Inverse Trigonometric Functions
Part - 2

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\frac{2\pi}{3}$

Reason (R): $\sin^{-1}(\sin\theta)=\theta$, if $\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Ans. Option (D) is correct.

Explanation:

The principal value branch of $\sin^{-1}x$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Let $x=\sin\theta\Rightarrow\theta=\sin^{-1}x$

$\sin^{-1}(\sin\theta)=\sin^{-1}x=\theta$

$\sin^{-1}(\sin\theta)=\theta$, if $\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$.

Hence R is true.

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)\neq\frac{2\pi}{3}$, since $\frac{2\pi}{3}\notin\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Hence A is false.

Q. 2. Assertion (A): Range of $\tan^{-1}x$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

Reason (R): Domain of $\tan^{-1}x$ is R.

Ans. Option (B) is correct.

Explanation: Domain of tan x is the set $\{x : x\in R$ and $x\neq(2n+1)\frac{\pi}{2}, n\in Z\}$ and Range is R.

$\Rightarrow$ tan x is not defined for odd multiples of $\frac{\pi}{2}$.

If we restrict the domain of tangent function to $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$, then it is one-one and onto with its range as R. Actually tan x restricted to any of the intervals $\left(\frac{-3\pi}{2},\frac{-\pi}{2}\right),\left(\frac{-\pi}{2},\frac{\pi}{2}\right),\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ etc., is bijective and its range is R.

Thus $\tan^{-1}x$ can be defined as a function whose domain is R and range could be any of the intervals $\left(\frac{-3\pi}{2},\frac{-\pi}{2}\right),\left(\frac{-\pi}{2},\frac{\pi}{2}\right),\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ and soon.

$\therefore$ Both A and R are true but R is not correct explanation of A.

Q. 3. Assertion (A): Principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$

Reason (R): Principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation:

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\sin^{-1}\left(\sin\frac{\pi}{4}\right)$$
$$=\frac{\pi}{4}$$
$$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)=y$$
$$\cot y=\frac{-1}{\sqrt{3}}$$
$$=-\cot\left(\frac{\pi}{3}\right)$$
$$=\cot\left(\pi-\frac{\pi}{3}\right)$$
$$=\cot\left(\frac{2\pi}{3}\right)$$
$$\Rightarrow\quad\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)=\frac{2\pi}{3}$$

Hence Assertion is correct and Reason is incorrect.

Q. 4. Assertion (A): Range of $\cot^{-1}x$ is $(0,\pi)$

Reason (R): Domain of $\tan^{-1}x$ is R.

Ans. Option (B) is correct.

Q. 5. Assertion (A): Principal value of $\cos^{-1}(1)$ is π

Reason (R): Value of cos $0°$ is 1

Ans. Option (D) is correct.

Explanation: In case of Assertion:

$$\cos^{-1}(1)=y$$
$$\cos y=1$$
$$\cos y=\cos 0° \qquad [\because\cos 0°=1]$$
$$\therefore\quad y=0$$

$\Rightarrow$ Principal value of $\cos^{-1}(1)$ is 0

Hence Assertion is in correct.

Reason is correct.