Relations and Functions
Part - 1

Assertion-Reasoning MCQs

Directions (Q. Nos. 46-60) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true; R is False.
(d) A is false; R is true.

46. **Assertion (A)** The relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y)$: y is divisible by $x\}$ is not an equivalence relation.

Reason (R) The relation R will be an equivalence relation, if it is reflexive, symmetric and transitive.

47. **Assertion (A)** If R is the relation defined in set $\{1,2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$, then R is reflexive

Reason (R) The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric.

48. Let R be any relation in the set A of human beings in a town at a particular time.

Assertion (A) If $R = \{(x, y) : x$ is wife of $y\}$, then R is reflexive.

Reason (R) If $R = \{(x, y) : x$ is father of $y\}$, then R is neither reflexive nor symmetric nor transitive.

49. **Assertion (A)** If $A = \{x \in z : 0 \leq x \leq 12\}$ and R is the relation in A given by $R = \{(a, b) : a = b$. Then, the set of all elements related to 1 is $\{1, 2\}$.

Reason (**R**) If R_1 and R_2 are equivalence relation in a set A, then $R_1 \cap R_2$ is an equivalence relation.

50. If R is the relation in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is even$\}$,

Assertion (**A**) R is an equivalence relation.

Reason (**R**) All elements of $\{1, 3, 5\}$ are related to all elements of $\{2, 4\}$.

51. **Assertion** (**A**) The function $f : R^* \to R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R^* is the set of all non-zero real numbers.

Reason (**R**) The function $g : N \to R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto.

52. **Assertion** (**A**) Let A and B be sets. Then, the function $f : A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is bijective.

Reason (**R**) A function f is said to be bijective, if it is both one-one and onto.

53. **Assertion** (**A**) The modulus function $f : R \to R$ given by $f(x) = |x|$ is neither one-one nor onto.

Reason (**R**) The signum function $f : R \to R$ given by $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is bijective.

54. **Assertion** (**A**) Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 are ± 4.

Reason (**R**) A function $f : A \to B$ is called a one-one function, if distinct elements of A have distinct images in B.

55. **Assertion** (**A**) The function $f : R \to R$ given by $f(x) = x^3$ is injective.

Reason (**R**) The function $f : X \to Y$ is injective, if $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in X$.

56. **Assertion** (**A**) The function $f(x) = x^2 + bx + c$, where b and c are real constants, describes onto mapping.

Reason (**R**) Let $A = \{1, 2, 3, \ldots, n\}$ and $B = \{a, b\}$. Then, the number of surjections from A into B is $2^n - 2$.

57. **Assertion** (**A**) Let a relation R defined from set $A = \{1, 2, 5, 6\}$ to A is $R = \{(1, 1), (1, 6), (6, 1)\}$, then R is symmetric relation.

Reason (**R**) A relation R in set A is called symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$ for every $a, b \in A$.

58. **Assertion** (**A**) The $f : R \to R$ given by $f(x) = [x] + x$ is one-one onto.

Reason (**R**) A function is said to be one-one and onto, if each element has unique image and range of $f(x)$ is equal to codomain of $f(x)$.

59. **Assertion** (**A**) Let $A = \{2, 4, 6\}$ and $B = \{3, 5, 7, 9\}$ and defined a function $f = \{(2, 3), (4, 5), (6, 7)\}$ from A to B. Then, f is not onto.

Reason (**R**) A function $f : A \to B$ is said to be onto, if every element of B is the image of some elements of A under f.

60. **Assertion** (**A**) Let a relation R defined from set B to B such that $B = \{1, 2, 3, 4\}$ and $R = \{1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$, then R is transitive.

Reason (**R**) A relation R in set A is called transitive, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R, \forall\, a, b, c \in A$

ANSWER KEY

Assertion-Reasoning MCQs

46. (a) *47. (d)* *48. (d)* *49. (d)* *50. (c)* *51. (c)* *52. (a)* *53. (c)* *54. (b)* *55. (a)*
56. (d) *57. (a)* *58. (a)* *59. (a)* *60. (a)*

SOLUTION

46. **Assertion** Here, $R = \{(x, y) : y$ is divisible by $x\}$ is a relation in the set $A = \{1, 2, 3, 4, 5, 6\}$.

For reflexive, we know that x is divisible by x, which is true for all $x \in A$.

$\therefore (x, x) \in R$ for all $x \in A$.

So, R is reflexive.

For symmetry, we observe that 6 is divisible by 2 i.e. $(2, 6) \in R$ but 2 is not divisible by 6 i.e. $(6, 2) \notin R$.

So, R is not symmetric.

For transitivity, let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow y$ is divisible by x and z is divisible by y.

$\Rightarrow z$ is divisible by x.

$\Rightarrow (x, z) \in R$

e.g. 2 is divisible by 1 and 4 is divisible by 2. So, 4 is divisible by 1. So, R is transitive.

Hence, R is not an equivalence relation.

47. **Assertion** Let $A = \{1, 2, 3, 4, 5, 6\}$

A relation R is defined on set A is

$$R = \{(a, b) : b = a + 1\}$$

$\therefore \quad R = \{(1, 2,), (2, 3), (3, 4), (4, 5), (5, 6)\}$

Now, $6 \in A$ but $(6, 6) \notin R$

$\therefore$ R is not reflexive.

Reason Given set $A = \{1, 2, 3\}$

A relation R on A is defined as

$$R = \{(1, 2), (2, 1)\}$$

$\because (1, 2) \in R$ and $(2, 1) \in R$.

So, R is symmetric.

48. **Assertion** Here R is not reflexive: as x cannot be wife of x.

Reason Here, R is not reflexive; as x cannot be father of x, for any x. R is not symmetric as if x is father of y, then y cannot be father of x. R is not transitive as if x is father of y and y is father of z, then x is grandfather (not father) of z.

49. **Assertion** The elements that are related to 1 will be those elements from set A which are equal to 1.

Hence, the set of elements related to 1 is {1}.

Reason Since, R_1 and R_2 are equivalence relations, therefore $(a, a) \in R_1$, $(a, a) \in R_2$, $\forall\ a \in A$.

This implies that $(a, a) \in R_1 \cap R_2, \forall a$.

Hence, $R_1 \cap R_2$ is reflexive.

Further, $(a, b) \in R_1 \cap R_2 \Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2 \Rightarrow (b, a) \in R_1$ and $(b, a) \in R_2$ $\Rightarrow (b, a) \in R_1 \cap R_2$.

Hence, $R_1 \cap R_2$ is symmetric.

Similarly , $(a, b) \in R_1 \cap R_2$ and $(b, c) \in R_1 \cap R_2$ $\Rightarrow (a, c) \in R_1$ and $(a, c) \in R_2 \Rightarrow (a, c) \in R_1 \cap R_2$.

This implies that $R_1 \cap R_2$ is transitive.

Hence, $R_1 \cap R_2$ is an equivalence relation.

50. **Assertion** Given that, $A = \{1, 2, 3, 4, 5\}$,

$$R = \{(a, b) : |a - b| \text{ is even}\}$$

Let $a \in A \Rightarrow |a - a| = 0$ (which is even),$\forall\ a$

So, R is reflexive.

Let $(a, b) \in R \Rightarrow |a - b|$ is even.

$\Rightarrow |a - b| = |-(b - a)| = |b - a|$, therefore $|b - a|$ is also even.

$\Rightarrow (b, a) \in R$. So, R is symmetric.

Now , let $(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow |a - b|$ is even and $|b - c|$ is even.

$\Rightarrow (a - b)$ is even and $(b - c)$ is even.

$\Rightarrow (a - c) = (a - b) + (b - c)$ is even

[$\because$ sum of two even integers is even]

$\Rightarrow |a - c|$ is even $\Rightarrow (a, c) \in R$.

So, R is transitive.

Hence, R is an equivalence relation.

Reason Also, no element of the {1, 3, 5} can be related to any element of {2, 4}, as all elements of {1, 3, 5} are odd and all elements of {2, 4} are even.

So, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

Hence Reason is not correct.

51. **Assertion** It is given that $f : R^* \to R^*$ is defined by

$$f(x) = \frac{1}{x}$$

For one-one, $f(x) = f(y)$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{y}$$

$$\Rightarrow \quad x = y$$

Therefore, f is one-one.

For onto, it is clear that for $y \in R^*$, there exists $x = \frac{1}{y} \in R^*$ (exists as $y \neq 0$) such that

$$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y$$

Therefore, f is onto. Thus, the given function (f) is one-one and onto.

Reason Now, consider function $g : N \to R^*$ defined by $g(x) = \frac{1}{x}$.

We have, $g(x_1) = g(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$$\Rightarrow \quad x_1 = x_2$$

Therefore, g is one-one.

Further, it is clear that g is not onto as for $1 \cdot 2 \in R^*$, there does not exist any x in N such that $g(x) = 1 \cdot 2 \Rightarrow \frac{1}{x} = 1 \cdot 2$

$$\Rightarrow \quad x = \frac{1}{1 \cdot 2} \notin N \text{ (domain)}$$

Hence, function g is one-one but not onto.

52. **Assertion** Here, $f : A \times B \to B \times A$ is defined as $f(a, b) = (b, a)$.

Let $(a_1, b_1), (a_2, b_2) \in A \times B$ such that

$$f(a_1, b_1) = f(a_2, b_2)$$

$$\Rightarrow \quad (b_1, a_1) = (b_2, a_2)$$

$$\Rightarrow \quad b_1 = b_2 \text{ and } a_1 = a_2$$

$$\Rightarrow \quad (a_1, b_1) = (a_2, b_2)$$

Therefore, f is one-one.

Now, let $(b, a) \in B \times A$ be any element.

Then, there exists $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$. [definition of f]

Therefore, f is onto. Hence, f is bijective.

53. **Assertion** Here, $f : R \to R$ is given by

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

It is seen that $f(-1) = |-1| = 1, f(1) = |1| = 1$

Therefore, $f(-1) = f(1)$ but $-1 \neq 1$

Therefore, f is not one-one.

Now, consider $-1 \in R$

It is known that $f(x)=|x|$ is always non-negative.

Thus, there does not exist any element x in domain R such that $f(x)=|x|=-1$.

Therefore, f is not onto.

Hence, the modulus function is neither one-one nor onto.

Reason $f: R \to R, f(x)=\begin{cases} 1, & \text{if} \quad x>0 \\ 0, & \text{if} \quad x=0 \\ -1, & \text{if} \quad x<0 \end{cases}$

It is seen that $f(1)=f(2)=1$ but $1 \neq 2$.
Therefore, f is not one-one.

Now, as $f(x)$ takes only three values (1, 0 or -1), therefore for the element -2 in codomain R, there does not exist any x in domain R such that $f(x)=-2$.

Therefore, f is not onto. Hence, the Signum function is neither one-one nor onto.

54. **Assertion** Consider $x^2+1=17$

$\Rightarrow \quad x^2=16$

$\Rightarrow \quad x=\pm 4$

Hence, pre-images of 17 are ± 4.

Hence, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

55. **Assertion** Here, $f: R \to R$ is given as $f(x)=x^3$.

Suppose $f(x)=f(y)$

where $x, y \in R$

$\Rightarrow \quad x^3=y^3 \quad \ldots(\text{i})$

Now, we try to show that $x=y$.

Suppose $x \neq y$, their cubes will also not be equal.

$$x^3 \neq y^3$$

However, this will be a contradiction to Eq. (i).

Therefore, $x=y$. Hence, f is injective.

Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

56. **Assertion** Given function is

$$f(x)=x^2+bx+c.$$

It is a quadratic equation in x.

So, we will get a parabola either downward or upward.

Hence, it is a many-one mapping and not onto mapping.

Hence, it is neither one-one nor onto mapping.

Reason Total number of functions

$$=(n(B))^{n(A)}=2^n.$$

Clearly, a function will not be onto if all elements of A map to either a or b.

57. **Assertion** We have, $A=\{1, 2, 5, 6\}$
and $R=\{(1, 1), (1, 6), (6, 1)\}$

Here, $(1, 6) \in R$

$\Rightarrow \quad (6, 1) \in R$

Hence, R is symmetric relation.

58. **Assertion** Since, greatest integer $[x]$ gives only integer value.

But $f(x)=[x]+x$ gives all real values and there is no repeated value of $f(x)$ for any value of x.

Hence, $f(x)$ is one-one and onto.

59. **Assertion** Given that,

$A=\{2, 4, 6\}$,

$B=\{3, 5, 7, 9\}$

and $\quad R=\{(2, 3), (4, 5), (6, 7)\}$

Here, $f(2)=3$, $f(4)=5$ and $f(6)=7$

It can be seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one but f is not onto as $9 \in B$ does not have a pre-image in A.

Hence, both Assertion and Reason are true, but Reason is not a correct explanation of Assertion.

60. **Assertion** We have, $B=\{1, 2, 3, 4\}$
and $R=\{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$

Here, $(1, 3), (3, 1) \in R$

$\Rightarrow \quad (1, 1) \in R$

Hence, R is transitive.