

Chapter – 6

Triangles

Similar Figures

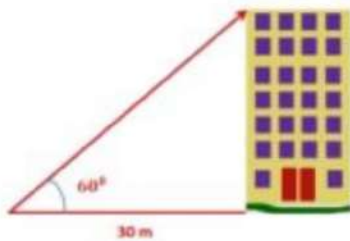
Introduction

Applications of triangles in the real world are,

The triangular shape is the strongest shape and thus used extensively in architecture.

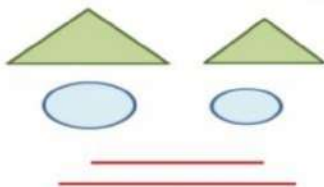
Triangles form the base unit of engineering structures like bridges.

Trigonometry, which is calculations with triangles can be used to measure the height of a building or a mountain.



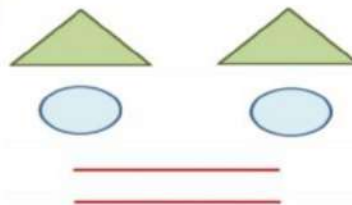
Similar Figures

- Two figures are said to be similar if they have the same shape but not necessarily the same size.



Congruent Figures

- Two figures are congruent if they have the same shape and size.



If two figures are similar, then we can shrink or stretch the figure without changing its shape to obtain another similar shape of the figure.

Concept of similarity is used to measure the heights or distance of objects like mountain, planets.

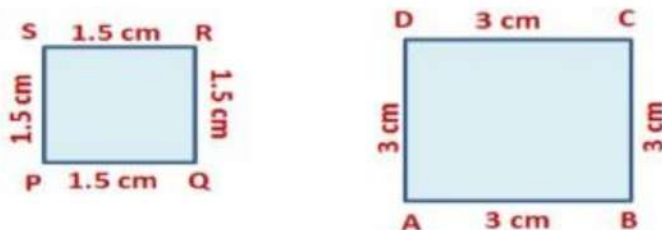
All congruent figures are similar but similar figures need not be congruent.

Two polygons of the same number of sides are similar if

- all the corresponding angles are equal.
- all the corresponding sides are in the same ratio

Ratio of corresponding sides is referred to as the scale factor (Representative Fraction) for the polygons.

Let's consider an example, here are two quadrilaterals PQRS and ABCD.



As the corresponding angles and ratio of corresponding sides are equal, the two quadrilaterals PQRS and ABCD are similar.

We write $PQRS \sim ABCD$

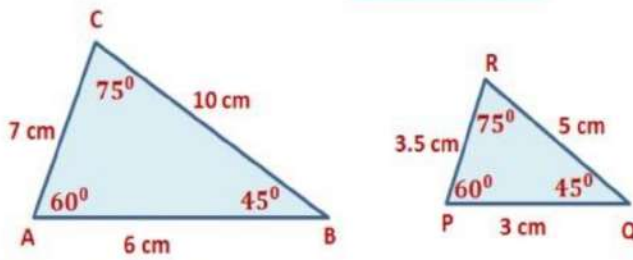
The symbol ' $\sim$ ' stands for 'is similar to'

Similarity of triangles

Similarity of Triangles

Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides are in the same proportion.

For example here are two triangles, ΔABC and ΔPQR



In $\triangle ABC$ and $\triangle PQR$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

$$\frac{AB}{PQ} = \frac{6}{3} = \frac{2}{1}, \frac{BC}{QR} = \frac{10}{5} = \frac{2}{1}, \frac{AC}{PR} = \frac{7}{3.5} = \frac{2}{1}$$

$\triangle ABC$ and $\triangle PQR$ are similar as the ratio of their corresponding sides is same and corresponding angles are equal.

- If **corresponding angles of two triangles are equal**, then they are known as **equiangular triangles**.
- The ratio of any two corresponding sides in equiangular triangle is always the same.

Basic Proportionality Theorem or Thales Theorem

Theorem 1: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: In $\triangle ABC$, $DE \parallel BC$ and DE intersect AB at D and AC at E .

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and CD and draw $EN \perp AB$ and $DM \perp AC$

Proof: As $EN \perp AB$, EN is the height of $\triangle ADE$ and $\triangle DBE$

We know, Area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

Now, area of $\triangle ADE = \frac{1}{2} \times AD \times EN$

$$\text{Area of } \Delta BDE = \frac{1}{2} \times DB \times EN$$

$$\text{Area of } \Delta ADE = \frac{1}{2} \times AE \times DM$$

$$\text{Area of } \Delta DEC = \frac{1}{2} \times EC \times DM$$

The ratio of area of triangles, ΔADE and ΔBDE

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times EC \times DM} = \frac{AD}{DB} \rightarrow \text{Eq 1}$$

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \rightarrow \text{Eq 2}$$

According to the property of triangles, the triangles drawn between the same parallel lines and on the same base have equal areas.

Therefore, the area of ΔBDE = area of $\Delta DEC \rightarrow \text{Eq 3}$

From equations 1, 2 and 3 we have, $\frac{AD}{DB} = \frac{AE}{EC}$

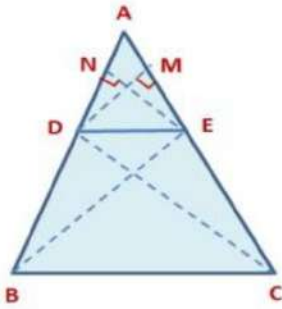
Theorem 2: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

(Converse of Basic Proportionality)

Given: A ΔABC and a line DE , intersecting AB at D and AC at E such that,

$$\frac{AD}{DB} = \frac{AE}{EC} \rightarrow \text{Eq 1}$$

To Prove: $DE \parallel BC$.



We will assume that in ΔABC , DE is not parallel to BC . If DE is not parallel to BC , we draw DE' parallel to BC .

Applying Basic Proportionality Theorem we get,

$$\frac{AD}{DB} = \frac{AE'}{E'C} \rightarrow \text{Eq 2}$$

We will assume that in ΔABC , DE is not parallel to BC . If DE is not parallel to BC , we draw DE' parallel to BC .

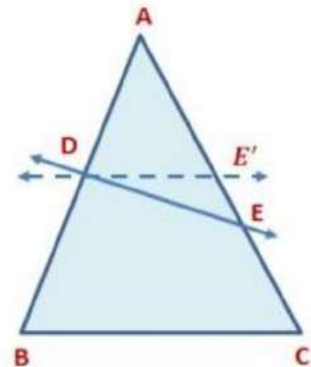
Applying Basic Proportionality Theorem we get,

$$\frac{AD}{DB} = \frac{AE'}{E'C} \rightarrow \text{Eq 2}$$

From Eq 1 and Eq 2, we get

$$\frac{AE}{EC} = \frac{AE'}{E'C} \rightarrow \text{Eq 3}$$

Adding 1 to both sides of Eq 3, we get



$$\frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1 \Rightarrow \frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C}$$

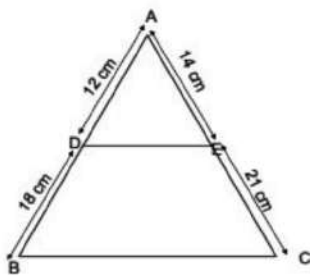
$$\frac{AC}{EC} = \frac{AC}{E'C} \Rightarrow EC = E'C$$

Therefore, we can say that the point E and F coincide. Hence, DE is also parallel to BC.

Therefore, we can say that the point E and F coincide. Hence, DE is also parallel to BC.

Example: If D and E are points on the respective sides AB and AC of ΔABC such that $AD = 12$ cm, $BD = 18$ cm, $AE = 14$ cm, $EC = 21$ cm

Prove that $DE \parallel BC$.



In ΔABC ,

$$\frac{AD}{DB} = \frac{12}{18} = \frac{2}{3} \text{ and } \frac{AE}{EC} = \frac{14}{21} = \frac{2}{3}$$

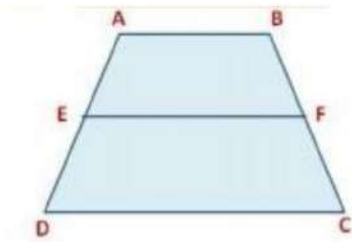
$$\text{Now, } \frac{AD}{DB} = \frac{AE}{EC}$$

$DE \parallel BC$ (By Converse of basic proportionality theorem)

Example: ABCD is a trapezium with $AB \parallel DC$. E and F are two points on non-parallel sides AD and BC respectively, such that EF is parallel to AB. Show

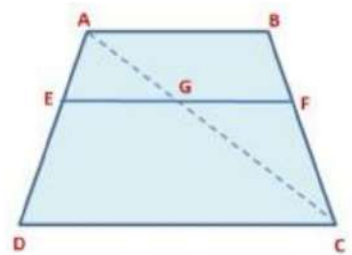
that $\frac{AE}{ED} = \frac{BF}{FC}$.

(REFERENCE: NCERT)



In trapezium ABCD, $AB \parallel DC$ and $EF \parallel AB$.

Now, join AC to intersect EF at G.



Here, $EF \parallel AB$

($\because$ lines parallel to the same line are parallel to each other)

In $\triangle ADC$, $EG \parallel DC$ ($\because EF \parallel DC$)

So, $\frac{AE}{ED} = \frac{AG}{GC} \rightarrow$ Eq 1 (By Basic Proportionality Theorem)

In $\triangle ABC$, $GF \parallel AB$ ($\because EF \parallel AB$)

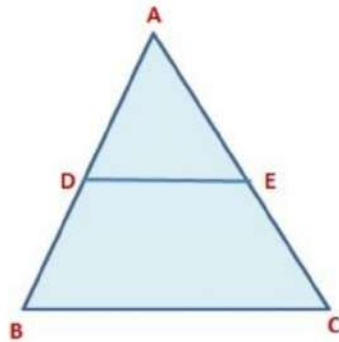
Similarly, $\frac{CG}{AG} = \frac{CF}{BF}$ (By Basic Proportionality Theorem)

On taking reciprocal of the terms we get, $\frac{AG}{GC} = \frac{BF}{CF} \rightarrow$ Eq 2

From Eq 1 and Eq 2 we get,

$$\frac{AE}{ED} = \frac{BF}{CF}$$

Example: In the given figure, $DE \parallel BC$. If $AD = 3$ cm. $DB = 4$ cm and $AE = 9$ cm, find EC.



In $\triangle ABC$, $DE \parallel BC$

So, $\frac{AD}{DB} = \frac{AE}{EC}$ (By Basic Proportionality Theorem)

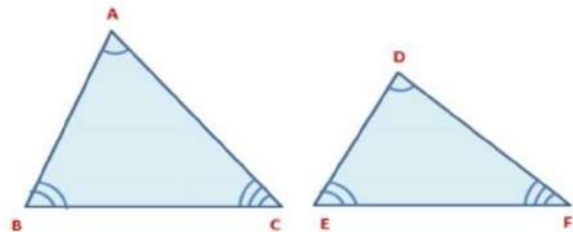
$$\frac{3}{4} = \frac{9}{EC} \Rightarrow EC = \frac{4 \times 9}{3} = 12$$

$\therefore EC = 12 \text{ cm}$

Criteria for similarity of triangles AAA Criterion

Criteria for Similarity of Triangles

$\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$
$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
We write similarity of these two triangles as $\triangle ABC \sim \triangle DEF$



AAA Similarity Criterion

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

We can prove this theorem by taking two triangles ABC and DEF.

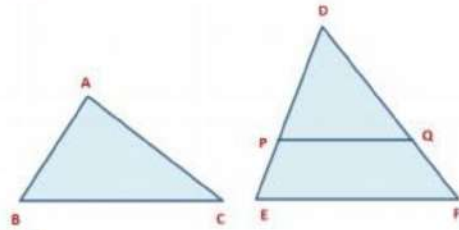
Given: $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

To Prove: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ and then $\Delta ABC \sim \Delta DEF$

Construction: Cut $DP=AB$ and $DQ=AC$.

Join PQ .

Proof: In ΔABC and ΔDEF



$AB=DP$	By Construction
$AC=DQ$	By Construction
$\angle A = \angle D$	Given
$\Delta ABC \cong \Delta DEF$	By SAS Congruency Rule

$\therefore \angle B = \angle P$ (by CPCT)

But $\angle B = \angle E$ (given)

$\therefore \angle P = \angle E \Rightarrow PQ \parallel EF$ ($\because$ Corresponding angles are equal)

$\therefore \frac{DP}{PE} = \frac{DQ}{QF}$ (By Basic Proportionality Theorem)

On taking reciprocal we get,

$$\frac{PE}{DP} = \frac{QF}{DQ}$$

Adding 1 to both sides, we get

$$\begin{aligned} \frac{PE}{DP} + 1 &= \frac{QF}{DQ} + 1 \Rightarrow \frac{PE + DP}{DP} = \frac{QF + DQ}{DQ} \Rightarrow \frac{DE}{DP} = \frac{DF}{DQ} \\ \Rightarrow \frac{DE}{AB} &= \frac{DF}{AC} \quad (\because DP=AB \text{ and } DQ=AC) \end{aligned}$$

On taking reciprocal we get,

$$\frac{AB}{DE} = \frac{AC}{DF}$$

Similarly, $\frac{AB}{DE} = \frac{BC}{EF}$ and, so $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

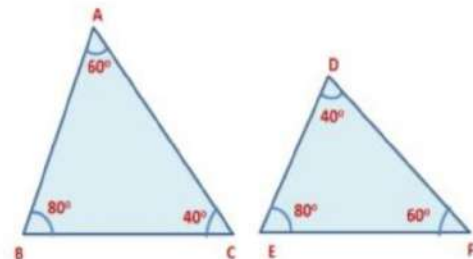
Hence, $\Delta ABC \sim \Delta DEF$

If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angle will also be equal. Therefore AAA similarity criterion can be considered as AA similarity criterion.

Example: State whether the given pair of triangles are similar or not. In the case of similarity, mention the criterion.

In ΔABC and ΔFED ,

$\angle A = \angle F = 60^\circ$
$\angle B = \angle E = 80^\circ$
$\angle C = \angle D = 40^\circ$
$\therefore \Delta ABC \sim \Delta FED$ by AAA similarity criterion

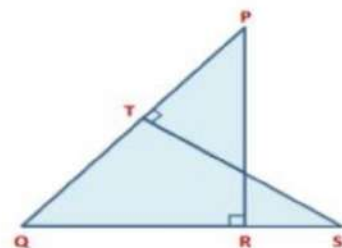


Example: In the given figure, PQR and QST are two right-angled triangles, right-angled at R and T, respectively. Prove that $QR \times QS = QP \times QT$

In ΔPRQ and ΔSTQ

$\angle PRQ = \angle QTS$	90°
$\angle PQR = \angle SQT$	Common angle
$\Delta PQR \sim \Delta QST$	By AA Similarity Criterion

Therefore, $\frac{QR}{QT} = \frac{QP}{QS}$



(Corresponding sides of similar triangles are proportional)

$\Rightarrow QR \times QS = QP \times QT$ Hence proved

Example: A girl of height 90 cm is walking away from the base of a lamp post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, then find the length of her shadow after 4s.

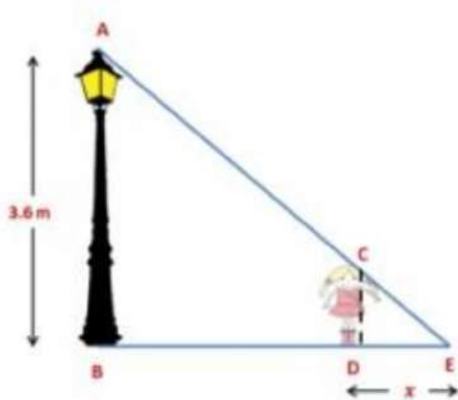
(REFERENCE: NCERT)

Let AB be the lamp-post, CD be the height of the girl and D be the position of the girl after 4 s.

Distance travelled by girl in 4 s = $1.2 \times 4 = 4.8$ m

Here, AB = 3.6 m, CD = 90 cm = 0.9 m, BD = 4.8

In $\triangle ABE$ and $\triangle CDE$



$\angle B = \angle D$	90°
$\angle E = \angle E$	Common angle
$\triangle ABE \sim \triangle CDE$	By AA Similarity Criterion

Therefore, $\frac{BE}{DE} = \frac{AB}{CD}$ (Corresponding sides of similar triangles are proportional)

$$\frac{4.8 + x}{x} = \frac{3.6}{0.9} \Rightarrow \frac{4.8 + x}{x} = 4 \Rightarrow 4x = 4.8 + x \Rightarrow 3x = 4.8$$

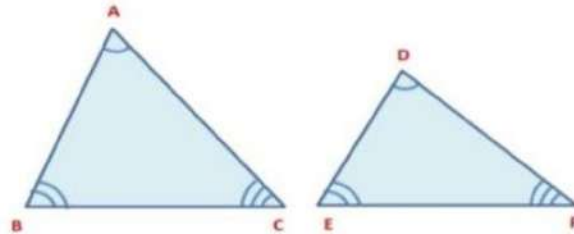
$$x = 1.6$$

Hence, the length of her shadow is 1.6 m after 4 s.

Criteria for similarity of triangles - SSS Criterion

Criteria for Similarity of Triangles

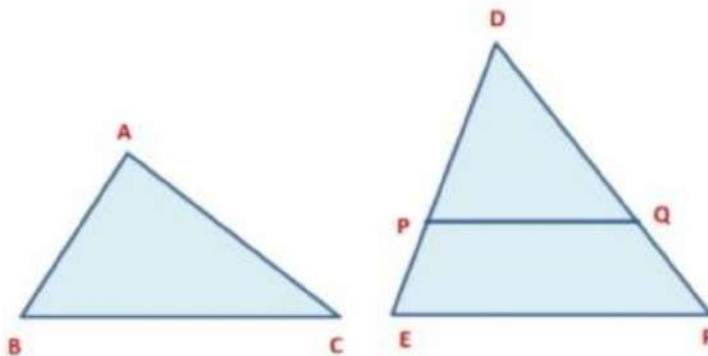
$\angle A = \angle D, \angle B = \angle E, \text{ and } \angle C = \angle F$
$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
We write similarity of these two triangles as $\triangle ABC \sim \triangle DEF$



SSS Similarity Criterion

If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

We can prove this theorem by taking two triangles ABC and DEF.



Given: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} < 1 \rightarrow Eq1$

To Prove: $\angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F$ and then $\triangle ABC \sim \triangle DEF$

CONSTRUCTION : Cut $DP = AB$ and $DQ = AC$. Join PQ .

Proof:

It is given that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{DP}{DE} = \frac{DQ}{DF}$ ($\because DP = AB$ and $DQ = AC$)

$\Rightarrow PQ \parallel EF$ (By Converse of Basic Proportionality Theorem)

$$\therefore \angle DPQ = \angle E \text{ and } \angle DQP = \angle F \rightarrow \text{Eq 2}$$

($\because$ Corresponding angles are equal)

In ΔDPQ and ΔDEF

$$\angle DPQ = \angle E \text{ and } \angle DQP = \angle F$$

$\therefore \Delta DPQ \sim \Delta DEF$ (By AA Similarity criterion)

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF} \rightarrow \text{Eq 3}$$

(Corresponding sides of similar triangles are proportional)

Putting $AB = DP$ and $AC = DQ$ in Eq 1 we get,

$$\frac{DP}{DE} = \frac{DQ}{DF} = \frac{BC}{EF} \rightarrow \text{Eq 4}$$

In ΔABC and ΔDPQ ,

$BC = PQ$	From Eq 2 and Eq 3
$AB = DP$	By Construction
$AC = DQ$	By Construction
$\Delta ABC \sim \Delta DPQ$	By SSS Congruency Rule

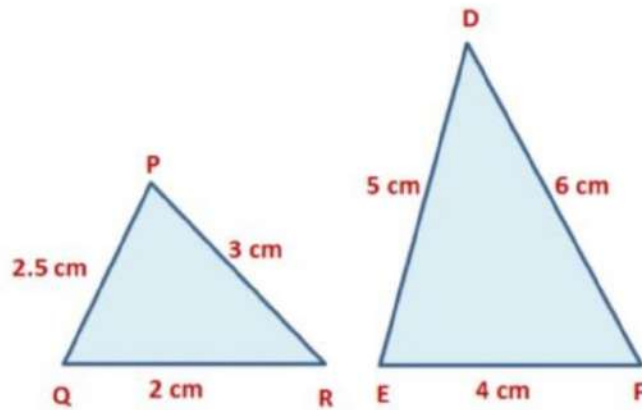
So, $\angle A = \angle D$, $\angle B = \angle DPQ$ and $\angle C = \angle DQP$ (By CPCT)

$$\Rightarrow \angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F \text{ (From Eq 2)}$$

$\therefore \Delta ABC \sim \Delta DEF$ (Hence Proved)

Example: For the given pair of triangles, state the similarity criterion used and write the relation in symbolic form.

In ΔPQR and ΔDEF ,



In Δ PQR and Δ DEF,

$$\frac{PQ}{DE} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{PR}{DF} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{QR}{EF} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{PQ}{DE} = \frac{PR}{DF} = \frac{QR}{EF} = \frac{1}{2}$$

∴ Δ PQR ~ Δ DEF by SSS Similarity Criterion

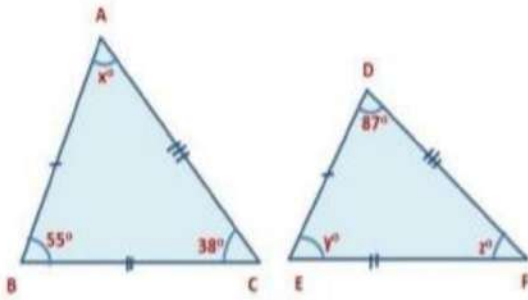
Example: Find the value of x° , y° , z° in the given pair of triangles.

Given: AB = DE, BC = EF and AC = DF

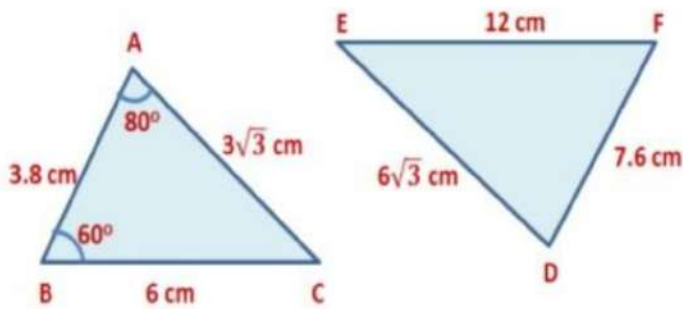
∴ Δ ABC ~ Δ DEF by SSS Similarity Criterion

So, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

$$x^\circ = 87^\circ, y^\circ = 55^\circ, z^\circ = 38^\circ$$



Example: In the given figure, find $\angle F$.



In $\triangle ABC$ and $\triangle DEF$,

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{1}{2}$$

$\triangle ABC \sim \triangle DEF$ by SSS criterion of similarity

$\Rightarrow \angle A = \angle D, \angle B = \angle F$ and $\angle C = \angle E$

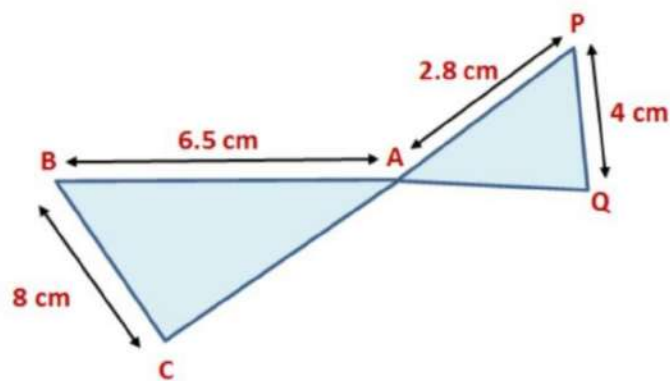
$\therefore \angle D = 80^\circ, \angle F = 60^\circ$

Hence $\angle F = 60^\circ$

Example: In the given figure, $\triangle ACB \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm, $AP = 2.8$ cm, find CA and AQ .

(REFERENCE: NCERT)

Here, $\triangle ACB \sim \triangle APQ$



$$\Rightarrow \frac{AC}{AP} = \frac{CB}{AQ} = \frac{AB}{AQ}$$

$$\Rightarrow \frac{AC}{AP} = \frac{CB}{PQ} \text{ and } \frac{CB}{PQ} = \frac{AB}{AQ}$$

$$\Rightarrow \frac{AC}{2.8} = \frac{8}{4} \Rightarrow AC = \frac{2.8 \times 8}{4} = 5.6$$

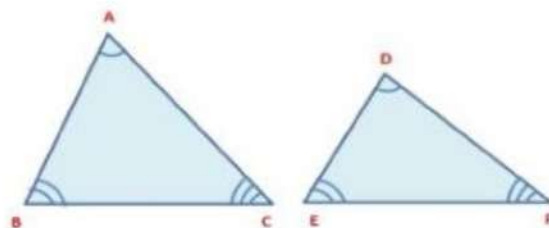
$$\Rightarrow \frac{8}{4} = \frac{6.5}{AQ} = AQ = \frac{4 \times 6.5}{8} = 3.25$$

AC = 5.6 cm and AQ = 3.25 cm

Criteria for similarity of triangles - SAS Criterion

Criteria for Similarity of Triangles

$\angle A = \angle D, \angle B = \angle E, \text{ and } \angle C = \angle F$
$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
We write similarity of these two triangles as $\triangle ABC \sim \triangle DEF$



SAS Similarity Criterion

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

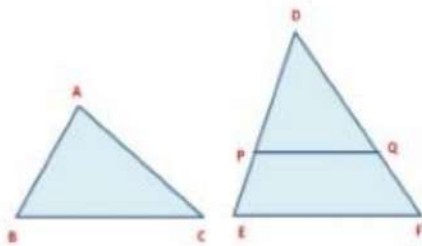
We can prove this theorem by taking two triangles ABC and DEF.

Given: $\frac{AB}{DE} = \frac{AC}{DF} < 1 \rightarrow \text{Eq 1}$ and $\angle A = \angle D \rightarrow \text{Eq 2}$

To Prove: $\Delta ABC \sim \Delta DEF$

Construction: Cut $DP=AB$ and $DQ= AC$. Join PQ.

Proof: In ΔABC and ΔDPQ



$AB=DP$	By Construction
$AC=DQ$	By Construction
$\angle A = \angle D$	Given
$\Delta ABC \cong \Delta DPQ \rightarrow \text{Eq3}$	By SAS Congruency Rule

From Eq1, $\frac{AB}{DE} = \frac{AC}{DF} \Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ ($\because DP=AB$ and $DQ=AC$)

$\Rightarrow PQ \parallel EF$ (By Converse of Basic Proportionality Theorem)

$\angle DPQ = \angle E$ and $\angle DQP = \angle F \rightarrow \text{Eq 2}$

($\because$ Corresponding angles are equal)

$\therefore \Delta DPQ \sim \Delta DEF$ (By AA Similarity criterion) $\rightarrow \text{Eq4}$

From Eq 3 and Eq4 we get,

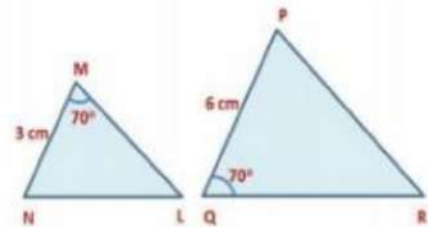
$\Delta ABC \sim \Delta DEF$

(Since two congruent triangles are always similar but its converse is not true)

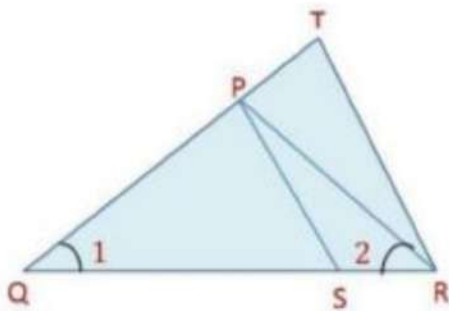
Example: For the given pair of triangles, state the similarity criterion used and write the relation in symbolic form.

In $\triangle MNL$ and $\triangle QPR$

$\angle M = \angle Q = 70^\circ$
$\frac{MN}{PQ} = \frac{1}{2}$
$\frac{ML}{QR} = \frac{1}{2}$
Then, $\frac{MN}{PQ} = \frac{ML}{QR}$
$\therefore \triangle MNL \sim \triangle QPR$ by SAS similarity Criterion



Example: In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\triangle PQS \sim \triangle TQR$.



In $\triangle PQR$, $\angle 1 = \angle 2$ (given)

$PR = PQ$

(Sides opposite to equal angles of a triangle are also equal)

$$\frac{QR}{QS} = \frac{QT}{PR} \text{ (Given)}$$

$$\text{Now, } PR = PQ \Rightarrow \frac{QR}{QS} = \frac{QT}{PQ} \rightarrow \text{Eq 1}$$

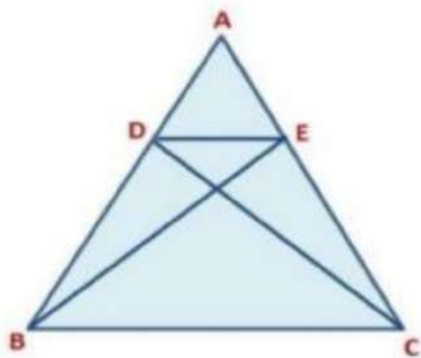
$$\text{Taking reciprocal of Eq 1 we get, } \frac{QR}{QS} = \frac{PQ}{QT} \rightarrow \text{Eq 2}$$

In ΔPQS and ΔTQR ,

$\angle PQS = \angle TQR$	Common
$\frac{QR}{QS} = \frac{QT}{PQ}$	From Eq 2
$\Delta PQS \sim \Delta TQR$	By SAS similarity criterion

Example: In the given figure, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.

(REFERENCE: NCERT)



It is given that $\Delta ABE \cong \Delta ACD$

$\Rightarrow AB = AC$ and $AE = AD$

(Corresponding parts of congruent triangles)

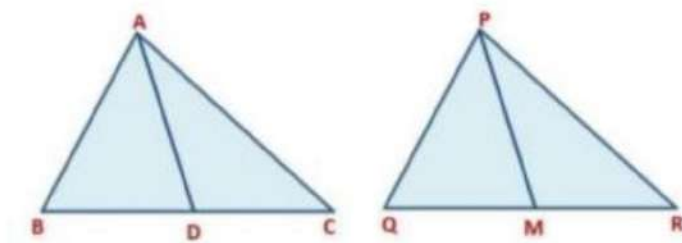
$$\frac{AB}{AC} = 1 \text{ and } \frac{AD}{AE} = 1 \Rightarrow \frac{AB}{AC} = \frac{AD}{AE} \rightarrow \text{Eq 1}$$

In ΔADE and ΔABC , we have $\frac{AD}{AE} = \frac{AB}{AC}$ (From Eq 1)

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \text{ and } \angle DAE = \angle BAC \text{ (common angle)}$$

$\Delta ADE \sim \Delta ABC$ by SAS similarity criterion

Example: Sides AB and BC and median AD of ΔABC are respectively proportional to sides PQ and QR and median PM of ΔPQR . Show that $\Delta ABC \sim \Delta PQR$.



It is given that BC and AD are medians

of ΔABC and ΔPQR respectively,

$$\text{and } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

Now, $\frac{AB}{PQ} = \frac{1/2 BC}{1/2 QR} = \frac{AD}{PM}$ (multiplying both numerator and denominator of the middle term by $1/2$)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

(Median bisects the opposite side, $1/2 BC = BD$ and $1/2 QR = QM$)

$\therefore \Delta ADB \sim \Delta PMQ$ (by SSS Similarity Criterion)

$\Rightarrow \angle B = \angle Q \rightarrow \text{Eq 1}$

Area of similar triangles

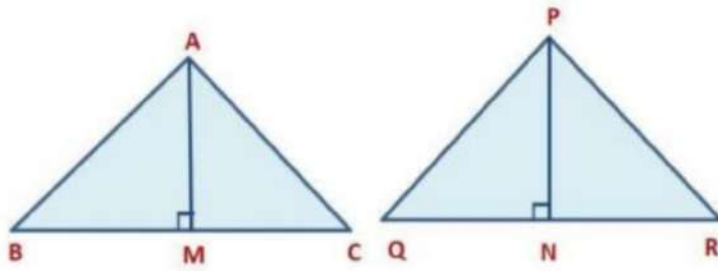
Areas of Similar Triangles

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given: $\Delta ABC \sim \Delta PQR$

$$\text{To prove : } \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

CONSTRUCTION : Draw altitudes AM and PN of ΔABC and ΔPQR .



Proof:

$$ar(\triangle ABC) = \frac{1}{2} \times BC \times AM$$

$$ar(\triangle PQR) = \frac{1}{2} \times QR \times PN$$

$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN} \rightarrow Eq1$$

In $\triangle ABM$ and $\triangle PQN$

$\angle B = \angle Q$	As $\triangle ABC \sim \triangle PQR$
$\angle M = \angle N$	Each is of 90°
$\triangle ABM \sim \triangle PQN$	By AA similarity criterion

$$\therefore \frac{AM}{PN} = \frac{AB}{PQ} \rightarrow Eq 2 \text{ also } \triangle ABC \sim \triangle PQR \text{ (given)}$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \rightarrow Eq 3$$

$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{AB}{PQ} \times \frac{AM}{PN} \quad (\because \frac{BC}{QR} = \frac{AB}{PQ})$$

$$= \frac{AB}{PQ} \times \frac{AB}{PQ} \quad (\text{From Eq 2})$$

$$= \left(\frac{AB}{PQ}\right)^2$$

Using Eq 3, we get:

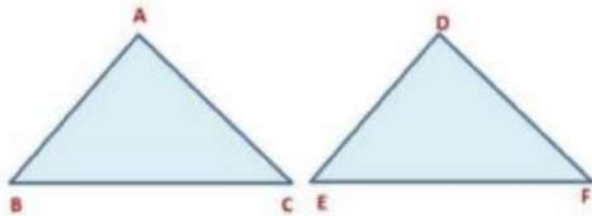
$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{AC}{PR}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

Example: Let $\triangle ABC \sim \triangle DEF$ and their areas be respectively 64 cm² and 121 cm².

If $EF = 15.4$ cm, then find BC .

(REFERENCE: NCERT)

Given: $\triangle ABC \sim \triangle DEF$



$$\therefore \frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \left(\frac{BC}{EF}\right)^2$$

(By a theorem of the area of similar triangles)

$$\begin{aligned} \frac{64}{121} &= \left(\frac{BC}{15.4}\right)^2 \Rightarrow \left(\frac{8}{11}\right)^2 = \left(\frac{BC}{15.4}\right)^2 \Rightarrow \frac{8}{11} = \frac{BC}{15.4} \Rightarrow BC \\ &= \frac{8 \times 15.4}{11} \end{aligned}$$

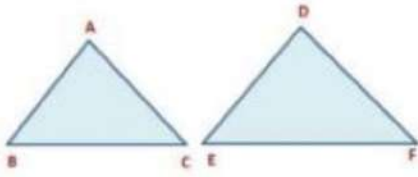
$$BC = 11.2 \text{ cm}$$

Example: If the areas of two similar triangles are equal, then prove that they are congruent.

Let the two triangles be $\triangle ABC$ and $\triangle DEF$

It is given that $\triangle ABC \sim \triangle DEF$ and

$$ar(\triangle ABC) = ar(\triangle DEF)$$



Therefore, $\frac{ar(\triangle ABC)}{ar(\triangle DEF)} = 1$

$$\left(\frac{AB}{DE}\right)^2 = \left(\frac{AC}{DF}\right)^2 = \left(\frac{BC}{EF}\right)^2$$

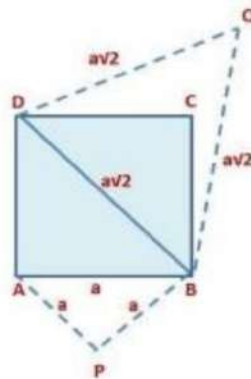
(By a theorem of the area of similar triangles)

$\Rightarrow AB = DE, BC = EF$ and $CA = FD$

$\therefore \triangle ABC \cong \triangle DEF$ (By SSS Congruency)

Example: Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of an equilateral triangle described on one of its diagonals.

(REFERENCE: NCERT)



Let ABCD be a square of length a .

We know, $BD^2 = AB^2 + AD^2 \Rightarrow BD^2 = a^2 + a^2 = 2a^2$

$$BD = \sqrt{2a^2}$$

Now, construct two equilateral triangles described on one side of the square and another described on the diagonal of the square.

$\therefore \Delta PAB \sim \Delta QBD$ (Equilateral triangles are similar)

$$\therefore \frac{\text{ar}(\Delta PAB)}{\text{ar}(\Delta QBD)} = \left(\frac{AB}{BD}\right)^2 = \frac{a^2}{(a\sqrt{2})^2} = \frac{a^2}{2a^2} = \frac{1}{2}$$

(By a theorem of the area of similar triangles)

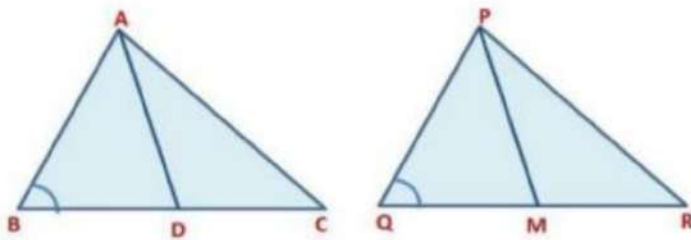
$$\text{ar}(\Delta PAB) = \frac{1}{2} \text{ar}(\Delta QBD)$$

Example: Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

(REFERENCE: NCERT)

Let ΔABC and ΔPQR be the two similar triangles.

Let AD and PM be the median of ΔABC



And ΔPQR respectively.

We know that $\Delta ABC \sim \Delta PQR$

$\Rightarrow \angle B = \angle Q \rightarrow \text{Eq 1}$ (Corresponding angles are equal)

$$\frac{AB}{PQ} = \frac{BC}{QR} \quad (\text{Ratio of corresponding sides are equal})$$

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$$

(As D is the mid-point of BC and M is the mid-point of QR)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \rightarrow \text{Eq 2}$$

In $\triangle ABD$ and $\triangle PQM$,

$\angle ABD = \angle PQM$	From Eq 1
$\frac{AB}{PQ} = \frac{BD}{QM} \rightarrow \text{Eq 2}$	The ratio of corresponding sides are equal
$\triangle ABD \sim \triangle PQM$	By SAS similarity criterion

Now, $\frac{\text{ar}(\triangle PAB)}{\text{ar}(\triangle QBD)} = \left(\frac{AB}{BD}\right)^2$ (By a theorem of the area of similar triangles)

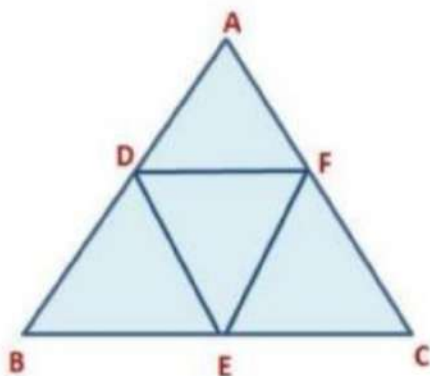
$$\therefore \frac{\text{ar}(\triangle PAB)}{\text{ar}(\triangle QBD)} = \left(\frac{AD}{PM}\right)^2 = (\text{ADPM})^2 \text{ (From Eq 3)}$$

Example: D, E, and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$. Find the ratio of the areas of $\triangle DEF$ and $\triangle ABC$.

(REFERENCE: NCERT)

It is given that D, E, and F are the mid-points of sides AB, BC, and CA. Join the points D, E, and F.

We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and its length is equal to the half of the length of the third side (Mid-Point Theorem)



$$DF = \frac{1}{2}BC, DE = \frac{1}{2}CA, \text{ and } EF = \frac{1}{2}AB \rightarrow \text{Eq 1}$$

In $\triangle DEF$ and $\triangle CAB$,

$$\frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} = \frac{1}{2} \quad (\text{From Eq 1})$$

$\triangle DEF \sim \triangle CAB$ (by SSS Similarity Criterion)

$$\text{Now, } \frac{\text{ar}(\triangle DEF)}{\text{ar}(\triangle CAB)} = \left(\frac{DE}{CA}\right)^2 \quad (\text{By a theorem of the area of similar triangles})$$

$$= \frac{\left(\frac{1}{2}CA\right)^2}{CA} = \frac{1}{4}$$

$$\therefore \frac{\text{ar}(\triangle DEF)}{\text{ar}(\triangle ABC)} = \frac{1}{4}$$

Pythagoras Theorem

Pythagoras Theorem is also known as the Baudhayan Theorem. Now we will prove this theorem using the concept of similarity of two triangles formed by the perpendicular to the hypotenuse from the opposite vertex of the right-angled triangle.

Theorem 1: If a perpendicular is drawn from the vertex of the right angle of a right-angled triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and each other.

Given: A right triangle ABC, right-angled at B. Let BD be the perpendicular to the hypotenuse AC.

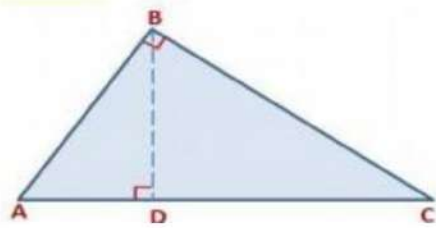
To Prove:

i) $\triangle ADB \sim \triangle ABC$

ii) $\triangle BDC \sim \triangle ABC$

iii) $\triangle ADB \sim \triangle BDC$

Proof: In $\triangle ADB$ and $\triangle ABC$



$\angle A = \angle A$	Common Angle
$\angle ADB = \angle ABC$	90°
$\triangle ADB \sim \triangle ABC$	By AA similarity criterion

Similarly, $\triangle BDC \sim \triangle ABC$, $\triangle ADB \sim \triangle BDC$

We will apply this theorem in proving the Pythagoras Theorem:

Pythagoras Theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

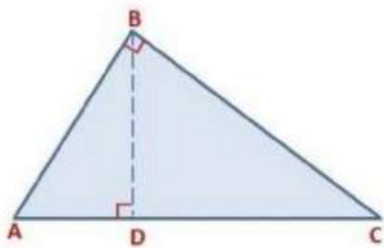
Given: A right triangle ABC right angled at B

To Prove: $AC^2 = AB^2 + BC^2$

Construction: Draw $BD \perp AC$

Proof: As ABC is a right-angled triangle and $BD \perp AC$.

$\therefore \triangle ADB \sim \triangle ABC$ (By theorem 1)



So, $\frac{AD}{AB} = \frac{AB}{AC}$

(Corresponding sides of similar triangles are proportional)

$$\Rightarrow AD \cdot AC = AB^2 \rightarrow \text{Eq 1}$$

Also, $\triangle BDC \sim \triangle ABC$ (By theorem 1)

$$\text{So, } \frac{CD}{BC} = \frac{BC}{AC}$$

(Corresponding sides of similar triangles are proportional)

$$\Rightarrow CD \cdot AC = BC^2 \rightarrow \text{Eq 2}$$

Adding Eq 1 and Eq 2 we get,

$$AD \cdot AC + CD \cdot AC = AB^2 + BC^2 \Rightarrow AC(AD + CD) = AB^2 + BC^2$$

$$AC \cdot AC = AB^2 + BC^2$$

$$(\because AC = AD + CD)$$

$$AB^2 + BC^2 = AC^2$$

The above theorem was given by an ancient Indian Mathematician Baudhayan in the following form:

The diagonal of a rectangle produces by itself the same area as produced by its both sides (i.e. length and breadth). This theorem is referred to as the Baudhayan Theorem.

The converse of Pythagoras Theorem

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Given: A triangle ABC in which $AC^2 = AB^2 + BC^2$

To Prove: $\angle B = 90^\circ$

Construction: Construct a ΔPQR right-angled at Q, such that $PQ = AB$ and $QR = BC$

Proof: From ΔPQR we have,

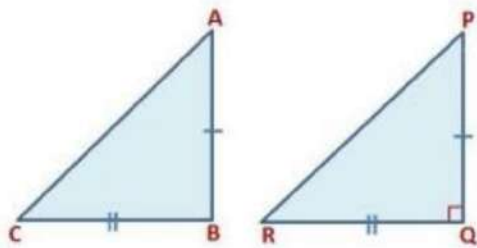
$$PR^2 = PQ^2 + QR^2$$

(Using Pythagoras theorem as $\angle Q = 90^\circ$)

$$\text{Now, } PR^2 = AB^2 + BC^2 \rightarrow \text{Eq 1}$$

(By Construction $PQ = AB$ and $QR = BC$)

$$\text{and } AC^2 = AB^2 + BC^2 \rightarrow \text{Eq 2 (Given)}$$



$\therefore AC = PR$ (From Eq 1 and Eq 2)

In $\triangle ABC \sim \triangle PQR$

$AB = PQ$	By Construction
$BC = QR$	By Construction
$AC = PR$	Proved above
$\triangle ABC \cong \triangle PQR$	By SSS congruence rule

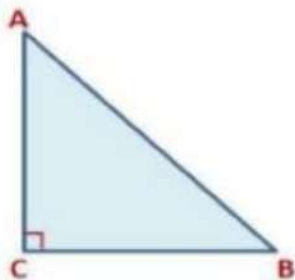
$\therefore \angle B = \angle Q$ (By CPCT)

But $\angle Q = 90^\circ$ (By Construction)

So, $\angle B = 90^\circ$

Example: In an isosceles right-angled triangle, if the hypotenuse is $7\sqrt{2}$ cm, then find the length of the sides of the triangle.

Let ABC be an isosceles right-angled triangle, right-angled at B with $AB = BC$.



Now, $AC = 7\sqrt{2}$ cm

By Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = AB^2 + AB^2 \text{ (Given } AB = BC \text{)}$$

$$AC^2 = 2AB^2 \Rightarrow (7\sqrt{2})^2 = 2AB^2 \Rightarrow 98 = 2AB^2 \Rightarrow 49 = AB^2$$

$$AB = 7 \text{ cm}$$

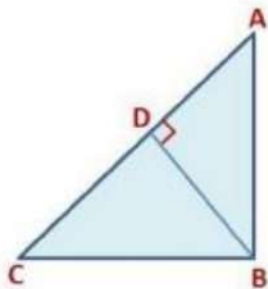
$$\therefore AB = BC = 7 \text{ cm}$$

Example: In the given figure, if $AD \perp BC$, prove that
 $AB^2 + CD^2 = BD^2 + AC^2$

In right-angled $\triangle BDA$, $AB^2 = AD^2 + BD^2 \rightarrow \text{Eq 1}$
 (By Pythagoras Theorem)

In right-angled $\triangle CDA$, $AC^2 = CD^2 + AD^2 \rightarrow \text{Eq 2}$
 (By Pythagoras Theorem)

Subtracting Eq 2 from Eq 1 we get,
 $AB^2 - AC^2 = AD^2 + BD^2 - (CD^2 + AD^2)$
 $AB^2 - AC^2 = AD^2 + BD^2 - CD^2 - AD^2$
 $\Rightarrow AB^2 - AC^2 = BD^2 - CD^2$
 $AB^2 + CD^2 = BD^2 + AC^2$

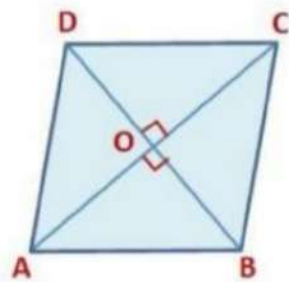


Example: Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

(REFERENCE: NCERT)

Let ABCD be rhombus in which $AB = BC = CD = DA$ Diagonals AC and BD are right angle bisectors of each other at O.

In $\triangle AOB$, $\angle AOB = 90^\circ$



$$OA = \frac{1}{2} AC \quad (\because OA = OC) \rightarrow \text{Eq 1}$$

$$\text{And } OB = \frac{1}{2} BD \quad (\because OB = OD) \rightarrow \text{Eq 2}$$

Using Pythagoras theorem, we get

$$OA^2 + OB^2 = AB^2$$

From Eq 1 and Eq 2

$$\left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 = AB^2 \Rightarrow AC^2 + BD^2 = 4AB^2$$

$$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

$$(\because AB = BC = CD = DA)$$

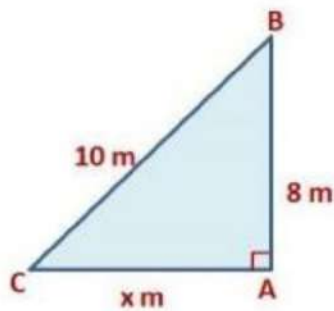
Example: A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

(REFERENCE: NCERT)

Let AB be the position of a window from the ground, then the height of the window, $AB = 8$ m and length of the ladder $BC = 10$ m

Let $AC = x$ m be the distance of the foot of the ladder from the base of the wall.

In ΔBAC , using Pythagoras Theorem we get,



$$BC^2 = AC^2 + AB^2$$

$$10^2 = x^2 + 8^2 \Rightarrow 100 = x^2 + 64 \Rightarrow 36 = x^2$$

$$x = 6$$

$$AC = 6 \text{ m}$$

$\therefore$ The distance of the foot of the ladder from the base of the wall is 6 m.

The converse of Pythagoras Theorem

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Given: A triangle ABC in which $AC^2 = AB^2 + BC^2$

To Prove: $\angle B = 90^\circ$

Construction: Construct a ΔPQR right-angled at Q, such that $PQ = AB$ and $QR = BC$

Proof: From ΔPQR we have,