

Chapter 1. Rational and Irrational Numbers

Selina ICSE Solutions for Class 10 Maths Chapter 1 Rational and Irrational Numbers

Exercise 1(A)

Solution 1:

(i)

Any rational number between x and y

is given as $\frac{x+y}{2}$.

Thus the rational number between

$$\begin{aligned}\frac{3}{8} \text{ and } \frac{7}{12} &= \frac{\frac{3}{8} + \frac{7}{12}}{2} \\ &= \frac{\frac{9+14}{24}}{2} \\ &= \frac{23}{24 \times 2} \\ &= \frac{23}{48}\end{aligned}$$

Similarly the rational number between

$$\begin{aligned}\frac{3}{8} \text{ and } \frac{23}{48} &= \frac{\frac{3}{8} + \frac{23}{48}}{2} \\ &= \frac{\frac{18}{48} + \frac{23}{48}}{2} \\ &= \frac{\frac{18+23}{48}}{2} \\ &= \frac{41}{96}\end{aligned}$$

Thus the rational numbers

between $\frac{3}{8}$ and $\frac{7}{12}$ are: $\frac{23}{48}, \frac{41}{96}$

Thus, we have, $\frac{3}{8} < \frac{41}{96} < \frac{23}{48} < \frac{7}{12}$

(ii)

Any rational number between x and y

is given as $\frac{x+y}{2}$.

Thus the rational number between

$$\begin{aligned}\frac{1}{3} \text{ and } \frac{1}{4} &= \frac{\frac{1}{3} + \frac{1}{4}}{2} \\ &= \frac{\frac{4+3}{12}}{2} \\ &= \frac{7}{12 \times 2} \\ &= \frac{7}{24}\end{aligned}$$

Similarly, the rational number between

$$\begin{aligned}\frac{7}{24} \text{ and } \frac{1}{4} &= \frac{\frac{7}{24} + \frac{1}{4}}{2} \\ &= \frac{\frac{7}{24} + \frac{6}{24}}{2} \\ &= \frac{13}{24 \times 2} \\ &= \frac{13}{48}\end{aligned}$$

Thus, the rational numbers

between $\frac{1}{3}$ and $\frac{1}{4}$ are $\frac{7}{24}$ and $\frac{13}{48}$

Thus, we have $\frac{1}{3} < \frac{7}{24} < \frac{13}{48} < \frac{1}{4}$

Solution 2:

(i)

L.C.M of 5 and 7 = 35

$$\frac{2}{5} \text{ and } \frac{3}{7} = \frac{2 \times 7}{5 \times 7} \text{ and } \frac{3 \times 5}{5 \times 7} = \frac{14}{35} \text{ and } \frac{15}{35}$$

However, to find more rational numbers let us multiply the numerator and denominator by multiples of 35.

$$\text{Thus, we have } \frac{2}{5} = \frac{2 \times 7 \times 5}{5 \times 7 \times 5} = \frac{70}{175}$$

$$\text{and } \frac{3}{7} = \frac{3 \times 5 \times 5}{7 \times 5 \times 5} = \frac{75}{175}$$

$$\text{Since } \frac{70}{175} < \frac{75}{175}$$

$$\text{Thus, we have } \frac{70}{175} < \frac{71}{175} < \frac{72}{175} < \frac{73}{175} < \frac{74}{175} < \frac{75}{175}$$

$$\text{Thus, we have } \frac{2}{5} < \frac{71}{175} < \frac{72}{175} < \frac{73}{175} < \frac{3}{7}.$$

(ii)

L.C.M of 11 and 16 = 176

$$\frac{4}{11} \text{ and } \frac{9}{16} = \frac{4 \times 16}{11 \times 16} \text{ and } \frac{9 \times 11}{16 \times 11} = \frac{64}{176} \text{ and } \frac{99}{176}$$

$$\text{Since } \frac{64}{176} < \frac{99}{176}$$

$$\text{Thus, we have } \frac{64}{176} < \frac{65}{176} < \frac{66}{176} < \frac{67}{176} < \frac{99}{176}.$$

Thus, the three rational numbers

between $\frac{4}{11}$ and $\frac{9}{16}$ are given below:

$$\frac{4}{11} < \frac{65}{176} < \frac{66}{176} < \frac{67}{176} < \frac{9}{16}$$

Solution 3:

(i)

Both 5 and -2 are integers as well as rational numbers.

Since the set of integers is the subset of rational numbers, we have $-2 < -1 < 0 < 1 < 2 < 3 < 4 < 5$.

Thus, any three rational numbers between 5 and -2 are given below:

-2, -1 and 0

(ii)

$$-\frac{3}{4} \text{ and } \frac{1}{2}$$

L.C.M of 4 and 2 = 4

$$-\frac{3}{4} \text{ and } \frac{1}{2} = \frac{-3}{4} \text{ and } \frac{2}{4}$$

$$\text{Since } \frac{-3}{4} < \frac{2}{4}$$

$$\text{Thus, we have, } \frac{-3}{4} < \frac{-2}{4} < \frac{-1}{4} < 0 < \frac{1}{4} < \frac{2}{4}$$

Thus, the three rational numbers

between $-\frac{3}{4}$ and $\frac{1}{2}$ are given below:

$$\frac{-3}{4} < \frac{-2}{4} < \frac{-1}{4} < \frac{1}{4} < \frac{2}{4}$$

Solution 4:

Given rational numbers are 5 and 8.

Here, $5 < 8$.

$$\Rightarrow x = 5 \text{ and } y = 8$$

To insert 4 rational numbers between 5 and 8, $n = 4$

$$\Rightarrow d = \frac{y - x}{n + 1} = \frac{8 - 5}{4 + 1} = \frac{3}{5}$$

Hence,

$$x + d = 5 + \frac{3}{5} = \frac{25 + 3}{5} = \frac{28}{5} = 5\frac{3}{5}$$

$$x + 2d = 5 + 2 \times \frac{3}{5} = 5 + \frac{6}{5} = \frac{25 + 6}{5} = \frac{31}{5} = 6\frac{1}{5}$$

$$x + 3d = 5 + 3 \times \frac{3}{5} = 5 + \frac{9}{5} = \frac{25 + 9}{5} = \frac{34}{5} = 6\frac{4}{5}$$

$$x + 4d = 5 + 4 \times \frac{3}{5} = 5 + \frac{12}{5} = \frac{25 + 12}{5} = \frac{37}{5} = 7\frac{2}{5}$$

$\therefore$ Required rational numbers are $5\frac{3}{5}$, $6\frac{1}{5}$, $6\frac{4}{5}$ and $7\frac{2}{5}$.

Solution 5:

Given rational numbers are $\frac{1}{3}$ and $\frac{5}{9}$.

Here, $\frac{1}{3} < \frac{5}{9}$.

$$\Rightarrow x = \frac{1}{3} \text{ and } y = \frac{5}{9}$$

To insert 5 rational numbers between $\frac{1}{3}$ and $\frac{5}{9}$, $n = 5$

$$\Rightarrow d = \frac{y - x}{n + 1} = \frac{\frac{5}{9} - \frac{1}{3}}{5 + 1} = \frac{\frac{5 - 3}{9}}{6} = \frac{2}{9} \times \frac{1}{6} = \frac{1}{27}$$

Hence,

$$x + d = \frac{1}{3} + \frac{1}{27} = \frac{9 + 1}{27} = \frac{10}{27}$$

$$x + 2d = \frac{1}{3} + 2 \times \frac{1}{27} = \frac{9 + 2}{27} = \frac{11}{27}$$

$$x + 3d = \frac{1}{3} + 3 \times \frac{1}{27} = \frac{9 + 3}{27} = \frac{12}{27} = \frac{4}{9}$$

$$x + 4d = \frac{1}{3} + 4 \times \frac{1}{27} = \frac{9 + 4}{27} = \frac{13}{27}$$

$$x + 5d = \frac{1}{3} + 5 \times \frac{1}{27} = \frac{9 + 5}{27} = \frac{14}{27}$$

$\therefore$ Required rational numbers are $\frac{10}{27}$, $\frac{11}{27}$, $\frac{4}{9}$, $\frac{13}{27}$ and $\frac{14}{27}$.

Solution 6:

Given rational numbers are 4.6 and 8.4

$$4.6 < 8.4$$

$$\Rightarrow \frac{46}{10} < \frac{84}{10}$$

$$\Rightarrow \frac{46}{10} < \frac{46+84}{10+10} < \frac{84}{10}$$

$$\Rightarrow \frac{46}{10} < \frac{130}{20} < \frac{84}{10}$$

$$\Rightarrow \frac{46}{10} < \frac{46+130}{10+20} < \frac{130}{20} < \frac{130+84}{20+10} < \frac{84}{10}$$

$$\Rightarrow \frac{46}{10} < \frac{176}{30} < \frac{130}{20} < \frac{214}{30} < \frac{84}{10}$$

$$\Rightarrow \frac{46}{10} < \frac{46+176}{10+30} < \frac{176}{30} < \frac{176+130}{30+20} < \frac{130}{20} < \frac{130+214}{20+30} < \frac{214}{30} < \frac{84}{10}$$

$$\Rightarrow \frac{46}{10} < \frac{222}{40} < \frac{176}{30} < \frac{306}{50} < \frac{130}{20} < \frac{344}{50} < \frac{214}{30} < \frac{84}{10}$$

$$\Rightarrow 4.6 < 5.6 < 5.9 < 6.1 < 6.5 < 6.9 < 7.1 < 8.4$$

$\therefore$ Required rational numbers are 5.6, 5.9, 6.1, 6.5, 6.9 and 7.1

Solution 7:

Given rational numbers are 1 and 2.

Here, $1 < 2$.

$$\Rightarrow x = 1 \text{ and } y = 2$$

To insert 7 rational numbers between 1 and 2, $n = 7$

$$\Rightarrow d = \frac{y-x}{n+1} = \frac{2-1}{7+1} = \frac{1}{8}$$

Hence,

$$x + d = 1 + \frac{1}{8} = \frac{8+1}{8} = \frac{9}{8} = 1\frac{1}{8}$$

$$x + 2d = 1 + 2 \times \frac{1}{8} = \frac{8+2}{8} = \frac{10}{8} = \frac{5}{4} = 1\frac{1}{4}$$

$$x + 3d = 1 + 3 \times \frac{1}{8} = \frac{8+3}{8} = \frac{11}{8} = 1\frac{3}{8}$$

$$x + 4d = 1 + 4 \times \frac{1}{8} = \frac{8+4}{8} = \frac{12}{8} = \frac{3}{2} = 1\frac{1}{2}$$

$$x + 5d = 1 + 5 \times \frac{1}{8} = \frac{8+5}{8} = \frac{13}{8} = 1\frac{5}{8}$$

$$x + 6d = 1 + 6 \times \frac{1}{8} = \frac{8+6}{8} = \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4}$$

$$x + 7d = 1 + 7 \times \frac{1}{8} = \frac{8+7}{8} = \frac{15}{8} = 1\frac{7}{8}$$

$\therefore$ Required rational numbers are $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{3}{8}$, $1\frac{1}{2}$, $1\frac{5}{8}$, $1\frac{3}{4}$ and $1\frac{7}{8}$.

Solution 8:

Given rational numbers are 1.8 and 3.6

Here, $1.8 < 3.6$

$\Rightarrow x = 1.8$ and $y = 3.6$

To insert 8 rational numbers between 1.8 and 3.6, $n = 8$

$$\Rightarrow d = \frac{y - x}{n + 1} = \frac{3.6 - 1.8}{8 + 1} = \frac{1.8}{9} = 0.2$$

Hence,

$$x + d = 1.8 + 0.2 = 2.0$$

$$x + 2d = 1.8 + 2 \times 0.2 = 1.8 + 0.4 = 2.2$$

$$x + 3d = 1.8 + 3 \times 0.2 = 1.8 + 0.6 = 2.4$$

$$x + 4d = 1.8 + 4 \times 0.2 = 1.8 + 0.8 = 2.6$$

$$x + 5d = 1.8 + 5 \times 0.2 = 1.8 + 1.0 = 2.8$$

$$x + 6d = 1.8 + 6 \times 0.2 = 1.8 + 1.2 = 3.0$$

$$x + 7d = 1.8 + 7 \times 0.2 = 1.8 + 1.4 = 3.2$$

$$x + 8d = 1.8 + 8 \times 0.2 = 1.8 + 1.6 = 3.4$$

$\therefore$ Required rational numbers are 2.0, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2 and 3.4.

Solution 9:

Consider the given numbers: $-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3}$ and $\frac{11}{18}$

The L.C.M of 9, 12, and 18 is 36

Thus the given numbers are:

$$\begin{aligned} -\frac{5}{9}, \frac{7}{12}, -\frac{2}{3} \text{ and } \frac{11}{18} &= -\frac{5 \times 4}{9 \times 4}, \frac{7 \times 3}{12 \times 3}, -\frac{2 \times 12}{3 \times 12} \text{ and } \frac{11 \times 2}{18 \times 2} \\ &= -\frac{20}{36}, \frac{21}{36}, -\frac{24}{36} \text{ and } \frac{22}{36} \end{aligned}$$

Thus the numbers in ascending order are shown below:

$$-\frac{24}{36}, -\frac{20}{36}, \frac{21}{36} \text{ and } \frac{22}{36}$$

Thus the given numbers in ascending order are shown below:

$$-\frac{2}{3}, -\frac{5}{9}, \frac{7}{12} \text{ and } \frac{11}{18}$$

We need to find the difference between the largest and smallest of the above numbers.

$$\begin{aligned} \text{Thus, difference} &= \frac{11}{18} - \left(-\frac{2}{3}\right) \\ &= \frac{11}{18} + \frac{2}{3} \\ &= \frac{11}{18} + \frac{2 \times 6}{3 \times 6} \\ &= \frac{11}{18} + \frac{12}{18} \\ &= \frac{11+12}{18} \\ &= \frac{23}{18} \end{aligned}$$

We need to express this fraction as a decimal, correct to one decimal place.

Thus, we have $\frac{23}{18} = 1.2\bar{7} \approx 1.3$.

Solution 10:

Consider the given numbers: $\frac{5}{8}$, $-\frac{3}{16}$, $-\frac{1}{4}$ and $\frac{17}{32}$.

The LCM of 8, 16, 4 and 32 is 32.

Thus, the given numbers are given below:

$$\begin{aligned}\frac{5}{8}, -\frac{3}{16}, -\frac{1}{4} \text{ and } \frac{17}{32} &= \frac{5 \times 4}{8 \times 4}, -\frac{3 \times 2}{16 \times 2}, -\frac{1 \times 8}{4 \times 8} \text{ and } \frac{17 \times 1}{32 \times 1} \\ &= \frac{20}{32}, -\frac{6}{32}, -\frac{8}{32} \text{ and } \frac{17}{32}\end{aligned}$$

Thus, the numbers in descending order are shown below:

$$\frac{20}{32}, \frac{17}{32}, -\frac{6}{32} \text{ and } -\frac{8}{32}.$$

Thus, the given numbers in descending order are listed below:

$$\frac{5}{8}, \frac{17}{32}, -\frac{3}{16} \text{ and } -\frac{1}{4}.$$

We need to find the sum of the

largest and the smallest of the above numbers.

$$\begin{aligned}\text{Thus, sum} &= \frac{5}{8} + \left(-\frac{1}{4}\right) \\ &= \frac{5}{8} - \frac{1}{4} \\ &= \frac{5}{8} - \frac{1 \times 2}{4 \times 2} \\ &= \frac{5}{8} - \frac{2}{8} \\ &= \frac{3}{8}\end{aligned}$$

We need to express this fraction as a decimal, correct to two decimal places.

$$\text{Thus, we have } \frac{3}{8} = 0.375 \approx 0.38.$$

Exercise 1(B)**Solution 1:**

In a recurring decimal, if all the digits in the decimal part are not repeating, it is called a mixed recurring decimal and if all the digits in the decimal part are repeating, it is called a pure recurring decimal.

Thus, we have

(i) $0.\overline{083}$: Pure recurring decimal

(ii) $0.0\overline{83}$: Mixed recurring decimal

(iii) $0.\overline{227}$: Pure recurring decimal

(iv) $3.5\overline{4}$: Mixed recurring decimal

(v) $2.\overline{81}$: Pure recurring decimal

Solution 2:

$$(i) \frac{4}{15} = 0.26666... = 0.\overline{26}$$

$$(ii) \frac{2}{7} = 0.285714285714..... = 0.\overline{285714}$$

$$(iii) \frac{4}{9} = 0.44444..... = 0.\overline{4}$$

$$(iv) \frac{5}{24} = 0.2083333..... = 0.208\overline{3}$$

$$(v) \frac{8}{13} = 0.615384615384.... = 0.\overline{615384}$$

Solution 3(i):

Given decimal number is $0.5\overline{3}$

$$x = 0.5\overline{3} \dots (1)$$

The number of digits after the decimal point which do not have bar on them is 1.

Thus multiplying both sides of equation (1) by $10^1 = 10$

$$\Rightarrow 10x = 5.\overline{3} \dots (2)$$

$\therefore$ The right hand side of the number is only the repeating decimal part.

And the number of repeating decimal parts is 1.

Thus, multiplying both sides of equation (2) by $10^1 = 10$

$$100x = 53.\overline{3} \dots (3)$$

Subtracting equation (2) from equation (3), we have,

$$90x = 48$$

$$\Rightarrow x = \frac{48}{90}$$

$$\Rightarrow x = \frac{8}{15}$$

$$\therefore 0.5\overline{3} = \frac{8}{15}$$

Solution 3(ii):

Given decimal number is $0.2\overline{27}$

$$x = 0.2\overline{27} \quad \dots(1)$$

The number of digits after the decimal point which do not have the bar on them is 1.

Thus, multiplying both sides of equation (1) by $10^1 = 10$

$$\Rightarrow 10x = 2.\overline{27} \quad \dots(2)$$

$\therefore$ The right hand side of the number is only the repeating decimal part.

The number of repeating decimal parts is 2.

Thus, multiplying both sides of equation (2) by $10^2 = 100$.

$$1000x = 227.\overline{27} \quad \dots(3)$$

Subtracting equation (2) from equation (3), we have

$$990x = 225$$

$$\Rightarrow x = \frac{225}{990}$$

$$\Rightarrow x = \frac{5}{22}$$

$$\therefore 0.2\overline{27} = \frac{5}{22}$$

Solution 3(iii):

Given decimal number is $0.2\overline{104}$

$$x = 0.2\overline{104} \quad \dots(1)$$

The number of digits after the decimal point which do not have the bar on them is 1.

Thus, multiplying both sides of equation (1) by $10^1 = 10$

$$\Rightarrow 10x = 2.\overline{104} \quad \dots(2)$$

$\therefore$ The right hand side of the number is only the repeating decimal part.

The number of repeating decimal parts is 3.

Thus, multiplying both sides of equation (2) by $10^3 = 1000$

$$10000x = 2104.\overline{104} \quad \dots(3)$$

Subtracting equation (2) from equation (3), we have

$$9990x = 2102$$

$$\Rightarrow x = \frac{2102}{9990}$$

$$\Rightarrow x = \frac{1051}{4995}$$

$$\therefore 0.2\overline{104} = \frac{1051}{4995}$$

Solution 3(iv):

Given decimal number is $3.\dot{5}\dot{2}$

Now, $3.\dot{5}\dot{2} = 3 + 0.\dot{5}\dot{2}$

For $0.\dot{5}\dot{2}$, numerator = $52 - 5 = 47$

And, denominator = 90

$$\therefore 3.\dot{5}\dot{2} = 3 + 0.\dot{5}\dot{2}$$

$$= 3 + \frac{47}{90}$$

$$= 3\frac{47}{90}$$

Solution 3(v):

Given decimal number is $2.24\overline{689}$

Now, $2.24\overline{689} = 2 + 0.24\overline{689}$

For $0.24\overline{689}$, numerator = $24689 - 24 = 24665$

And, denominator = 99900

$$\therefore 2.24\overline{689} = 2 + 0.24\overline{689}$$

$$= 2 + \frac{24665}{99900}$$

$$= 2 + \frac{4933}{19980}$$

$$= 2\frac{4933}{19980}$$

Solution 3(vi):

Given decimal number is $0.\overline{572}$

For $0.\overline{572}$, numerator = $572 - 0 = 572$

And, denominator = 999

$$\therefore 0.\overline{572} = \frac{572}{999}$$

Solution 3(vii):

Given decimal number is $0.15\dot{8}$

For $0.15\dot{8}$, numerator = $158 - 15 = 143$

And, denominator = 900

$$\therefore 0.15\dot{8} = \frac{143}{900}$$

Solution 3(viii):

Given decimal number is $0.03\overline{84}$

For $0.03\overline{84}$, numerator = $0384 - 03 = 381$

And, denominator = 9990

$$\therefore 0.03\overline{84} = \frac{381}{9990} = \frac{127}{3330}$$

Solution 4:

$$\frac{1}{7} = 0.142857142857 \dots = 0.\overline{142857}$$

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$$

Solution 5(i):

Given number is $\frac{7}{16}$

Since $16 = 2 \times 2 \times 2 \times 2 = 2^4 = 2^4 \times 5^0$

i.e. 16 can be expressed as $2^m \times 5^n$

$\therefore \frac{7}{16}$ is convertible into the terminating decimal.

Solution 5(ii):

Given number is $\frac{23}{125}$

Since $125 = 5 \times 5 \times 5 = 5^3 = 2^0 \times 5^3$

i.e. 125 can be expressed as $2^m \times 5^n$

$\therefore \frac{23}{125}$ is convertible into the terminating decimal.

Solution 5(iii):

Given number is $\frac{9}{14}$

Since $14 = 2 \times 7 = 2^1 \times 7^1$

i.e. 14 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{9}{14}$ is not convertible into the terminating decimal.

Solution 5(iv):

Given number is $\frac{32}{45}$

Since $45 = 3 \times 3 \times 5 = 3^2 \times 5^1$

i.e. 45 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{32}{45}$ is not convertible into the terminating decimal.

Solution 5(v):

Given number is $\frac{43}{50}$

Since $50 = 2 \times 5 \times 5 = 2^1 \times 5^2$

i.e. 50 can be expressed as $2^m \times 5^n$

$\therefore \frac{43}{50}$ is convertible into the terminating decimal.

Solution 5(vi):

Given number is $\frac{17}{40}$

Since $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$

i.e. 40 can be expressed as $2^m \times 5^n$

$\therefore \frac{17}{40}$ is convertible into the terminating decimal.

Solution 5(vii):

Given number is $\frac{61}{75}$

Since $75 = 3 \times 5 \times 5 = 3^1 \times 5^2$

i.e. 75 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{61}{75}$ is not convertible into the terminating decimal.

Solution 5(viii):

Given number is $\frac{123}{250}$

Since $250 = 2 \times 5 \times 5 \times 5 = 2^1 \times 5^3$

i.e. 250 can be expressed as $2^m \times 5^n$

$\therefore \frac{123}{250}$ is convertible into the terminating decimal.

Exercise 1(C)

Solution 1:

$$\begin{aligned} \text{(i)} \quad (2 + \sqrt{2})^2 &= 2^2 + 2(2)(\sqrt{2}) + (\sqrt{2})^2 \\ &= 4 + 4\sqrt{2} + 2 = 6 + 4\sqrt{2} \end{aligned}$$

Irrational

$$\begin{aligned} \text{(ii)} \quad (3 - \sqrt{3})^2 &= (3)^2 - 2(3)(\sqrt{3}) + (\sqrt{3})^2 \\ &= 9 - 6\sqrt{3} + 3 \\ &= 12 - 6\sqrt{3} = 6(2 - \sqrt{3}) \end{aligned}$$

Irrational

$$\begin{aligned} \text{(iii)} \quad (5 + \sqrt{5})(5 - \sqrt{5}) &= (5)^2 - (\sqrt{5})^2 \\ &= 25 - 5 = 20 \end{aligned}$$

Rational

$$\begin{aligned} \text{(iv)} \quad (\sqrt{3} - \sqrt{2})^2 &= (\sqrt{3})^2 - 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2 \\ &= 3 - 2\sqrt{6} + 2 = 5 - 2\sqrt{6} \end{aligned}$$

Irrational

$$\text{(v)} \quad \left(\frac{3}{2\sqrt{2}}\right)^2 = \frac{(3)^2}{(2\sqrt{2})^2} = \frac{9}{4 \times 2} = \frac{9}{8} \text{ Rational}$$

$$\text{(vi)} \quad \left(\frac{\sqrt{7}}{6\sqrt{2}}\right)^2 = \frac{(\sqrt{7})^2}{(6\sqrt{2})^2} = \frac{7}{36 \times 2} = \frac{7}{72} \text{ Rational}$$

Solution 2:

$$\begin{aligned} \text{(i)} \quad \left(\frac{3\sqrt{5}}{5}\right)^2 &= \frac{3^2(\sqrt{5})^2}{5^2} \\ &= \frac{9 \times 5}{25} \\ &= \frac{9}{5} \\ &= 1\frac{4}{5} \end{aligned}$$

(ii)

$$\begin{aligned} (\sqrt{3} + \sqrt{2})^2 &= (\sqrt{3})^2 + 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2 \\ &= 3 + 2\sqrt{6} + 2 = 5 + 2\sqrt{6} \end{aligned}$$

(iii)

$$\begin{aligned} (\sqrt{5} - 2)^2 &= (\sqrt{5})^2 - 2(\sqrt{5})(2) + (2)^2 \\ &= 5 - 4\sqrt{5} + 4 \\ &= 9 - 4\sqrt{5} \end{aligned}$$

(iv)

$$\begin{aligned} (3 + 2\sqrt{5})^2 &= 3^2 + 2(3)(2\sqrt{5}) + (2\sqrt{5})^2 \\ &= 9 + 12\sqrt{5} + 20 \\ &= 29 + 12\sqrt{5} \end{aligned}$$

Solution 3:

(i) False

(ii) $2\sqrt{4} + 2 = 2 \times 2 + 2 = 4 + 2 = 6$ which is true

(iii) $3\sqrt{7} - 2\sqrt{7} = \sqrt{7}$ True.

(iv) False because

$$\frac{2}{7} = 0.285714$$

which is recurring and non-terminating and hence it is rational

(v) True because $\frac{5}{11} = 0.\overline{45}$ which is recurring and non-terminating

(vi) True

(vii) False

(viii) True.

Solution 4:

Given Universal set is

$$\left\{-6, -5\frac{3}{4}, -\sqrt{4}, -\frac{3}{5}, -\frac{3}{8}, 0, \frac{4}{5}, 1, 1\frac{2}{3}, \sqrt{8}, 3.01, \pi, 8.47\right\}$$

(i)

We need to find the set of rational numbers.

Rational numbers are numbers of the form $\frac{p}{q}$, where $q \neq 0$.

$$U = \left\{-6, -5\frac{3}{4}, -\sqrt{4}, -\frac{3}{5}, -\frac{3}{8}, 0, \frac{4}{5}, 1, 1\frac{2}{3}, \sqrt{8}, 3.01, \pi, 8.47\right\}$$

Clearly, $-5\frac{3}{4}$, $-\frac{3}{5}$, $-\frac{3}{8}$, $\frac{4}{5}$ and $1\frac{2}{3}$ are of the form $\frac{p}{q}$.

Hence, they are rational numbers.

Since the set of integers is a subset of rational numbers,

-6 , 0 and 1 are also rational numbers.

Thus, decimal numbers 3.01 and 8.47 are also rational numbers

because they are terminating decimals.

Hence, from the above set, the set of rational

numbers is Q , and $Q = \left\{-6, -5\frac{3}{4}, -\frac{3}{5}, -\frac{3}{8}, 0, \frac{4}{5}, 1, 1\frac{2}{3}, 3.01, 8.47\right\}$

(ii)

We need to find the set of irrational numbers.

Irrational numbers are numbers which are not rational.

From the above subpart, the set of rational

numbers is Q ,

$$\text{and } Q = \left\{-6, -5\frac{3}{4}, -\frac{3}{5}, -\frac{3}{8}, 0, \frac{4}{5}, 1, 1\frac{2}{3}, 3.01, 8.47\right\}$$

Set of irrational numbers is the set of complement of the rational numbers over real numbers.

Here the set of irrational numbers is $U - Q = \{\sqrt{8}, \pi\}$

(iii)

We need to find the set of integers.

Set of integers consists of zero, the natural numbers and their additive inverses.

The set of integers is $\mathbb{Z}$

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Here the set of integers is $U \cap \mathbb{Z} = \{-6, \sqrt{4}, 0, 1\}$.

(iv)

We need to find the set of non-negative integers.

Set of non-negative integers consists of zero and the natural numbers.

The set of non-negative integers is $\mathbb{Z}^+$ and

$$\mathbb{Z}^+ = \{0, 1, 2, 3, \dots\}$$

Here the set of integers is $U \cap \mathbb{Z}^+ = \{0, 1\}$

Solution 5:

$$\begin{array}{r} 1.73209\dots \\ 1 \overline{) 3.0000000000} \\ \underline{-1} \\ 27 \overline{) 200} \\ \underline{-189} \\ 343 \overline{) 1100} \\ \underline{-1029} \\ 3462 \overline{) 7100} \\ \underline{-6924} \\ 346409 \overline{) 17160000} \\ \underline{-311841} \\ 144815900\dots \end{array}$$

$\Rightarrow \sqrt{3} = 1.73209\dots$ which is an irrational number.

$$\begin{array}{r} 2.23606\dots \\ 1 \overline{) 5.0000000000\dots} \\ \underline{-4} \\ 42 \overline{) 100} \\ \underline{-84} \\ 443 \overline{) 1600} \\ \underline{-1329} \\ 4466 \overline{) 27100} \\ \underline{-26796} \\ 447206 \overline{) 3040000} \\ \underline{-2683236} \\ 356764\dots \end{array}$$

$\sqrt{5} = 2.23606\dots$ which is an irrational number.

Solution 6:

Let us suppose that $\sqrt{3}$ and $\sqrt{5}$ are rational numbers

$$\therefore \sqrt{3} = \frac{a}{b} \text{ and } \sqrt{5} = \frac{x}{y} \text{ (Where } a, b \in \mathbb{Z} \text{ and } b, y \neq 0, x, y)$$

Squaring both sides

$$3 = \frac{a^2}{b^2}, 5 = \frac{x^2}{y^2}$$

$$3b^2 = a^2, 5y^2 = x^2 \quad \dots (*)$$

$\Rightarrow a^2$ and x^2 are odd as $3b^2$ and $5y^2$ are odd.

$\Rightarrow a$ and x are odd....(1)

Let $a = 3c, x = 5z$

$$a^2 = 9c^2, x^2 = 25z^2$$

$$3b^2 = 9c^2, 5y^2 = 25z^2 \text{ (From equation } (*) \text{)}$$

$$\Rightarrow b^2 = 3c^2, y^2 = 5z^2$$

$\Rightarrow b^2$ and y^2 are odd as $3c^2$ and $5z^2$ are odd.

$\Rightarrow b$ and y are odd...(2)

From equation (1) and (2) we get a, b, x, y are odd integers.

i.e., a, b , and x, y have common factors 3 and 5 this contradicts our assumption that $\frac{a}{b}$ and $\frac{x}{y}$ are rational i.e., a, b and x, y do not have any common factors other than.

$$\Rightarrow \frac{a}{b} \text{ and } \frac{x}{y} \text{ is not rational}$$

$$\Rightarrow \sqrt{3} \text{ and } \sqrt{5} \text{ are irrational.}$$

Solution 7:

$\sqrt{3} + 5$ and $\sqrt{5} - 3$ are irrational numbers whose sum is irrational.

$$(\sqrt{3} + 5) + (\sqrt{5} - 3) = \sqrt{3} + \sqrt{5} + 5 - 3 = \sqrt{3} + \sqrt{5} + 2 \text{ which is irrational.}$$

Solution 8:

$\sqrt{3} + 5$ and $4 - \sqrt{3}$ are two irrational numbers whose sum is rational.

$$(\sqrt{3} + 5) + (4 - \sqrt{3}) = \sqrt{3} + 5 + 4 - \sqrt{3} = 9$$

Solution 9:

$\sqrt{3} + 2$ and $\sqrt{2} - 3$ are two irrational numbers whose difference is irrational.

$$(\sqrt{3} + 2) - (\sqrt{2} - 3) = \sqrt{3} + 2 - \sqrt{2} + 3 = \sqrt{3} - \sqrt{2} + 5 \text{ which is irrational.}$$

Solution 10:

$\sqrt{5} - 3$ and $\sqrt{5} + 3$ are irrational numbers whose difference is rational.

$$(\sqrt{5} - 3) - (\sqrt{5} + 3) = \sqrt{5} - 3 - \sqrt{5} - 3 = -6 \text{ which is rational.}$$

Solution 11:

Consider two irrational numbers $(5 + \sqrt{2})$ and $(\sqrt{5} - 2)$

Thus, the product, $(5 + \sqrt{2}) \times (\sqrt{5} - 2) = 5\sqrt{5} - 10 + \sqrt{10} - 2\sqrt{2}$ is irrational.

Solution 12:

$(\sqrt{3} + \sqrt{2})$ and $(\sqrt{3} - \sqrt{2})$ are irrational numbers whose product is rational.

$$(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$$

Solution 13:

$$(i) 3\sqrt{5} = \sqrt{3^2 \times 5} = \sqrt{45}, 4\sqrt{3} = \sqrt{4^2 \times 3} = \sqrt{48}$$

$$\text{and } 45 < 48 \therefore \sqrt{45} < \sqrt{48} \Rightarrow 3\sqrt{5} < 4\sqrt{3}$$

$$(ii) 2\sqrt[3]{5} = \sqrt[3]{2^3 \times 5} = \sqrt[3]{40}, 3\sqrt[3]{2} = \sqrt[3]{3^3 \times 2} = \sqrt[3]{54}$$

$$\text{and } 40 < 54 \Rightarrow \sqrt[3]{40} < \sqrt[3]{54}$$

$$\Rightarrow 2\sqrt[3]{5} < 3\sqrt[3]{2}$$

$$(iii) 6\sqrt{5} = \sqrt{6^2 \times 5} = \sqrt{180}$$

$$7\sqrt{3} = \sqrt{7^2 \times 3} = \sqrt{147}$$

$$8\sqrt{2} = \sqrt{8^2 \times 2} = \sqrt{128}$$

$$\text{and } 128 < 147 < 180$$

$$\therefore \sqrt{128} < \sqrt{147} < \sqrt{180}$$

$$\Rightarrow 8\sqrt{2} < 7\sqrt{3} < 6\sqrt{5}$$

Solution 14:

$$(i) 2\sqrt[4]{6} = \sqrt[4]{2^4 \times 6} = \sqrt[4]{96}$$

$$3\sqrt[4]{2} = \sqrt[4]{3^4 \times 2} = \sqrt[4]{162}$$

$$\text{Since } 162 > 96$$

$$\Rightarrow \sqrt[4]{162} > \sqrt[4]{96}$$

$$\Rightarrow 3\sqrt[4]{2} > 2\sqrt[4]{6}$$

$$(ii) 7\sqrt{3} = \sqrt{7^2 \times 3} = \sqrt{141}$$

$$3\sqrt{7} = \sqrt{3^2 \times 7} = \sqrt{63}$$

$$141 > 63 \Rightarrow \sqrt{141} > \sqrt{63}$$

$$\Rightarrow 7\sqrt{3} > 3\sqrt{7}$$

Solution 15:

$$(i) \sqrt[6]{15} = (15)^{\frac{1}{6}} \text{ and } \sqrt[4]{12} = (12)^{\frac{1}{4}}$$

Make powers $\frac{1}{6}$ and $\frac{1}{4}$ same

L.C.M. of 6,4 is 12

$$\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$$

$$\text{and } \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

$$\Rightarrow \sqrt[6]{15} = (15)^{\frac{1}{6}} = (15)^{\frac{2}{12}} = (15^2)^{\frac{1}{12}} = (225)^{\frac{1}{12}}$$

$$\text{and } \sqrt[4]{12} = (12)^{\frac{1}{4}} = (12)^{\frac{3}{12}} = (12^3)^{\frac{1}{12}} = (1728)^{\frac{1}{12}}$$

$$\Rightarrow 1272 > 225$$

$$\Rightarrow (1728)^{\frac{1}{12}} > (225)^{\frac{1}{12}}$$

$$\Rightarrow \sqrt[4]{12} > \sqrt[6]{15}$$

$$(ii) \sqrt{24} = (24)^{\frac{1}{2}} \text{ and } \sqrt[3]{35} = (35)^{\frac{1}{3}}$$

L.C.M. of 2 and 3 is 6.

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}, \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

$$\Rightarrow (24)^{\frac{1}{2}} = (24)^{\frac{3}{6}} = (24^3)^{\frac{1}{6}} = (13824)^{\frac{1}{6}}$$

$$(35)^{\frac{1}{3}} = (35)^{\frac{2}{6}} = (35^2)^{\frac{1}{6}} = (1225)^{\frac{1}{6}}$$

$$\Rightarrow 13824 > 1225$$

$$\Rightarrow (13824)^{\frac{1}{6}} > \sqrt[3]{35}$$

$$\Rightarrow \sqrt{24} > \sqrt[3]{35}$$

Solution 16:

We know that $5 = \sqrt{25}$ and $6 = \sqrt{36}$.

Thus consider the numbers,

$$\sqrt{25} < \sqrt{26} < \sqrt{27} < \sqrt{28} < \sqrt{29} < \sqrt{30} < \sqrt{31} < \sqrt{32} < \sqrt{33} < \sqrt{34} < \sqrt{35} < \sqrt{36}$$

Therefore, any two irrational numbers between 5 and 6 is $\sqrt{27}$ and $\sqrt{28}$

Solution 17:

We know that $2\sqrt{5} = \sqrt{4 \times 5} = \sqrt{20}$ and $3\sqrt{3} = \sqrt{27}$

Thus, we have, $\sqrt{20} < \sqrt{21} < \sqrt{22} < \sqrt{23} < \sqrt{24} < \sqrt{25} < \sqrt{26} < \sqrt{27}$

So any five irrational numbers between $2\sqrt{5}$ and $3\sqrt{3}$ are:

$$\sqrt{21}, \sqrt{22}, \sqrt{23}, \sqrt{24} \text{ and } \sqrt{26}$$

Solution 18:

We want rational numbers a/b and c/d such that: $\sqrt{2} < a/b < c/d < \sqrt{3}$

Consider any two rational numbers between 2 and 3 such that they are perfect squares.

Let us take 2.25 and 2.56 as $\sqrt{2.25} = 1.5$ and $\sqrt{2.56} = 1.6$

Thus we have,

$$\sqrt{2} < \sqrt{2.25} < \sqrt{2.56} < \sqrt{3}$$

$$\Rightarrow \sqrt{2} < 1.5 < 1.6 < \sqrt{3}$$

$$\Rightarrow \sqrt{2} < \frac{15}{10} < \frac{16}{10} < \sqrt{3}$$

$$\Rightarrow \sqrt{2} < \frac{3}{2} < \frac{8}{5} < \sqrt{3}$$

Therefore any two rational numbers between $\sqrt{2}$ and $\sqrt{3}$ are: $\frac{3}{2}$ and $\frac{8}{5}$

Solution 19:

Consider some rational numbers between 3 and 5 such that they are perfect squares.

Let us take, 3.24, 3.61, 4, 4.41 and 4.84 as

$$\sqrt{3.24} = 1.8, \sqrt{3.61} = 1.9, \sqrt{4} = 2, \sqrt{4.41} = 2.1 \text{ and } \sqrt{4.84} = 2.2$$

Thus we have,

$$\sqrt{3} < \sqrt{3.24} < \sqrt{3.61} < \sqrt{4} < \sqrt{4.41} < \sqrt{4.84} < \sqrt{5}$$

$$\Rightarrow \sqrt{3} < 1.8 < 1.9 < 2 < 2.1 < 2.2 < \sqrt{5}$$

$$\Rightarrow \sqrt{3} < \frac{18}{10} < \frac{19}{10} < 2 < \frac{21}{10} < \frac{22}{10} < \sqrt{5}$$

$$\Rightarrow \sqrt{3} < \frac{9}{5} < \frac{19}{10} < 2 < \frac{21}{10} < \frac{11}{5} < \sqrt{5}$$

Therefore, any three rational numbers between $\sqrt{3}$ and $\sqrt{5}$ are:

$$\frac{9}{5}, \frac{19}{10} \text{ and } \frac{21}{10}$$

Exercise 1(D)

Solution 1:

(i) $\sqrt{180} = \sqrt{2 \times 2 \times 5 \times 3 \times 3} = 6\sqrt{5}$ Which is irrational

$\therefore \sqrt{180}$ is a surds

(ii) $\sqrt[4]{27} = \sqrt[4]{3 \times 3 \times 3}$ Which is irrational

$\therefore \sqrt[4]{27}$ is a surds

(iii) $\sqrt[5]{128} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2\sqrt[5]{4}$

$\therefore \sqrt[5]{128}$ is a surds

(iv) $\sqrt[3]{64} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2} = 4$ which is rational

$\therefore \sqrt[3]{64}$ is not a surds

(v) $\sqrt[3]{25}\sqrt[3]{40} = \sqrt[3]{5 \times 5 \times 2 \times 2 \times 2 \times 5} = 2 \times 5 = 10$

$\therefore \sqrt[3]{25}\sqrt[3]{40}$ is not a surds

(vi) $\sqrt[3]{-125} = \sqrt[3]{-5 \times -5 \times -5} = -5$

$\therefore$ is not a surds

(vii) $\sqrt{\pi}$ not a surds as π is irrational

(viii) $\sqrt{3 + \sqrt{2}}$ is not a surds because $3 + \sqrt{2}$ is irrational.

Solution 2:

(i) $5\sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$ which is rational

$\therefore$ lowest rationalizing factor is $\sqrt{2}$

(ii) $\sqrt{24} = \sqrt{2 \times 2 \times 2 \times 3} = 2\sqrt{6}$

$\therefore$ lowest rationalizing factor is $\sqrt{6}$

(iii) $(\sqrt{5} - 3)(\sqrt{5} + 3) = (\sqrt{5})^2 - (3)^2 = 5 - 9 = -4$

$\therefore$ lowest rationalizing factor is $(\sqrt{5} + 3)$

(iv) $7 - \sqrt{7}$

$(7 - \sqrt{7})(7 + \sqrt{7}) = 49 - 7 = 42$

Therefore, lowest rationalizing factor is $(7 + \sqrt{7})$

(v) $\sqrt{18} - \sqrt{50}$

$\sqrt{18} - \sqrt{50} = \sqrt{2 \times 3 \times 3} - \sqrt{5 \times 5 \times 2}$

$= 3\sqrt{2} - 5\sqrt{2} = -2\sqrt{2}$

$\therefore$ lowest rationalizing factor is $\sqrt{2}$

(vi) $\sqrt{5} - \sqrt{2}$

$(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 3$

Therefore lowest rationalizing factor is $\sqrt{5} + \sqrt{2}$

(vii) $\sqrt{13} + 3$

$(\sqrt{13} + 3)(\sqrt{13} - 3) = (\sqrt{13})^2 - 3^2 = 13 - 9 = 4$

Its lowest rationalizing factor is $\sqrt{13} - 3$

$$(viii) 15 - 3\sqrt{2}$$

$$\begin{aligned} 15 - 3\sqrt{2} &= 3(5 - \sqrt{2}) \\ &= 3(5 - \sqrt{2})(5 + \sqrt{2}) \\ &= 3 \times [5^2 - (\sqrt{2})^2] \\ &= 3 \times [25 - 2] \\ &= 3 \times 23 \\ &= 69 \end{aligned}$$

Its lowest rationalizing factor is $5 + \sqrt{2}$

$$(ix) 3\sqrt{2} + 2\sqrt{3}$$

$$\begin{aligned} 3\sqrt{2} + 2\sqrt{3} &= (3\sqrt{2} + 2\sqrt{3})(3\sqrt{2} - 2\sqrt{3}) \\ &= (3\sqrt{2})^2 - (2\sqrt{3})^2 \\ &= 9 \times 2 - 4 \times 3 \\ &= 18 - 12 \\ &= 6 \end{aligned}$$

its lowest rationalizing factor is $3\sqrt{2} - 2\sqrt{3}$

Solution 3:

(i)

$$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

(ii)

$$\frac{2\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2}{5} \sqrt{15}$$

(iii)

$$\begin{aligned} \frac{1}{\sqrt{3} - \sqrt{2}} \times \left(\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} \right) &= \frac{\sqrt{3} + \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{\sqrt{3} + \sqrt{2}}{3 - 2} \\ &= \sqrt{3} + \sqrt{2} \end{aligned}$$

(iv)

$$\begin{aligned} \frac{3}{\sqrt{5} + \sqrt{2}} \times \left(\frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} \right) &= \frac{3(\sqrt{5} - \sqrt{2})}{(\sqrt{5})^2 - (\sqrt{2})^2} = \frac{3(\sqrt{5} - \sqrt{2})}{5 - 2} \\ &= \sqrt{5} - \sqrt{2} \end{aligned}$$

(v)

$$\begin{aligned} \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \times \left(\frac{2 - \sqrt{3}}{2 - \sqrt{3}} \right) &= \frac{(2 - \sqrt{3})^2}{(2)^2 - (\sqrt{3})^2} = \frac{4 + 3 - 4\sqrt{3}}{4 - 3} \\ &= \frac{7 - 4\sqrt{3}}{1} = 7 - 4\sqrt{3} \end{aligned}$$

(vi)

$$\begin{aligned} \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} &= \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - (1)^2} = \frac{3 + 1 + 2\sqrt{3}}{3 - 1} = \frac{4 + 2\sqrt{3}}{2} \\ &= \frac{2(2 + \sqrt{3})}{2} = 2 + \sqrt{3} \end{aligned}$$

(vii)

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{3+2-2\sqrt{6}}{3-2}$$
$$= 5 - 2\sqrt{6}$$

(viii)

$$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$$
$$= \frac{6+5-2\sqrt{30}}{(\sqrt{6})^2 - (\sqrt{5})^2} = \frac{11-2\sqrt{30}}{6-5} = 11 - 2\sqrt{30}$$

(ix)

$$\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{(2\sqrt{5}+3\sqrt{2})^2}{(2\sqrt{5})^2 - (3\sqrt{2})^2}$$
$$= \frac{4 \times 5 + 9 \times 2 + 12\sqrt{10}}{20 - 18}$$
$$= \frac{20 + 18 + 12\sqrt{10}}{2} = \frac{38 + 12\sqrt{10}}{2} = \frac{2(19 + 6\sqrt{10})}{2}$$
$$= 19 + 6\sqrt{10}$$

Solution 4:

$$(i) \frac{2 + \sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = a + b\sqrt{3}$$

$$\frac{(2 + \sqrt{3})^2}{(2)^2 - (\sqrt{3})^2} = a + b\sqrt{3}$$

$$\frac{4 + 3 + 4\sqrt{3}}{4 - 3} = a + b\sqrt{3}$$

$$7 + 4\sqrt{3} = a + b\sqrt{3}$$

$$a = 7, b = 4$$

$$(ii) \frac{\sqrt{7} - 2}{\sqrt{7} + 2} \times \frac{\sqrt{7} - 2}{\sqrt{7} - 2} = a\sqrt{7} + b$$

$$\frac{(\sqrt{7} - 2)^2}{(\sqrt{7})^2 - (2)^2} = a\sqrt{7} + b$$

$$\frac{7 + 4 - 4\sqrt{7}}{7 - 4} = a\sqrt{7} + b$$

$$\frac{11 - 4\sqrt{7}}{3} = a\sqrt{7} + b$$

$$a = \frac{-4}{3}, b = \frac{11}{3}$$

$$(iii) \frac{3}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = a\sqrt{3} - b\sqrt{2}$$

$$\frac{3(\sqrt{3} + \sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2} = a\sqrt{3} - b\sqrt{2}$$

$$\frac{3(\sqrt{3} + \sqrt{2})}{3 - 2} = a\sqrt{3} - b\sqrt{2}$$

$$(3\sqrt{3} + 3\sqrt{2}) = a\sqrt{3} - b\sqrt{2}$$

$$\Rightarrow a = 3, b = -3$$

$$(iv) \frac{5 + 3\sqrt{2}}{5 - 3\sqrt{2}} \times \frac{5 + 3\sqrt{2}}{5 + 3\sqrt{2}} = a + b\sqrt{2}$$

$$\frac{(5 + 3\sqrt{2})^2}{(5)^2 - (3\sqrt{2})^2} = a + b\sqrt{2}$$

$$\frac{25 + 18 + 30\sqrt{2}}{25 - 18} = a + b\sqrt{2}$$

$$\frac{43 + 30\sqrt{2}}{7} = a + b\sqrt{2}$$

$$a = \frac{43}{7}, b = \frac{30}{7}$$

Solution 5:

(i)

$$\frac{22}{2\sqrt{3}+1} + \frac{17}{2\sqrt{3}-1}$$

$$\frac{22(2\sqrt{3}-1) + 17(2\sqrt{3}+1)}{(2\sqrt{3}+1)(2\sqrt{3}-1)} = \frac{44\sqrt{3} - 22 + 34\sqrt{3} + 17}{(2\sqrt{3})^2 - 1}$$

$$= \frac{78\sqrt{3} - 5}{12 - 1} = \frac{78\sqrt{3} - 5}{11}$$

(ii)

$$\frac{\sqrt{2}}{\sqrt{6}-2} - \frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}} = \frac{\sqrt{2}(\sqrt{6}+\sqrt{2}) - \sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6})^2 - (\sqrt{2})^2}$$

$$= \frac{\sqrt{12} + 2 - \sqrt{18} + \sqrt{6}}{6 - 2} = \frac{2\sqrt{3} + 2 - 3\sqrt{2} + \sqrt{6}}{4}$$

Solution 6:

$$(i) x^2 = \left(\frac{\sqrt{5}-2}{\sqrt{5}+2} \right)^2 = \frac{5+4-4\sqrt{5}}{5+4+4\sqrt{5}} = \frac{9-4\sqrt{5}}{9+4\sqrt{5}}$$

$$= \frac{9-4\sqrt{5}}{9+4\sqrt{5}} \times \frac{(9-4\sqrt{5})}{(9-4\sqrt{5})} = \frac{(9-4\sqrt{5})^2}{(9)^2 - (4\sqrt{5})^2}$$

$$= \frac{81+80-72\sqrt{5}}{81-80} = 161-72\sqrt{5}$$

$$(ii) y^2 = \left(\frac{\sqrt{5}+2}{\sqrt{5}-2} \right)^2 = \frac{5+4+4\sqrt{5}}{5+4-4\sqrt{5}} = \frac{9+4\sqrt{5}}{9-4\sqrt{5}}$$

$$= \frac{9+4\sqrt{5}}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}} = \frac{(9+4\sqrt{5})^2}{(9)^2 - (4\sqrt{5})^2} = \frac{81+80+72\sqrt{5}}{81-80}$$

$$= 161+72\sqrt{5}$$

$$(iii) xy = \frac{(\sqrt{5}-2)(\sqrt{5}+2)}{(\sqrt{5}+2)(\sqrt{5}-2)} = 1$$

$$(iv) x^2 + y^2 + xy = 161 - 72\sqrt{5} + 161 + 72\sqrt{5} + 1$$

$$= 322 + 1 = 323$$

Solution 7:

$$(i) m = \frac{1}{3 - 2\sqrt{2}}$$

$$= \frac{1}{3 - 2\sqrt{2}} \times \frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}}$$

$$= \frac{3 + 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2}$$

$$= \frac{3 + 2\sqrt{2}}{9 - 8}$$

$$= 3 + 2\sqrt{2}$$

$$\Rightarrow m^2 = (3 + 2\sqrt{2})^2$$

$$= (3)^2 + 2 \times 3 \times 2\sqrt{2} + (2\sqrt{2})^2$$

$$= 9 + 12\sqrt{2} + 8$$

$$= 17 + 12\sqrt{2}$$

$$(ii) n = \frac{1}{3 + 2\sqrt{2}}$$

$$= \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$$

$$= \frac{3 - 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2}$$

$$= \frac{3 - 2\sqrt{2}}{9 - 8}$$

$$= 3 - 2\sqrt{2}$$

$$\Rightarrow n^2 = (3 - 2\sqrt{2})^2$$

$$= (3)^2 - 2 \times 3 \times 2\sqrt{2} + (2\sqrt{2})^2$$

$$= 9 - 12\sqrt{2} + 8$$

$$= 17 - 12\sqrt{2}$$

$$(iii) mn = (3 + 2\sqrt{2})(3 - 2\sqrt{2}) = (3)^2 - (2\sqrt{2})^2 = 9 - 8 = 1$$

Solution 8:

$$(i) \frac{1}{x} = \frac{1}{2\sqrt{3} + 2\sqrt{2}} \times \frac{2\sqrt{3} - \sqrt{2}}{2\sqrt{3} - 2\sqrt{2}} = \frac{2\sqrt{3} - 2\sqrt{2}}{12 - 8}$$

$$= \frac{2(\sqrt{3} - \sqrt{2})}{4} = \frac{\sqrt{3} - \sqrt{2}}{2}$$

$$(ii) x + \frac{1}{x} = 2\sqrt{3} + 2\sqrt{2} + \frac{\sqrt{3} - \sqrt{2}}{2}$$

$$= 2(\sqrt{3} + \sqrt{2}) + \frac{(\sqrt{3} - \sqrt{2})}{2}$$

$$= \frac{4(\sqrt{3} + \sqrt{2}) + (\sqrt{3} - \sqrt{2})}{2}$$

$$= \frac{4\sqrt{3} + 4\sqrt{2} + \sqrt{3} - \sqrt{2}}{2}$$

$$= \frac{5\sqrt{3} + 3\sqrt{2}}{2}$$

$$(iii) \left(x + \frac{1}{x}\right)^2 = \left(\frac{5\sqrt{3} + 3\sqrt{2}}{2}\right)^2 = \frac{75 + 18 + 30\sqrt{6}}{4}$$

$$= \frac{93 + 30\sqrt{6}}{4}$$

Solution 9:

Given that $x = 1 - \sqrt{2}$

We need to find the value of $\left(x - \frac{1}{x}\right)^3$.

Since $x = 1 - \sqrt{2}$, we have

$$\frac{1}{x} = \frac{1}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}}$$

$$\Rightarrow \frac{1}{x} = \frac{1 + \sqrt{2}}{1^2 - (\sqrt{2})^2} \quad [\text{Since } (a-b)(a+b) = a^2 - b^2]$$

$$\Rightarrow \frac{1}{x} = \frac{1 + \sqrt{2}}{1 - 2}$$

$$\Rightarrow \frac{1}{x} = \frac{1 + \sqrt{2}}{-1}$$

$$\Rightarrow \frac{1}{x} = -(1 + \sqrt{2}) \dots (1)$$

$$\text{Thus, } \left(x - \frac{1}{x}\right) = (1 - \sqrt{2}) - (-(1 + \sqrt{2}))$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 1 - \sqrt{2} + 1 + \sqrt{2}$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^3 = 2^3$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^3 = 8$$

Solution 10:

$$\text{Given } x = 5 - 2\sqrt{6}$$

We need to find $x^2 + \frac{1}{x^2}$:

Since $x = 5 - 2\sqrt{6}$, we have

$$\frac{1}{x} = \frac{1}{5 - 2\sqrt{6}}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} \times \frac{5 + 2\sqrt{6}}{5 + 2\sqrt{6}}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{(5 - 2\sqrt{6})(5 + 2\sqrt{6})}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{5^2 - (2\sqrt{6})^2}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{25 - 24}$$

$$\Rightarrow \frac{1}{x} = 5 + 2\sqrt{6} \dots (1)$$

$$\text{Thus, } \left(x - \frac{1}{x}\right) = (5 - 2\sqrt{6}) - (5 + 2\sqrt{6})$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 5 - 2\sqrt{6} - 5 - 2\sqrt{6}$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = -4\sqrt{6} \dots (2)$$

Now consider $\left(x - \frac{1}{x}\right)^2$:

Thus

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2x \times \frac{1}{x} \quad [\text{since } (a - b)^2 = a^2 - 2ab + b^2]$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = x^2 + \frac{1}{x^2} \dots (3)$$

Thus, from equations (2) and (3), we have

$$x^2 + \frac{1}{x^2} = (-4\sqrt{6})^2 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 96 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 98$$

Solution 11:

$$\begin{aligned}
\text{L.H.S.} &= \frac{1}{3-2\sqrt{2}} - \frac{1}{2\sqrt{2}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} \\
&= \frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} \\
&= \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} \\
&\quad - \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} \\
&= \frac{3+\sqrt{8}}{(3)^2-(\sqrt{8})^2} - \frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8})^2-(\sqrt{7})^2} + \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2} - \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2} + \frac{\sqrt{5}+2}{(\sqrt{5})^2-(2)^2} \\
&= \frac{3+\sqrt{8}}{9-8} - \frac{\sqrt{8}+\sqrt{7}}{8-7} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4} \\
&= 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2 \\
&= 3+2 \\
&= 5 \\
&= \text{R.H.S.}
\end{aligned}$$

Solution 12:

$$\begin{aligned}
&\frac{1}{\sqrt{3}-\sqrt{2}+1} \\
&= \frac{1}{(\sqrt{3}-\sqrt{2})+1} \times \frac{(\sqrt{3}-\sqrt{2})-1}{(\sqrt{3}-\sqrt{2})-1} \\
&= \frac{\sqrt{3}-\sqrt{2}-1}{(\sqrt{3}-\sqrt{2})^2-(1)^2} \\
&= \frac{\sqrt{3}-\sqrt{2}-1}{(\sqrt{3})^2-2\sqrt{6}+(\sqrt{2})^2-1} \\
&= \frac{\sqrt{3}-\sqrt{2}-1}{3-2\sqrt{6}+2-1} \\
&= \frac{\sqrt{3}-\sqrt{2}-1}{4-2\sqrt{6}} \\
&= \frac{(\sqrt{3}-\sqrt{2})-1}{2(2-\sqrt{6})} \\
&= \frac{\sqrt{3}-\sqrt{2}-1}{2(2-\sqrt{6})} \times \frac{2+\sqrt{6}}{2+\sqrt{6}} \\
&= \frac{2\sqrt{3}-2\sqrt{2}-2+\sqrt{18}-\sqrt{12}-\sqrt{6}}{2[(2)^2-(\sqrt{6})^2]} \\
&= \frac{2\sqrt{3}-2\sqrt{2}-2+3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{2(4-6)} \\
&= \frac{\sqrt{2}-2-\sqrt{6}}{2(-2)} \\
&= \frac{\sqrt{2}-2-\sqrt{6}}{-4} \\
&= \frac{1}{4}(2+\sqrt{6}-\sqrt{2})
\end{aligned}$$

Solution 13(i):

$$\sqrt{2} = 1.4 \text{ and } \sqrt{3} = 1.7$$

$$\begin{aligned} & \frac{1}{\sqrt{3} - \sqrt{2}} \\ &= \frac{1}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} \\ &= \frac{\sqrt{3} + \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ &= \frac{\sqrt{3} + \sqrt{2}}{3 - 2} \\ &= \sqrt{3} + \sqrt{2} \\ &= 1.7 + 1.4 \\ &= 3.1 \end{aligned}$$

Solution 13(ii):

$$\sqrt{2} = 1.4 \text{ and } \sqrt{3} = 1.7$$

$$\begin{aligned} \text{(ii)} \quad & \frac{1}{3 + 2\sqrt{2}} \\ &= \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}} \\ &= \frac{3 - 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2} \\ &= \frac{3 - 2\sqrt{2}}{9 - 8} \\ &= 3 - 2\sqrt{2} \\ &= 3 - 2(1.4) \\ &= 3 - 2.8 \\ &= 0.2 \end{aligned}$$