

Chapter 4. Expansions (Including Substitution)

Exercise 4(A)

Solution 1:

We know that

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\begin{aligned}(2a+b)^2 &= 4a^2 + b^2 + 2 \times 2a \times b \\ &= 4a^2 + b^2 + 4ab\end{aligned}$$

We know that

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\begin{aligned}(3a+7b)^2 &= 9a^2 + 49b^2 + 2 \times 3a \times 7b \\ &= 9a^2 + 49b^2 + 42ab\end{aligned}$$

We know that

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned}(3a-4b)^2 &= 9a^2 + 16b^2 - 2 \times 3a \times 4b \\ &= 9a^2 + 16b^2 - 24ab\end{aligned}$$

We know that

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned}\left(\frac{3a}{2b} - \frac{2b}{3a}\right)^2 &= \left(\frac{3a}{2b}\right)^2 + \left(\frac{2b}{3a}\right)^2 - 2 \times \frac{3a}{2b} \times \frac{2b}{3a} \\ &= \frac{9a^2}{4b^2} + \frac{4b^2}{9a^2} - 2\end{aligned}$$

Solution 2:

- $(101)^2$

$$(101)^2 = (100 + 1)^2$$

We know that

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$\begin{aligned}\therefore (100 + 1)^2 &= 100^2 + 1^2 + 2 \times 100 \times 1 \\ &= 10000 + 1 + 200 \\ &= 10,201\end{aligned}$$

- $(502)^2$

$$(502)^2 = (500 + 2)^2$$

We know that

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$\begin{aligned}\therefore (500 + 2)^2 &= 500^2 + 2^2 + 2 \times 500 \times 2 \\ &= 250000 + 4 + 2000 \\ &= 2,52,004\end{aligned}$$

- $(97)^2$

$$(97)^2 = (100 - 3)^2$$

We know that

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned}\therefore (100 - 3)^2 &= 100^2 + 3^2 - 2 \times 100 \times 3 \\ &= 10000 + 9 - 600 \\ &= 9,409\end{aligned}$$

- $(998)^2$

$$(998)^2 = (1000 - 2)^2$$

We know that

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned}\therefore (1000 - 2)^2 &= 1000^2 + 2^2 - 2 \times 1000 \times 2 \\ &= 1000000 + 4 - 4000 \\ &= 9,96,004\end{aligned}$$

Solution 3:

(i)

$$\left(\frac{7}{8}x + \frac{4}{5}y\right)^2$$

We know that

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$\begin{aligned}\therefore \left(\frac{7}{8}x + \frac{4}{5}y\right)^2 &= \left(\frac{7}{8}x\right)^2 + \left(\frac{4}{5}y\right)^2 + 2 \times \frac{7}{8}x \times \frac{4}{5}y \\ &= \frac{49x^2}{64} + \frac{16y^2}{25} + \frac{7xy}{5}\end{aligned}$$

(ii)

$$\left(\frac{2x}{7} - \frac{7y}{4}\right)^2$$

We know that

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned}\therefore \left(\frac{2x}{7} - \frac{7y}{4}\right)^2 &= \left(\frac{2}{7}x\right)^2 + \left(\frac{7}{4}y\right)^2 - 2 \times \frac{2}{7}x \times \frac{7}{4}y \\ &= \frac{4x^2}{49} + \frac{49y^2}{16} - xy\end{aligned}$$

Solution 4:

(i) Consider the given expression:

Let us expand the first term: $\left(\frac{a}{2b} + \frac{2b}{a}\right)^2$

We know that

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\begin{aligned}\therefore \left(\frac{a}{2b} + \frac{2b}{a}\right)^2 &= \left(\frac{a}{2b}\right)^2 + \left(\frac{2b}{a}\right)^2 + 2 \times \frac{a}{2b} \times \frac{2b}{a} \\ &= \frac{a^2}{4b^2} + \frac{4b^2}{a^2} + 2 \dots (1)\end{aligned}$$

Let us expand the second term: $\left(\frac{a}{2b} - \frac{2b}{a}\right)^2$

We know that

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned}\therefore \left(\frac{a}{2b} - \frac{2b}{a}\right)^2 &= \left(\frac{a}{2b}\right)^2 + \left(\frac{2b}{a}\right)^2 - 2 \times \frac{a}{2b} \times \frac{2b}{a} \\ &= \frac{a^2}{4b^2} + \frac{4b^2}{a^2} - 2 \dots (2)\end{aligned}$$

Thus from (1) and (2), the given expression is

$$\begin{aligned}\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2 - 4 &= \frac{a^2}{4b^2} + \frac{4b^2}{a^2} + 2 - \frac{a^2}{4b^2} - \frac{4b^2}{a^2} + 2 - 4 \\ &= 0\end{aligned}$$

(ii) Consider the given expression:

Let us expand the first term: $(4a + 3b)^2$

We know that

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$\begin{aligned}\therefore (4a + 3b)^2 &= (4a)^2 + (3b)^2 + 2 \times 4a \times 3b \\ &= 16a^2 + 9b^2 + 24ab \dots (1)\end{aligned}$$

Let us expand the second term: $(4a - 3b)^2$

We know that

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned}\therefore (4a - 3b)^2 &= (4a)^2 + (3b)^2 - 2 \times 4a \times 3b \\ &= 16a^2 + 9b^2 - 24ab \dots (2)\end{aligned}$$

Thus from (1) and (2), the given expression is

$$\begin{aligned}(4a + 3b)^2 - (4a - 3b)^2 + 48ab \\ = 16a^2 + 9b^2 + 24ab - 16a^2 - 9b^2 + 24ab + 48ab \\ = 96ab\end{aligned}$$

Solution 5:

We know that

$$(a + b)^2 = a^2 + b^2 + 2ab$$

and

$$(a - b)^2 = a^2 + b^2 - 2ab$$

Rewrite the above equation, we have

$$\begin{aligned}(a - b)^2 &= a^2 + b^2 + 2ab - 4ab \\ &= (a + b)^2 - 4ab \dots (1)\end{aligned}$$

Given that $a + b = 7$; $ab = 10$

Substitute the values of $(a + b)$ and (ab)

in equation (1), we have

$$\begin{aligned}(a - b)^2 &= (7)^2 - 4(10) \\ &= 49 - 40 = 9\end{aligned}$$

$$\Rightarrow a - b = \pm\sqrt{9}$$

$$\Rightarrow a - b = \pm 3$$

Solution 6:

We know that

$$(a-b)^2 = a^2 + b^2 - 2ab$$

and

$$(a+b)^2 = a^2 + b^2 + 2ab$$

Rewrite the above equation, we have

$$\begin{aligned}(a+b)^2 &= a^2 + b^2 - 2ab + 4ab \\ &= (a-b)^2 + 4ab \dots (1)\end{aligned}$$

Given that $a-b = 7$; $ab=18$

Substitute the values of $(a-b)$ and (ab)

in equation (1), we have

$$\begin{aligned}(a+b)^2 &= (7)^2 + 4(18) \\ &= 49 + 72 = 121\end{aligned}$$

$$\Rightarrow a+b = \pm\sqrt{121}$$

$$\Rightarrow a+b = \pm 11$$

Solution 7:

(i)

We know that

$$(x+y)^2 = x^2 + y^2 + 2xy$$

and

$$(x-y)^2 = x^2 + y^2 - 2xy$$

Rewrite the above equation, we have

$$\begin{aligned}(x-y)^2 &= x^2 + y^2 + 2xy - 4xy \\ &= (x+y)^2 - 4xy \dots (1)\end{aligned}$$

Given that $x+y = \frac{7}{2}$; $xy=\frac{5}{2}$

Substitute the values of $(x+y)$ and (xy)

in equation (1), we have

$$\begin{aligned}(x-y)^2 &= \left(\frac{7}{2}\right)^2 - 4\left(\frac{5}{2}\right) \\ &= \frac{49}{4} - 10 = \frac{9}{4}\end{aligned}$$

$$\Rightarrow x-y = \pm\sqrt{\frac{9}{4}}$$

$$\Rightarrow x-y = \pm\frac{3}{2} \dots (2)$$

(ii)

We know that

$$x^2 - y^2 = (x + y)(x - y) \dots (3)$$

From equation (2) we have,

$$x - y = \pm \frac{3}{2}$$

Thus equation (3) becomes,

$$x^2 - y^2 = \left(\frac{7}{2}\right)\left(\pm \frac{3}{2}\right) \quad [\text{given } x + y = \frac{7}{2}]$$

$$\Rightarrow x^2 - y^2 = \pm \frac{21}{4}$$

Solution 8:

(i)

We know that

$$(a-b)^2 = a^2 + b^2 - 2ab$$

and

$$(a+b)^2 = a^2 + b^2 + 2ab$$

Rewrite the above equation, we have

$$\begin{aligned}(a+b)^2 &= a^2 + b^2 - 2ab + 4ab \\ &= (a-b)^2 + 4ab \dots (1)\end{aligned}$$

Given that $a-b = 0.9$; $ab=0.36$

Substitute the values of $(a-b)$ and (ab)

in equation (1), we have

$$\begin{aligned}(a+b)^2 &= (0.9)^2 + 4(0.36) \\ &= 0.81 + 1.44 = 2.25\end{aligned}$$

$$\Rightarrow a+b = \pm\sqrt{2.25}$$

$$\Rightarrow a+b = \pm 1.5 \dots (2)$$

(ii)

We know that

$$a^2 - b^2 = (a+b)(a-b) \dots (3)$$

From equation (2) we have,

$$a+b = \pm 1.5$$

Thus equation (3) becomes,

$$a^2 - b^2 = (\pm 1.5)(0.9) \quad [\text{given } a-b = 0.9]$$

$$\Rightarrow a^2 - b^2 = \pm 1.35$$

Solution 9:

(i)

We know that

$$(a-b)^2 = a^2 + b^2 - 2ab$$

Rewrite the above identity as,

$$a^2 + b^2 = (a-b)^2 + 2ab \dots (1)$$

Similarly, we know that,

$$(a+b)^2 = a^2 + b^2 + 2ab$$

Rewrite the above identity as,

$$a^2 + b^2 = (a+b)^2 - 2ab \dots (2)$$

Adding the equations (1) and (2), we have

$$2(a^2 + b^2) = (a-b)^2 + 2ab + (a+b)^2 - 2ab$$

$$\Rightarrow 2(a^2 + b^2) = (a-b)^2 + (a+b)^2$$

$$\Rightarrow (a^2 + b^2) = \frac{1}{2}[(a-b)^2 + (a+b)^2] \dots (3)$$

Given that $a+b = 6$; $a-b=4$

Substitute the values of $(a+b)$ and $(a-b)$

in equation (3), we have

$$(a^2 + b^2) = \frac{1}{2}[(4)^2 + (6)^2]$$

$$= \frac{1}{2}[16 + 36]$$

$$= \frac{52}{2}$$

$$\Rightarrow a^2 + b^2 = 26 \dots (4)$$

(ii)

From equation (4), we have

$$a^2 + b^2 = 26$$

Consider the identity

$$(a-b)^2 = a^2 + b^2 - 2ab \dots (5)$$

Substitute the value $a-b = 4$ and $a^2 + b^2 = 26$

in the above equation, we have

$$(4)^2 = 26 - 2ab$$

$$\Rightarrow 2ab = 26 - 16$$

$$\Rightarrow 2ab = 10$$

$$\Rightarrow ab = 5$$

Solution 10:

(i)

We know that

$$(a+b)^2 = a^2 + b^2 + 2ab$$

and

$$(a-b)^2 = a^2 + b^2 - 2ab$$

Thus,

$$\begin{aligned}\left(a + \frac{1}{a}\right)^2 &= a^2 + \frac{1}{a^2} + 2 \times a \times \frac{1}{a} \\ &= a^2 + \frac{1}{a^2} + 2 \dots (1)\end{aligned}$$

Given that $a + \frac{1}{a} = 6$; Substitute in equation (1), we have

$$(6)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 36 - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 34 \dots (2)$$

Similarly, consider

$$\begin{aligned}\left(a - \frac{1}{a}\right)^2 &= a^2 + \frac{1}{a^2} - 2 \times a \times \frac{1}{a} \\ &= a^2 + \frac{1}{a^2} - 2 \\ &= 34 - 2 \text{ [from (2)]}\end{aligned}$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = 32$$

$$\Rightarrow a - \frac{1}{a} = \pm\sqrt{32}$$

$$\Rightarrow a - \frac{1}{a} = \pm 4\sqrt{2} \dots (3)$$

(ii)

We need to find $a^2 - \frac{1}{a^2}$:

$$\text{We know that, } a^2 - \frac{1}{a^2} = \left(a - \frac{1}{a}\right)\left(a + \frac{1}{a}\right)$$

$$a - \frac{1}{a} = \pm 4\sqrt{2}; a + \frac{1}{a} = 6$$

Thus,

$$a^2 - \frac{1}{a^2} = (\pm 4\sqrt{2})(6)$$

$$\Rightarrow a^2 - \frac{1}{a^2} = \pm 24\sqrt{2}$$

Solution 11:

(i)

We know that

$$(a+b)^2 = a^2 + b^2 + 2ab$$

and

$$(a-b)^2 = a^2 + b^2 - 2ab$$

Thus,

$$\begin{aligned}\left(a - \frac{1}{a}\right)^2 &= a^2 + \frac{1}{a^2} - 2 \times a \times \frac{1}{a} \\ &= a^2 + \frac{1}{a^2} - 2 \dots (1)\end{aligned}$$

Given that $a - \frac{1}{a} = 8$; Substitute in equation (1), we have

$$(8)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 64 + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 66 \dots (2)$$

Similarly, consider

$$\begin{aligned}\left(a + \frac{1}{a}\right)^2 &= a^2 + \frac{1}{a^2} + 2 \times a \times \frac{1}{a} \\ &= a^2 + \frac{1}{a^2} + 2 \\ &= 66 + 2 \text{ [from (2)]}\end{aligned}$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 68$$

$$\Rightarrow a + \frac{1}{a} = \pm 2\sqrt{17}$$

$$\Rightarrow a + \frac{1}{a} = \pm 2\sqrt{17} \dots (3)$$

(ii)

We need to find $a^2 - \frac{1}{a^2}$:

We know that, $a^2 - \frac{1}{a^2} = \left(a - \frac{1}{a}\right)\left(a + \frac{1}{a}\right)$

$$a - \frac{1}{a} = 8; a + \frac{1}{a} = \pm 2\sqrt{17}$$

Thus,

$$a^2 - \frac{1}{a^2} = (\pm 2\sqrt{17})(8)$$

$$\Rightarrow a^2 - \frac{1}{a^2} = \pm 16\sqrt{17}$$

Solution 12:

(i)

Consider the given equation

$$a^2 - 3a + 1 = 0$$

Rewrite the given equation, we have

$$a^2 + 1 = 3a$$

$$\Rightarrow \frac{a^2 + 1}{a} = 3$$

$$\Rightarrow \frac{a^2}{a} + \frac{1}{a} = 3$$

$$\Rightarrow a + \frac{1}{a} = 3 \dots (1)$$

(ii)

We need to find $a^2 + \frac{1}{a^2}$:

We know the identity, $(a+b)^2 = a^2 + b^2 + 2ab$

$$\therefore \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2 \dots (2)$$

From equation (1), we have,

$$a + \frac{1}{a} = 3$$

Thus equation (2), becomes,

$$(3)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow 9 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 7$$

Solution 13:

(i)

Consider the given equation

$$a^2 - 5a - 1 = 0$$

Rewrite the given equation, we have

$$a^2 - 1 = 5a$$

$$\Rightarrow \frac{a^2 - 1}{a} = 5$$

$$\Rightarrow \frac{a^2}{a} - \frac{1}{a} = 5$$

$$\Rightarrow a - \frac{1}{a} = 5 \dots (1)$$

(ii)

We need to find $a + \frac{1}{a}$:

We know the identity, $(a - b)^2 = a^2 + b^2 - 2ab$

$$\therefore \left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow (5)^2 = a^2 + \frac{1}{a^2} - 2 \quad [\text{from (1)}]$$

$$\Rightarrow 25 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 27 \dots\dots(2)$$

Now consider the identity $(a+b)^2 = a^2 + b^2 + 2ab$

$$\therefore \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 27 + 2 \quad [\text{from (2)}]$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 29$$

$$\Rightarrow a + \frac{1}{a} = \pm\sqrt{29} \dots\dots(3)$$

Solution 14:

Given that $(3x+4y) = 16$ and $xy=4$

We need to find $9x^2 + 16y^2$.

We know that

$$(a+b)^2 = a^2 + b^2 + 2ab$$

Consider the square of $3x+4y$:

$$\therefore (3x+4y)^2 = (3x)^2 + (4y)^2 + 2 \times 3x \times 4y$$

$$\Rightarrow (3x+4y)^2 = 9x^2 + 16y^2 + 24xy \dots (1)$$

Substitute the values of $(3x+4y)$ and xy in the above equation (1), we have

$$(3x+4y)^2 = 9x^2 + 16y^2 + 24xy$$

$$\Rightarrow (16)^2 = 9x^2 + 16y^2 + 24(4)$$

$$\Rightarrow 256 = 9x^2 + 16y^2 + 96$$

$$\Rightarrow 9x^2 + 16y^2 = 160$$

Solution 15:

Given x is 2 more than y , so $x = y + 2$

Sum of squares of x and y is 34, so $x^2 + y^2 = 34$.

Replace $x = y + 2$ in the above equation and solve for y .

$$\text{We get } (y+2)^2 + y^2 = 34$$

$$2y^2 + 4y - 30 = 0$$

$$y^2 + 2y - 15 = 0$$

$$(y+5)(y-3) = 0$$

$$\text{So } y = -5 \text{ or } 3$$

$$\text{For } y = -5, x = -3$$

$$\text{For } y = 3, x = 5$$

Product of x and y is 15 in both cases.

Solution 16:

Let the two positive numbers be a and b .

Given difference between them is 5 and sum of squares is 73.

$$\text{So } a - b = 5, a^2 + b^2 = 73$$

Squaring on both sides gives

$$(a - b)^2 = 5^2$$

$$a^2 + b^2 - 2ab = 25$$

$$\text{but } a^2 + b^2 = 73$$

$$\text{so } 2ab = 73 - 25 = 48$$

$$ab = 24$$

So, the product of numbers is 24.

Exercise 4(B)**Solution 1:**

(i)

$$(a - b)^3 = a^3 - 3ab(a - b) - b^3$$

$$\begin{aligned}(3a - 2b)^3 &= (3a)^3 - 3 \times 3a \times 2b(3a - 2b) - (2b)^3 \\ &= 27a^3 - 18ab(3a - 2b) - 8b^3 \\ &= 27a^3 - 54a^2b + 36ab^2 - 8b^3\end{aligned}$$

(ii)

$$(a + b)^3 = a^3 + 3ab(a + b) + b^3$$

$$\begin{aligned}(5a + 3b)^3 &= (5a)^3 + 3 \times 5a \times 3b(5a + 3b) + (3b)^3 \\ &= 125a^3 + 45ab(5a + 3b) + 27b^3 \\ &= 125a^3 + 225a^2b + 135ab^2 + 27b^3\end{aligned}$$

(iii)

$$\begin{aligned}(a+b)^3 &= a^3 + 3ab(a+b) + b^3 \\ \left(2a + \frac{1}{2a}\right)^3 &= (2a)^3 + 3 \times 2a \times \frac{1}{2a} \times \left(2a + \frac{1}{2a}\right) + \left(\frac{1}{2a}\right)^3 \\ &= 8a^3 + 3\left(2a + \frac{1}{2a}\right) + \frac{1}{8a^3} \\ \left(2a + \frac{1}{2a}\right)^3 &= 8a^3 + 6a + \frac{3}{2a} + \frac{1}{8a^3}\end{aligned}$$

(iv)

$$\begin{aligned}(a-b)^3 &= a^3 - 3ab(a-b) - b^3 \\ \left(3a - \frac{1}{a}\right)^3 &= (3a)^3 - 3 \times 3a \times \frac{1}{a} \left(3a - \frac{1}{a}\right) - \left(\frac{1}{a}\right)^3 \\ &= 27a^3 - 9\left(3a - \frac{1}{a}\right) - \frac{1}{a^3} \\ &= 27a^3 - 27a + \frac{9}{a} - \frac{1}{a^3}\end{aligned}$$

Solution 2:

(i)

$$\begin{aligned}a^2 + \frac{1}{a^2} &= 47 \\ \left(a + \frac{1}{a}\right)^2 &= a^2 + \frac{1}{a^2} + 2 \\ \Rightarrow \left(a + \frac{1}{a}\right)^2 &= 47 + 2 \\ \Rightarrow \left(a + \frac{1}{a}\right)^2 &= 49 \\ \Rightarrow a + \frac{1}{a} &= \pm\sqrt{49} \\ \Rightarrow a + \frac{1}{a} &= \pm 7 \dots (1)\end{aligned}$$

(ii)

$$\begin{aligned}\left(a + \frac{1}{a}\right)^3 &= a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) \\ \Rightarrow a^3 + \frac{1}{a^3} &= \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right) \\ \Rightarrow a^3 + \frac{1}{a^3} &= (\pm 7)^3 - 3(\pm 7) \quad [\text{from (1)}] \\ \Rightarrow a^3 + \frac{1}{a^3} &= \pm 322\end{aligned}$$

Solution 3:

(i)

$$\begin{aligned}a^2 + \frac{1}{a^2} &= 18 \\ \left(a - \frac{1}{a}\right)^2 &= a^2 + \frac{1}{a^2} - 2 \\ \Rightarrow \left(a - \frac{1}{a}\right)^2 &= 18 - 2 \\ \Rightarrow \left(a - \frac{1}{a}\right)^2 &= 16 \\ \Rightarrow a - \frac{1}{a} &= \pm\sqrt{16} \\ \Rightarrow a - \frac{1}{a} &= \pm 4 \dots (1)\end{aligned}$$

(ii)

$$\begin{aligned}\left(a - \frac{1}{a}\right)^3 &= a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right) \\ \Rightarrow a^3 - \frac{1}{a^3} &= \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right) \\ \Rightarrow a^3 - \frac{1}{a^3} &= (\pm 4)^3 + 3(\pm 4) \quad [\text{from (1)}] \\ \Rightarrow a^3 - \frac{1}{a^3} &= \pm 76\end{aligned}$$

Solution 4:

Given that $a + \frac{1}{a} = p \dots (1)$

$$\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow a^3 + \frac{1}{a^3} = (p)^3 - 3(p) \text{ [from (1)]}$$

$$\Rightarrow a^3 + \frac{1}{a^3} = p(p^2 - 3)$$

Solution 5:

Given that $a+2b=5$;

We need to find $a^3 + 8b^3 + 30ab$:

Now consider the cube of $a+2b$:

$$\begin{aligned}(a+2b)^3 &= a^3 + (2b)^3 + 3 \times a \times 2b \times (a+2b) \\ &= a^3 + 8b^3 + 6ab \times (a+2b)\end{aligned}$$

$$5^3 = a^3 + 8b^3 + 6ab \times (5) \text{ [} \because a+2b=5 \text{]}$$

$$125 = a^3 + 8b^3 + 30ab$$

Thus the value of $a^3 + 8b^3 + 30ab$ is 125.

Solution 6:

Given that $\left(a + \frac{1}{a}\right)^2 = 3$

$$\Rightarrow a + \frac{1}{a} = \pm\sqrt{3} \dots (1)$$

We need to find $a^3 + \frac{1}{a^3}$:

Consider the identity,

$$\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow a^3 + \frac{1}{a^3} = (\pm\sqrt{3})^3 - 3(\pm\sqrt{3}) \text{ [from (1)]}$$

$$\Rightarrow a^3 + \frac{1}{a^3} = \pm 3\sqrt{3} - 3(\pm\sqrt{3})$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 0$$

Solution 7:

Given that $a+2b+c=0$;

$$\Rightarrow a+2b = -c \dots (1)$$

Now consider the expansion of $(a+2b)^3$:

$$(a+2b)^3 = (-c)^3$$

$$a^3 + (2b)^3 + 3 \times a \times 2b \times (a+2b) = -c^3$$

$$\Rightarrow a^3 + 8b^3 + 3 \times a \times 2b \times (-c) = -c^3 \text{ [from (1)]}$$

$$\Rightarrow a^3 + 8b^3 - 6abc = -c^3$$

$$\Rightarrow a^3 + 8b^3 + c^3 = 6abc$$

Hence proved.

Solution 8:

Property is if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

(i) $a = 13, b = -8$ and $c = -5$

$$13^3 + (-8)^3 + (-5)^3 = 3(13)(-8)(-5) = 1560$$

(ii) $a = 7, b = 3, c = -10$

$$7^3 + 3^3 + (-10)^3 = 3(7)(3)(-10) = -630$$

(iii) $a = 9, b = -5, c = -4$

$$9^3 - 5^3 - 4^3 = 9^3 + (-5)^3 + (-4)^3 = 3(9)(-5)(-4) = 540$$

(iv) $a = 38, b = -26, c = -12$

$$38^3 + (-26)^3 + (-12)^3 = 3(38)(-26)(-12) = 35568$$

Solution 9:

(i)

$$a - \frac{1}{a} = 3$$

$$\left(a - \frac{1}{a}\right)^2 = 9$$

$$a^2 + \frac{1}{a^2} = 9 + 2 = 11$$

(ii)

$$a - \frac{1}{a} = 3$$

$$\left(a - \frac{1}{a}\right)^3 = 27$$

$$a^3 + \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right) = 27$$

$$a^3 + \frac{1}{a^3} = 27 + 9 = 36$$

Solution 10:

(i)

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = (4)^2 + 2 \quad [\because a - \frac{1}{a} = 4]$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 18 \dots (1)$$

(ii)

We know that

$$\begin{aligned} a^4 + \frac{1}{a^4} &= \left(a^2 + \frac{1}{a^2}\right)^2 - 2 \\ &= (18)^2 - 2 \quad [\text{from (1)}] \\ &= 324 - 2 \end{aligned}$$

$$\Rightarrow a^4 + \frac{1}{a^4} = 322$$

(iii)

$$\left(a - \frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right)$$

$$\Rightarrow a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right)$$

$$\Rightarrow a^3 - \frac{1}{a^3} = (4)^3 + 3(4) \quad [\because a - \frac{1}{a} = 4]$$

$$\Rightarrow a^3 - \frac{1}{a^3} = 64 + 12$$

$$\Rightarrow a^3 - \frac{1}{a^3} = 76$$

Solution 11:

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = (2)^2 - 2 \quad [\because x + \frac{1}{x} = 2]$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 2 \dots (1)$$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$\Rightarrow x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$\Rightarrow x^3 + \frac{1}{x^3} = (2)^3 - 3(2) \quad [\because x + \frac{1}{x} = 2]$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 8 - 6$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 2 \dots (2)$$

We know that

$$\begin{aligned} x^4 + \frac{1}{x^4} &= \left(x^2 + \frac{1}{x^2}\right)^2 - 2 \\ &= (2)^2 - 2 \quad [\text{from (1)}] \\ &= 4 - 2 \end{aligned}$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 2 \dots (3)$$

Thus from equations (1), (2) and (3), we have

$$x^2 + \frac{1}{x^2} = x^3 + \frac{1}{x^3} = x^4 + \frac{1}{x^4}$$

Solution 12:

Given that $2x - 3y = 10$, $xy = 16$

$$\therefore (2x - 3y)^3 = (10)^3$$

$$\Rightarrow 8x^3 - 27y^3 - 3(2x)(3y)(2x - 3y) = 1000 \Rightarrow 8x^3 - 27y^3 - 18xy(2x - 3y) = 1000$$

$$\Rightarrow 8x^3 - 27y^3 - 18(16)(10) = 1000$$

$$\Rightarrow 8x^3 - 27y^3 - 2880 = 1000$$

$$\Rightarrow 8x^3 - 27y^3 = 1000 + 2880$$

$$\Rightarrow 8x^3 - 27y^3 = 3880$$

Solution 13:

(i)

$$(3x + 5y + 2z)(3x - 5y + 2z)$$

$$= \{(3x + 2z) + (5y)\} \{(3x + 2z) - (5y)\}$$

$$= (3x + 2z)^2 - (5y)^2$$

$$\{\text{since } (a + b)(a - b) = a^2 - b^2\}$$

$$= 9x^2 + 4z^2 + 2 \times 3x \times 2z - 25y^2$$

$$= 9x^2 + 4z^2 + 12xz - 25y^2$$

$$= 9x^2 + 4z^2 - 25y^2 + 12xz$$

(ii)

$$(3x - 5y - 2z)(3x - 5y + 2z)$$

$$= \{(3x - 5y) - (2z)\} \{(3x - 5y) + (2z)\}$$

$$= (3x - 5y)^2 - (2z)^2 \{\text{since } (a + b)(a - b) = a^2 - b^2\}$$

$$= 9x^2 + 25y^2 - 2 \times 3x \times 5y - 4z^2$$

$$= 9x^2 + 25y^2 - 30xy - 4z^2$$

$$= 9x^2 + 25y^2 - 4z^2 - 30xy$$

Solution 14:

Given sum of two numbers is 9 and their product is 20.

Let the numbers be a and b.

$$a + b = 9$$

$$ab = 20$$

Squaring on both sides gives

$$(a+b)^2 = 9^2$$

$$a^2 + b^2 + 2ab = 81$$

$$a^2 + b^2 + 40 = 81$$

So sum of squares is $81 - 40 = 41$

Cubing on both sides gives

$$(a + b)^3 = 9^3$$

$$a^3 + b^3 + 3ab(a + b) = 729$$

$$a^3 + b^3 + 60(9) = 729$$

$$a^3 + b^3 = 729 - 540 = 189$$

So the sum of cubes is 189.

Solution 15:

Cubing on both sides gives

$$(x - y)^3 = 5^3$$

$$x^3 - y^3 - 3xy(x - y) = 125$$

$$x^3 - y^3 - 72(5) = 125$$

$$x^3 - y^3 = 125 + 360 = 485$$

So, difference of their cubes is 485.

Cubing both sides, we get

$$(x + y)^3 = 11^3$$

$$x^3 + y^3 + 3xy(x + y) = 1331$$

$$x^3 + y^3 = 1331 - 72(11) = 1331 - 792 = 539$$

So, sum of their cubes is 539.

Exercise 4(C)**Solution 1:**

$$\begin{aligned} \text{(i)} \quad (x + 8)(x + 10) &= x^2 + (8 + 10)x + 8 \times 10 \\ &= x^2 + 18x + 80 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (x + 8)(x - 10) &= x^2 + (8 - 10)x + 8 \times (-10) \\ &= x^2 - 2x - 80 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (x - 8)(x + 10) &= x^2 - (8 - 10)x - 8 \times 10 \\ &= x^2 + 2x - 80 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (x - 8)(x - 10) &= x^2 - (8 + 10)x + 8 \times 10 \\ &= x^2 - 18x + 80 \end{aligned}$$

Solution 2:

$$\begin{aligned}
 \text{(i)} \quad \left(2x - \frac{1}{x}\right)\left(3x + \frac{2}{x}\right) &= (2x)(3x) - \left(\frac{1}{x}\right)(3x) + \left(\frac{2}{x}\right)(2x) - \left(\frac{1}{x}\right)\left(\frac{2}{x}\right) \\
 &= 6x^2 - (3 - 2) - \frac{2}{x^2} \\
 &= 6x^2 - (-1) - \frac{2}{x^2} \\
 &= 6x^2 + 1 - \frac{2}{x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \left(3a + \frac{2}{b}\right)\left(2a - \frac{3}{b}\right) &= (3a)(2a) + \left(\frac{2}{b}\right)(2a) - \left(\frac{3}{b}\right)(3a) - \left(\frac{2}{b}\right)\left(\frac{3}{b}\right) \\
 &= 6a^2 + \left(\frac{4}{b} - \frac{9}{b}\right)a - \frac{6}{b^2} \\
 &= 6a^2 + \left(-\frac{5}{b}\right)a - \frac{6}{b^2} \\
 &= 6a^2 - \frac{5a}{b} - \frac{6}{b^2}
 \end{aligned}$$

Solution 3:

$$\begin{aligned}
 \text{(i)} \quad (x + y + z)^2 &= x^2 + y^2 + z^2 + 2(x)(y) + 2(y)(z) + 2(z)(x) \\
 &= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad (x - 2y + 2)^2 &= x^2 + (2y)^2 + (2)^2 - 2(x)(2y) - 2(2y)(2) + 2(2)(x) \\
 &= x^2 + 4y^2 + 4 - 4xy - 8y + 4x
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad (5a - 3b + c)^2 &= (5a)^2 + (3b)^2 + (c)^2 - 2(5a)(3b) - 2(3b)(c) + 2(c)(5a) \\
 &= 25a^2 + 9b^2 + c^2 - 30ab - 6bc + 10ca
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad (5x - 3y - 2)^2 &= (5x)^2 + (3y)^2 + (2)^2 - 2(5x)(3y) + 2(3y)(2) - 2(2)(5x) \\
 &= 25x^2 + 9y^2 + 4 - 30xy + 12y - 20x
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad \left(x - \frac{1}{x} + 5\right)^2 &= (x)^2 + \left(\frac{1}{x}\right)^2 + (5)^2 - 2(x)\left(\frac{1}{x}\right) - 2\left(\frac{1}{x}\right)(5) + 2(5)(x) \\
 &= x^2 + \frac{1}{x^2} + 25 - 2 - \frac{10}{x} + 10x \\
 &= x^2 + \frac{1}{x^2} + 23 - \frac{10}{x} + 10x
 \end{aligned}$$

Solution 4:

We know that

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \dots (1)$$

Given that, $a^2 + b^2 + c^2 = 50$ and $a+b+c=12$.

We need to find $ab + bc + ca$:

Substitute the values of $(a^2 + b^2 + c^2)$ and $(a+b+c)$ in the identity (1), we have

$$(12)^2 = 50 + 2(ab + bc + ca)$$

$$\Rightarrow 144 = 50 + 2(ab + bc + ca)$$

$$\Rightarrow 94 = 2(ab + bc + ca)$$

$$\Rightarrow ab + bc + ca = \frac{94}{2}$$

$$\Rightarrow ab + bc + ca = 47$$

Solution 5:

We know that

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \dots (1)$$

Given that, $a^2 + b^2 + c^2 = 35$ and $ab + bc + ca = 23$.

We need to find $a + b + c$:

Substitute the values of $(a^2 + b^2 + c^2)$ and $(ab + bc + ca)$ in the identity (1), we have

$$(a+b+c)^2 = 35 + 2(23)$$

$$\Rightarrow (a+b+c)^2 = 81$$

$$\Rightarrow a+b+c = \pm\sqrt{81}$$

$$\Rightarrow a+b+c = \pm 9$$

Solution 6:

We know that

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \dots (1)$$

Given that, $a+b+c = p$ and $ab + bc + ca = q$.

We need to find $a^2 + b^2 + c^2$:

Substitute the values of $(ab + bc + ca)$ and $(a+b+c)$ in the identity (1), we have

$$(p)^2 = a^2 + b^2 + c^2 + 2(q)$$

$$\Rightarrow p^2 = a^2 + b^2 + c^2 + 2q$$

$$\Rightarrow a^2 + b^2 + c^2 = p^2 - 2q$$

Solution 7:

$$a^2 + b^2 + c^2 = 50 \text{ and } ab + bc + ca = 47$$

$$\text{Since } (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$\therefore (a+b+c)^2 = 50 + 2(47)$$

$$\Rightarrow (a+b+c)^2 = 50 + 94 = 144$$

$$\Rightarrow a+b+c = \sqrt{144} = \pm 12$$

$$\therefore a+b+c = \pm 12$$

Solution 8:

$$x + y - z = 4 \text{ and } x^2 + y^2 + z^2 = 30$$

Since $(x + y - z)^2 = x^2 + y^2 + z^2 + 2(xy - yz - zx)$, we have

$$(4)^2 = 30 + 2(xy - yz - zx)$$

$$\Rightarrow 16 = 30 + 2(xy - yz - zx)$$

$$\Rightarrow 2(xy - yz - zx) = -14$$

$$\Rightarrow xy - yz - zx = \frac{-14}{2} = -7$$

$$\therefore xy - yz - zx = -7$$

Exercise 4(D)**Solution 1:**

$$\text{Given that } x^3 + 4y^3 + 9z^3 = 18xyz \text{ and } x + 2y + 3z = 0$$

$$\therefore x + 2y = -3z, 2y + 3z = -x \text{ and } 3z + x = -2y$$

Now

$$\begin{aligned} \frac{(x+2y)^2}{xy} + \frac{(2y+3z)^2}{yz} + \frac{(3z+x)^2}{zx} &= \frac{(-3z)^2}{xy} + \frac{(-x)^2}{yz} + \frac{(-2y)^2}{zx} \\ &= \frac{9z^2}{xy} + \frac{x^2}{yz} + \frac{4y^2}{zx} \\ &= \frac{x^3 + 4y^3 + 9z^3}{xyz} \end{aligned}$$

$$\text{Given that } x^3 + 4y^3 + 9z^3 = 18xyz$$

$$\therefore \frac{(x+2y)^2}{xy} + \frac{(2y+3z)^2}{yz} + \frac{(3z+x)^2}{zx} = \frac{18xyz}{xyz} = 18$$

Solution 2:

(i)

Given that $a + \frac{1}{a} = m$;

Now consider the expansion of $\left(a + \frac{1}{a}\right)^2$:

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow m^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = m^2 - 2$$

Now consider the expansion of $\left(a - \frac{1}{a}\right)^2$:

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = m^2 - 2 - 2$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = m^2 - 4$$

$$\Rightarrow \left(a - \frac{1}{a}\right) = \pm\sqrt{m^2 - 4} \dots (1)$$

(ii)

$$\begin{aligned} a^2 - \frac{1}{a^2} &= \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right) \quad [\text{since } a^2 - b^2 = (a+b)(a-b)] \\ &= m\left(\pm\sqrt{m^2 - 4}\right) \\ &= \pm m\sqrt{m^2 - 4} \end{aligned}$$

Solution 3:

$$\begin{aligned} &(2x^2 - 8)(x - 4)^2 \\ &= (2x^2 - 8)(x^2 - 8x + 16) \\ &= 4x^4 - 16x^3 + 32x^2 - 8x^2 + 64x - 128 \\ &= 4x^4 - 16x^3 + 24x^2 + 64x - 128 \end{aligned}$$

Hence,

coefficient of $x^3 = -16$

coefficient of $x^2 = 24$

constant term = -128

Solution 4:

Given that

$$x^2 + \frac{1}{9x^2} = \frac{25}{36}$$

$$\Rightarrow x^2 + \frac{1}{(3x)^2} = \frac{25}{36} \dots (1)$$

Now consider the expansion of $\left(x + \frac{1}{3x}\right)^2$:

$$\left(x + \frac{1}{3x}\right)^2 = x^2 + \frac{1}{(3x)^2} + 2 \times x \times \frac{1}{3x}$$

$$\Rightarrow = x^2 + \frac{1}{(3x)^2} + \frac{2}{3}$$

$$\Rightarrow = \frac{25}{36} + \frac{2}{3} \quad [\text{from (1)}]$$

$$\Rightarrow = \frac{25 + 24}{36}$$

$$\Rightarrow = \frac{49}{36}$$

$$\Rightarrow x + \frac{1}{3x} = \pm \sqrt{\frac{49}{36}}$$

$$\Rightarrow x + \frac{1}{3x} = \pm \frac{7}{6} \dots (2)$$

Solution 5:

(i)

$$2(x^2 + 1) = 5x$$

$$(x^2 + 1) = \frac{5}{2}x$$

Dividing by x , we have

$$\frac{(x^2 + 1)}{x} = \frac{5}{2}$$

$$\Rightarrow \left(x + \frac{1}{x}\right) = \frac{5}{2} \dots (1)$$

Now consider the expansion of $\left(x + \frac{1}{x}\right)^2$:

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow \left(\frac{5}{2}\right)^2 = x^2 + \frac{1}{x^2} + 2 \text{ [from (1)]}$$

$$\Rightarrow \left(\frac{5}{2}\right)^2 - 2 = x^2 + \frac{1}{x^2}$$

$$\Rightarrow \frac{25}{4} - 2 = x^2 + \frac{1}{x^2}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \frac{25-8}{4}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \frac{17}{4} \dots (2)$$

Now consider the expansion of $\left(x - \frac{1}{x}\right)^2$:

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = \frac{17}{4} - 2 \text{ [from (2)]}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = \frac{17-8}{4}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = \pm \frac{3}{2} \dots (3)$$

(ii)

We know that,

$$\left(x^3 - \frac{1}{x^3}\right) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

$$\therefore \left(x^3 - \frac{1}{x^3}\right) = \left(\pm \frac{3}{2}\right)^3 + 3\left(\pm \frac{3}{2}\right) \text{ [from (3)]}$$

$$= \pm \frac{27}{8} + \frac{9}{2}$$

$$\Rightarrow \left(x^3 - \frac{1}{x^3}\right) = \pm \frac{27+36}{8}$$

$$\Rightarrow \left(x^3 - \frac{1}{x^3}\right) = \pm \frac{63}{8}$$

Solution 6:

$$a^2 + b^2 = 34, ab = 12$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$= 34 + 2 \times 12 = 34 + 24 = 58$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$= 34 - 2 \times 12 = 34 - 24 = 10$$

$$(i) 3(a + b)^2 + 5(a - b)^2$$

$$= 3 \times 58 + 5 \times 10 = 174 + 50$$

$$= 224$$

$$(ii) 7(a - b)^2 - 2(a + b)^2$$

$$= 7 \times 10 - 2 \times 58 = 70 - 116 = -46$$

Solution 7:

$$\text{Given } 3x - \frac{4}{x} = 4;$$

$$\text{We need to find } 27x^3 - \frac{64}{x^3}$$

Let us now consider the expansion of $\left(3x - \frac{4}{x}\right)^3$:

$$\left(3x - \frac{4}{x}\right)^3 = 27x^3 - \frac{64}{x^3} - 3 \times 3x \times \frac{4}{x} \left(3x - \frac{4}{x}\right)$$

$$\Rightarrow (4)^3 = 27x^3 - \frac{64}{x^3} - 144 \quad [\text{Given: } 3x - \frac{4}{x} = 4]$$

$$\Rightarrow 64 + 144 = 27x^3 - \frac{64}{x^3}$$

$$\Rightarrow 27x^3 - \frac{64}{x^3} = 208$$

Solution 8:

Given that $x^2 + \frac{1}{x^2} = 7$

We need to find the value of $7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x}$

Consider the given equation:

$$x^2 + \frac{1}{x^2} - 2 = 7 - 2 \text{ [subtract 2 from both the sides]}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 5$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = \pm\sqrt{5} \dots (1)$$

$$\begin{aligned} \therefore 7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} &= 7x^3 - \frac{7}{x^3} + 8x - \frac{8}{x} \\ &= 7\left(x^3 - \frac{1}{x^3}\right) + 8\left(x - \frac{1}{x}\right) \dots (2) \end{aligned}$$

Now consider the expansion of $\left(x - \frac{1}{x}\right)^3$:

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$\Rightarrow x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

$$\Rightarrow x^3 - \frac{1}{x^3} = (\sqrt{5})^3 + 3(\sqrt{5}) \dots (3)$$

Now substitute the value of $x^3 - \frac{1}{x^3}$ in equation (2), we have

$$\begin{aligned} 7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} &= 7\left(x^3 - \frac{1}{x^3}\right) + 8\left(x - \frac{1}{x}\right) \\ \Rightarrow 7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} &= 7\left[(\sqrt{5})^3 + 3(\sqrt{5})\right] + 8[\sqrt{5}] \text{ [from (3)]} \\ \Rightarrow 7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} &= 7[5(\sqrt{5}) + 3(\sqrt{5})] + 8[\sqrt{5}] \\ \Rightarrow 7x^3 + 8x - \frac{7}{x^3} - \frac{8}{x} &= 64\sqrt{5} \end{aligned}$$

Solution 9:

$$\text{Given } x = \frac{1}{x-5};$$

By cross multiplication,

$$\Rightarrow x(x-5) = 1 \Rightarrow x^2 - 5x = 1 \Rightarrow x^2 - 1 = 5x$$

Dividing both sides by x ,

$$\frac{x^2 - 1}{x} = 5$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 5 \dots (1)$$

Now consider the expansion of $\left(x - \frac{1}{x}\right)^2$:

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow (5)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 25 + 2 = 27 \dots (2)$$

Let us consider the expansion of $\left(x + \frac{1}{x}\right)^2$:

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 27 + 2 \quad [\text{from (2)}]$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 29$$

$$\Rightarrow \left(x + \frac{1}{x}\right) = \pm\sqrt{29} \dots (3)$$

We know that

$$\begin{aligned} x^2 - \frac{1}{x^2} &= \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) \\ &= (\pm\sqrt{29})(5) \quad [\text{from equations (1) and (3)}] \end{aligned}$$

$$\Rightarrow x^2 - \frac{1}{x^2} = \pm 5\sqrt{29}$$

Solution 10:

Given $x = \frac{1}{5-x}$;

By cross multiplication,

$$\Rightarrow x(5-x) = 1 \Rightarrow x^2 - 5x = -1 \Rightarrow x^2 + 1 = 5x$$

Dividing both sides by x ,

$$\frac{x^2 + 1}{x} = 5$$

$$\Rightarrow \left(x + \frac{1}{x}\right) = 5 \dots (1)$$

We know that

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$= (5)^3 - 3(5) \quad [\text{from equation (1)}]$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 125 - 15 = 110$$

Solution 11:

Given that $3a + 5b + 4c = 0$

$$3a + 5b = -4c$$

Cubing both sides,

$$(3a + 5b)^3 = (-4c)^3$$

$$\Rightarrow (3a)^3 + (5b)^3 + 3 \times 3a \times 5b(3a + 5b) = -64c^3$$

$$\Rightarrow 27a^3 + 125b^3 + 45ab \times (-4c) = -64c^3$$

$$\Rightarrow 27a^3 + 125b^3 - 180abc = -64c^3$$

$$\Rightarrow 27a^3 + 125b^3 + 64c^3 = 180abc$$

Hence proved.

Solution 12:

Let a, b be the two numbers

$$\therefore a + b = 7 \text{ and } a^3 + b^3 = 133$$

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$\Rightarrow (7)^3 = 133 + 3ab(7)$$

$$\Rightarrow 343 = 133 + 21ab \Rightarrow 21ab = 343 - 133 = 210$$

$$\Rightarrow 21ab = 210 \Rightarrow ab = 10$$

$$\text{Now } a^2 + b^2 = (a + b)^2 - 2ab$$

$$= 7^2 - 2 \times 10 = 49 - 20 = 29$$

Solution 13:

$$(i) 4x^2 + ax + 9 = (2x + 3)^2$$

Comparing coefficients of x terms, we get

$$ax = 12x$$

$$\text{so, } a = 12$$

$$(ii) 4x^2 + ax + 9 = (2x - 3)^2$$

Comparing coefficients of x terms, we get

$$ax = -12x$$

$$\text{so, } a = -12$$

$$(iii) 9x^2 + (7a - 5)x + 25 = (3x + 5)^2$$

Comparing coefficients of x terms, we get

$$(7a - 5)x = 30x$$

$$7a - 5 = 30$$

$$7a = 35$$

$$a = 5$$

Solution 14:

Given

$$\frac{x^2+1}{x} = \frac{10}{3}$$

$$x + \frac{1}{x} = \frac{10}{3}$$

Squaring on both sides, we get

$$x^2 + \frac{1}{x^2} + 2 = \frac{100}{9}$$

$$x^2 + \frac{1}{x^2} = \frac{100-18}{9} = \frac{82}{9}$$

$$x - \frac{1}{x} = \sqrt{\left(x + \frac{1}{x}\right)^2 - 4} = \sqrt{\frac{100}{9} - 4} = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

$$\therefore x - \frac{1}{x} = \frac{8}{3}$$

Cubing both sides, we get

$$\left(x - \frac{1}{x}\right)^3 = \frac{512}{27}$$

$$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = \frac{512}{27}$$

$$x^3 - \frac{1}{x^3} = \frac{512}{27} + 8 = \frac{512+216}{27} = \frac{728}{27}$$

Solution 15:

Given difference between two positive numbers is 4 and difference between their cubes is 316.

Let the positive numbers be a and b

$$a - b = 4$$

$$a^3 - b^3 = 316$$

Cubing both sides,

$$(a - b)^3 = 64$$

$$a^3 - b^3 - 3ab(a - b) = 64$$

$$\text{Given } a^3 - b^3 = 316$$

$$\text{So } 316 - 64 = 3ab(4)$$

$$252 = 12ab$$

$$\text{So } ab = 21; \text{ product of numbers is 21}$$

Squaring both sides, we get

$$(a - b)^2 = 16$$

$$a^2 + b^2 - 2ab = 16$$

$$a^2 + b^2 = 16 + 42 = 58$$

Sum of their squares is 58.

Exercise 4(E)

Solution 1:

Using identity:

$$(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$$

$$(i) (x + 6)(x + 4)(x - 2)$$

$$= x^3 + (6 + 4 - 2)x^2 + [6 \times 4 + 4 \times (-2) + (-2) \times 6]x + 6 \times 4 \times (-2)$$

$$= x^3 + 8x^2 + (24 - 8 - 12)x - 48$$

$$= x^3 + 8x^2 + 4x - 48$$

$$(ii) (x - 6)(x - 4)(x + 2)$$

$$= x^3 + (-6 - 4 + 2)x^2 + [-6 \times (-4) + (-4) \times 2 + 2 \times (-6)]x + (-6) \times (-4) \times 2$$

$$= x^3 - 8x^2 + (24 - 8 - 12)x + 48$$

$$= x^3 - 8x^2 + 4x + 48$$

$$(iii) (x - 6)(x - 4)(x - 2)$$

$$= x^3 + (-6 - 4 - 2)x^2 + [-6 \times (-4) + (-4) \times (-2) + (-2) \times (-6)]x + (-6) \times (-4) \times (-2)$$

$$= x^3 - 12x^2 + (24 + 8 + 12)x - 48$$

$$= x^3 - 12x^2 + 44x - 48$$

$$(iv) (x + 6)(x - 4)(x - 2)$$

$$= x^3 + (6 - 4 - 2)x^2 + [6 \times (-4) + (-4) \times (-2) + (-2) \times 6]x + 6 \times (-4) \times (-2)$$

$$= x^3 - 0x^2 + (-24 + 8 - 12)x + 48$$

$$= x^3 - 28x + 48$$

Solution 2:

$$\begin{aligned}
 \text{(i)} \quad (2x + 3y)(4x^2 - 6xy + 9y^2) &= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2] \\
 &= (2x)^3 + (3y)^3 \\
 &= 8x^3 + 27y^3
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \left(3x - \frac{5}{x}\right)\left(9x^2 + 15 + \frac{25}{x^2}\right) &= \left(3x - \frac{5}{x}\right)\left((3x)^2 + (3x)\left(\frac{5}{x}\right) + \left(\frac{5}{x}\right)^2\right) \\
 &= (3x)^3 - \left(\frac{5}{x}\right)^3 \\
 &= 27x^3 - \frac{125}{x^3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \left(\frac{a}{3} - 3b\right)\left(\frac{a^2}{9} + ab + 9b^2\right) &= \left(\frac{a}{3} - 3b\right)\left(\left(\frac{a}{3}\right)^2 + \left(\frac{a}{3}\right)(3b) + (3b)^2\right) \\
 &= \left(\frac{a}{3}\right)^3 - (3b)^3 \\
 &= \frac{a^3}{27} - 27b^3
 \end{aligned}$$

Solution 3:

Using identity: $(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$

$$\begin{aligned}
 \text{(i)} \quad (104)^3 &= (100 + 4)^3 \\
 &= (100)^3 + (4)^3 + 3 \times 100 \times 4(100 + 4) \\
 &= 1000000 + 64 + 1200 \times 104 \\
 &= 1000000 + 64 + 124800 \\
 &= 1124864
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad (97)^3 &= (100 - 3)^3 \\
 &= (100)^3 - (3)^3 - 3 \times 100 \times 3(100 - 3) \\
 &= 1000000 - 27 - 900 \times 97 \\
 &= 1000000 - 27 - 87300 \\
 &= 912673
 \end{aligned}$$

Solution 4:

$$\frac{(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3}{(x - y)^3 + (y - z)^3 + (z - x)^3}$$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$$\text{Now, } x^2 - y^2 + y^2 - z^2 + z^2 - x^2 = 0$$

$$\Rightarrow (x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3 = 3(x^2 - y^2)(y^2 - z^2)(z^2 - x^2) \quad \dots(1)$$

$$\text{And, } x - y + y - z + z - x = 0$$

$$\Rightarrow (x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x) \quad \dots(2)$$

Now,

$$\begin{aligned} & \frac{(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3}{(x - y)^3 + (y - z)^3 + (z - x)^3} \\ &= \frac{3(x^2 - y^2)(y^2 - z^2)(z^2 - x^2)}{3(x - y)(y - z)(z - x)} \quad \dots[\text{From (1) and (2)}] \\ &= \frac{(x - y)(x + y)(y - z)(y + z)(z - x)(z + x)}{(x - y)(y - z)(z - x)} \\ &= (x + y)(y + z)(z + x) \end{aligned}$$

Solution 5:

$$(i) \frac{0.8 \times 0.8 \times 0.8 + 0.5 \times 0.5 \times 0.5}{0.8 \times 0.8 - 0.8 \times 0.5 + 0.5 \times 0.5}$$

Let $0.8 = a$ and $0.5 = b$

Then, the given expression becomes

$$\frac{a \times a \times a + b \times b \times b}{a \times a - a \times b + b \times b}$$

$$= \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$= \frac{(a + b)(a^2 - ab + b^2)}{a^2 - ab + b^2}$$

$$= a + b$$

$$= 0.8 + 0.5$$

$$= 1.3$$

$$(ii) \frac{1.2 \times 1.2 + 1.2 \times 0.3 + 0.3 \times 0.3}{1.2 \times 1.2 \times 1.2 - 0.3 \times 0.3 \times 0.3}$$

Let $1.2 = a$ and $0.3 = b$

Then, the given expression becomes

$$\frac{a \times a + a \times b + b \times b}{a \times a \times a - b \times b \times b}$$

$$= \frac{a^2 + ab + b^2}{a^3 - b^3}$$

$$= \frac{a^2 + ab + b^2}{(a - b)(a^2 + ab + b^2)}$$

$$= \frac{1}{a - b}$$

$$= \frac{1}{1.2 - 0.3}$$

$$= \frac{1}{0.9}$$

$$= \frac{10}{9}$$

$$= 1\frac{1}{9}$$

Solution 6:

$$a^3 - 8b^3 + 27c^3 = a^3 + (-2b)^3 + (3c)^3$$

Since $a - 2b + 3c = 0$, we have

$$a^3 - 8b^3 + 27c^3 = a^3 + (-2b)^3 + (3c)^3$$

$$= 3(a)(-2b)(3c)$$

$$= -18abc$$

Solution 7:

$$x + 5y = 10$$

$$\Rightarrow (x + 5y)^3 = 10^3$$

$$\Rightarrow x^3 + (5y)^3 + 3(x)(5y)(x + 5y) = 1000$$

$$\Rightarrow x^3 + (5y)^3 + 3(x)(5y)(10) = 1000$$

$$= x^3 + (5y)^3 + 150xy = 1000$$

$$= x^3 + (5y)^3 + 150xy - 1000 = 0$$

Solution 8:

$$x = 3 + 2\sqrt{2}$$

$$(i) \quad \frac{1}{x} = \frac{1}{3 + 2\sqrt{2}}$$

$$= \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$$

$$= \frac{3 - 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2}$$

$$= \frac{3 - 2\sqrt{2}}{9 - 8}$$

$$\therefore \frac{1}{x} = 3 - 2\sqrt{2} \quad \dots (1)$$

$$(ii) \quad x - \frac{1}{x} = (3 + 2\sqrt{2}) - (3 - 2\sqrt{2}) \quad \dots [\text{From (1)}]$$

$$= 3 + 2\sqrt{2} - 3 + 2\sqrt{2}$$

$$\therefore x - \frac{1}{x} = 4\sqrt{2} \quad \dots (2)$$

$$(iii) \quad \left(x - \frac{1}{x}\right)^3 = (4\sqrt{2})^3 \quad \dots [\text{From (2)}]$$

$$= 64 \times 2\sqrt{2}$$

$$= 128\sqrt{2}$$

$$(iv) \quad x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

$$= 128\sqrt{2} + 3(4\sqrt{2})$$

$$= 128\sqrt{2} + 12\sqrt{2}$$

$$= 140\sqrt{2}$$

Solution 9:

$$a + b = 11 \text{ and } a^2 + b^2 = 65$$

$$\text{Now, } (a + b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow (11)^2 = 65 + 2ab$$

$$\Rightarrow 121 = 65 + 2ab$$

$$\Rightarrow 2ab = 56$$

$$\Rightarrow ab = 28$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$= (11)(65 - 28)$$

$$= 11 \times 37$$

$$= 407$$