Determinants
Part - 1

Assertion-Reasoning MCQs

Directions (Q. Nos. 55-69) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true; R is False.
(d) A is false; R is true.

55. **Assertion** **(A)** If $A = \begin{bmatrix} 2 & 1+2i \\ 1-2i & 7 \end{bmatrix}$, then $\det(A)$ is real.

Reason **(R)** If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, a_{ij} being complex numbers, then $|A|$ is always real.

56. **Assertion** **(A)** If $A = \begin{bmatrix} 1 & 2 \\ 5 & -1 \end{bmatrix}$, then $|A| = -11$.

Reason **(R)** If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, then $|A| = a_{11}a_{22} - a_{21}a_{12}$.

57. **Assertion** **(A)** If $\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 1 & -2 & 3 \\ 5 & 3 & 8 \end{vmatrix}$, then $\Delta = -12$.

Reason **(R)** If we expand the determinant either by any row or by any column, then the value of determinant always be same.

58. **Assertion** **(A)** If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then $x = \pm 6$.

Reason **(R)** If A and B are matrices of order 3 and $|A| = 4$, $|B| = 6$, then $|2AB| = 192$.

59. Assertion (A) Determinant of a skew-symmetric matrix of order 3 is zero.

Reason (R) For any matrix $A, |A^T| = |A|$ and $|-A| = -|A|$.

60. Assertion (A) The points $A\,(a,\, b+c)$, $B\,(b,\, c+a)$ and $C\,(c,\, a+b)$ are collinear.

Reason (R) Area of a triangle with three collinear points is zero.

61. Assertion (A) The equation of the line joining $A(1, 3)$ and $B(0, 0)$ is given by $y = 3x$.

Reason (R) The area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) in the form of determinant is

$$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

62. Assertion (A) Minor of an element of a determinant of order $n (n \geq 2)$ is a determinant of order n.

Reason (R) If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to $\frac{1}{|A|}$.

63. Assertion (A) $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ where, A_{ij} is cofactor of a_{ij}.

Reason (R) $\Delta =$ Sum of the products of elements of any row (or column) with their corresponding cofactors.

64. Assertion (A) The matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular.

Reason (R) A square matrix A is said to be singular, if $|A| = 0$.

65. Assertion (A) If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, then $|A| = 0$.

Reason (R) $|\text{adj}\, A| = |A|^{n-1}$, where n is order of matrix.

66. Let A be 2×2 matrix.

Assertion (A) adj (adj A) $= A$

Reason (R) $|\text{adj}\, A| = |A|$

67. Assertion (A) If $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix}$.

Reason (R) If $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} \frac{2}{13} & -\frac{5}{13} \\ \frac{3}{13} & -\frac{1}{13} \end{bmatrix}$.

68. Assertion (A) If A is a 3×3 non-singular matrix, then $|A^{-1} \text{adj}\, A| = |A|$.

Reason (R) If A and B both are invertible matrices such that B is inverse of A, then $AB = BA = I$.

69. Assertion (A) The system of equations $2x - y = -2$; $3x + 4y = 3$ has unique solution and $x = -\frac{5}{11}$ and $y = \frac{12}{11}$.

Reason (R) The system of equations $AX = B$ has a unique solution, if $|A| \neq 0$.

ANSWER KEY

Assertion-Reasoning MCQs

55. (c)	56. (a)	57. (a)	58. (b)	59. (c)	60. (a)	61. (c)	62. (d)	63. (a)	64. (a)
65. (b)	66. (b)	67. (d)	68. (b)	69. (a)					

SOLUTION

55. Given, $A = \begin{bmatrix} 2 & 1+2i \\ 1-2i & 7 \end{bmatrix}$

$\therefore \quad |A| = 14 - (1-2i)(1+2i)$

$= 14 - [1+4] = 14 - 5 = 9$

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, a_{ij} being complex numbers,

then $|A|$ may be real or complex.

Hence, Assertion is true and Reason is false.

56. We have, $A = \begin{bmatrix} 1 & 2 \\ 5 & -1 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 1 & 2 \\ 5 & -1 \end{vmatrix}$

$= 1(-1) - 5(2)$

$= -1 - 10$

$= -11$

57. Given, $\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 1 & -2 & 3 \\ 5 & 3 & 8 \end{vmatrix}$

$= 1\{-16-9\} + 1\{3+10\}$

$= -25 + 13$

$= -12$ [expanding along R_1]

58. **Assertion** $\because \begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow \quad 2x^2 - 40 = 18 + 14$

$\Rightarrow \quad 2x^2 = 32 + 40$

$\Rightarrow \quad x^2 = \frac{72}{2} = 36$

$\therefore \quad x = \pm 6$

Reason We know that, $|AB| = |A| \cdot |B|$

$\therefore \quad |2AB| = 8|AB|$

$= 8|A| \cdot |B|$

$= 8 \times 4 \times 6$

$= 192$

59. **Assertion** Determinant of a skew-symmetric matrix of odd order is zero.

$\therefore$ Assertion is true.

Reason For any matrix A, $|A^T| = A$

and $\quad |-A| = |A|$

[when A is of even order]

and $\quad |-A| = -|A|$

[when A is of odd order]

$\therefore$ Reason is false.

60. We know that, the area of triangle with three collinear points is zero.

Now, consider area of

$$\Delta ABC = \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$= \frac{1}{2} | a\{(c+a) \times 1 - (a+b) \times 1\} - (b+c)\{b \times 1 - 1 \times c\} + 1\{b \times (a+b) - (c+a) \times c\}|$$

$$= \frac{1}{2} | a(c+a-a-b) - (b+c)(b-c) + 1(ab + b^2 - c^2 - ac)|$$

$$= \frac{1}{2} | ac - ab - b^2 + c^2 + ab + b^2 - c^2 - ac|$$

$$= \frac{1}{2} \times 0 = 0$$

Since, area of $\Delta ABC = 0$.

Hence, points $A(a, b+c)$, $B(b, c+a)$ and $C(c, a+b)$ are collinear.

61. **Assertion** Let $P(x, y)$ be any point on AB.

Then, area of ΔABP is zero.

[since, the three points are collinear]

$$\therefore \quad \frac{1}{2} \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

This gives $\frac{1}{2}(3x - y) = 0$

or $\quad y = 3x$

which is the equation of required line AB.

Reason The area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

62. **Assertion** Minor of an element of a determinant of order $n (n \geq 2)$ is a determinant of order $n - 1$.

So, Assertion is false.

Reason We know, $AA^{-1} = I$

$\therefore \quad |AA^{-1}| = |I| \Rightarrow |A||A^{-1}| = 1$

$\Rightarrow \quad |A^{-1}| = \frac{1}{|A|}$

So, Reason is true.

63. By expanding the determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \text{ along } R_1, \text{ we have}$$

$$\Delta = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$,

where A_{ij} is cofactor of a_{ij}.

= Sum of products of elements of R_1 with their corresponding cofactors

64. The determinant of the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is

$$|A| = \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix} = 8 - 8 = 0$$

Hence, A is a singular matrix.

65. Assertion The given matrix is

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$$

Then, $|A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

By expanding along R_1 (first row), we get

$$|A| = 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

$$= 1(-9+12) - 1(-18+15) - 2(8-5)$$

$$= 1(3) - 1(-3) - 2(3) = 3 + 3 - 6 = 0,$$

which is a true statement.

Reason $|\text{adj}(A)| = |A|^{n-1}$ is a true statement.

Hence, both Assertion and Reason are true but Reason is not a correct explanation of Assertion.

66. Assertion $\because$ adj (adj A)

$$= |A|^{n-2} A = |A|^{2-2} A \quad [\because n = 2]$$

$$= |A|^0 A = A$$

Reason $|\text{adj } A| = |A|^{n-1} = |A|^{2-1} \quad [\because n = 2]$

$$= |A|$$

Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

67. Assertion Let $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$.

We have, $|A| = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = 6 - (-8) = 14$

Cofactors of $|A|$ are

$A_{11} = 3$, $A_{12} = -4$, $A_{21} = 2$ and $A_{22} = 2$.

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

$$\text{Now,} \quad A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} \frac{3}{14} & \frac{2}{14} \\ -\frac{4}{14} & \frac{2}{14} \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & \frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} \end{bmatrix}$$

Reason Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$.

We have, $|A| = \begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix} = -2 - (-15) = 13$

Now, cofactors of $|A|$ are

$A_{11} = 2$, $A_{12} = 3$, $A_{21} = -5$ and $A_{22} = -1$.

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}' = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

$$\text{Now,} \quad A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{13} & -\frac{5}{13} \\ \frac{3}{13} & -\frac{1}{13} \end{bmatrix}$$

68. Assertion $|A^{-1} \text{ adj } A| = |A^{-1}| \cdot |\text{adj } A|$

$[\because |AB| = |A| |B|]$

$= |A|^{-1} |\text{adj } A| \quad [\because |A^{-1}| = |A|^{-1}]$

$= |A|^{-1} \cdot |A|^2$

[$\because$ A is a 3×3 non-singular matrix, so $|\text{adj } A| = |A|^2$]

$= |A|$

Reason It is a true statement. Hence, both Assertion and Reason are true but Reason is not a correct explanation of Assertion.

69. The given system can be written as

$$AX = B,$$

where $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Here, $|A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 2 \times 4 - (-3) = 11 \neq 0$

Thus, A is non-singular.

Therefore, its inverse exists.

Therefore, the given system has a unique solution given by $X = A^{-1} B$.

Now, $\operatorname{adj}(A) = \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$

$$\therefore \qquad A^{-1} = \frac{1}{|A|} (\operatorname{adj} A) = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

Now, $X = A^{-1} B = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$$= \frac{1}{11} \begin{bmatrix} -8 + 3 \\ 6 + 6 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$$

Hence, $x = \frac{-5}{11}$ and $y = \frac{12}{11}$

Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.