Integrals

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): $\int \frac{dx}{x^2+2x+3} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+c$

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

Ans. Option (A) is correct.

Explanation:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c.$$

This is a standard integral and hence R is true.

$$\int \frac{dx}{x^2+2x+3} = \int \frac{dx}{(x+1)^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+c$$

Hence A is true and R is the correct explanation for A.

Q. 2. Assertion(A): $\int e^x[\sin x - \cos x]\,dx = e^x \sin x + C$

Reason (R): $\int e^x[f(x)+f'(x)]\,dx = e^x f(x)+c$

Ans. Option (D) is correct.

Explanation:

$$\begin{aligned}\int e^x[f(x)+f'(x)]dx &= \int e^x f(x)dx + \int e^x f'(x)dx \\ &= f(x)e^x - \int f'(x)e^x dx + \int f'(x)e^x dx \\ &= e^x f(x)+c\end{aligned}$$

Hence R is true.

$$\begin{aligned}\int e^x(\sin x - \cos x)dx &= e^x(-\cos x)+c \\ &= -e^x\cos x + c\end{aligned}$$

$$\left[\because \frac{d}{dx}(-\cos x) = \sin x\right]$$

Hence A is false.

Q. 3. Assertion (A): $\int x^x(1+\log x)dx = x^x + c$

Reason (R): $\frac{d}{dx}(x^x) = x^x(1+\log x)$

Ans. Option (A) is correct.

Explanation : Let $y = x^x$

$\Rightarrow \log y = x\log x$

Differentiating w.r.t. x

$$\begin{aligned}\Rightarrow \frac{1}{y}\frac{dy}{dx} &= x\left(\frac{1}{x}\right)+\log x(1) \\ \frac{dy}{dx} &= y(1+\log x) \\ &= x^x(1+\log x)\end{aligned}$$

Hence R is true.

Since $\frac{d}{dx}(x^x) = x^x(1+\log x)$

$\int x^x(1+\log x)dx = x^x + c$

Using the concept of anti-derivative, A is true. R is the correct explanation for A.

Q. 4. Assertion (A): $\int x^2 dx = \frac{x^3}{3}+c$

Reason (R): $\int e^{x^2}dx = e^{x^{3/3}}+c$

Ans. Option (C) is correct.

Explanation:

Since $\int x^n\,dx = \frac{x^{n+1}}{n+1}+c,$

$$\int x^2 dx = \frac{x^{2+1}}{2+1}+c = \frac{x^3}{3}+c$$

$\therefore$ A is true.

$\int e^{x^2}\,dx$ is a function which can not be integrated.

$\therefore$ R is false.

Q. 5. Assertion (A): $\int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x}dx = \frac{\pi}{4}$

Reason (R): $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}dx = \frac{\pi}{4}$

Ans. Option (A) is correct.

Explanation:

Let $$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx \quad \text{...(i)}$$

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\therefore \quad I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-x\right) dx}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}$$

$$I = \int \frac{\cos x}{\cos x + \sin x}\, dx \quad \text{...(ii)}$$

Adding equations (i) + (ii),

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\, dx$$

$$= \int_0^{\pi/2} 1\, dx$$

$$= [x]_0^{\pi/2}$$

$$= \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$

Hence R is true.
From (ii), A is also true.
R is the correct explanation for A.

Q. 6. Assertion (A): $\int_{-3}^{3} (x^3+5)dx = 30$

Reason (R): $f(x) = x^3 + 5$ is an odd function.

Ans. Option (C) is correct.

Explanation:

Let $$f(x) = x^3 + 5$$

$$f(-x) = (-x)^3 + 5$$

$$= -x^3 + 5$$

$f(x)$ is neither even nor odd. Hence R is false.

$$\int_{-3}^{3} x^3\, dx = 0 \quad [\because x^3 \text{ is odd}]$$

$$\int_{-3}^{3} 5\, dx = 5[x]_{-3}^{3} = 30$$

$$\therefore \int_{-3}^{3} (x^3+5)dx = 0 + 30 = 30$$

Hence A is true.

Q. 7. Assertion (A): $\frac{d}{dx}\left[\int_0^{x^2} \frac{dt}{t^2+4}\right] = \frac{2x}{x^4+4}$

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

Ans. Option (A) is correct.

Explanation:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c.$$

This is a standard integral and hence true.
So R is true.

$$\int_0^{x^2} \frac{dt}{t^2+4} = \left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^{x^2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x^2}{2}\right)$$

$$\frac{d}{dx}\left[\int_0^{x^2} \frac{dt}{t^2+4}\right] = \frac{d}{dx}\left[\frac{1}{2}\tan^{-1}\left(\frac{x^2}{2}\right)\right]$$

$$= \frac{1}{2} \times \frac{1}{1+\frac{x^4}{4}} \times \frac{2x}{2}$$

$$= \frac{x}{2} \times \frac{4}{4+x^4}$$

$$= \frac{2x}{4+x^4}$$

Hence A is true and R is the correct explanation for A.

Q. 8. Assertion (A): $\int_{-1}^{1} (x^3 + \sin x + 2)dx = 0$

Reason (R):

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function i.e.,} (-x) = f(x) \\ 0, & \text{if } f(x) \text{ is an odd function i.e., } f(-x) = -f(x) \end{cases}$$

Ans. Option (D) is correct.

Explanation:

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function i.e.,} (-x) = f(x) \\ 0, & \text{if } f(x) \text{ is an odd function i.e., } f(-x) = -f(x) \end{cases}$$

This is a property of the definite integrals and hence R is true.

$$\int_{-1}^{1} (x^3 + \sin x + 2)dx$$

$$= \underbrace{\int_{-1}^{1} (x^3 + \sin x)dx}_{\text{Odd function}} + \underbrace{\int_{-1}^{1} 2dx}_{\text{Even function}}$$

$$= 0 + 2[x]_{-1}^{1}$$

$$= 2 \times 2$$

$$= 4$$

Hence A is false.