Probability

Exercise 25(A)

Question 1.

A coin is tossed once. Find the probability of:
(i) getting a tail
(ii) not getting a tail

Solution:

Sample Space = {H, T}
n(s) = 2
(i) A = Event of getting a tail = {T}
n(A)= 1

Therefore, the probability of getting a tail = $\frac{n(A)}{n(s)} = \frac{1}{2}$

(ii) Not getting a tail means getting a heads.
Event of getting a heads = {H}
n(A) = 1

Therefore, the probability of getting a tail = $\frac{n(A)}{n(s)} = \frac{1}{2}$

Question 2.

A bag contains 3 white, 5 black and 2 red balls, all of the same shape and size. A ball is drawn from the bag without looking into it, find the probability that the ball drawn is:
(i) a black ball.
(ii) a red ball.
(iii) a white ball.
(iv) not a red ball.
(v) not a black ball.

Solution:

Total number of balls = 3 + 5 + 2 = 10

Total number of events = P(n) = 10

(i) There are 5 black balls

Favourable number of events = P(A) = 5

Hence, P(getting a black ball) = $\frac{P(A)}{P(n)} = \frac{5}{10} = \frac{1}{2}$

(ii) There are 2 red balls

Favourable number of events = P(A) = 2

Hence, P(getting a red ball) = $\frac{P(A)}{P(n)} = \frac{2}{10} = \frac{1}{5}$

(iii) There are 3 white balls

Favourable number of events = P(A) = 3

Hence, P(getting a white ball) = $\frac{P(A)}{P(n)} = \frac{3}{10}$

(iv) There are 3 + 5 = 8 balls which are not red

Favourable number of events = P(A) = 8

Hence, P(not getting a red ball) = $\frac{P(A)}{P(n)} = \frac{8}{10} = \frac{4}{5}$

(v) There are 3 + 2 = 5 balls which are not black

Favourable number of events = P(A) = 5

Hence, P(not getting a black ball) = $\frac{P(A)}{P(n)} = \frac{5}{10} = \frac{1}{2}$

Question 3.

In a single throw of a die, find the probability of getting a number:

(i) greater than 4.

(ii) less than or equal to 4.

(iii) not greater than 4.

Solution:

Sample space = {1, 2, 3, 4, 5, 6}

n(s) = 6

(i) E = event of getting a number greater than 4 = {5, 6}

n (E) = 2

Probability of a number greater than 4 = $\frac{n(E)}{n(s)} = \frac{2}{6} = \frac{1}{3}$

(ii) E=event of getting a number less than or equal to 4={1, 2, 3, 4}

n(E) = 4

Probability of a number less than or equal to 4 = $\frac{n(E)}{n(s)} = \frac{4}{6} = \frac{2}{3}$

(iii) E=event of getting a number not greater than 4={1, 2, 3, 4}

n(E) = 4

Probability of a number not greater than 4 = $\frac{n(E)}{n(s)} = \frac{4}{6} = \frac{2}{3}$

Question 4.
In a single throw of a die, find the probability that the number:
(i) will be an even number.
(ii) will not be an even number.
(iii) will be an odd number.
Solution:
Sample space = {1, 2, 3, 4, 5, 6}
n(s) = 6
(i) E = event of getting an even number = {2, 4, 6}
n(E) = 3
Probability of a getting an even number = $\frac{n(E)}{n(s)} = \frac{3}{6} = \frac{1}{2}$
(ii) E = event of not getting an even number = {1, 3, 5}
n(E) = 3
Probability of a not getting an even number = $\frac{n(E)}{n(s)} = \frac{3}{6} = \frac{1}{2}$
(iii) E = event of getting an odd number = {1, 3, 5}
n(E) = 3
Probability of a getting an odd number = $\frac{n(E)}{n(s)} = \frac{3}{6} = \frac{1}{2}$

Question 5.
From a well shuffled deck of 52 cards, one card is drawn. Find the probability that the card drawn will:
(i) be a black card.
(ii) not be a red card.
(iii) be a red card.
(iv) be a face card.
(v) be a face card of red colour.
Solution:
Total number of cards = 52
Total number of outcomes = P(s) = 52
There are 13 cards of each type. The cards of heart and diamond are red in colour. Spade and diamond are black. So, there are 26 red cards and 26 black cards.
(i) Number of black cards in a deck = 26
P(E) = favourable outcomes for the event of drawing a black card = 26

Probability of drawing a black card = $\frac{P(E)}{P(s)} = \frac{26}{52} = \frac{1}{2}$

(ii) Number of red cards in a deck = 26
Therefore, number of non-red cards = 52 -26 = 26
P(E) = favourable outcomes for the event of not drawing a red card = 26

Probability of not drawing a red card = $\frac{P(E)}{P(s)} = \frac{26}{52} = \frac{1}{2}$

(iii) Number of red cards in a deck = 26
P(E) = favourable outcomes for the event of drawing a red card = 26

Probability of drawing a red card = $\frac{P(E)}{P(s)} = \frac{26}{52} = \frac{1}{2}$

(iv) There are 52 cards in a deck of cards, and 12 of these cards are face cards (4 kings, 4 queens, and 4 jacks).
P(E) = 12

Probability of drawing a face card = $\frac{P(E)}{P(s)} = \frac{12}{52} = \frac{3}{13}$

(v) There are 26 red cards in a deck, and 6 of these cards are face cards (2 kings, 2 queens, and 2 jacks).
P(E) = 6

Probability of drawing a red face card = $\frac{P(E)}{P(s)} = \frac{6}{52} = \frac{3}{26}$

Question 6.
(i) If A and B are two complementary events then what is the relation between P(A) and P(B)?
(ii) If the probability of happening an event A is 0.46. What will be the probability of not happening of the event A?
Solution:
(i) Two complementary events, taken together, include all the outcomes for an experiment and the sum of the probabilities of all outcomes is 1.
P(A) + P(B) = 1
(ii) P(A) = 0.46
Let P(B) be the probability of not happening of event A
We know,
P(A) + P(B) = 1
P(B) = 1 – P(A)
P(B) = 1 – 0.46
P(B) = 0.54
Hence the probability of not happening of event A is 0.54

Question 7.
In a T.T. match between Geeta and Ritu, the probability of the winning of Ritu is 0.73. Find the probability of:
(i) winning of Geeta
(ii) not winning of Ritu
Solution:

(i) Winning of Geeta is a complementary event to winning of Ritu
Therefore,
P(winning of Ritu) + P(winning of Geeta) = 1
P(winning of Geeta) = 1 – P(winning of Ritu)
P(winning of Geeta) = 1 – 0.73
P(winning of Geeta) = 0.27
(ii) Not winning of Ritu is a complementary event to winning of Ritu
Therefore,
P(winning of Ritu) + P(not winning of Ritu) = 1
P(not winning of Ritu) = 1 – P(winning of Ritu)
P(not winning of Ritu) = 1 – 0.73
P(not winning of Ritu) = 0.27

Question 8.
In a race between Mahesh and John, the probability that John will lose the race is 0.54.
Find the probability of:
(i) winning of Mahesh
(ii) winning of John
Solution:
(i) But if John looses, Mahesh wins
Hence, probability of John losing the race = Probability of Mahesh winning the race since it is a race between these two only
Therefore, P(winning of Mahesh) = 0.54
(ii) P(winning of Mahesh) + P(winning of John) = 1
0.54 + P(winning of John) = 1
P(winning of John) = 1 – 0.54
P(winning of John) = 0.46

Question 9.
(i) Write the probability of a sure event
(ii) Write the probability of an event when impossible
(iii) For an event E, write a relation representing the range of values of P(E)
Solution:

(i) The probability of a sure event is 1 i.e. P(S) = 1 where 'S' is the sure event.

Proof: In a sure event n(E) = n(S)

[Since Number of elements in Event 'E' will be equal to the number of element in sample-space.]

By definition of Probability :

$P(S) = n(E) / n(S) = 1$

$P(S) = 1$

(ii) The probability of an impossible event is '0' i.e. P (S) = 0

Proof: Since E has no element, n(E) = 0

From definition of Probability:

$P(S) = n(E) / n(S) = 0 / n(S)$

$P(S) = 0$

(iii) The probability of an event lies between '0' and '1'.

i.e. $0 \leq P(E) \leq 1$.

Proof: Let 'S' be the sample space and 'E' be the event.

Then

$0 \leq n(E) \leq n(S)$

$0 / n(S) \leq n(E) / n(S) \leq n(S) / n(S)$

or $0 \leq P(E) \leq 1$

The number of elements in ‘E’ can’t be less than ‘0’ i.e. negative and greater than the number of elements in S.

Question 10.
In a single throw of die, find the probability of getting:
(i) 5
(ii) 8
(iii) a number less than 8
(iv) a prime number
Solution:

Sample space = {1, 2, 3, 4, 5, 6}

n(S) = 6

(i) E = event of getting a 5 on a throw of die = {5}

n(E) = 1

Probability of getting a 5 = $P(S) = \frac{n(E)}{n(S)} = \frac{1}{6}$

(ii) There are only six possible outcomes in a single throw of a die. If we want to find probability of 8 to come up, then in that case number of possible or favourable outcome is 0 (zero)

n(E) = 0

Probability of getting a 8 = $P(S) = \frac{n(E)}{n(S)} = \frac{0}{6} = 0$

(iii) If we consider to find the probability of number less than 8, then all six cases are favourable

n(E) = 6

Probability of getting a number less than8 = $P(S) = \frac{n(E)}{n(S)} = \frac{6}{6} = 1$

(iv) E = event of getting a prime number = {2, 3, 5,}

n(E) = 3

Probability of getting a prime number = $P(S) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

Question 11.

A die is thrown once. Find the probability of getting:
(i) an even number
(ii) a number between 3 and 8
(iii) an even number or a multiple of 3

Solution:

Sample space = {1, 2, 3, 4, 5, 6}

n(S) = 6

(i) E = the possible even numbers = {2, 4, 6}

n(E) = 3

Probability of getting an even number = $P(S) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(ii) E = the possible even numbers between 3 and 8 = {4, 5, 6}

n(E) = 3

Probability of getting an even number between 3 and 8 =

$P(S) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(iii) E = the event of getting an even number or a multiple of 3 = {2, 3, 4, 6}

n(E) = 4

Probability of getting an even number or a multiple of 3 = $P(S) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

Question 12.

Which of the following cannot be the probability of an event?

(i) 3/5

(ii) 2.7

(iii) 43%

(iv) -0.6

(v) -3.2

(vi) 0.35

Solution:

The probability of an event lies between '0' and '1' i.e. $0 \leq P(E) \leq 1$.

(i) $\frac{3}{5} = 0.6$

$\because 0 \leq 0.6 \leq 1$

Hence, it can be the probability of an event.

(ii) 2.7

$\because 0 \leq 1 \leq 2.7$

Hence, it cannot be the probability of an event.

(iii) $43\% = \frac{43}{100} = 0.43$

$0 \leq 0.43 \leq 1$

Hence, it can be the probability of an event.

(iv) -0.6

$-0.6 \leq 0 \leq 1$

Hence, it cannot be the probability of an event.

(v) -3.2

$-3.2 \leq 0 \leq 1$

Hence, it cannot be the probability of an event.

(vi) 0.35

$0 \leq 0.35 \leq 1$

Hence, it can be the probability of an event.

Question 13.

A bag contains six identical black balls. A child withdraws one ball from the bag without looking into it. What is the probability that he takes out:

(i) a white ball

(ii) a black ball

Solution:

Possible number of outcomes = 6 = number of balls in the bag

n(S) = 6

(i) E = event of drawing a white ball = number of white balls in the bag = 0

n(E) = 0

Probability of drawing a white ball = $P(S) = \frac{n(E)}{n(S)} = \frac{0}{6} = 0$

(ii) E = event of drawing a black ball = number of black balls in the bag = 6

n(E) = 6

Probability of drawing a black ball = $P(S) = \frac{n(E)}{n(S)} = \frac{6}{6} = 1$

Question 14.

A single letter is selected at random from the word ‘Probability’. Find the probability that it is a vowel.

Solution:

Possible outcomes = S = {'P', 'r', 'o', 'b', 'a', 'b', 'i', 'l', 'i', 't', 'y'}

n(S) = 11

Event of selection of vowels = E = {'o', 'a', 'i', 'i'}

n(E) = 4

Probability of selection of a vowel = $P(S) = \frac{n(E)}{n(S)} = \frac{4}{11}$

Question 15.

Ramesh chooses a date at random in January for a party.

January					
Mon		6	13	20	27
Tue		7	14	21	28
Wed	1	8	15	22	29
Thurs	2	9	16	23	30
Fri	3	10	17	24	31
Sat	4	11	18	25	
Sun	5	12	19	26	

Find the probability that he chooses:

(i) a Wednesday.

(ii) a Friday.

(iii) a Tuesday or a Saturday.

Solution:

Number of possible outcomes = number of days in the month =31

n(S) = 31

(i) E = event of selection of a Wednesday = {1, 8, 15, 22, 29}

n(E) = 5

Probability of selection of a Wednesday = $P(S) = \frac{n(E)}{n(S)} = \frac{5}{31}$

(ii) E = event of selection of a Friday = {3, 10, 17, 24, 31}

n(E) = 5

Probability of selection of a Friday = $P(S) = \frac{n(E)}{n(S)} = \frac{5}{31}$

(iii) E = event of selection of a Tuesday or a Saturday = {4, 7, 11, 14, 18, 21, 25, 28}

n(E) = 8

Probability of selection of a Tuesday or a Saturday = $P(S) = \frac{n(E)}{n(S)} = \frac{8}{31}$

Exercise 25(B)

Question 1.

Nine cards (identical in all respects) are numbered 2 to 10. A card is selected from them at random. Find the probability that the card selected will be:
(i) an even number
(ii) a multiple of 3
(iii) an even number and a multiple of 3
(iv) an even number or a multiple of 3

Solution:

There are 9 cards from which one card is drawn.

Total number of elementary events = n(S) = 9

(i) From numbers 2 to 10, there are 5 even numbers i.e. 2, 4, 6, 8, 10

Favorable number of events = n(E) = 5

Probability of selecting a card with an even number = $\frac{n(E)}{n(S)} = \frac{5}{9}$

(ii) From numbers 2 to 10, there are 3 numbers which are multiples of 3 i.e. 3, 6, 9

Favorable number of events = n(E) = 3

Probability of selecting a card with a multiple of 3=

$$\frac{n(E)}{n(S)} = \frac{3}{9} = \frac{1}{3}$$

(iii) From numbers 2 to 10, there is one number which is an even number as well as multiple of 3 i.e. 6

Favorable number of events = n(E) = 1

Probability of selecting a card with a number which is an even number as well as multiple of 3 = $\frac{n(E)}{n(S)} = \frac{1}{9}$

(iv) From numbers 2 to 10, there are 7 numbers which are even numbers or a multiple of 3 i.e. 2, 3, 4, 6, 8, 9, 10
Favorable number of events = n(E) = 7

Probability of selecting a card with a number which is an even number or a multiple of 3 = $\frac{n(E)}{n(S)} = \frac{7}{9}$

Question 2.

Hundred identical cards are numbered from 1 to 100. The cards The cards are well

shuffled and then a card is drawn. Find the probability that the number on card drawn is:
(i) a multiple of 5
(ii) a multiple of 6
(iii) between 40 and 60
(iv) greater than 85
(v) less than 48

Solution:

There are 100 cards from which one card is drawn.

Total number of elementary events = n(S) = 100

(i) From numbers 1 to 100, there are 20 numbers which are multiple of 5 i.e. {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100} Favorable number of events = n(E) = 20

Probability of selecting a card with a multiple of 5 =

$$\frac{n(E)}{n(S)} = \frac{20}{100} = \frac{1}{5}$$

(ii) From numbers 1 to 100, there are 16 numbers which are multiple of 6 i.e. {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96} Favorable number of events = n(E) = 16

Probability of selecting a card with a multiple of 6 =

$$\frac{n(E)}{n(S)} = \frac{16}{100} = \frac{4}{25}$$

(iii) From numbers 1 to 100, there are 19 numbers which are between 40 and 60 i.e. {41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59} Favorable number of events = n(E) = 19

Probability of selecting a card between 40 and 60=

$$\frac{n(E)}{n(S)} = \frac{19}{100}$$

(iv) From numbers 1 to 100, there are 15 numbers which are greater than 85 i.e. {86, 87, ..., 98, 99, 100} Favorable number of events = n(E) = 15

Probability of selecting a card with a number greater than 85 =

$$\frac{n(E)}{n(S)} = \frac{15}{100} = \frac{3}{20}$$

(v) From numbers 1 to 100, there are 47 numbers which are less than 48 i.e. {1, 2, ..., 46, 47} Favorable number of events = n(E) = 47

Probability of selecting a card with a number less than 48 =

$$\frac{n(E)}{n(S)} = \frac{47}{100}$$

Question 3.

From 25 identical cards, numbered 1, 2, 3, 4, 5, ……, 24, 25: one card is drawn at random. Find the probability that the number on the card drawn is a multiple of:
(i) 3
(ii) 5
(iii) 3 and 5
(iv) 3 or 5

Solution:

There are 25 cards from which one card is drawn.

Total number of elementary events = n(S) = 25

(i) From numbers 1 to 25, there are 8 numbers which are multiple of 3 i.e. {3, 6, 9, 12, 15, 18, 21, 24} Favorable number of events = n(E) = 8

Probability of selecting a card with a multiple of 3 =

$$\frac{n(E)}{n(S)} = \frac{8}{25}$$

(ii) From numbers 1 to 25, there are 5 numbers which are multiple of 5 i.e. {5, 10, 15, 20, 25} Favorable number of events = n(E) = 5

Probability of selecting a card with a multiple of 5 =

$$\frac{n(E)}{n(S)} = \frac{5}{25} = \frac{1}{5}$$

(iii) From numbers 1 to 25, there is only one number which is multiple of 3 and 5 i.e. {15} Favorable number of events = n(E) = 1

Probability of selecting a card with a multiple of 3 and 5 =

$$\frac{n(E)}{n(S)} = \frac{1}{25}$$

(iv) From numbers 1 to 25, there are 12 numbers which are multiple of 3 or 5 i.e. {3, 5, 6, 9, 10, 12, 15, 18, 20,21, 24, 25} Favorable number of events = n(E) = 12

Probability of selecting a card with a multiple of 3 or 5 =

$$\frac{n(E)}{n(S)} = \frac{12}{25}$$

Question 4.

A die is thrown once. Find the probability of getting a number:

(i) less than 3

(ii) greater than or equal to 4

(iii) less than 8

(iv) greater than 6

Solution:

In throwing a dice, total possible outcomes = {1, 2, 3, 4, 5, 6}

n(S) = 6

(i) On a dice, numbers less than 3 = {1, 2}

n(E) = 2

Probability of getting a number less than 3 = $\frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

(ii) On a dice, numbers greater than or equal to 4 = {4, 5, 6}

n(E) = 3

Probability of getting a number greater than or equal to 4 =

$\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(iii) On a dice, numbers less than 8 = {1, 2, 3, 4, 5, 6}

n(E) = 6

Probability of getting a number less than 8 = $\frac{n(E)}{n(S)} = \frac{6}{6} = 1$

(iv) On a dice, numbers greater than 6 = 0

n(E) = 0

Probability of getting a number greater than 6 = $\frac{n(E)}{n(S)} = \frac{0}{6} = 0$

Question 5.

A book contains 85 pages. A page is chosen at random. What is the probability that the sum of the digits on the page is 8?

Solution:

Number of pages in the book = 85

Number of possible outcomes = n(S) = 85

Out of 85 pages, pages that sum up to 8 = {8, 17, 26, 35, 44, 53, 62, 71, 80}

pages that sum up to 8 = n(E) = 9

Probability of choosing a page with the sum of digits on the page equals 8 = $\frac{n(E)}{n(S)} = \frac{9}{85}$

Question 6.
A pair of dice is thrown. Find the probability of getting a sum of 10 or more, if 5 appears on the first die.
Solution:

In throwing a dice, total possible outcomes = {1, 2, 3, 4, 5, 6}

n(S) = 6

For two dice, n(S) = 6 × 6 = 36

Favorable cases where the sum is 10 or more with 5 on 1^{st} die = {(5, 5), (5, 6)}

Event of getting the sum is 10 or more with 5 on 1^{st} die =n(E) = 2

Probability of getting a sum of 10 or more with 5 on 1^{st} die =

$$\frac{n(E)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

Question 7.
If two coins are tossed once, what is the probability of getting:
(i) both heads.
(ii) at least one head.
(iii) both heads or both tails.
Solution:

When two coins are tossed together possible number of outcomes = {HH, TH, HT, TT}

n(S) = 4

(i) E = event of getting both heads = {HH}

n(E) = 1

Probability of getting both heads = $\frac{n(E)}{n(S)} = \frac{1}{4}$

(ii) E = event of getting at least one head = {HH, TH, HT}

n(E) = 3

Probability of getting at least one head = $\frac{n(E)}{n(S)} = \frac{3}{4}$

(iii) E = event of getting both heads or both tails = {HH, TT}

n(E) = 2

Probability of getting both heads or both tails = $\frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Question 8.
Two dice are rolled together. Find the probability of getting:
(i) a total of at least 10.
(ii) a multiple of 2 on one die and an odd number on the other die.
Solution:
In throwing a dice, total possible outcomes = {1, 2, 3, 4, 5, 6}

n(S) = 6

For two dice, n(S) = 6 × 6 = 36

(i) E = event of getting a total of at least 10 = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

n(E) = 6

Probability of getting a total of at least 10 = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) E = event of getting a multiple of 2 on one die and an odd number on the other = {(2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5), (1, 2), (3, 2), (5, 2), (1, 4), (3, 4), (5, 4), (1, 6), (3, 6), (5, 6)}

n(E) = 18

Probability of getting a multiple of 2 on one die and an odd number on the other = $\frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

Question 9.
A card is drawn from a well shuffled pack of 52 cards. Find the probability that the card drawn is:
(i) a spade(v) Jack or queen
(ii) a red card(vi) ace and king
(iii) a face card(vii) a red and a king
(iv) 5 of heart or diamond(viii) a red or a king
Solution:

Number of possible outcomes when card is drawn from pack of 52 cards = 52

n(S) = 52

(i) Number of spade cards = 13 = E = event of drawing a spade

n(E) = 13

Probability of drawing a spade = $\frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

(ii) Number of red cards(hearts + diamonds) = 26 = E = event of drawing a red card

n(E) = 26

Probability of drawing a red card = $\frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

(iii) Number of face cards(4 kings + 4 queens + 4 Jacks)= 12 = E = Event of drawing a face card

n(E) = 12

Probability of drawing a face card = $\frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

(iv) E = event of drawing a 5 of heart or of diamond = {5H, 5D}

n(E) = 2

Probability of drawing a 5 of heart or of diamond =

$$\frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(v) E = event of drawing a jack or a queen = {JH, JS, JD, JC, QH, QS, QD, QC}

n(E) = 8

Probability of drawing a jack or a queen = $\frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$

(vi) A card cannot be both an ace as well as a king.

E =event of drawing an ace and a king = 0

n(E) = 0

Probability of drawing an ace and a king = $\frac{n(E)}{n(S)} = \frac{0}{52} = 0$

(vii) E = event of drawing a red and a king = {KH, KD }

n(E) = 2

Probability of drawing a red and a king = $\frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

(viii) E = event of drawing a red or a king = 26 red cards (13 h + 13D) + 2 black kings[since 26 red cards contain 2 red ki

n(E) = 28

Probability of drawing a red or a king = $\frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}$

Question 10.
A bag contains 16 colored balls. Six are green, 7 are red and 3 are white. A ball is chosen, without looking into the bag. Find the probability that the ball chosen is:
(i) red(v) green or red
(ii) not red(vi) white or green
(iii) white(vii) green or red or white
(iv) not white
Solution:

Balls in the bag = 16 = Number of balls that could be drawn

n(S) = 16

(i) E = Event of drawing a red ball = number of red balls = 7

n(E) = 7

Probability of drawing a red ball = $\frac{n(E)}{n(S)} = \frac{7}{16}$

(ii) Not a red ball = 16 - number of red balls = 16 -7 = 9 = E

n(E) = 9

Probability of not drawing a red ball = $\frac{n(E)}{n(S)} = \frac{9}{16}$

(iii) E = Event of drawing a white ball = number of white balls = 3

n(E) = 3

Probability of drawing a white ball = $\frac{n(E)}{n(S)} = \frac{3}{16}$

(iv) Not a white ball = 16 - number of white balls = 16-3 = 13=E

n(E) = 13

Probability of not drawing a white ball = $\frac{n(E)}{n(S)} = \frac{13}{16}$

(v) E = Event of drawing a green or a red ball = number of green balls + number of red balls = 6 + 7 = 13

n(E) = 13

Probability of drawing a green or a red ball = $\frac{n(E)}{n(S)} = \frac{13}{16}$

(vi) E = Event of drawing a green or a white ball = number of green balls + number of white balls = 6 + 3 = 9

n(E) = 9

Probability of drawing a green or a white ball = $\frac{n(E)}{n(S)} = \frac{9}{16}$

(vii) E = Event of drawing a green or a white or a red ball = number of green balls + number of white balls + number of re

n(E) = 16

Probability of drawing a green or a white or a red ball =

$$\frac{n(E)}{n(S)} = \frac{16}{16} = 1$$

Question 11.

A ball is drawn at random from a box containing 12 white, 16 red and 20 green balls.
Determine the probability that the ball drawn is:
(i) white(iii) not green
(ii) red(iv) red or white

Solution:

Total number of balls in the box = 48

Total possible outcomes on drawing a ball = 48

n(S) = 48

(i) Event of drawing a white ball = E = 12

n(E) = 12

Probability of drawing a white ball = $\frac{n(E)}{n(S)} = \frac{12}{48} = \frac{1}{4}$

(ii) Event of drawing a red ball = E = 16

n(E) = 16

Probability of drawing a red ball = $\frac{n(E)}{n(S)} = \frac{16}{48} = \frac{1}{3}$

(iii) Event of drawing a green ball = E = 2
n(E) = 20

Probability of drawing a green ball = $\frac{n(E)}{n(S)} = \frac{20}{48} = \frac{5}{12}$

Probability of not drawing a green ball = 1 - $\frac{5}{12}$

$= \frac{12-5}{12} = \frac{7}{12}$

(iv) red or a white ball = 12 + 16 = 28 balls

Event of drawing a red or white ball = E = 28

n(E) = 28

Probability of not drawing a green ball = $\frac{n(E)}{n(S)} = \frac{28}{48} = \frac{7}{12}$

Question 12.
A card is drawn from a pack of 52 cards. Find the probability that the card drawn is:
(i) a red card
(ii) a black card
(iii) a spade
(iv) an ace
(v) a black ace
(vi) ace of diamonds
(vii) not a club
(viii) a queen or a jack
Solution:
Number of possible outcomes when card is drawn from pack of 52 cards = 52

n(S) = 52

(i) Number of red cards(hearts + diamonds) = 26 = E

n(E) = 26

Probability of drawing a red card = $\frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

(ii) Number of black cards(spade + clubs) = 26 = E

n(E) = 26

Probability of drawing a black card = $\frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

(iii) Number of spade cards = 13 = E = event of drawing a spade

n(E) = 13

Probability of drawing a spade = $\frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

(iv) Number of ace cards = 4 = E = event of drawing an ace

n(E) = 4

Probability of drawing an ace = $\frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

(v) Number of black ace cards = 2 = E = event of drawing an ace

n(E) = 2

Probability of drawing a black ace = $\frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

(vi) There is only one ace of diamonds.

E = event of drawing an ace of diamonds

n(E) = 1

Probability of drawing an ace of diamonds = $\frac{n(E)}{n(S)} = \frac{1}{52}$

(vii) Number of club cards = 13 = E = event of drawing a club card

n(E) = 13

Probability of drawing a club card = $\frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

Probability of not drawing a club card = 1 - $\frac{1}{4}$

$= \frac{4-1}{4} = \frac{3}{4}$

(viii) E = event of drawing a jack or a queen = {JH, JS, JD, JC, QH, QS, QD, QC}

n(E) = 8

Probability of drawing a jack or a queen = $\frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$

Question 13.

Thirty identical cards are marked with numbers 1 to 30. If one card is drawn at random, find the probability that it is:
(i) a multiple of 4 or 6
(ii) a multiple of 3 and 5
(iii) a multiple of 3 or 5

Solution:

There are 30 cards from which one card is drawn.

Total number of elementary events = n(S) = 30

(i) From numbers 1 to 30, there are 10 numbers which are multiple of 4 or 6 i.e. {4, 6, 8, 12, 16, 18, 20, 24, 28, 30} Favorable number of events = n(E) = 10

Probability of selecting a card with a multiple of 4 or 6 =

$$\frac{n(E)}{n(S)} = \frac{10}{30} = \frac{1}{3}$$

(ii) From numbers 1 to 30, there are 2 numbers which are multiple of 3 and 5 i.e. {15, 30} Favorable number of events = n(E) = 2

Probability of selecting a card with a multiple of 3 and 5 =

$$\frac{n(E)}{n(S)} = \frac{2}{30} = \frac{1}{15}$$

(iii) From numbers 1 to 30, there are 14 numbers which is multiple of 3 or 5 i.e. {3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25, 27, 30} Favorable number of events = n(E) = 14

Probability of selecting a card with a multiple of 3 or 5 =

$$\frac{n(E)}{n(S)} = \frac{14}{30} = \frac{7}{15}$$

Question 14.

In a single throw of two dice, find the probability of:
(i) a doublet
(ii) a number less than 3 on each dice
(iii) an odd number as a sum
(iv) a total of at most 10
(v) an odd number on one dice and a number less than or equal to 4 on the other dice.

Solution:

The number of possible outcomes is 6 × 6 = 36. We write them as given below :

1,11,21,31,41,51,6

2,12,22,32,42,52,6

3,13,23,33,43,53,6

4,14,24,34,44,54,6

5,15,25,35,45,55,6

6,16,26,36,46,56,6

n(S) = 36

(i) E = Event of getting a doublet = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

n(E) = 6

Probability of getting a doublet = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) E = Event of getting a number less than 3 on each dice

= {(1, 1), (1, 2), (2, 1), (2, 2)}

n(E) = 4

Probability of getting a number less than 3 on each dice = $\frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

(iii) E = Event of getting an odd number as a sum = {(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4
(6, 3), (6, 5)}

n(E) = 18

Probability of getting an odd number as sum = $\frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

(iv) E = Event of getting a total of at most 10 =
{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)

(3,1), (3,2), (3,3), (3,4), (3,5) ,(3,6)

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

(5,1), (5,2),(5,3), (5,4), (5,5)

(6,1), (6,2), (6,3), (6,4)}

Therefore total number of favorable ways = 33 = n(E)

Probability of getting a total of at most 10 = $\frac{n(E)}{n(S)} = \frac{33}{36} = \frac{11}{12}$

(v) E = Event of getting an odd number on dice 1 and a number less than or equal to 4 on dice 2 =

{(1,1), (1,2), (1,3), (1,4), (1, 5)

(2, 1), (2, 3), (2, 5)

(3,1), (3,2), (3,3), (3,4), (3, 5)

(4, 1), (4, 3), (4, 5)

(5,1), (5,2),(5,3), (5,4)}

Therefore total number of favorable ways = 20 = n(E)

Probability of getting an odd number on dice 1 and a number less than or equal to 4 on dice 2 = $\frac{n(E)}{n(S)} = \frac{20}{36} = \frac{5}{9}$

Exercise 25(C)

Question 1.

A bag contains 3 red balls, 4 blue balls and 1 yellow ball, all the balls being identical in shape and size. If a ball is taken out of the bag without looking into it; find the probability that the ball is:
(i) yellow
(ii) red
(iii) not yellow
(iv) neither yellow nor red

Solution:

Total number of balls in the bag = 3+4+1 = 8 balls

Number of possible outcomes = 8 = n(S)

(i) Event of drawing a yellow ball = {Y}

n(E) = 1

Probability of drawing a yellow ball = $\frac{n(E)}{n(S)} = \frac{1}{8}$

(ii) Event of drawing a red ball = {R, R, R}

n(E) = 3

Probability of drawing a red ball = $\frac{n(E)}{n(S)} = \frac{3}{8}$

(iii) Probability of not drawing a yellow ball = 1 - Probability of drawing a yellow ball

Probability of not drawing a yellow ball = 1 - $\frac{1}{8}$

$= \frac{8-1}{8} = \frac{7}{8}$

(iv) Neither yellow ball nor red ball means a blue ball

Event of not drawing a yellow or red ball = E = 4

n(E) = 4

Probability of not drawing a yellow or red ball = $\frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

Question 2.

A dice is thrown once. What is the probability of getting a number:
(i) greater than 2?
(ii) less than or equal to 2?

Solution:

Number of possible outcomes when dice is thrown = {1,2,3,4,5,6}

n(S) = 6

(i) Event of getting a number greater than 2 = E = {3, 4, 5, 6}

n(E) = 4

Probability of getting a number greater than 2 = $\frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

(ii) Event of getting a number less than or equal to 2 = E = {1, 2}

n(E) = 2

Probability of getting a number less than or equal to 2 =

$$\frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Question 3.

From a well shuffled deck of 52 cards, one card is drawn. Find the probability that the card drawn is:

(i) a face card

(ii) not a face card

(iii) a queen of black card

(iv) a card with number 5 or 6

(v) a card with number less than 8

(vi) a card with number between 2 and 9

Solution:

Total number of possible outcomes = 52

n(S) = 52

(i) No. of face cards in a deck of 52 cards = 12 (4 kings, 4 queens and 4 jacks)

Event of drawing a face cards = E = (4 kings, 4 queens and 4 jacks)

n(E) = 12

Probability of drawing a face card = $\frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

(ii) Probability of not drawing a face card = 1 - probability of drawing a face card

Probability of not drawing a face card = $1 - \frac{3}{13}$

$= \frac{13-3}{13} = \frac{10}{13}$

(iii) Event of drawing a queen of black color ={ Q(spade), Q(club)} = E

n(E) = 2

Probability of drawing a queen of black color =

$\frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

(iv) Event of drawing a card with number 5 or 6 = E = {5H, 5D, 5S, 5C, 6H, 6D, 6S, 6C}

n(E) = 8

Probability of drawing a card with number 5 or 6 =

$\frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$

(v) Numbers less than 8 = { 2, 3, 4, 5, 6, 7}

Event of drawing a card with number less than 8 = E = {6H cards, 6D cards, 6S cards, 6C cards}

n(E) = 24

Probability of drawing a card with number less than 8 =

$\frac{n(E)}{n(S)} = \frac{24}{52} = \frac{6}{13}$

(vi) Number between 2 and 9 = {3, 4, 5, 6, 7, 8}

Event of drawing a card with number between 2 and 9 = E = {6H cards, 6D cards, 6S cards, 6C cards}

n(E) = 24

Probability of drawing a card with number between 2 and 9 =

$\frac{n(E)}{n(S)} = \frac{24}{52} = \frac{6}{13}$

Question 4.

In a match between A and B:

(i) the probability of winning of A is 0.83. What is the probability of winning of B?

(ii) the probability of losing the match is 0.49 for B. What is the probability of winning of A?

Solution:

(i) Probability of winning of A + Probability of losing of A = 1

Probability of losing of A = Probability of winning of B

Therefore,

Probability of winning of A + Probability of winning of B = 1

0.83 + Probability of winning of B = 1

Probability of winning of B = 1 - 0.83 = 0.17

(ii) Probability of winning of B + Probability of losing of B = 1

Probability of losing of B = Probability of winning of A

Therefore,

Probability of winning of A = 0.49

Question 5.

A and B are friends. Ignoring the leap year, find the probability that both friends will have:

(i) different birthdays?

(ii) the same birthday?

Solution:

Out of the two friends, A's birthday can be any day of the year. Now, B's birthday can also be any day of 365 days in the year.

We assume that these 365 outcomes are equally likely.

(i) If A's birthday is different from B's, the number of favourable outcomes for his birthday is 365 - 1 = 364

So, P (A's birthday is different from B's birthday) = $\frac{364}{365}$

(ii) P(A and B have the same birthday)

= 1 - P (both have different birthdays)

= $1 - \frac{364}{365}$ [Using P(E') = 1 - P(E)]

= $\frac{1}{365}$

Question 6.

A man tosses two different coins (one of Rs 2 and another of Rs 5) simultaneously. What is the probability that he gets:

(i) at least one head?

(ii) at most one head?

Solution:

When two coins are tossed simultaneously, the possible outcomes are {(H, H), (H, T), (T, H), (T, T)}

n(S) = 4

(i) The outcomes favourable to the event E, 'at least one head' are

{(H, H), (H, T), (T, H)}

So, the number of outcomes favourable to E is 3 = n(E)

Therefore, P(E) = $\frac{n(E)}{n(S)} = \frac{3}{4}$

(ii) The outcomes favourable to the event E, 'at most one head'

are {(T, H), (H, T), (T, T)}

So, the number of outcomes favourable to E is 3 = n(E)

Therefore, P(E) = $\frac{n(E)}{n(S)} = \frac{3}{4}$

Question 7.

A box contains 7 red balls, 8 green balls and 5 white balls. A ball is drawn at random from the box. Find the probability that the ball is:
(i) white
(ii) neither red nor white.

Solution:

Total number of balls in the box = 7+8+5 = 20 balls

Total possible outcomes = 20 = n(S)

(i) Event of drawing a white ball = E = number of white balls

n(E) = 5

Probability of drawing a white ball = $\frac{n(E)}{n(S)} = \frac{5}{20} = \frac{1}{4}$

(ii) Neither red ball nor white ball = green ball

Event of not drawing a red or white ball = E = number of green ball

n(E) = 8

Probability of drawing a white ball = $\frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$

Question 8.
All the three face cards of spades are removed from a well shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting:
(i) a black face card
(ii) a queen
(iii) a black card
Solution:

Total number of cards = 52

3 face cards of spades are removed

Remaining cards = 52 - 3 = 49 = number of possible outcomes

n(S) = 49

(i) Number of black face cards left = 3 face cards of club

Event of drawing a black face card = E = 3

n(E) = 3

Probability of drawing a black face card = $\frac{n(E)}{n(S)} = \frac{3}{49}$

(ii) Number of queen cards left = 3

Event of drawing a black face card = E = 3

n(E) = 3

Probability of drawing a queen card = $\frac{n(E)}{n(S)} = \frac{3}{49}$

(iii) Number of black cards left = 23 cards (13 club + 10 spade)

Event of drawing a black card = E = 23

n(E) = 23

Probability of drawing a black card = $\frac{n(E)}{n(S)} = \frac{23}{49}$

Question 9.
In a musical chairs game, a person has been advised to stop playing the music at any time within 40 seconds after its start. What is the probability that the music will stop within the first 15 seconds?

Solution:

Total result = 0 sec to 40 sec

Total possible outcomes = 40

n(S) = 40

Favorable results = 0 sec to 15 sec

Favorable outcomes = 15

n(E) = 15

Probability that the music will stop in first 15 sec = $\frac{n(E)}{n(S)} = \frac{15}{40} = \frac{3}{8}$

Question 10.

In a bundle of 50 shirts, 44 are good, 4 have minor defects and 2 have major defects. What is the probability that:

(i) it is acceptable to a trader who accepts only a good shirt?

(ii) it is acceptable to a trader who rejects only a shirt with major defects?

Solution:

Total number of shirts = 50

Total number of elementary events = 50 = n(S)

(i) Since trader accepts only good shirts and number of good shirts = 44

Event of accepting good shirts = 44 = n(E)

Probability of accepting a good shirt = $\frac{n(E)}{n(S)} = \frac{44}{50} = \frac{22}{25}$

(ii) Since trader rejects shirts with major defects only and number of shirts with major defects = 2

Event of accepting shirts = 50 - 2 = 48 = n(E)

Probability of accepting shirts = $\frac{n(E)}{n(S)} = \frac{48}{50} = \frac{24}{25}$

Question 11.

Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is:

(i) 8

(ii) 13

(iii) less than or equal to 12

Solution:

The number of possible outcomes = $6 \times 6 = 36$.

(i) The outcomes favourable to the event 'the sum of the two numbers is 8' = E ={(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

The number of outcomes favourable to E = n(E) = 5.

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$

(ii) There is no outcome favourable to the event E = 'the sum of two numbers is 13'.

n(E) = 0

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{0}{36}$

(iii) All the outcomes are favourable to the event E = 'sum of two numbers $\le$ 12'.

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{36}{36} = 1$

Question 12.

Which of the following cannot be the probability of an event?

(i) 3/7

(ii) 0.82

(iii) 37%

(iv) -2.4

Solution:

We know that probability of an event E is $0 \le P(E) \le 1$

(i) Since $0 \le \frac{3}{7} \le 1$

Therefore, $\frac{3}{7}$ can be a probability of an event.

(ii) Since $0 \le 0.82 \le 1$

Therefore, 0.82 can be a probability of an event.

(iii)Since $0 \le 37\% = \left(\frac{37}{100}\right) \le 1$

Therefore, 37% can be a probability of an event.

(iv) Since $-2.4 < 0$

Therefore, -2.4 cannot be a probability of an event.

Question 13.
If P(E) = 0.59; find P(not E)
Solution:
P(E) + P(not E) = 1
0.59 + P(not E) = 1
P(not E) = 1 – 0.59 = 0.41

Question 14.
A bag contains a certain number of red balls. A ball is drawn. Find the probability that the ball drawn is:
(i) black
(ii) red
Solution:
Total possible outcomes = number of red balls.

(i) Number of favorable outcomes for black balls = 0

P(black ball) = 0

(ii) Number of favorable outcomes for red balls = number of red balls

P(red ball) =
$$\frac{\text{number of favorable outcomes}}{\text{total possible outcomes}} = \frac{\text{number or red balls}}{\text{number of red balls}} = 1$$

Question 15.
The probability that two boys do not have the same birthday is 0.897. What is the probability that the two boys have the same birthday?
Solution:
P(do not have the same birthday)+P(have same birthday) = 1
0.897 + P(have same birthday) = 1
P(have same birthday) = 1 – 0.897
P(have same birthday) = 0.103

Question 16.
A bag contains 10 red balls, 16 white balls and 8 green balls. A ball is drawn out of the bag at random. What is the probability that the ball drawn will be:
(i) not red?
(ii) neither red nor green?
(iii) white or green?

Solution:

Total number of possible outcomes = 10+16+8 = 34 balls

n(S) = 34

(i) Favorable outcomes for not a red ball = favorable outcomes for white or green ball

Number of favorable outcomes for white or green ball = 16+8=24 = n(E)

Probability for not drawing a red ball = $\frac{n(E)}{n(S)} = \frac{24}{34} = \frac{12}{17}$

(ii) Favorable outcomes for neither a red nor a green ball = favorable outcomes for white ball

Number of favorable outcomes for white ball = 16 = n(E)

Probability for not drawing a red or green ball = $\frac{n(E)}{n(S)} = \frac{16}{34} = \frac{8}{17}$

(iii) Number of favorable outcomes for white or green ball = 16+8=24 = n(E)

Probability for drawing a white or green ball = $\frac{n(E)}{n(S)} = \frac{24}{34} = \frac{12}{17}$

Question 17.

A bag contains twenty Rs 5 coins, fifty Rs 2 coins and thirty Re 1 coins. If it is equally likely that one of the coins will fall down when the bag is turned upside down, what is the probability that the coin:

(i) will be a Re 1 coin?

(ii) will not be a Rs 2 coin?

(iii) will neither be a Rs 5 coin nor be a Re 1 coin?

Solution:

Total number of coins = 20+50+30 = 100

Total possible outcomes = 100 = n(S)

(i) Number of favorable outcomes for Re 1 coins = 30 = n(E)

Probability(Re 1 coin) = $\frac{n(E)}{n(S)} = \frac{30}{100} = \frac{3}{10}$

(ii) Number of favorable outcomes for not a Rs 2 coins = number of favorable outcomes for Re 1 or Rs 5 coins = 30+20 = 50 = n(E)

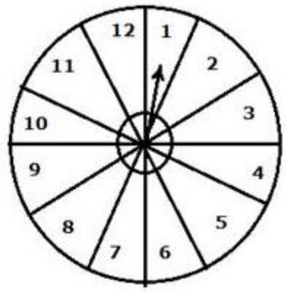
Probability(not Rs 2 coin) = $\frac{n(E)}{n(S)} = \frac{50}{100} = \frac{1}{2}$

(iii) number of favorable outcomes for neither Re 1 nor Rs 5 coins = Number of favorable outcomes for Rs 2 coins = 50 = n(E)

Probability(neither Re 1 nor Rs 5 coin) = $\frac{n(E)}{n(S)} = \frac{50}{100} = \frac{1}{2}$

Question 18.

A game consists of spinning arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; as shown below.



If the outcomes are equally likely, find the probability that the pointer will point at:

(i) 6

(ii) an even number

(iii) a prime number

(iv) a number greater than 8

(v) a number less than or equal to 9

(vi) a number between 3 and 11

Solution:

Total number of possible outcomes = 12

(i) Number of favorable outcomes for 6 = 1

P(the pointer will point at 6) = $\frac{1}{12}$

(ii) Favorable outcomes for an even number are 2, 4, 6, 8, 10, 12

Number of favorable outcomes = 6

P(the pointer will be at an even number) = $\frac{6}{12} = \frac{1}{2}$

(iii) Favorable outcomes for a prime number are 2, 3, 5, 7, 11

Number of favorable outcomes = 5

P(the pointer will be at a prime number) = $\frac{5}{12}$

(iv) Favorable outcomes for a number greater than 8 are 9, 10, 11, 12

Number of favorable outcomes = 4

P(the pointer will be at a number greater than 8) = $\frac{4}{12} = \frac{1}{3}$

(v) Favorable outcomes for a number less than or equal to 9 are 1, 2, 3, 4, 5, 6, 7, 8, 9

Number of favorable outcomes = 9

P(the pointer will be at a number less than or equal to 9) =

$\frac{9}{12} = \frac{3}{4}$

(vi) Favorable outcomes for a number between 3 and 11 are 4, 5, 6, 7, 8, 9, 10

Number of favorable outcomes = 7

P(the pointer will be at a number between 3 and 11) = $\frac{7}{12}$

Question 19.
One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting:
(i) a queen of red color
(ii) a black face card
(iii) the jack or the queen of the hearts
(iv) a diamond
(v) a diamond or a spade
Solution:
Total possible outcomes = 52

(i) Number queens of red color = 2

Number of favorable outcomes = 2

P(queen of red color) = $\frac{2}{52} = \frac{1}{26}$

(ii) Number of black cards = 26

Number of black face cards = 6

Number of favorable outcomes = 6

P(black face card) = $\frac{6}{52} = \frac{3}{26}$

(iii) Favorable outcomes for jack or queen of hearts = 1 jack + 1 queen

Number of favorable outcomes = 2

P(jack or queen of hearts) = $\frac{2}{52} = \frac{1}{26}$

(iv) Number of favorable outcomes for a diamond = 13

Number of favorable outcomes = 13

P(getting a diamond) = $\frac{13}{52} = \frac{1}{4}$

(v) Number of favorable outcomes for a diamond or a spade = 13 + 13 = 26

Number of favorable outcomes = 26

P(getting a diamond or a spade) = $\frac{26}{52} = \frac{1}{2}$

Question 20.
From a deck of 52 cards, all the face cards are removed and then the remaining cards are shuffled. Now one card is drawn from the remaining deck. Find the probability that the card drawn is:

(i) a black card
(ii) 8 of red color
(iii) a king of black color

Solution:

There are 12 face cards in a deck.

Therefore, possible number of outcomes = 52 - 12 = 40

(i) number of favorable outcomes for black cards = 26 cards - 6 face cards = 20

P(a black card) = $\frac{20}{40} = \frac{1}{2}$

(ii) number of favorable outcomes for 8 of red color = 2

P(getting a card with 8 of red color) = $\frac{2}{40} = \frac{1}{20}$

(iii) Since all face cards are removed

Number of favorable outcomes for a king of black color = 0

P(getting a king of black color) = $\frac{0}{40} = 0$

Question 21.

Seven cards:- the eight, the nine, the ten, jack, queen, king and ace of diamonds are well shuffled. One card is then picked up at random.
(i) What is the probability that the card drawn is the eight or the king?
(ii) If the king is drawn and put aside, what is the probability that the second card picked up is:
a) an ace? b) a king?

Solution:

Total number of possible outcomes = 7

(i) Number of favorable outcomes for the card is 8 or the king = 2

P(card is 8 or the king) = $\frac{2}{7}$

(ii) a) If a king is drawn and put aside, then total possible outcomes = 6

Number of favorable outcomes for an ace = 1

P(card is an ace) = $\frac{1}{6}$

b) Now, for second pick number of king = 0

Number of favorable outcomes for a king = 0

P(card is a king) = $\frac{0}{6} = 0$

Question 22.
A box contains 150 bulbs out of which 15 are defective. It is not possible to just look at a bulb and tell whether or not it is defective. One bulb is taken out at random from this box. Calculate the probability that the bulb taken out is:
(i) a good one
(ii) a defective one
Solution:

Total number of possible outcomes = 150

(i) out of 150 bulbs, 15 are defective

Number of bulbs which are good = 150 - 15 = 135

P(taking out a good bulb) = $\frac{135}{150} = \frac{9}{10}$

(ii)Number of bulbs which are defective = 15

P(taking out a defective bulb) = $\frac{15}{150} = \frac{1}{10}$

Question 23.
(i) 4 defective pens are accidentally mixed with 16 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is drawn at random from the lot. What is the probability that the pen is defective?
(ii) Suppose the pen drawn in (i) is defective and is not replaced. Now one more pen is drawn at random from the rest. What is the probability that this pen is:
a) defective
b) not defective?
Solution:

(i) Total number of pens = 4 + 16 = 20

Total possible outcomes = 20

Number of defective pens = 4

P(defective pen) = $\frac{4}{20} = \frac{1}{5}$

(ii) If defective pen drawn in first draw is not replaced, total possible outcomes = 20 - 1 = 19

a) Number of defective pens = 3

P(defective pens) = $\frac{3}{19}$

b) Number of not defective pens = 16

P(not defective pens) = $\frac{16}{19}$

Question 24.

A bag contains 100 identical marble stones which are numbered 1 to 100. If one stone is drawn at random from the bag, find the probability that it bears:
(i) a perfect square number
(ii) a number divisible by 4
(iii) a number divisible by 5
(iv) a number divisible by 4 or 5
(v) a number divisible by 4 and 5

Solution:

Total number of possible outcomes = 100

(i) Numbers which are perfect squares = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Number of favorable outcomes = 10

P(a perfect square) = $\frac{10}{100} = \frac{1}{10}$

(ii) Numbers which are divisible by 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100

Number of favorable outcomes = 25

P(number divisible by 4) = $\frac{25}{100} = \frac{1}{4}$

(iii) Numbers which are divisible by 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

Number of favorable outcomes = 20

P(number divisible by 5) = $\frac{20}{100} = \frac{1}{5}$

(iv) Numbers which are divisible by 4 or 5 = 4, 5, 8, 10, 12, 15, 16, 20, 24, 25, 28, 30, 32, 35, 36, 40, 44, 45, 48, 50, 52, 55, 56, 60, 64, 65, 68, 70, 72, 75, 76, 80, 84, 85, 88, 90, 92, 95, 96, 100

Number of favorable outcomes = 40

P(number divisible by 4 or 5) = $\frac{40}{100} = \frac{2}{5}$

(v) Numbers which are divisible by 4 and 5 = 20, 40, 60, 80, 100

Number of favorable outcomes = 5

P(number divisible by 4 and 5) = $\frac{5}{100} = \frac{1}{20}$

Question 25.

A circle with diameter 20 cm is drawn somewhere on a rectangular piece of paper with length 40 cm and width 30 cm. This paper is kept horizontal on table top and a die, very small in size, is dropped on the rectangular paper without seeing towards it. If the die falls and lands on paper only, find the probability that it will fall and land:
(i) inside the circle
(ii) outside the circle

Solution:

Diameter of the circle = 20 cm

Radius = 10 cm

Area of circle = $\pi r^2 = \frac{22}{7} \times 10 \times 10 = \frac{2200}{7} cm^2$

Length of paper = 40 cm

Width of paper = 30 cm

Area of paper = 1200 cm^2

Total possible outcomes = area of rectangular paper

(i) Since paper is kept on table top and die falls and lands on paper.

Number of favorable outcomes = area of circle.

P(inside the circle) = $\frac{\text{Area of circle}}{\text{Area of rectangular paper}} = \frac{2200/7}{1200} = \frac{11}{42}$

(ii) P(outside the circle) = 1 - P(inside the circle)

$= 1 - \frac{11}{42}$

$= \frac{31}{42}$

Question 26.

Two dice (each bearing numbers 1 to 6) are rolled together. Find the probability that the sum of the numbers on the upper-most faces of two dice is:

(i) 4 or 5
(ii) 7, 8 or 9
(iii) between 5 and 8
(iv) more than 10
(v) less than 6

Solution:

When two dice are rolled, total number of possible outcomes = 36

(i) Favorable outcomes for the sum of numbers 4 or 5 are:

{(1, 3), (1, 4), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}

Number of favorable outcomes = 7

P(getting a sum of 4 or 5) = $\frac{7}{36}$

(ii) Favorable outcomes for the sum of numbers 7, 8 or 9 are:

{(1, 6), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 3), (4, 4), (4, 5), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3)}

Number of favorable outcomes = 15

P(getting a sum of 7, 8 or 9) = $\frac{15}{36} = \frac{5}{12}$

(iii) Favorable outcomes for the sum of numbers between 5 and 8 i.e. 6 or 7 are:

{(1, 5), (1, 6), (2, 4), (2, 5), (3, 3), (3, 4), (4, 2), (4, 3), (5, 1), (5, 2), (6, 1)}

Number of favorable outcomes = 11

P(getting a sum of 6 or 7) = $\frac{11}{36}$

(iv) Favorable outcomes for the sum of numbers more than 10 i.e. 11 or 12 are:

{(5, 6), (6, 5), (6, 6)}

Number of favorable outcomes = 3

P(getting a sum of numbers more than 10) = $\frac{3}{36} = \frac{1}{12}$

(v) Favorable outcomes for the sum of numbers less than 6 I.e. 2, 3, 4 or 5 are:

{(1,1), (1, 2), (1, 3), (1, 4), (2,1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}

Number of favorable outcomes = 10

P(getting a sum of less than 6) = $\frac{10}{36} = \frac{5}{18}$

Question 27.

Three coins are tossed together. Write all the possible outcomes. Now, find the probability of getting:

(i) exactly two heads

(ii) at least two heads
(iii) at most two heads
(iv) all tails
(v) at least one tail

Solution:

When three coins are tossed, possible outcomes are:

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

Total possible outcomes = 8

(i) Favorable outcomes for exactly two heads = HHT, THH, HTH

Number of favorable outcomes = 3

P(exactly two heads) = $\frac{3}{8}$

(ii) Favorable outcomes for at least two heads = HHT, THH, HTH, HHH

Number of favorable outcomes = 4

P(at least two heads) = $\frac{4}{8} = \frac{1}{2}$

(iii) Favorable outcomes for at most two heads = HHT, THH, HTH, HTT, THT, TTH, TTT

Number of favorable outcomes = 7

P(at most two heads) = $\frac{7}{8}$

(iv) Favorable outcomes for all tails = TTT

Number of favorable outcomes = 1

P(all tails) = $\frac{1}{8}$

(v) Favorable outcomes for at least one tails = HHT, THH, HTH, HTT, THT, TTH, TTT

Number of favorable outcomes = 7

P(at least one tail) = $\frac{7}{8}$

Question 28.

Two dice are thrown simultaneously. What is the probability that:
(i) 4 will not come up either time?
(ii) 4 will come up at least once?

Solution:

When two dice are thrown, total possible outcomes = 36

(i) Favorable outcomes for 4 will not come up either time:

{(1,1), (1,2), (1,3), (1,5), (1,6)

(2,1), (2,2), (2,3), (2,5), (2,6)

(3,1), (3,2), (3,3), (3,5), (3,6)

(5,1), (5,2), (5,3), (5,5), (5,6)

(6,1), (6,2), (6,3), (6,5), (6,6)}

Number of favorable outcomes = 25

P(4 will not come up) = $\frac{25}{36}$

(ii) P(4 will come up once) = 1 - P(4 will not come up either time)

P(4 will come up once) = 1 - $\frac{25}{36}$

P(4 will come up once) = $\frac{36-25}{36} = \frac{11}{36}$

Question 29.

Cards marked with numbers 1, 2, 3 ……… 20 are well shuffled and a card is drawn at random. What is the probability that the number on the card is:

(i) a prime number

(ii) divisible by 3

(iii) a perfect square

Solution:

Total possible outcomes = 20

(i) Favorable outcomes for a prime number = 2, 3, 5, 7, 11, 13, 17, 19

Number of favorable outcomes = 8

P(a prime number) = $\frac{8}{20} = \frac{2}{5}$

(ii) Favorable outcomes for a number divisible by 3 = 3, 6, 9, 12, 15, 18

Number of favorable outcomes = 6

P(divisible by 3) = $\frac{6}{20} = \frac{3}{10}$

(iii) Favorable outcomes for a perfect square = 1, 4, 9, 16

Number of favorable outcomes = 4

P(a perfect square) = $\frac{4}{20} = \frac{1}{5}$

Question 30.
Offices in Delhi are open for five days in a week (Monday to Friday). Two employees of an office remain absent for one day in the same particular week. Find the probability that they remain absent on:
(i) the same day
(ii) consecutive day
(iii) different days
Solution:

Total number of possible outcomes = 5 x 5 = 25

The possible outcomes are:

MM, MT, MW, MTh, MF, TM, TT, TW, TTh, TF, WM, WT, WW, WTh, WF, ThM, ThT, ThW, ThTh, ThF, FM, FT, FW, FTh, FF

(i) Favorable outcomes for two employees remaining absent on same day are: MM, TT, WW, ThTh, FF

Number of favorable outcomes = 5

P(same day) = $\frac{5}{25} = \frac{1}{5}$

(ii) Favorable outcomes for two employees remaining absent on consecutive days : MT, TM, TW, WT, WTh, ThW, ThF, FTh

Number of favorable outcomes = 8

P(consecutive days) = $\frac{8}{25}$

(iii) P(absent on diff days) = 1 - P(absent on same days)

$= 1 - \frac{1}{5}$

$= \frac{4}{5}$

Question 31.
A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball; find the number of black balls in the box.
Solution:

Let the number of black balls in the box be x

Total number of balls in the box = x+30

P(drawing a black ball) = $\frac{x}{x+30}$

P(drawing a white ball = $\frac{30}{x+30}$

But, P(drawing a black ball) = $\frac{2}{5}$ x P(drawing a white ball)

$$\frac{x}{x+30} = \frac{2}{5} \times \frac{30}{x+30}$$
$$\frac{x}{x+30} = \frac{12}{x+30}$$
$$x = 12$$

Number of black balls in the box = 12

Question 32.
From a pack of 52 playing cards, all cards whose numbers are multiples of 3 are removed. A card is now drawn at random. What is the probability that the card drawn is
(i) A face card (King, Jack or Queen)
(ii) An even numbered red card?
Solution:

No. of total cards = 52

Cards removed of 4 colours of multiples of 3

= 3, 6, 9 = 4 × 3 = 12

Remaining cards = 52 - 12 = 40

(i) No. of face cards = 12 cards

$\Rightarrow$ Probability $P(E) = \frac{12}{40} = \frac{3}{10}$

(ii) An even number red cards = 2,4,8,10 cards = 2×4 = 8

$\Rightarrow$ Probability $P(E) = \frac{8}{40} = \frac{1}{5}$