Chapter – 10

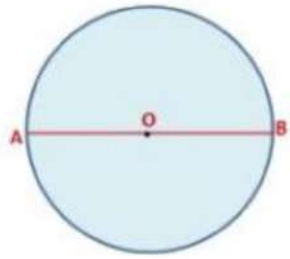
Circles

Introduction to Circles

Introduction
We use circles extensively in our daily life. Tires of a vehicle, football, wall clock, Camera lenses, cake, and pies are all circular.

A circle is a collection of all points in a plane that are at a constant distance (radius) from the fixed point (centre).

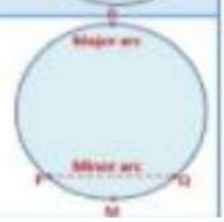
O	Center of the Circle
OA	Radius of the Circle
AB	Diameter of the Circle

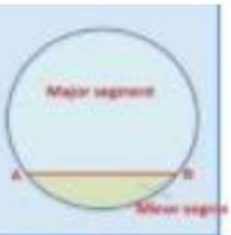
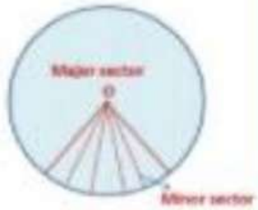


1. The diameter of a circle is twice the radius.

2. Two or more circles having the same Center are called concentric circles.

Important Terms Related to Circle

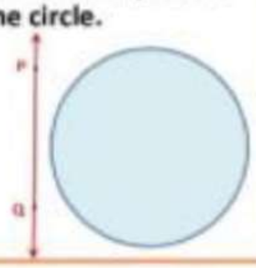
Chord	It is the line segment joining any two points on the circumference of the circle. Diameter is the longest chord of the circle.	
Arc	A continuous piece of a circle is called an arc. In the given figure, P and Q are two points on the circle which divide it into parts, called the arcs. $\overparen{QRP}$ is the major arc $\overparen{PMQ}$ is the minor arc	

Segment	The region between a chord and either of its arcs is called a segment of the circle. Minor Segment – The segment formed between minor arc and the chord. Major Segment - The segment formed between major arc and the chord. 
Sector	The region between an arc and the two radii, joining the ends of the arc to the Center, is called a sector. Minor Sector – The sector formed by a minor arc Major Sector – The sector formed by a major arc 

Let us consider a circle and a line PQ. Three possible cases can arise according to the position of the line PQ with respect to the circle.

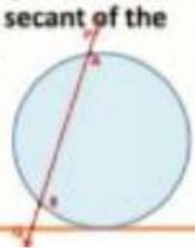
Non Intersecting Line

- There is no common point between the circle and the line PQ
- The line PQ is said to be non - intersecting line with respect to the circle.



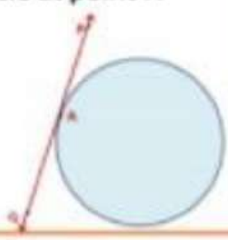
Secant

- There are two common points A and B between the circle and the line PQ.
- As line PQ is intersecting the circle at two points,so it is called secant of the circle.



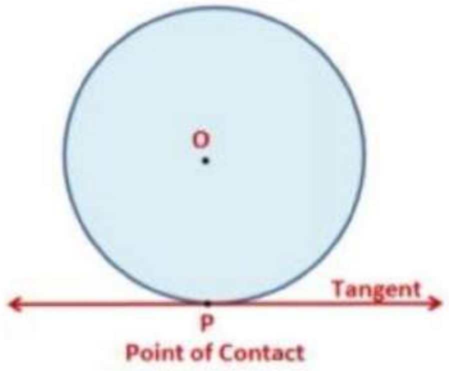
Tangent

- There is only one common point between the circle and the line PQ.
- The line AB is said to be the tangent to the circle at point P.



Tangent to a Circle

Tangent to a Circle



• A tangent to a circle is a line that intersects the circle at one point only.

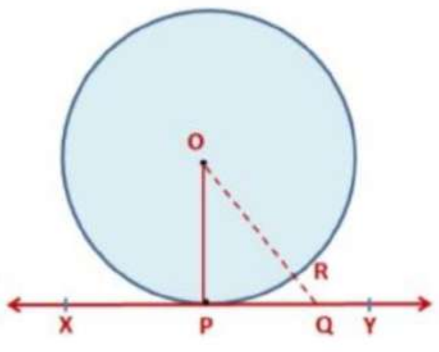
The word tangent is derived from the Latin word 'tangere' which means to touch and was introduced by the Danish mathematician Thomas Fineke in 1583.

• The common point of the tangent and the circle is called the point of contact and the tangent touches the circle at the common point.

• The tangent to a circle is a special case of the secant when the two endpoints of its corresponding chord coincide.

• There can only be a maximum of two parallel tangents which can be drawn to the opposite sides of the center.

Theorem: The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Given: A circle with center O and a tangent XY to the circle at a point P.

To Prove: OP ⊥ XY

Construction: Take point Q on XY other than P and join OQ.

Proof: Here, point Q must lie outside the circle because if it lies inside the circle then XY will become a secant and not a tangent to the circle.

Therefore, OQ is longer than the radius OP of the circle.

That is, $OQ > OP$

Now, this is true for every point on the line XY except the point P.

OP is the shortest of all the distances of the point O to the points of XY. So OP is perpendicular to XY. (OP ⊥ XY)

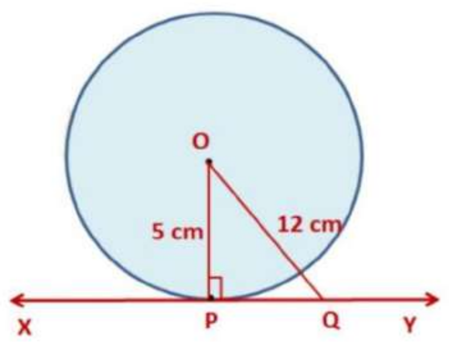
By above theorem we can conclude that at any point on a circle there can only be one tangent.

The line containing the radius through the point of contact is also sometimes called the 'normal' to the circle at the point.

Example: A tangent PQ at a point P of a circle of radius 5 cm meets a line through the center O at a point Q, so that $OQ = 12$ cm. Find the length of PQ.

(REFERENCE: NCERT)

We know that tangent at any point of a circle is perpendicular to the radius through the point of contact.



Here, OP ⊥ XY

In right-angled Δ OPQ

$OQ^2 = OP^2 + PQ^2$

(By Pythagoras Theorem)

$\Rightarrow 12^2 = 5^2 + PQ^2$

$\Rightarrow 144 = 25 + PQ^2 \Rightarrow 119 = PQ^2$

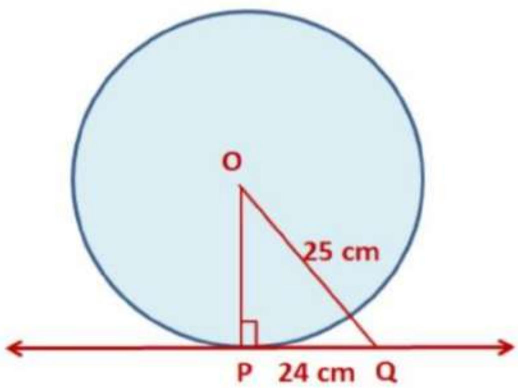
$PQ = \sqrt{119}$ cm

Example: Find the radius of a circle, if the length of the tangent from a point at a distance of 25 cm from the centre of the circle, is 16 cm.

Let PQ be a tangent drawn from point Q to the circle with centre O

such that OQ = 25 cm and PQ = 24 cm.

We know that tangent at any point of a circle is perpendicular to the radius through the point of contact.



In right-angled Δ OPQ, OP ⊥ PQ,

$OQ^2 = OP^2 + PQ^2$

(By Pythagoras Theorem)

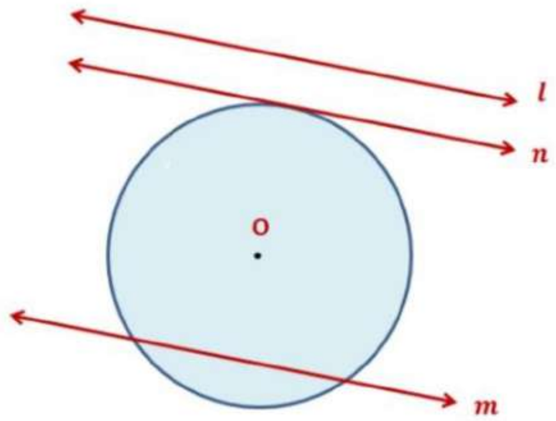
$\Rightarrow 25^2 = OP^2 + 24^2$

$\Rightarrow 625 = OP^2 + 576 \Rightarrow 49 = OP^2$

$OP = 7cm$

The radius of the circle is 7 cm.

Example: Draw a circle and two lines parallel to a given line such that one is a tangent and the other a secant to the circle.



Step 1: Draw a circle with centre O and a line l.

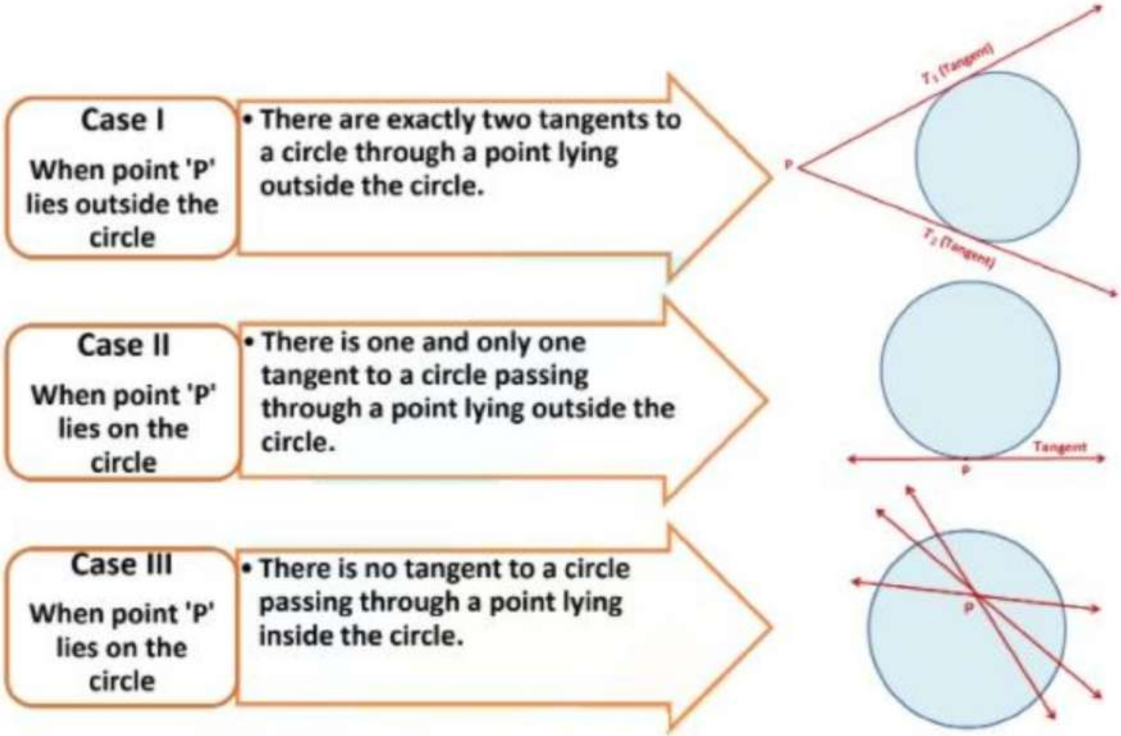
Step 2: Draw two lines parallel to l, such that one line is a tangent to the circle and the other line is secant to the circle.

Here, line m is the secant and line n is the tangent.

Tangent from a Point on a Circle

Number of Tangents from a point on a circle

The number of tangents that can be drawn from a point on a circle depends upon the position of the point with respect to the circle.



The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of the tangent from the point P to the circle.

Theorem 2: The lengths of tangents drawn from an external point to a circle are equal.

Given: A circle with center O and a point P lying outside the circle.

Let PQ and PR be the two tangents from the point P to the circle.

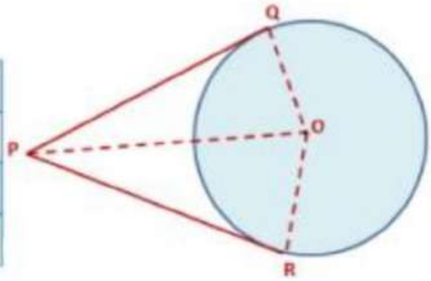
To Prove: PQ = PR

Construction: Join OP, OQ, and OR

Proof: We know that a tangent at any point of a circle is perpendicular to the radius through the point of contact. Then ∠OQP and ∠ORP are right angles.

In ΔOQP and Δ ORP,

OQ = OR	**Radii of the same circle**
OP = OP	**Common**
ΔOQP ≅ Δ ORP	**RHS Congruence Criterion**
∴ PQ = PR	**CPCT**



This theorem can also be proved by using the Pythagoras Theorem as follows:

In right-angled ΔOQP,

$OP^2 = OQ^2 + PQ^2$

(By Pythagoras Theorem)

$PQ^2 = OP^2 - OQ^2 \Longrightarrow PQ^2 = OP^2 - OR^2 \rightarrow$ Eq 1

($\because$ OQ = OR Radii of a circle)

In right-angled ΔORP,

$OP^2 = OR^2 + PR^2$

(By Pythagoras Theorem)

$PR^2 = OP^2 - OR^2 \rightarrow$ Eq 2

From Eq 1 and Eq 2, we get

PQ = PR

From the above theorem,

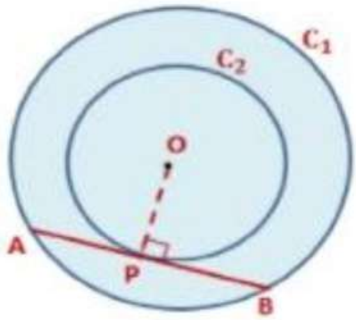
∠OPQ = ∠OPR (∵ ΔOQP ≅ Δ ORP)

⟹OP is the angle bisector of ∠QPR. Thus, the centre lies on the bisector of the angle between the two tangents.

Example: Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

(REFERENCE: NCERT)

Given: We have two concentric circles C_1 and C_2 with center O and a chord AB of the larger circle C_1 which touches the smaller circle C_2 at point P.



To Prove: AP = BP

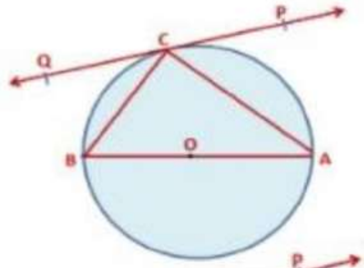
Construction: Join OP.

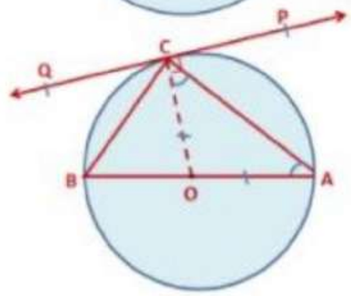
Proof: Chord AB of the larger circle C_1 touches the smaller circle C_2 at point P, so it is a tangent to C_2 and OP is the radius.

We know that a tangent at any point of a circle is perpendicular to the radius through the point of contact. So, $OP \perp AB$.

Now, AB is a chord of the circle C_1 and $OP \perp AB$. Therefore, OP is the bisector of the chord AB, as the perpendicular from the centre bisects the chord, i.e., AP = BP.

Example: In the following figure, PQ is a tangent at a point C to circle with centre O. If AB is diameter and $\angle CAB = 60°$, then find $\angle PCA$.





Here, $\angle CAB = 60^{\circ}$. Join OC.

Now, $OA = OC$ (Radii of the circle)

$\angle OCA = \angle OAC$ (angles opposite to equal sides of a triangle are equal)

$\angle OCA = \angle OAC = 60^{\circ} \rightarrow$ Eq 1

We know that a tangent at any point of a circle is perpendicular to the radius through the point of contact. So, $OC \perp PQ$.

$\Rightarrow \angle OCP = 90^{\circ}$

$\Rightarrow \angle OCA + \angle PCA = 90^{\circ} \Rightarrow 60^{\circ} + \angle PCA = 90^{\circ}$

(Using Eq 1)

$\angle PCA = 90^{\circ} - 60^{\circ} = 30^{\circ}$

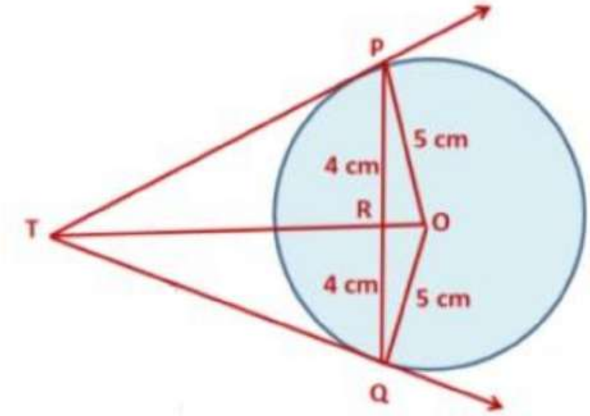
Example: PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of TP.

(REFERENCE: NCERT)

OT is a perpendicular bisector of PQ. Therefore, $PR = RQ = 4$ cm.

Let $TR = y$

In right triangle ORP, we have



$OP^2 = OR^2 + RP^2$

$OR^2 = OP^2 - RP^2 \Longrightarrow OR^2 = 5^2 - 4^2$

$OR2 = 5^2 - 4^2$

$OR^2 = 25 - 16 = 9 \Longrightarrow OR = 3$ cm

In right triangles PRT and OPT, we have

$TP^2 = TR^2 + PR^2$ and $OT^2 = TP^2 + OP^2$

$\Longrightarrow OT^2 = TR^2 + PR^2 + OP^2$ (Substituting the value of TP^2)

$\Longrightarrow (y + 3)^2 = y^2 + 4^2 + 5^2$

$\Longrightarrow (y + 3)^2 = y^2 + 41 \Longrightarrow y^2 + 6y + 9 = y^2 + 41$

$\Longrightarrow 6y = 32 \Longrightarrow y = \frac{32}{6} = \frac{16}{3}$

$\Longrightarrow TR = \frac{16}{3}$

$\therefore TP^2 = TR^2 + PR^2 \Longrightarrow TP^2 = (163)^2 + 4^2 = \frac{256}{9} + 16$

$TP^2 = \frac{256 + 144}{9} = \frac{400}{9} \Rightarrow TP = \frac{20}{3}$ cm

Example: A quadrilateral ABCD is drawn to circumscribe a circle.

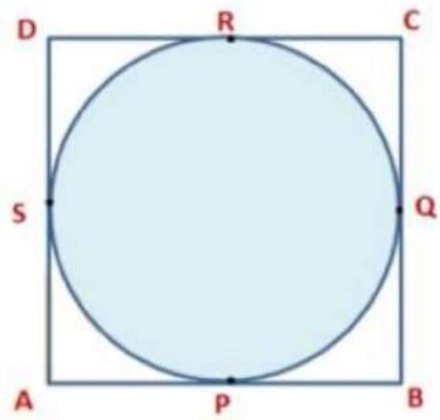
Prove that $AB + CD = AD + BC$.

(REFERENCE : NCERT)

Given: A quadrilateral ABCD circumscribing a circle

To prove: $AB + CD = AD + BC$

Proof: We know that lengths of tangents drawn from an external point to a circle are equal.



$\therefore AP = AS$ ($\because$ A is the external point) $\rightarrow$ Eq 1

$BP = BQ$ ($\because$ B is the external point) $\rightarrow$ Eq 2

$CR = CQ$ ($\because$ C is the external point) $\rightarrow$ Eq 3

$DR = DS$ ($\because$ D is the external point) $\rightarrow$ Eq 4

On adding Eq 1, 2, 3 and 4 we get,

$AP + BP + CR + DR = AS + BQ + CQ + DS$

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$AB + CD = AD + BC$