2 A+\sin^2 A\right)=r^2=\text{RHS}$

Question 7:

If $\frac{\cos A}{\cos B}=m$ and $\frac{\cos A}{\sin B}=n$ show that:

$(m^2+n^2)\cos^2 B=n^2$.

Solution 7:

LHS = $(m^2+n^2)\cos^2 B$

$=\left(\frac{\cos^2 A}{\cos^2 B}+\frac{\cos^2 A}{\sin^2 B}\right)\cos^2 B$

$$= \left(\frac{\cos^2 A \sin^2 B + \cos^2 A \cos^2 B}{\cos^2 B \sin^2 B}\right)\cos^2 B$$

$$= \left(\frac{\cos^2 A \sin^2 B + \cos^2 A \cos^2 B}{\sin^2 B}\right)$$

$$= \frac{\cos^2 A\left(\sin^2 B + \cos^2 B\right)}{\sin^2 B}$$

$$= \frac{\cos^2 A}{\sin^2 B}$$

$$= n^2$$

Hence, $(m^2 + n^2)\cos^2 B = n^2$.

EXERCISE 21 (C)

Question 1:

(i) $\frac{\cos 22^\circ}{\sin 68^\circ}$ (ii) $\frac{\tan 47^\circ}{\cot 43^\circ}$

(iii) $\frac{\sec 75^\circ}{\operatorname{cosec} 15^\circ}$ (iv) $\frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ}$

(v) $\cos^2 40° + \cos^2 50°$

(vi) $\sec^2 18° - \cot^2 72°$

(vii) sin15° cos75° + cos15° sin 75°

(viii) sin42° sin 48° − cos42° cos48°

Solution 1:

(i) $\frac{\cos 22^\circ}{\sin 68^\circ} = \frac{\cos\left(90^\circ - 68^\circ\right)}{\sin 68^\circ} = \frac{\sin 68^\circ}{\sin 68^\circ} = 1$

(ii) $\frac{\tan 47^\circ}{\cot 43^\circ} = \frac{\tan\left(90^\circ - 43^\circ\right)}{\cot 43^\circ} = \frac{\cot 43^\circ}{\cot 43^\circ} = 1$

(iii) $\frac{\sec 75^\circ}{\operatorname{cosec} 15^\circ} = \frac{\sec\left(90^\circ - 15^\circ\right)}{\operatorname{cosec} 15^\circ} = \frac{\operatorname{cosec} 15^\circ}{\operatorname{cosec} 15^\circ} = 1$

(iv) $\frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ}$

$$= \frac{\cos\left(90^\circ - 35^\circ\right)}{\sin 35^\circ} + \frac{\cot\left(90^\circ - 55^\circ\right)}{\tan 55^\circ}$$

$= \frac{\sin 35^\circ}{\sin 35^\circ} + \frac{\tan 55^\circ}{\tan 55^\circ}$

$= 1 + 1 = 2$

(v) $\cos^2 40° + \cos^2 50°$

$= [\cos(90° - 50°)]^2 + \cos^2 50°$

$= \sin^2 50° + \cos^2 50°$

$= 1$

(vi) $\sec^2 18° - \cot^2 72°$

$= [\sec(90° - 72°)]^2 - \cot^2 72°$

$= \text{cosec}^2 72° - \cot^2 72°$

$= 1$

(vii) $\sin 15° \cos 75° + \cos 15° \sin 75°$

$= \sin(90° - 75°) \cos 75° + \cos(90° - 75°) \sin 75°$

$= \cos 75° \cos 75° + \sin 75° \sin 75°$

$= \cos^2 75° + \sin^2 75° = 1$

(viii) $\sin 42° \sin 48° - \cos 42° \cos 48°$

$= \sin(90° - 48°) \sin 48° - \cos(90° - 48°) \cos 48°$

$= \cos 48° \sin 48° - \sin 48° \cos 48° = 0$

Question 2:

Evaluate

(i) $\sin(90° - A) \cos A + \cos(90° - A) \sin A$

(ii) $\sin^2 35° + \sin^2 55°$

(iii) $\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$

(iv) $\frac{2\tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$

(v) $\cos^2 25° + \cos^2 65° - \tan^2 45°$

(vi) $\frac{\cos^2 32^\circ + \cos^2 58^\circ}{\sin^2 59^\circ + \sin^2 31^\circ}$

(vii) $\left(\frac{\sin 77^\circ}{\cos 13^\circ}\right)^2 + \left(\frac{\cos 77^\circ}{\sin 13^\circ}\right) - 2\cos^2 45$

(viii) $\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot 54^\circ}$

Solution 2:

(i) $\sin(90° - A) \cos A + \cos(90° - A) \sin A$

= cosA cosA + sinA sinA

= $\cos^2A + \sin^2A = 1$

(ii) $\sin^2 35° + \sin^2 55°$

= $[\sin(90° - 55°)]^2 + \sin^2 55°$

= $\cos^2 55o + \sin^2 55° = 1$

(iii) $\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$

$= \frac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} - 2$

$= \frac{\tan 36^\circ}{\tan 36^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2$

$= 1 + 1 - 2 = 0$

(iv) $\frac{2\tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$

$= \frac{2\tan(90^\circ - 37^\circ)}{\cot 37^\circ} - \frac{\cot(90^\circ - 10^\circ)}{\tan 10^\circ}$

$= \frac{2\cot 37^\circ}{\cot 37^\circ} - \frac{\tan 10^\circ}{\tan 10^\circ}$

$= 2 - 1 = 1$

(v) $\cos^2 25° + \cos^2 65° - \tan^2 45°$

$= \cos^2(90^\circ - 65^\circ) + \cos^2 65^\circ - \tan^2 45^\circ$

$= \sin^2 65^\circ + \cos^2 65^\circ - 1 = 1 - 1 = 0$

(vi) $\frac{\cos^2 32^\circ + \cos^2 58^\circ}{\sin^2 59^\circ + \sin^2 31^\circ}$

$= \frac{\cos^2(90^\circ - 58^\circ) + \cos^2 58^\circ}{\sin^2(90^\circ - 31^\circ) + \sin^2 31^\circ}$

$= \frac{\sin^2 58^\circ + \cos^2 58^\circ}{\cos^2 31^\circ + \sin^2 31^\circ}$

$= \frac{1}{1} = 1$

(vii) $\left(\frac{\sin 77^\circ}{\cos 13^\circ}\right)^2 + \left(\frac{\cos 77^\circ}{\sin 13^\circ}\right) - 2\cos^2 45$

$= \left[\frac{\sin(90^\circ - 13^\circ)}{\cos 13^\circ}\right] + \left[\frac{\cos(90^\circ - 13^\circ)}{\sin 13^\circ}\right] - 2\cos^2 45^\circ$

$= \left[\frac{\cos 13^\circ}{\cos 13^\circ}\right] + \left[\frac{\sin 13^\circ}{\sin 13^\circ}\right] - 2\left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 + 1 - 1 = 1$

(viii) $\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot 54^\circ}$

$\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot 54^\circ}$

$= \cos^2 26^\circ + \cos(90 - 26)^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\cot(90 - 36)^\circ}$

$= \cos^2 26^\circ + \sin 26^\circ \sin 26^\circ + \frac{\tan 36^\circ}{\tan 36^\circ}$

$= \cos^2 26^\circ + \sin^2 26^\circ + 1$

$= 1 + 1$

$= 2$

Question 3:

Show that:

(i) $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ = 1$

(ii) $\sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ = 2$

(iii) $\frac{\sin 26^\circ}{\sec 64^\circ} + \frac{\cos 26^\circ}{\operatorname{cosec} 64^\circ} = 1$

Solution 3:

(i) $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ$

$= \tan(90^\circ - 80^\circ)\tan(90^\circ - 75^\circ)\tan 75^\circ \tan 80^\circ$

$= \cot 80^\circ \cot 75^\circ \tan 75^\circ \tan 80^\circ$

$= 1[\text{As} \tan\theta, \cot\theta = 1]$

(ii) $\sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ = 2$

consider $\sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ$

$\Rightarrow \sin 42^\circ \sec(90^\circ - 42^\circ) + \cos 42^\circ \operatorname{cosec}(90^\circ - 42^\circ)$

$\Rightarrow \sin 42^\circ . \operatorname{cosec} 42^\circ + \cos 42^\circ \sec 42^\circ$

$\Rightarrow \sin 42^\circ, \frac{1}{\sin 42^\circ} + \cos 42^\circ \frac{1}{\cos 42^\circ}$

$\Rightarrow 1 + 1 = 2$

(iii) $\frac{\sin 26^\circ}{\sec 64^\circ} + \frac{\cos 26^\circ}{\operatorname{cosec} 64^\circ}$

$= \frac{\sin 26^\circ}{\sec(90^\circ - 26^\circ)} + \frac{\cos 26^\circ}{\operatorname{cosec}(90 - 26^\circ)}$

$= \frac{\sin 26^\circ}{\operatorname{cosec} 26^\circ} + \frac{\cos 26^\circ}{\sec 26^\circ}$

$\sin^2 26^\circ + \cos^2 26^\circ$

$= 1$

Question 4:

Express each of the following in terms of angles between 0° and 45°:

(i) $\sin 59^\circ + \tan 63^\circ$

(ii) $\operatorname{cosec} 68^\circ + \cot 72^\circ$

(iii) $\cos 74^\circ + \sec 67^\circ$

Solution 4:

(i) $\sin 59^\circ + \tan 63^\circ$

$= \sin(90 - 31)^\circ + \tan(90 - 27)^\circ$

$= \cos 31^\circ + \cot 27^\circ$

(ii) $\operatorname{cosec} 68^\circ + \cot 72^\circ$

$= \operatorname{cosec}(90 - 22)^\circ + \cot(90 - 18)^\circ$

$= \sec 22^\circ + \tan 18^\circ$

(iii) $\cos 74^\circ + \sec 67^\circ$

$= \cos(90 - 16)^\circ + \sec(90 - 23)^\circ$

$= \sin 16^\circ + \operatorname{cosec} 23^\circ$

Question 5:

Show that:

(i) $\frac{\sin A}{\sin(90^\circ - A)} + \frac{\cos A}{\cos(90^\circ - A)} = \sec A \operatorname{cosec} A$

(ii) $\sin A \cos A - \frac{\sin A \cos(90^\circ - A)\cos A}{\sec(90^\circ - A)} - \frac{\cos A \sin(90^\circ - A)\sin A}{\operatorname{cosec}(90^\circ - A)} = 0$

Solution 5:

(i) $\dfrac{\sin A}{\sin(90^\circ - A)} + \dfrac{\cos A}{\cos(90^\circ - A)}$

$= \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}$

$= \dfrac{\sin^2 A + \cos^2 A}{\cos A \sin A}$

$= \dfrac{1}{\cos A \sin A}$

$= \sec A \operatorname{cosec} A$

(ii) $\sin A \cos A - \dfrac{\sin A \cos(90^\circ - A)\cos A}{\sec(90^\circ - A)} - \dfrac{\cos A \sin(90^\circ - A)\sin A}{\operatorname{cosec}(90^\circ - A)}$

$= \sin A \cos A - \dfrac{\sin A \sin A \cos A}{\operatorname{cosec} A} - \dfrac{\cos A \cos A \sin A}{\sec A}$

$= \sin A \cos A - \sin^3 A \cos A - \cos^3 A \sin A$

$= \sin A \cos A - \sin A \cos A(\sin^2 A + \cos^2 A)$

$= \sin A \cos A - \sin A \cos A(1)$

$= 0$

Question 6:

For triangle ABC, show that:

(i) $\sin \dfrac{A+B}{2} = \cos \dfrac{c}{2}$

(ii) $\tan \dfrac{B+C}{2} = \cot \dfrac{A}{2}$

Solution 6:

(i) We know that for a triangle Δ ABC

$\angle A + \angle B + \angle C = 180^\circ$

$\dfrac{\angle B + \angle A}{2} = 90^\circ - \dfrac{\angle C}{2}$

$\text{Sin}\left(\dfrac{A+B}{2}\right) = \sin\left(90^\circ - \dfrac{C}{2}\right)$

$= \cos\left(\dfrac{c}{2}\right)$

(ii) We know that for a triangle Δ ABC

$\angle A + \angle B + \angle C = 180^\circ$

$\angle B + \angle C = 180^\circ - \angle A$

$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$

$\tan\left(\frac{B+C}{2}\right) = \tan\left(90^\circ - \frac{A}{2}\right)$

$= \cos\left(\frac{A}{2}\right)$

Question 7:

Evaluate:

(i) $3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(ii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$

(iii) $\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$

(iv) $\tan\left(55^\circ - A\right) - \cot\left(35^\circ + A\right)$

(v) $\operatorname{cosec}\left(65^\circ + A\right) - \sec\left(25^\circ - A\right)$

(vi) $2\frac{\tan 57}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2}\cos 45^\circ$

(vii) $\frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\frac{\sin^2 75^\circ}{\cos^2 15^\circ}$

(viii) $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$

(ix) $14\sin 30^\circ + 6\cos 60^\circ - 5\tan 45^\circ$

Solution 7:

(i) $3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

$= 3\frac{\sin\left(90^\circ - 18^\circ\right)}{\cos 18^\circ} - \frac{\sec\left(90^\circ - 58^\circ\right)}{\operatorname{cosec} 58^\circ}$

$= 3\frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} = 3 - 1 = 2$

(ii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$

$= 3\cos\left(90^\circ - 10^\circ\right)\operatorname{cosec} 10^\circ + 2\cos\left(90^\circ - 31^\circ\right)\operatorname{cosec} 31^\circ$

$= 3\sin 10^\circ \operatorname{cosec} 10^\circ + 2\sin 31^\circ \operatorname{cosec} 31^\circ$

$= 3 + 2 = 5$

(iii) $\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$

$= \frac{\sin\left(90^\circ - 10^\circ\right)}{\cos 10^\circ} + \sin\left(90^\circ - 31^\circ\right)\sec 31^\circ$

$= \frac{\cos 10^\circ}{\cos 10^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ}$

$= 1 + 1 = 2$

(iv) $\tan\left(55^\circ - A\right) - \cot\left(35^\circ + A\right)$

$= \tan\left[90^\circ - \left(35^\circ + A\right)\right] - \cot\left(35^\circ + A\right)$

$= \cot\left(35^\circ + A\right) - \cot\left(35^\circ + A\right)$

$= 0$

(v) $\operatorname{cosec}\left(65^\circ + A\right) - \sec\left(25^\circ - A\right)$

$= \operatorname{cosec}\left[90^\circ - \left(25^\circ - A\right)\right] - \sec\left(25^\circ - A\right)$

$= \sec\left(25^\circ - A\right) - \sec\left(25^\circ - A\right)$

$= 0$

(vi) $2\frac{\tan 57}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2}\cos 45^\circ$

$= 2\frac{\tan\left(90^\circ - 33^\circ\right)}{\cot 33^\circ} - \frac{\cot\left(90^\circ - 20^\circ\right)}{\tan 20^\circ} - \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$

$= 2\frac{\cot 33^\circ}{\cot 33^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 1$

$= 2 - 1 - 1$

$= 0$

(vii) $\frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\frac{\sin^2 75^\circ}{\cos^2 15^\circ}$

$= \frac{\left[\cot\left(90^\circ - 49^\circ\right)\right]^2}{\tan^2 49^\circ} - 2\frac{\left[\sin\left(90^\circ - 15^\circ\right)\right]^2}{\cos^2 15^\circ}$

$= \frac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2\frac{\cos^2 15^\circ}{\cos^2 15^\circ}$

$= 1 - 2 = -1$

(viii) $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$

$= \frac{\cos\left(90^\circ - 20^\circ\right)}{\sin 20^\circ} + \frac{\cos\left(90^\circ - 31^\circ\right)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2$

$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 2$

$= 1 + 1 - 2 = 0$

(ix) $14\sin 30^\circ + 6\cos 60^\circ - 5\tan 45^\circ$

$= 14\left(\frac{1}{2}\right) + 6\left(\frac{1}{2}\right) - 5(1)$

$= 7 + 3 - 5 = 5$

Question 8:

A triangle ABC is right angles at B; find the value of $\frac{\sec A.\operatorname{cosec} A - \tan A.\cot C}{\sin B}$

Solution 8:

Since, ABC is a right angled triangle, right angled at B.

So, A + C = 90°

$\frac{\sec A.\operatorname{cosec} A - \tan A.\cot C}{\sin B}$

$= \frac{\sec\left(90^\circ - C\right).\operatorname{cosec} C - \tan\left(90^\circ - C\right).\cot C}{\sin 90^\circ}$

$= \frac{\operatorname{cosec} C.\operatorname{cosec} C - \cot C.\cot C}{1}$

$= 1 \quad \left[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1\right]$

Question 9:

Find (in each case, given below) the value of x, if:

(i) $\sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

(ii) $\sin x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

(iii) $\cos x = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

(iv) $\tan x = \dfrac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$

(v) $\sin 2x = 2\sin\ 45^\circ \cos\ 45^\circ$

(vi) $\sin\ 3x = 2\sin 30^\circ \cos 30^\circ$

(vii) $\cos\left(2x - 6^\circ\right) = \cos^2 30^\circ - \cos^2 60^\circ$

Solution 9:

(i) $\sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$$\sin x = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\sin x = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \sin 30^\circ$$

Hence, $x = 30^\circ$

(ii) $\sin x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$$\sin x = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\sin x = \frac{3}{4} + \frac{1}{4} = 1 = \sin 90^\circ$$

Hence, $x = 90^\circ$

(iii) $\cos x = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$\cos x = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$\cos x = 0 = \cos\ 90^\circ$

Hence, $x = 90^\circ$

(iv) $\tan x = \dfrac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$

$$\tan x = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}.\frac{1}{\sqrt{3}}}$$

$$\tan x = \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Hence, $x = 30^\circ$

(v) $\sin 2x = 2\sin\ 45^\circ \cos\ 45^\circ$

$\sin\ 2x = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$\sin 2x = 1 = \sin\ 90^\circ$

$2x = 90^\circ$

Hence, $x = 45^\circ$

(vi) $3x = 2\sin 30^\circ \cos 30^\circ$

$\sin\ 3x = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\sin\ 3x = \frac{\sqrt{3}}{2} = \sin 60^\circ$

$3x = 60^\circ$

Hence, $x = 20^\circ$

(vii) $\cos\left(2x - 6^\circ\right) = \cos^2 30^\circ - \cos^2 60^\circ$

$\cos\left(2x-6\right) = \cos^2\left(90^\circ - 60^\circ\right) - \cos^2 60^\circ$

$\cos\left(2x-6\right) = \sin^2 60^\circ - \cos^2 60^\circ$

$\cos\left(2x-6\right) = 1 - 2\cos^2 60^\circ = 1 - 2\left(\frac{1}{2}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$

$\cos\left(2x-6\right) = \frac{1}{2}$

$\cos\left(2x-6\right) = \cos 60^\circ$

$\left(2x-6\right) = 60^\circ$

$2x = 66^\circ$

Hence, $x = 33^\circ$

Question 10:

In each case, given below, find the value of angle A, where $0° \le A \le 90°$.

(i) $\sin\left(90^\circ - 3A\right).\mathrm{cosec} 42^\circ = 1$

(ii) $\cos\left(90^\circ - A\right).\sec 77^\circ = 1$

Solution 10:

(i) $\sin\left(90^\circ - 3A\right).\mathrm{cosec} 42^\circ = 1$

$\cos 3A . \dfrac{1}{\sin 42^\circ} = 1$

$\cos 3A = \sin 42^\circ = \sin\left(90^\circ - 48^\circ\right) = \cos 48^\circ$

$3A = 48^\circ$

$A = 16^\circ$

(ii) $\cos\left(90^\circ - A\right).\sec 77^\circ = 1$

$\cos\left(90^\circ - A\right).\sec 77^\circ = 1$

$\sin A . \dfrac{1}{\cos 77^\circ} = 1$

$\sin A = \cos 77^\circ = \cos\left(90^\circ - 13^\circ\right) = \sin 13^\circ$

$A = 13^\circ$

Question 11:

Prove that:

(i) $\dfrac{\cos\left(90^\circ - \theta\right)\cos\theta}{\cot\theta} = 1 - \cos^2\theta$

(ii) $\dfrac{\sin\theta \sin\left(90^\circ - \theta\right)}{\cot\left(90^\circ - \theta\right)} = 1 - \sin^2\theta$

Solution 11:

(i) $\text{LHS} = \dfrac{\cos\left(90^\circ - \theta\right)\cos\theta}{\cot\theta} = \dfrac{\sin\theta\cos\theta}{\dfrac{\cos\theta}{\sin\theta}} = \sin^2\theta = 1 - \cos^2\theta$

(ii) $\text{LHS} = \dfrac{\sin\theta\sin\left(90^\circ - \theta\right)}{\cot\left(90^\circ - \theta\right)} = \dfrac{\sin\theta\cos\theta}{\tan\theta} = \dfrac{\sin\theta\cos\theta}{\dfrac{\sin\theta}{\cos\theta}} = \cos^2\theta = 1 - \sin^2\theta$

Question 12:

Evaluate:

$\dfrac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ}$

Solution 12:

$$\frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\cos ec^2 10^\circ - \tan^2 80^\circ}$$

$$= \frac{\sin 35^\circ.\cos\left(90^\circ - 35^\circ\right) + \cos 35^\circ.\sin\left(90^\circ - 35^\circ\right)}{\cos ec^2\left(90^\circ - 80^\circ\right) - \tan^2 80^\circ}$$

$$= \frac{\sin 35^\circ.\sin 35^\circ + \cos 35^\circ.\cos 35^\circ}{\sec^2 80^\circ - \tan^2 80^\circ}$$

$$= \frac{\sin^2 35^\circ + \cos^2 35^\circ}{\sec^2 80^\circ - \tan^2 80^\circ} = \frac{1}{1} = 1$$

EXERCISE. 21 (D)

Question 1:

Use tables to fins sine of:

(i) 21° (ii) 34° 42'

(iii) 47° 32' (iv) 62° 57'

(v) 10° 20' + 20° 45'

Solution 1:

(i) sin 21° = 0.3584

(ii) sin 34° 42'= 0.5693

(iii) sin 47° 32' = sin (47° 30' + 2') =0.7373 + 0.0004 = 0.7377

(iv) sin 62° 57' = sin (62° 54' + 3') = 0.8902 + 0.0004 = 0.8906

(v) sin (10° 20' + 20° 45') = sin 30° 65' = sin 31° 5' = 0.5150 + 0.0012 = 0.5162

Question 2:

Use tables to find cosine of:

(i) 2° 4’ (ii) 8° 12’

(iii) 26° 32’ (iv) 65° 41’

(v) 9° 23’ + 15° 54’

Solution 2:

(i) cos 2° 4’ = 0.9994 − 0.0001 = 0.9993

(ii) cos 8° 12’ = cos 0.9898

(iii) cos 26° 32’ = cos (26° 30’ + 2’) = 0.8949 − 0.0003 = 0.8946

(iv) cos 65° 41’ = cos (65° 36’ + 5’) = 0.4131 − 0.0013 = 0.4118
(v) cos (9° 23’ + 15° 54’) = cos 24° 77’ = cos 25° 17’ = cos (25° 12’ + 5’) = 0.9048 − 0.0006 = 0.9042

Question 3:

Use trigonometrical tables to find tangent of:
(i) 37° (ii) 42° 18' (iii) 17° 27'

Solution 3:

(i) tan 37° = 0.7536
(ii) tan 42° 18' = 0.9099
(iii) tan 17° 27' = tan (17° 24' + 3') = 0.3134 + 0.0010 = 0.3144

Question 4:

Use tables to find the acute angle θ, if the value of sin θ is:
(i) (i) 0.4848 (ii) 0.3827 (iii) 0.6525

Solution 4:

(i) From the tables, it is clear that sin 29° = 0.4848
Hence, θ = 29°
(ii) From the tables, it is clear that sin 22° 30' = 0.3827
Hence, θ = 22° 30'
(iii) From the tables, it is clear that sin 40° 42' = 0.6521
sin θ − sin 40° 42' = 0.6525 −; 0.6521 = 0.0004
From the tables, diff of 2' = 0.0004
Hence, θ = 40° 42' + 2' = 40° 44'

Question 5:

Use tables to find the acute angle θ, if the value of cos θ is:
(i) 0.9848 (ii) 0.9574 (iii) 0.6885

Solution 5:

(i) From the tables, it is clear that cos 10° = 0.9848
Hence, θ = 10°
(ii) From the tables, it is clear that cos 16° 48’ = 0.9573
cos θ − cos 16° 48’ = 0.9574 − 0.9573 = 0.0001

From the tables, diff of 1’ = 0.0001
Hence, θ = 16° 48’ − 1’ = 16° 47’
(iii) From the tables, it is clear that cos 46° 30’ = 0.6884
cos q − cos 46° 30’ = 0.6885 − 0.6884 = 0.0001
From the tables, diff of 1’ = 0.0002
Hence, θ = 46° 30’ − 1’ = 46° 29’

Question 6:

Use tables to find the acute angle θ, if the value of tan θ is:
(i) 0.2419 (ii) 0.4741 (iii) 0.7391

Solution 6:

(i) From the tables, it is clear that tan 13° 36’ = 0.2419
Hence, θ = 13° 36’
(ii) From the tables, it is clear that tan 25° 18’ = 0.4727
tan θ − tan 25° 18’ = 0.4741 − 0.4727 = 0.0014
From the tables, diff of 4’ = 0.0014
Hence, θ = 25° 18’ + 4’ = 25° 22’
(iii) From the tables, it is clear that tan 36° 24’ = 0.7373
tan θ − tan 36° 24’ = 0.7391 − 0.7373 = 0.0018
From the tables, diff of 4’ = 0.0018
Hence, θ = 36° 24’ + 4’ = 36° 28’

EXERCISE. 21(E)

Question 1:

1. Prove the following identities:

(i) $\frac{1}{\cos A + \sin A} + \frac{1}{\cos A - \sin A} = \frac{2\cos A}{2\cos^2 A - 1}$

(ii) $\operatorname{cosec} A - \cot A = \frac{\sin A}{1 + \cos A}$

(iii) $1 - \frac{\sin^2 A}{1 + \cos A} = \cos A$

(iv) $\frac{1 - \cos A}{\sin A} + \frac{\sin A}{1 - \cos A} = 2 \operatorname{cosec} A$

(v) $\frac{\cot A}{1-\tan A}+\frac{\tan A}{1-\cot A}=1+\tan A+\cot A$

(vi) $\frac{\cos A}{1-\tan A}+\tan A=\sec A$

(vii) $\frac{\sin A}{1-\cos A}-\cot A=\operatorname{cosec} A$

(viii) $\frac{\sin A-\cos A+1}{\sin A+\cos A-1}=\frac{\cos A}{1-\sin A}$

(ix) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\frac{\cos A}{1-\sin A}$

(x) $\sqrt{\frac{1-\cos A}{1+\cos A}}=\frac{\sin A}{1+\cos A}$

(xi) $\frac{1+(\sec A-\tan A)^2}{\operatorname{cosec} A(\sec A-\tan A)}=2\tan A$

(xii) $\frac{(\operatorname{cosec} A-\cot A)^2+1}{\sec A(\operatorname{cosec} A-\cot A)}=2\cot A$

(xiii) $\cot^2 A\left(\frac{\sec A-1}{1+\sin A}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)=0$

(xiv) $\frac{(1-2\sin^2 A)^2}{\cos^4 A-\sin^4 A}=2\cos^2 A-1$

(xv) $\sec^4 A(1-\sin^4 A)-2\tan^2 A=1$

(xvi) $\operatorname{cosec}^4 A(1-\cos^4 A)-2\cot^2 A=1$

(xvii) $(1+\tan A+\sec A)(1+\cot A-\operatorname{cosec} A)=2$

Solution 1:

(i) $\frac{1}{\cos A+\sin A}+\frac{1}{\cos A-\sin A}$

$=\frac{\cos A+\sin A+\cos A-\sin A}{(\cos A+\sin A)(\cos A-\sin A)}$

$=\frac{2\cos A}{\cos^2 A-\sin^2 A}$

$=\frac{2\cos A}{\cos^2 A-(1-\cos^2 A)}$

$=\frac{2\cos A}{2\cos^2 A-1}$

(ii) $\operatorname{cosec} A - \cot A$

$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

$= \frac{1 - \cos A}{\sin A}$

$= \frac{1 - \cos A}{\sin A} \times \frac{1 + \cos A}{1 + \cos A}$

$= \frac{1 - \cos^2 A}{\sin A\left(1 + \cos A\right)}$

$= \frac{\sin^2 A}{\sin A\left(1 + \cos A\right)}$

$= \frac{\sin A}{1 + \cos A}$

(iii) $1 - \frac{\sin^2 A}{1 + \cos A}$

$= \frac{1 + \cos A - \sin^2 A}{1 + \cos A}$

$= \frac{\cos A + \cos^2 A}{1 + \cos A}$

$= \frac{\cos A\left(1 + \cos A\right)}{1 + \cos A}$

$= \cos A$

(iv) $\frac{1 - \cos A}{\sin A} + \frac{\sin A}{1 - \cos A}$

$= \frac{\left(1 - \cos A\right)^2 + \sin^2 A}{\sin A\left(1 - \cos A\right)}$

$= \frac{1 + \cos^2 A - 2\cos A + \sin^2 A}{\sin A\left(1 - \cos A\right)}$

$= \frac{2 - 2\cos A}{\sin A(1 - \cos A)}$

$= \frac{2\left(1 - \cos A\right)}{\sin A(1 - \cos A)}$

$= 2\operatorname{cosec} A$

(v) $\frac{\cot A}{1 - \tan A} + \frac{\tan A}{1 - \cot A}$

$$= \frac{\frac{1}{\tan A}}{1-\tan A} + \frac{\tan A}{1-\frac{1}{\tan A}}$$

$$= \frac{1}{\tan A\left(1-\tan A\right)} + \frac{\tan^2 A}{\tan A - 1}$$

$$= \frac{1-\tan^3 A}{\tan A\left(1-\tan A\right)}$$

$$= \frac{\left(1-\tan A\right)\left(1+\tan A+\tan^2 A\right)}{\tan A\left(1-\tan A\right)}$$

$$= \frac{1+\tan A+\tan^2 A}{\tan A}$$

$$= \cot A + 1 + \tan A$$

(vi) $\frac{\cos A}{1+\sin A} + \tan A$

$$= \frac{\cos A}{1+\sin A} + \frac{\sin A}{\cos A}$$

$$= \frac{\cos^2 A + \sin A + \sin^2 A}{\left(1+\sin A\right)\cos A}$$

$$= \frac{1+\sin A}{\left(1+\sin A\right)\cos A}$$

$$= \frac{1}{\cos A}$$

$$= \sec A$$

(vii) $\frac{\sin A}{1-\cos A} - \cot A$

$$= \frac{\sin A}{1-\cos A} - \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A - \cos A + \cos^2 A}{\left(1-\cos A\right)\sin A}$$

$$= \frac{1-\cos A}{\left(1-\cos A\right)\sin A}$$

$$= \frac{1}{\sin A}$$

$$= \operatorname{cosec} A$$

(viii) $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$

$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{\sin A - (\cos A - 1)}{\sin A - (\cos A - 1)}$

$= \frac{(\sin A - \cos A + 1)^2}{\sin^2 A - (\cos A - 1)^2}$

$= \frac{\sin^2 A + \cos^2 A + 1 - 2\sin A\cos A - 2\cos A + 2\sin A}{\sin^2 A - \cos^2 A - 1 + 2\cos A}$

$= \frac{1 + 1 - 2\sin A\cos A - 2\cos A + 2\sin A}{-\cos^2 A - \cos^2 A + 2\cos A}$

$= \frac{2(1 - \cos A) + 2\sin(1 - \cos A)}{2\cos A(1 - \cos A)}$

$= \frac{1 + \sin A}{\cos A}$

$= \frac{1 + \sin A}{\cos A} \times \frac{1 - \sin A}{1 - \sin A}$

$= \frac{\cos^2 A}{\cos A(1 - \sin A)}$

$= \frac{\cos A}{1 - \sin A}$

(ix) $\sqrt{\frac{1 + \sin A}{1 - \sin A}}$

$= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 - \sin A}{1 - \sin A}}$

$= \sqrt{\frac{1 - \sin^2 A}{(1 - \sin A)^2}}$

$= \sqrt{\frac{\cos^2 A}{(1 + \sin A)^2}}$

$= \frac{\cos A}{1 - \sin A}$

(x) $\sqrt{\frac{1 - \cos A}{1 + \cos A}}$

$= \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 + \cos A}{1 + \cos A}}$

$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$

$= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$

$= \frac{\sin A}{1+\cos A}$

(xi) $\frac{1+(\sec A-\tan A)^2}{\operatorname{cosec}A(\sec A-\tan A)}$

$= \frac{(\sec^2 A-\tan^2 A)+(\sec A-\tan A)^2}{\operatorname{cosec}A(\sec A-\tan A)}$

$= \frac{(\sec A-\tan A)(\sec A+\tan A)+(\sec A+\tan A)^2}{\operatorname{cosec}A(\sec A-\tan A)}$

$= \frac{(\sec A+\tan A)+(\sec A-\tan A)}{\operatorname{cosec}A}$

$= \frac{2\sec A}{\operatorname{cosec}A}$

$= 2\frac{\frac{1}{\cos A}}{\frac{1}{\sin A}}$

$= 2\tan A$

(xii) $\frac{(\operatorname{cosec}A-\cot A)^2+1}{\sec A(\operatorname{cosec}A-\cot A)}$

$= \frac{(\operatorname{cosec}A-\cot A)^2+(\operatorname{cosec}^2 A-\cot^2 A)}{\sec A(\operatorname{cosec}A-\cot A)}$

$= \frac{(\operatorname{cosec}A-\cot A)^2+(\operatorname{cosec}A-\cot A)(\operatorname{cosec}A+\cot A)}{\sec A(\operatorname{cosec}A-\cot A)}$

$= \frac{(\operatorname{cosec}A-\cot A)+(\operatorname{cosec}A+\cot A)}{\sec A}$

$= \frac{2\operatorname{cosec}A}{\sec A}$

$= 2\cot A$

(xiii) $\cot^2 A\left(\frac{\sec A-1}{1+\sin A}\right)+\sec^2 A\left(\frac{\sec A-1}{1+\sec A}\right)$

$=\cot^2 A\left(\frac{\sec A-1}{1+\sin A}\times\frac{\sec A+1}{\sec A+1}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)$

$=\cot^2 A\left[\frac{\sec^2 A-1}{(1+\sin A)(\sec A+1)}\right]+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)$

$=\cot^2 A\left[\frac{\tan^2 A}{(1+\sin A)(\sec A+1)}\right]+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)$

$=\frac{1}{(1+\sin A)(\sec A+1)}+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)$

$=\frac{1+\sec^2 A(\sin A-1)(1+\sin A)}{(1+\sin A)(\sec A+1)}$

$=\frac{1+\sec^2 A(\sin^2 A-1)}{(1+\sin A)(\sec A+1)}$

$=\frac{1+\sec^2 A(-\cos^2 A)}{(1+\sin A)(\sec A+1)}$

$=\frac{1-1}{(1+\sin A)(\sec A+1)}$

$=0$

(xiv) $\frac{(1-2\sin^2 A)^2}{\cos^4 A-\sin^4 A}$

$=\frac{(1-2\sin^2 A)^2}{(\cos^2 A-\sin^2 A)(\cos^2 A+\sin^2 A)}$

$=\frac{(1-2\sin^2 A)^2}{1-\sin^2 A-\sin^2 A}$

$=\frac{(1-2\sin^2 A)^2}{1-2\sin^2 A}$

$=1-2\sin^2 A$

$=1-2(1-\cos^2 A)$

$=2\cos^2 A-1$

(xv) $\sec^4 A(1-\sin^4 A)-2\tan^2 A$

$$= \sec^4 A\left(1-\sin^2 A\right)\left(1+\sin^2 A\right)-2\tan^2 A$$
$$= \sec^4 A\left(\cos^2 A\right)\left(1+\sin^2 A\right)-2\tan^2 A$$
$$= \sec^2 A+\frac{\sin^2 A}{\cos^2 A}-2\tan^2 A$$
$$= \sec^2 A+\tan^2 A-2\tan^2 A$$
$$= \sec^2 A-\tan^2 A$$
$$=1$$

(xvi) $\operatorname{cosec}^4 A\left(1-\cos^4 A\right)-2\cot^2 A$

$$= \operatorname{cosec}^4 A\left(1-\cos^2 A\right)\left(1+\cos^2 A\right)-2\cot^2 A$$
$$= \operatorname{cosec}^4 A\left(\sin^2 A\right)\left(1+\cos^2 A\right)-2\cot^2 A$$
$$= \operatorname{cosec}^2 A\left(1+\cos^2 A\right)-2\cot^2 A$$
$$= \operatorname{cosec}^2 A+\frac{\cos^2 A}{\sin^2 A}-2\cot^2 A$$
$$= \operatorname{cosec}^2 A+\cot^2 A-2\cot^2 A$$
$$= \operatorname{cosec}^2 A-\cot^2 A$$
$$=1$$

(xvii) $\left(1+\tan A+\sec A\right)\left(1+\cot A-\operatorname{cosec} A\right)$

$$=1+\cot A-\operatorname{cosec} A+\tan A+1-\sec A+$$
$$\sec A+\operatorname{cosec} A-\operatorname{cosec} A\sec A$$
$$=2+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}-\frac{1}{\sin A\cos A}$$
$$=2+\frac{\cos^2 A+\sin^2 A}{\sin A\cos A}-\frac{1}{\sin A\cos A}$$
$$=2+\frac{1}{\sin A\cos A}-\frac{1}{\sin A\cos A}$$
$$=2$$

Question 2:

If sin A + cos A = p

and sec A + cosec A = q, then prove that: $q\left(p^2-1\right)=2p$

Solution 2:

$$q\left(p^2-1\right)=\left(\sec A+\operatorname{cosec} A\right)\left[\left(\sin A+\cos A\right)^2-1\right]$$

$$= (\sec A + \operatorname{cosec} A)\left[(\sin^2 A + \cos^2 A + 2\sin A\cos A) - 1\right]$$
$$= (\sec A + \operatorname{cosec} A)\left[(1 + 2\sin A\cos A) - 1\right]$$
$$= (\sec A + \operatorname{cosec} A)(2\sin A\cos A)$$
$$= 2\sin A + 2\cos A$$
$$= 2P$$

Question 3:

If x = a cos θ and y = b cot θ, show that:

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

Solution 3:

$$\frac{a^2}{x^2} - \frac{b^2}{y^2}$$
$$= \frac{a^2}{a^2\cos^2\theta} - \frac{b^2}{b^2\cot^2\theta}$$
$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$
$$= \frac{1 - \sin^2\theta}{\cos^2\theta}$$
$$= \frac{\cos^2\theta}{\cos^2\theta}$$
$$= 1$$

Question 4:

If sec A + tan A = p, show that:

$$\sin A = \frac{p^2 - 1}{p^2 + 1}$$

Solution 4:

$$\frac{p^2 - 1}{p^2 + 1}$$
$$= \frac{(\sec A + \tan A)^2 - 1}{(\sec A + \tan A)^2 + 1}$$

$$= \frac{\sec^2 A + \tan^2 A + 2\tan A \sec A - 1}{\sec^2 A + \tan^2 A + 2\tan A \sec A - 1}$$

$$= \frac{\tan^2 A + \tan^2 A + 2\tan A \sec A}{\sec^2 A + \sec^2 A + 2\tan A \sec A}$$

$$= \frac{2\tan^2 A + 2\tan A \sec A}{2\sec^2 A + 2\tan A \sec A}$$

$$= \frac{2\tan A(\tan A + \sec A)}{2\sec A(\tan A + \sec A)}$$

$$= \sin A$$

Question 5:

If tan A = n tan B and sin A = m sin B, prove that:

$$\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

Solution 5:

Given that, tan A = n tan B and sin A = m sin B.

$$\Rightarrow n = \frac{\tan A}{\tan B} \text{ and } m = \frac{\sin A}{\sin B}$$

$$\therefore \frac{m^2 - 1}{n^2 - 1}$$

$$= \frac{\left(\frac{\sin A}{\sin B}\right)^2 - 1}{\left(\frac{\tan A}{\tan B}\right)^2 - 1}$$

$$= \frac{\tan^2 B(\sin^2 A - \sin^2 B)}{\sin^2 B(\tan^2 A - \tan^2 B)}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 B\left(\frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}\right)}$$

$$= \frac{\cos^2 A(\sin^2 A - \sin^2 B)}{\sin^2 A\cos^2 B - (1 - \cos^2 B)\cos^2 A}$$

$$= \frac{\cos^2 A\left(1-\cos^2 A-1+\cos^2 B\right)}{\cos^2 B\left(\sin^2 A+\cos^2 A\right)-\cos^2 A}$$

$$= \frac{\cos^2 A\left(\cos^2 B-\cos^2 A\right)}{\cos^2 B-\cos^2 A}$$

$$= \cos^2 A$$

Question 6:

(i) If 2 sin A – 1 = 0, show that:

Sin 3A = 3 sin A – 4 $\sin^3$A

(ii) If 4 $\cos^2$ A – 3 = 0, Show that:

cos 3 A = 4 $\cos^3$ A – 3 cos A

Solution 6:

(i) 2 sinA − 1 = 0

$$\Rightarrow \sin A = \frac{1}{2}$$

We know $\sin\ 30^\circ = \frac{1}{2}$

So, A = 30°

$$\text{LHS} = \sin 3A = \sin 90^\circ = 1$$

$$\text{RHS} = 3\sin A - 4\sin^3 A$$

$$= 3\sin 30^\circ - 4\sin^3 30^\circ$$

$$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

LHS = RHS

(ii) $4\cos^2 A - 3 = 0$

$$\Rightarrow 4\cos^2 A = 3$$

$$\Rightarrow \cos^2 A = \frac{3}{4}$$

$$\Rightarrow \cos A = \frac{\sqrt{3}}{2}$$

We know $\cos\ 30^\circ = \frac{\sqrt{3}}{2}$

So, A = 30°

$\text{LHS} = \cos 3A = \cos 90^\circ = 0$

$\text{RHS} = 4\cos^3 A - 3\cos A$

$= 4\cos^3 30^\circ - 3\cos 30^\circ$

$= 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$

LHS = RHS

Question 7:

Evaluate

(i) $2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right) + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\sec 40^\circ}{\operatorname{co\,sec} 50^\circ}\right)$

(ii) $\sec 26^\circ \sin 64^\circ + \frac{\text{coe sec}\, 33^\circ}{\sec 57^\circ}$

(iii) $\frac{5\sin 66^\circ}{\cos 24^\circ} - \frac{2\cot 85^\circ}{\tan 5^\circ}$

(iv) $\cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ$

(v) $\sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$

(vi) $\frac{3\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(vii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$

(viii) $\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$

Solution 7:

(i) $2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right) + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\sec 40^\circ}{\operatorname{co\,sec} 50^\circ}\right)$

$= 2\left(\frac{\tan(90^\circ - 55^\circ)}{\cot 55^\circ}\right) + \left(\frac{\cot(90^\circ - 35^\circ)}{\tan 35^\circ}\right) - 3\left(\frac{\sec(90^\circ - 50^\circ)}{\operatorname{cosec} 50^\circ}\right)$

$= 2\left(\frac{\cot 55^\circ}{\cot 55^\circ}\right) + \left(\frac{\tan 35^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\operatorname{cosec} 50^\circ}{\operatorname{cosec} 50^\circ}\right)$

$= 2(1)^2 + 1^2 + -3$

$= 2 + 1 - 3$

$= 0$

(ii) $\sec 26^\circ \sin 64^\circ + \frac{\text{coe sec} 33^\circ}{\sec 57^\circ}$

$= \sec(90^\circ - 64^\circ)\sin 64^\circ + \frac{\text{co e sec}(90^\circ - 57^\circ)}{\sec 57^\circ}$

$= \text{cos ec} 64^\circ \sin 64^\circ + \frac{\sec 57^\circ}{\sec 57^\circ}$

$= 1 + 1 = 2$

(iii) $\frac{5\sin 66^\circ}{\cos 24^\circ} - \frac{2\cot 85^\circ}{\tan 5^\circ}$

$= \frac{5\sin(90^\circ - 24^\circ)}{\cos^\circ 24^\circ} - \frac{2\cot(90^\circ - 5^\circ)}{\tan 5^\circ}$

$= \frac{5\cos 24^\circ}{\cos 24^\circ} - \frac{2\tan 5^\circ}{\tan 5^\circ}$

$= 5 - 2 = 3$

(iv) $\cos 40^\circ \text{ cos ec} 50^\circ + \sin 50^\circ \sec 40^\circ$

$= \cos(90^\circ - 50^\circ)\text{cos ec} 50^\circ + \sin(90^\circ - 40^\circ)\sec 40^\circ$

$= \sin 50^\circ \text{ cos ec} 50^\circ + \cos 40^\circ \sec 40^\circ$

$= 1 + 1 = 2$

(v) $\sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$

$= \sin(90^\circ - 63^\circ)\sin 63^\circ - \cos 63^\circ \cos(90^\circ - 63^\circ)$

$= \cos 63^\circ \sin 63^\circ - \cos 63^\circ \sin 63^\circ$

$= 0$

(vi) $\frac{3\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\text{cos ec} 58^\circ}$

$= \frac{3\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\text{cos ec} 58^\circ}$

$= \frac{3\cos 18^\circ}{\cos 18^\circ} - \frac{\text{cos ec} 58^\circ}{\text{cos ec} 58^\circ}$

$= 3 - 1 = 2$

(vii) $3\cos 80^\circ \text{ cos ec} 10^\circ + 2\cos 59^\circ \text{ cos ec} 31^\circ$

$= 3\cos(90^\circ - 10^\circ)\text{cos ec} 10^\circ + 2\cos(90^\circ - 31^\circ)\text{cos ec} 31^\circ$

$= 3\sin 10^\circ \operatorname{cosec} 10^\circ + 2\sin 31^\circ \operatorname{cosec} 31^\circ$

$= 3 + 2 = 5$

(viii) $\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$

$= \frac{\cos(90^\circ - 15^\circ)}{\sin 15^\circ} + \frac{\sin(90^\circ - 78^\circ)}{\cos 78^\circ} - \frac{\cos(90^\circ - 72^\circ)}{\sin 72^\circ}$

$= \frac{\sin 15^\circ}{\sin 15^\circ} + \frac{\cos 78^\circ}{\cos 78^\circ} - \frac{\sin 72^\circ}{\sin 72^\circ}$

$= 1 + 1 - 1 = 1$

Question 8:

Prove that:

(i) $\tan(55^\circ + x) = \cot(35^\circ - x)$

(ii) $\sec(70^\circ - \theta) = \operatorname{cosec}(20^\circ + \theta)$

(iii) $\sin(28^\circ + A) = \cos(62^\circ - A)$

(iv) $\frac{1}{1+\cos(90^\circ - A)} + \frac{1}{1-\cos(90^\circ - A)} = 2\operatorname{cosec}^2(90^\circ - A)$

(v) $\frac{1}{1+\sin(90^\circ - A)} + \frac{1}{1-\sin(90^\circ - A)} = 2\sec^2(90^\circ - A)$

Solution 8:

(i) $\tan(55^\circ + x) = \tan[90^\circ - (35^\circ - x)] = \cot(35^\circ - x)$

(ii) $\sec(70^\circ - \theta) = \sec[90^\circ - (20^\circ + \theta)] = \operatorname{cosec}(20^\circ + \theta)$

(iii) $\sin(28^\circ + A) = \sin[90^\circ - 62^\circ - A] = \cos(62^\circ - A)$

(iv) $\frac{1}{1+\cos(90^\circ - A)} + \frac{1}{1-\cos(90^\circ - A)}$

$= \frac{1}{1+\sin A} + \frac{1}{1-\sin A}$

$= \frac{1-\sin A + 1 + \sin A}{(1+\sin A)(1-\sin A)}$

$= \frac{2}{1-\sin^2 A}$

$= \frac{2}{\cos^2 A}$

$= 2\sec^2 A$

$= 2\operatorname{cosec}^2(90^\circ - A)$

(v) $\frac{1}{1+\sin(90^\circ - A)} + \frac{1}{1-\sin(90^\circ - A)}$

$= \frac{1}{1+\cos A} + \frac{1}{1-\cos A}$

$= \frac{1-\cos A + 1 + \cos A}{(1+\cos A)(1-\cos A)}$

$= \frac{2}{1-\cos^2 A}$

$= 2\operatorname{cosec}^2 A$

$= 2\sec^2(90^\circ - A)$

Question 9:

If A and B are complementary angles, prove that:

(i) $\cot B + \cos B = \sec A \cos B(1+\sin B)$

(ii) $\cot A \cot B - \sin A \cos B - \cos A \sin B = 0$

(iii) $\operatorname{cosec}^2 A + \operatorname{cosec}^2 B = \operatorname{cosec}^2 A \operatorname{cosec}^2 B$

(iv) $\frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos B - \cos A}{\cos B + \cos A} = \frac{2}{2\sin^2 A - 1}$

Solution 9:

Since, A and B are complementary angles, A + B = 90°

(i) $\cot B + \cos B$

$= \cot(90^\circ - A) + \cos(90^\circ - A)$

$= \tan A + \sin A$

$= \frac{\sin A}{\cos A} + \sin A$

$= \frac{\sin A + \sin A \cos A}{\cos A}$

$= \frac{\sin A(1+\cos A)}{\cos A}$

$= \sec A \sin A (1 + \cos A)$

$= \sec A \sin(90^\circ - B)\left[1 + \cos(90^\circ - B)\right]$

$= \sec A \cos B (1 + \sin B)$

(ii) $\cot A \cot B - \sin A \cos B - \cos A \sin B$

$= \cot A \cot(90 - A) - \sin A \cos(90 - A) - \cos A \ \sin(90 - A)$

$= \cot A \tan A - \sin A \sin A - \cos A \cos A$

$= 1 - \left(\sin^2 A + \cos^2 A\right)$

$= 1 - 1$

$= 0$

(iii) $\operatorname{cosec}^2 A + \operatorname{cosec}^2 B$

$= \operatorname{cosec}^2 A + \left[\operatorname{cosec}(90 - A)\right]^2$

$= \operatorname{cosec}^2 A + \sec^2 A$

$= \dfrac{1}{\sin^2 A} + \dfrac{1}{\cos^2 A}$

$= \dfrac{\cos^2 A + \sin^2 A}{\sin^2 A \cos^2 A}$

$= \dfrac{1}{\sin^2 A \cos^2 A}$

= cosec^{2}A [sec(90Â° − B)]^2

= cosec^{2}A cosec^{2}B

(iv) $\dfrac{\sin A + \sin B}{\sin A - \sin B} + \dfrac{\cos B - \cos A}{\cos B + \cos A}$

$= \dfrac{\sin A + \sin B}{\sin A - \sin B} + \dfrac{\cos(90^\circ - A) - \cos(90^\circ - B)}{\cos(90^\circ - A) + \cos(90^\circ - B)}$

$= \dfrac{\sin A + \sin B}{\sin A - \sin B} + \dfrac{\sin A - \sin B}{\sin A + \sin B}$

$= \dfrac{(\sin A + \sin B)^2 + (\sin A - \sin B)^2}{(\sin A - \sin B)(\sin A + \sin B)}$

$= \dfrac{\sin^2 A + \sin^2 B + 2\sin A \sin B + \sin^2 A + \sin^2 B - 2\sin A}{\sin^2 A - \sin^2 B}$

$= 2\dfrac{\sin^2 A + \sin^2 B}{\sin^2 A - \sin^2 B}$

$$= 2\frac{\sin^2 A + \sin^2\left(90^\circ - A\right)}{\sin^2 A - \sin^2\left(90^\circ - A\right)}$$

$$= 2\frac{\sin^2 A + \cos^2 B}{\sin^2 A - \cos^2 B}$$

$$= \frac{2}{\sin^2 A - \left(1 - \sin^2 A\right)}$$

$$= \frac{2}{2\sin^2 A - 1}$$

Question 10:

Prove that

(i) $\frac{1}{\sin A - \cos A} - \frac{1}{\sin A + \cos A} = \frac{2\cos A}{2\sin^2 A - 1}$

(ii) $\frac{\cot^2 A}{\cos ecA - 1} - 1 = \cos ecA$

(iii) $\frac{\cos A}{1 + \sin A} = \sec A - \tan A$

(iv) $\cos A\left(1 + \cot A\right) + \sin A\left(1 + \tan A\right) = \sec A + \text{cosec } A$

(v) $\left(\sin A - \cos A\right)\left(1 + \tan A + \cot A\right) = = \frac{\sec A}{\cos ec^2 A} - \frac{\cos ecA}{\sec^2 A}$

(vi) $\sqrt{\sec^2 A + \cos ec^2 A} = \tan A + \cot A$

(vii) $\left(\sin A + \cos A\right)\left(\sec A + \cos ecA\right) = 2 + \sec A \cos ecA$

(viii) $\left(\tan A + \cot A\right)\left(\cos ecA - \sin A\right)\left(\sec A - \cos A\right) = 1$

(ix) $\cot^2 A - \cot^2 B = \frac{\cos^2 A - \cos^2 B}{\sin^2 A \sin^2 B} = \cos ec^2 A - \cos ec^2 B$

Solution 10:

(i) $\frac{1}{\sin A - \cos A} - \frac{1}{\sin A + \cos A}$

$$= \frac{\sin A + \cos A - \sin A + \cos A}{\left(\sin A - \cos A\right)\left(\sin A + \cos A\right)}$$

$$= \frac{2\cos A}{\sin^2 A - \cos^2 A}$$

$$= \frac{2\cos A}{\sin^2 A - \left(1 - \sin^2 A\right)}$$

$$= \frac{2\cos A}{2\sin^2 A - 1}$$

(ii) $\frac{\cot^2 A}{\operatorname{cosec} A - 1} - 1$

$$= \frac{\cot^2 A - \operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}$$

$$= \frac{-\operatorname{cosec} A + \operatorname{cosec}^2 A}{\operatorname{cosec} A - 1}$$

$$= \frac{\operatorname{cosec} A\left(\operatorname{cosec} A - 1\right)}{\operatorname{cosec} A - 1}$$

$$= \operatorname{cosec} A$$

(iii) $\frac{\cos A}{1 + \sin A}$

$$= \frac{\cos A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A}$$

$$= \frac{\cos A\left(1 - \sin A\right)}{1 - \sin^2 A}$$

$$= \frac{\cos A\left(1 - \sin A\right)}{\cos^2 A}$$

$$= \frac{1 - \sin A}{\cos A}$$

$$= \sec A - \tan A$$

(iv) $\cos A\left(1 + \cot A\right) + \sin A\left(1 + \tan A\right)$

$$= \cos A + \frac{\cos^2 A}{\sin A} + \sin A + \frac{\sin^2 A}{\cos A}$$

$$= \sin A + \frac{\cos^2 A}{\sin A} + \cos A + \frac{\sin^2 A}{\cos A}$$

$$= \left(\frac{\cos^2 A + \sin^2 A}{\sin A}\right) + \left(\frac{\cos^2 A + \sin^2 A}{\cos A}\right)$$

$$= \frac{1}{\sin A} + \frac{1}{\cos A}$$

$$= \operatorname{cosec} A + \sec A$$

(v) $\left(\sin A - \cos A\right)\left(1 + \tan A + \cot A\right)$

$$= \sin A + \frac{\sin^2 A}{\cos A} + \cos A - \cos A - \sin A - \frac{\cos^2 A}{\sin A}$$

$$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

$$= \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$$

(vi) LHS = $\sqrt{\sec^2 A + \operatorname{cosec}^2 A}$

$$= \sqrt{\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}}$$

$$= \sqrt{\frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A}}$$

$$= \sqrt{\frac{1}{\sin^2 A \cos^2 A}}$$

$$= \sqrt{\frac{1}{\sin A \cos A}}$$

RHS= tan A + cot A

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{1}{\sin A \cos A}$$

LHS = RHS

(vii) $(\sin A + \cos A)(\sec A + \operatorname{cosec} A)$

$$= \frac{\sin A}{\cos A} + 1 + 1 + \frac{\cos A}{\sin A}$$

$$= 2 + \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$$

$$= 2 + \frac{1}{\sin A \cos A}$$

$$= 2 + \sec A \operatorname{cosec} A$$

(viii) $(\tan A + \cot A)(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)\left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$=\left(\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}\right)\left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

$$=\left(\frac{1}{\sin A\cos A}\right)\left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$$

$$=1$$

(ix) $\cot^2 A-\cot^2 B$

$$=\frac{\cos^2 A}{\sin^2 A}-\frac{\cos^2 B}{\sin^2 B}$$

$$=\frac{\cos^2 A\sin^2 B-\cos^2 B\sin^2 A}{\sin^2 A\sin^2 B}$$

$$=\frac{\cos^2 A\left(1-\cos^2 B\right)-\cos^2 B\left(1-\cos^2 A\right)}{\sin^2 A\sin^2 B}$$

$$=\frac{\cos^2 A-\cos^2 A\cos^2 B-\cos^2 B+\cos^2 B\cos^2 A}{\sin^2 A\sin^2 B}$$

$$=\frac{\cos^2 A-\cos^2 B}{\sin^2 A\sin^2 B}$$

$$=\frac{1-\sin^2 A-1+\sin^2 B}{\sin^2 A\sin^2 B}$$

$$=\frac{-\sin^2 A+\sin^2 B}{\sin^2 A\sin^2 B}$$

$$=\frac{\sin^2 B}{\sin^2 A\sin^2 B}-\frac{\sin^2 A}{\sin^2 A\sin^2 B}$$

$$=\frac{1}{\sin^2 A}-\frac{1}{\sin^2 B}$$

$$=\operatorname{cosec}^2 A-\operatorname{cosec}^2 B$$

Question 11:

If $4\cos^2 A-3=0$ and $\leq A\leq 90°$, then prove that:

(i) $\sin 3A = 3\sin A-4\sin^3 A$

(ii) $\cos 3A = 4\cos^3 A-3\cos A$

Solution 11:

$4\cos^2 A-3=0$

$$\cos A=\frac{\sqrt{3}}{2}$$

We know $\cos 30^\circ = \frac{\sqrt{3}}{2}$

So, A= 30°

(i) $\text{LHS} = \sin 3A = \sin 90^\circ = 1$

$\text{RHS} = 3\sin A - 4\sin^3 A$

$= 3\sin 30^\circ - 4\sin^3 30^\circ$

$= 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3$

$= \frac{3}{2} - \frac{1}{2}$

$= 1$

LHS = RHS

(ii) $\text{LHS} = \cos 3A = \cos 90^\circ = 0$

$\text{RHS} = 4\cos^3 A - 3\cos A$

$= 4\cos^3 30^\circ - 3\cos 30^\circ$

$= 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$

LHS = RHS

Question 12:

Find A, if $0^\circ \le A \le 90^\circ$ and:

(i) $2\cos^2 A - 1 = 0$

(ii) $\sin 3A - 1 = 0$

(iii) $4\sin^2 A - 3 = 0$

(iv) $\cos^2 A - \cos A = 0$

(v) $2\cos^2 A + \cos A - 1 = 0$

Solution 12:

(i) $2\cos^2 A - 1 = 0$

$\Rightarrow \cos^2 A = \frac{1}{2}$

$\Rightarrow \cos A = \frac{1}{\sqrt{2}}$

We know $\cos\ 45^{\circ} = \frac{1}{\sqrt{2}}$

Hence, A = 45°

(ii) $\sin 3A - 1 = 0$

$\Rightarrow \sin 3A = 1$

We know $\sin\ 90^{\circ} = 1$

$\therefore 3A = 90^{\circ}$

Hence, $A = 30^{\circ}$

(iii) $4\sin^2 A - 3 = 0$

$\Rightarrow \sin^2 A = \frac{3}{4}$

$\Rightarrow \sin A = \frac{\sqrt{3}}{2}$

We know $\sin\ 60^{\circ} = \frac{\sqrt{3}}{2}$

Hence, A = 60°

(iv) $\cos^2 A - \cos A = 0$

$\Rightarrow \cos A(\cos A - 1) = 0$

$\Rightarrow \cos A = 0$ Or $\cos A = 1$

We know $\cos\ 90^{\circ} = 0$ and $\cos 0° = 1$

Hence, A = 90° or 0°

(v) $2\cos^2 A + \cos A - 1 = 0$

$\Rightarrow 2\cos^2 A + 2\cos A - \cos A - 1 = 0$

$\Rightarrow 2\cos A(\cos A + 1) - 1(\cos A + 1) = 0$

$\Rightarrow (2\cos A - 1)(\cos A + 1) = 0$

$\Rightarrow \cos A = \frac{1}{2}$ or $\cos A = -1$

We know $\cos\ 60^{\circ} = \frac{1}{2}$

We also know that for no value of $A(0^{\circ} \le A \le 90^{\circ})$, cos A = -1.

Hence, $A = 60^{\circ}$

Question 13:

If 0° < A < 90°; Find A, if:

(i) $\frac{\cos A}{1-\sin A}+\frac{\cos A}{1+\sin A}=4$

(ii) $\frac{\sin A}{\sec A-1}+\frac{\sin A}{\sec A+1}=2$

Solution 13:

(i) $\frac{\cos A}{1-\sin A}+\frac{\cos A}{1+\sin A}=4$

$\Rightarrow \frac{\cos A+\cos A\sin A+\cos A-\sin A\cos A}{(1-\sin A)(1+\sin A)}=4$

$\Rightarrow \frac{2\cos A}{1-\sin^2 A}=4$

$\Rightarrow \frac{2\cos A}{\cos^2 A}=4$

$\Rightarrow \frac{1}{\cos A}=2$

$\Rightarrow \cos A=\frac{1}{2}$

We know $\cos 60^\circ=\frac{1}{2}$

Hence, $A=60^\circ$

(ii) $\frac{\sin A}{\sec A-1}+\frac{\sin A}{\sec A+1}=2$

$\Rightarrow \frac{\sin A\sec A+\sin A+\sec A\sin A-\sin A}{(\sec A-1)(\sec A+1)}=2$

$\Rightarrow \frac{2\sin A\sec A}{\sec^2 A-1}=2$

$\Rightarrow \frac{\sin A\sec A}{\tan^2 A}=1$

$\Rightarrow \frac{\cos A}{\sin A}=1$

$\Rightarrow \cot A=1$

We know $\cot 45^\circ=1$

Hence, $A=45^\circ$

Question 14:

Prove that:

$$\left(\operatorname{cosec} A - \sin A\right)\left(\sec A - \cos A\right)\sec^2 A = \tan A$$

Solution 14:

L.H.S,

$$\left(\operatorname{cosec} A - \sin A\right)\left(\sec A - \cos A\right)\sec^2 A$$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\sec^2 A$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)\sec^2 A$$

$$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\sec^2 A$$

$$= \frac{\sin A}{\cos A} = \tan A = \text{R.H.S}$$