Matrices
Part - 2

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A = I$

Reason (R): AI = IA = A

Ans. Option (A) is correct.

> ***Explanation:*** AI = IA = A is true.
>
> Hence R is true.
>
> Given $A^2 = A,$
>
> $$\therefore \quad (I+A)^2 - 3A = I^2 + 2IA + A^2 - 3A$$
> $$= I + 2A + A - 3A$$
> $$= I$$
>
> Hence A is true.
>
> R is the correct explanation for A.

Q. 2. Assertion (A): $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a scalar matrix.

Reason (R): If all the elements of the principal diagonal are equal, it is called a scalar matrix.

Ans. Option (C) is correct.

> ***Explanation:*** In a scalar matrix the diagonal elements are equal and the non-diagonal elements are zero. Hence R is false.
>
> A is true since the diagonal elements are equal and the non-diagonal elements are zero.

Q. 3. Assertion (A): $(A + B)^2 \neq A^2 + 2AB + B^2$.

Reason (R): Generally $AB \neq BA$

Ans. Option (A) is correct.

> ***Explanation:*** For two matrices A and B, generally $AB \neq BA$.
>
> *i.e.*, matrix multiplication is not commutative.
>
> $\therefore$ R is true
>
> $$(A+B)^2 = (A+B)(A+B)$$
> $$= A^2 + AB + BA + B^2$$
> $$\neq A^2 + 2AB + B^2$$
>
> $\therefore$ A is true
>
> R is the correct explanation for A.

Q. 4. A and B are two matrices such that both AB and BA are defined.

Assertion (A): $(A + B)(A - B) = A^2 - B^2$

Reason (R): $(A + B)(A - B) = A^2 - AB + BA - B^2$

Ans. Option (D) is correct.

> ***Explanation:*** For two matrices A and B, even if both AB and BA are defined, generally $AB \neq BA$.
>
> $(A + B)(A - B) = A^2 - AB + BA - B^2$.
>
> Since $AB \neq BA$, $(A + B)(A - B) \neq A^2 - B^2$.
>
> Hence R is true and A is false.

Q. 5. Let A and B be two symmetric matrices of order 3.

Assertion (A): A(BA) and (AB)A are symmetric matrices.

Reason (R): AB is symmetric matrix if matrix multiplication of A with B is commutative.

Ans. Option (B) is correct.

Explanation: Generally $(AB)' = B'A'$

If $AB = BA$, then $(AB)' = (BA)' = A'B' = AB$

Since $(AB)' = AB$, AB is a symmetric matrix. Hence R is true.

$$A(BA) = (AB)A = ABA$$
$$(ABA)' = A'B'A' = ABA.$$

$A(BA)$ and $(AB)A$ are symmetric matrices. Hence A is true.

But R is not the correct explanation for A.

Q. 6. Assertion (A): If the matrix $P = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & 3 \end{bmatrix}$ is a symmetric matrix, then $a = \frac{-2}{3}$ and $b = \frac{3}{2}$.

Reason (R): If P is a symmetric matrix, then $P' = -P$.

Ans. Option (C) is correct.

Explanation: If P is a symmetric matrix, then $P' = P$.

Hence R is false.

As P is a symmetric matrix, $P' = P$

$$\therefore \quad \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

$\therefore$ By equality of matrices, $a = \frac{-2}{3}$ and $b = \frac{3}{2}$.

Hence A is true.

Q. 7. Assertion (A): If A is a symmetric matrix, then $B'AB$ is also symmetric.

Reason (R): $(ABC)' = C'B'A'$

Ans. Option (A) is correct.

Explanation: For three matrices A, B and C, if ABC is defined then $(ABC)' = C'B'A'$.

Hence R is true.

Given that A is symmetric $\Rightarrow A' = A$

$(B'AB)' = B'A'(B')' = B'AB$.

Hence A is true.

R is the correct explanation for A.

Q. 8. Assertion (A): If A and B are symmetric matrices, then $AB - BA$ is a skew symmetric matrix

Reason (R): $(AB)' = B'A'$

Ans. Option (A) is correct.

Explanation: $(AB)' = B'A' \Rightarrow$ R is true.

Given that A and B are symmetric matrices.

$\therefore A' = A$ and $B' = B$

$(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B' = BA - AB$

Since $(AB - BA)' = -(AB - BA)$,

$AB - BA$ is skew symmetric.

Hence A is true.

R is the correct explanation for A.