Application of Integrals

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

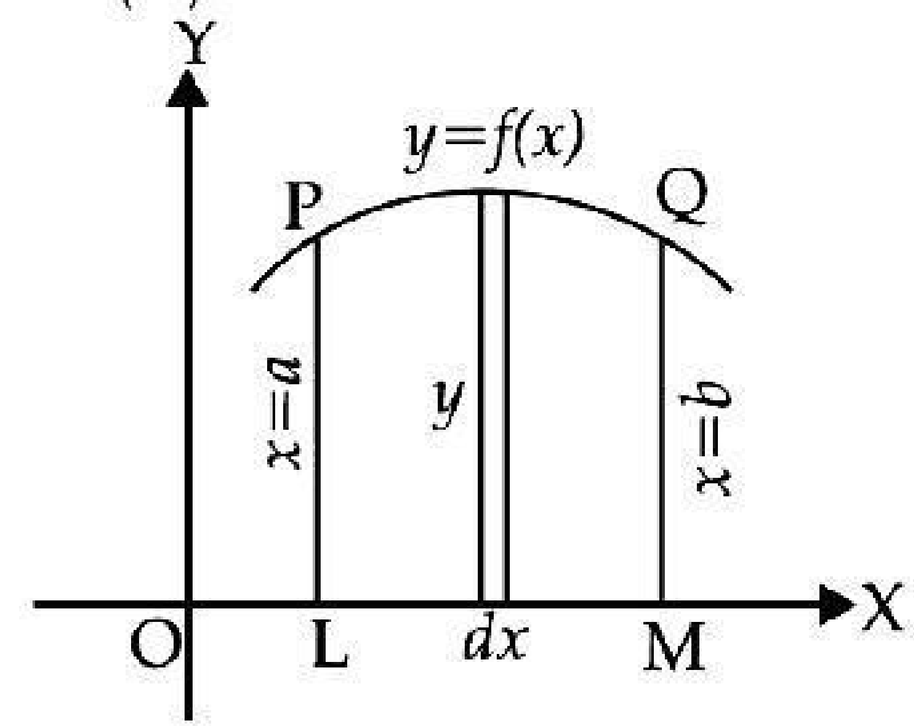
(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

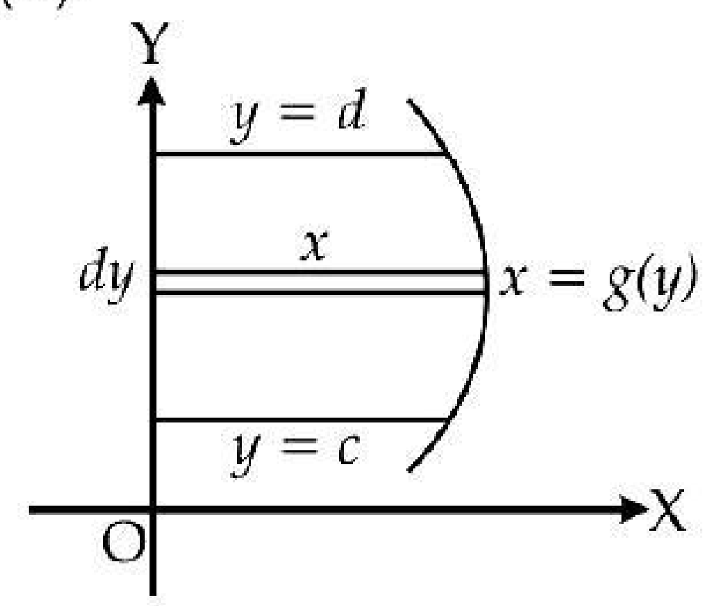
(D) A is false and R is True

Q. 1. Assertion (A):



The area of region $PQML = \int_a^b y\,dx = \int_a^b f(x)\,dx$

Reason (R):



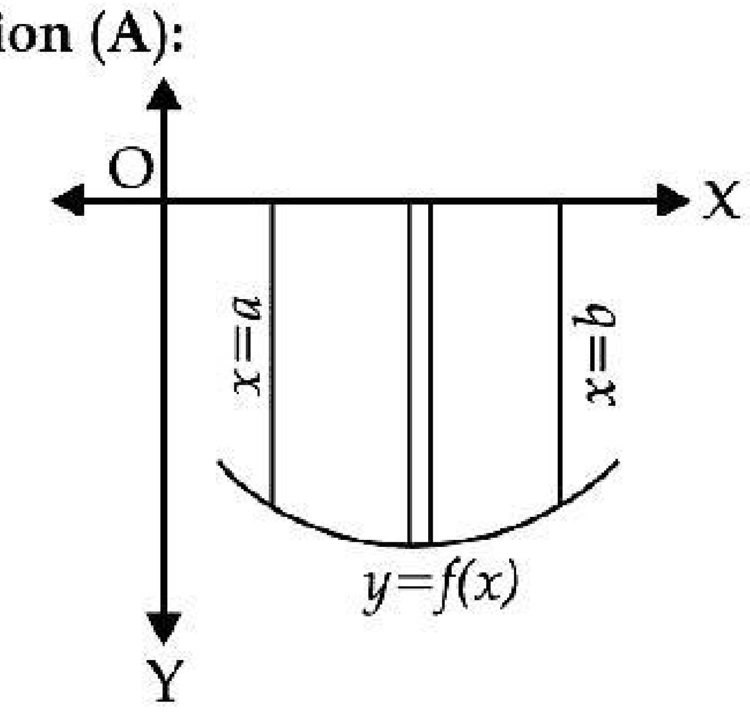
The area A of the region bounded by curve $x = g(y)$, y-axis and the lines $y = c$ and $y = d$ is given by

$A = \int_c^d x dy$

Ans. Option (B) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are individually correct.

Q. 2. Assertion (A):



$$\text{Area} = \left|\int_a^b f(x)dx\right|$$

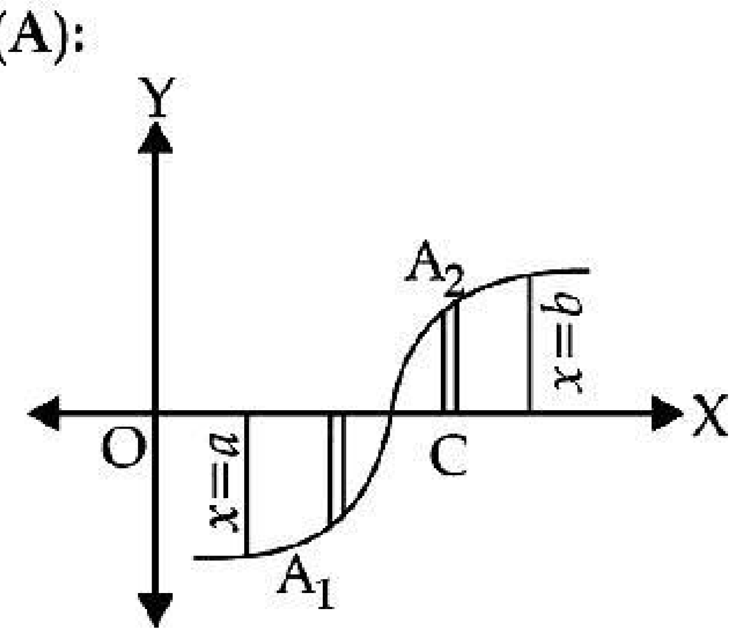
Reason (R): If the curve under consideration lies below x-axis, then $f(x) < 0$ from $x = a$ to $x = b$, the area bounded by the curve $y = f(x)$ and the ordinates $x = a$, $x = b$ and x-axis is negative. But, if the numerical value of the area is to be taken into consideration, then

$$\text{Area} - \left|\int_a^b f(x)dx\right|$$

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 3. Assertion (A):



$$\text{Area} = |A_1| + |A_2|$$

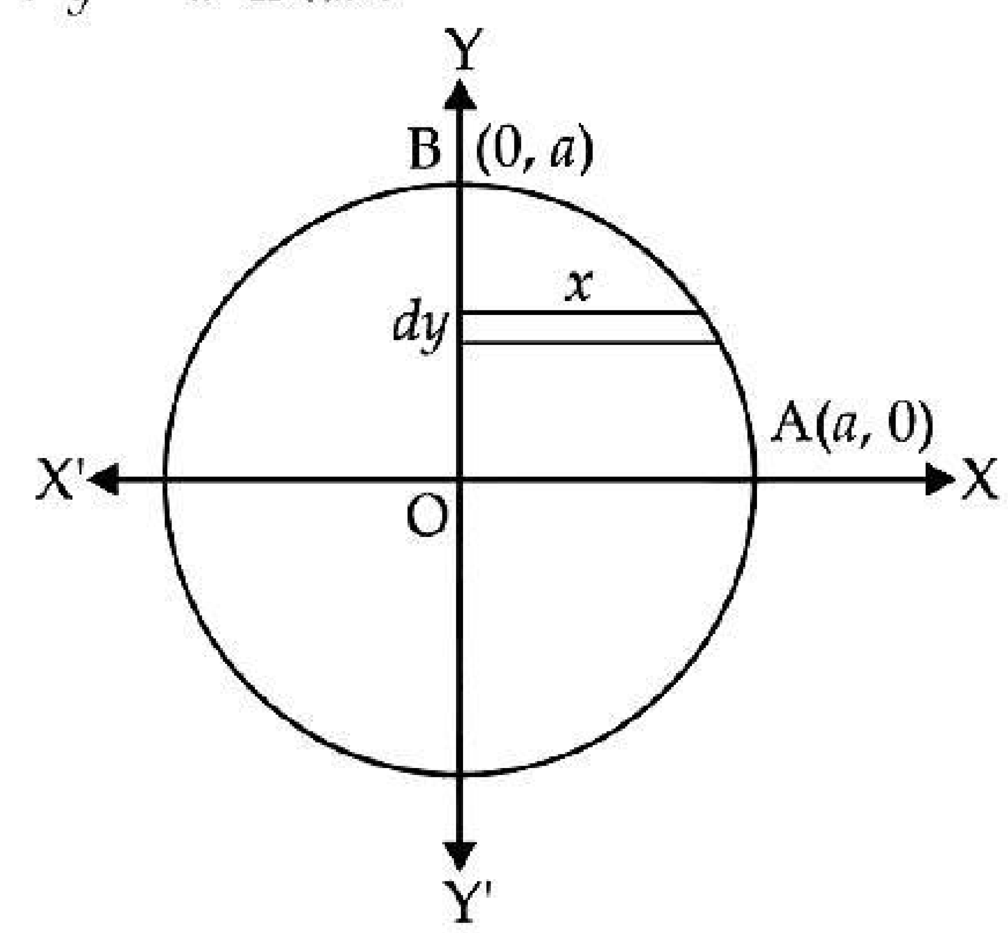
Reason (R): It may happen that some portion of the curve is above x-axis and some portion is below x-axis as shown in the figure. Let A_1 be the area below x-axis and A_2 be the area above the x-axis. Therefore, area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = a$ and $x = b$ is given by

$$\text{Area} = |A_1| + |A_2|$$

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 4. Assertion (A): The area enclosed by the circle $x^2 + y^2 = a^2$ is πa^2.



Reason (R): The area enclosed by the circle

$$= 4\int_0^a x dy$$

$$= 4\int_0^a \sqrt{a^2 - y^2}\, dy$$

$$= 4\left[\frac{y}{2}\sqrt{a^2 - y^2} + \frac{a^2}{2}\sin\ ^{1}\frac{y}{a}\right]_0^a$$

$$= 4\left[\left(\frac{a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}1\right) - 0\right]$$

$$= 4\frac{a^2}{2}\frac{\pi}{2}$$

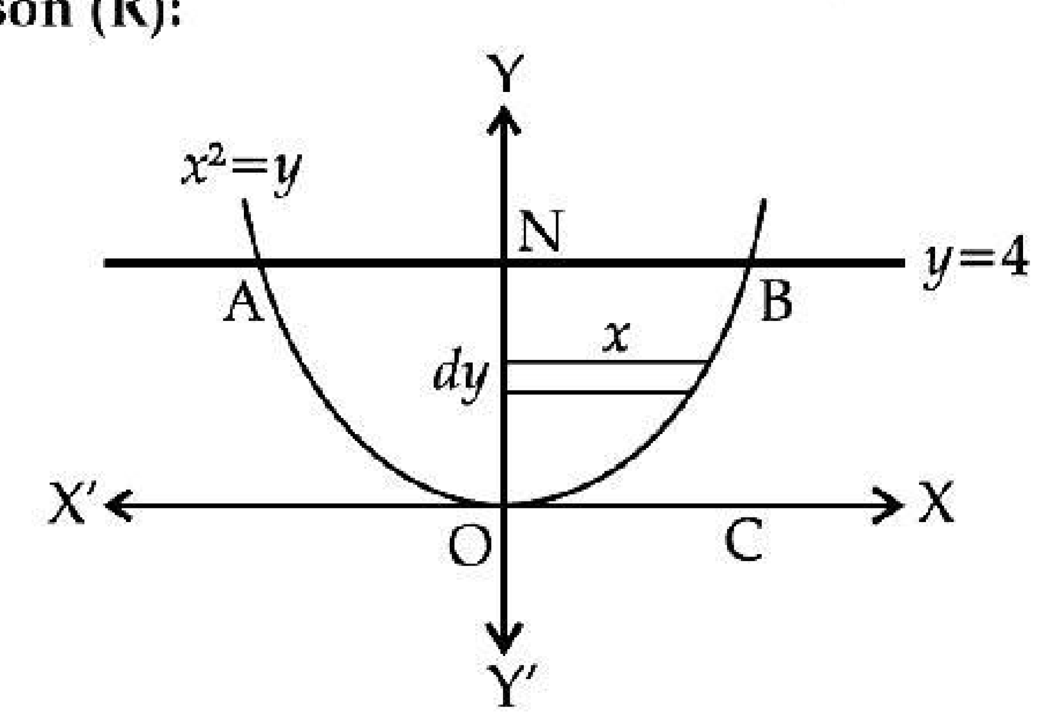
$$= \pi a^2$$

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 5. Assertion (A): The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is $\frac{3}{32}$.

Reason (R):



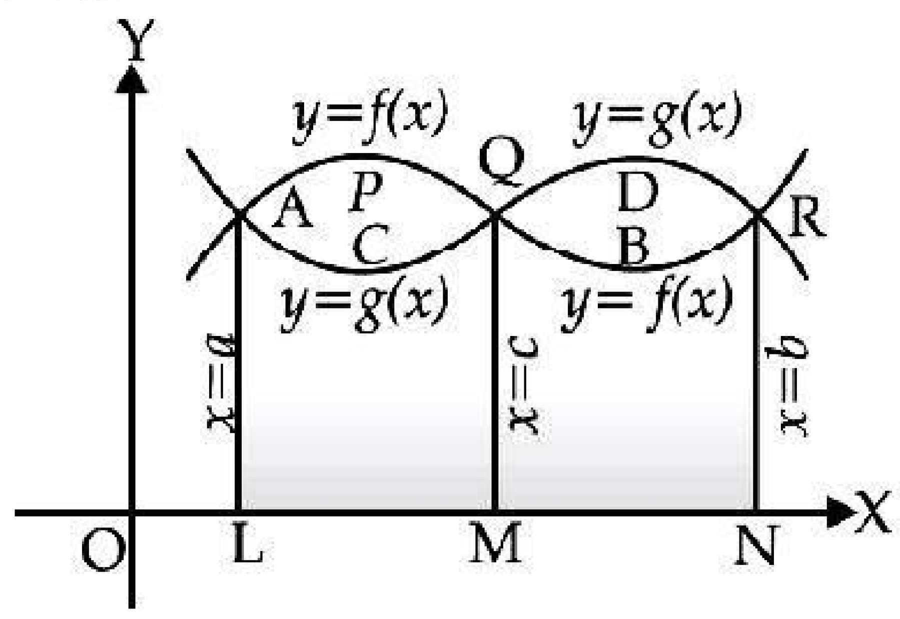
Since the given curve represented by the equation $y = x^2$ is a parabola symmetrical about y-axis only, therefore, from figure, the required area of the region $AOBA$ is given by

$$A = 2\int_0^4 x\,dy$$
$$= 2\int_0^4 \sqrt{y}\,dy$$
$$= 2\times\frac{2}{3}\left[y^{3/2}\right]_0^4$$
$$= \frac{4}{3}\times 8$$
$$= \frac{32}{3}$$

Ans. Option (D) is correct.

> ***Explanation:*** Assertion (A) is wrong. Reason (R) is the correct solution of Assertion (A).

Q. 6. Assertion (A): If the two curves $y = f(x)$ and $y = g(x)$ intersect at $x = a$, $x = c$ and $x = b$, such that $a < c < b$.

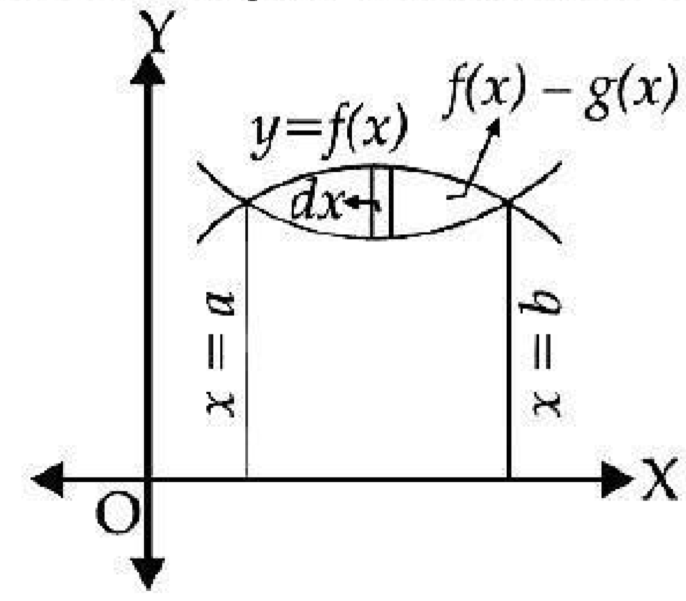


If $f(x) > g(x)$ in $[a, c]$ and $g(x) \leq f(x)$ in $[c, b]$, then Area of the regions bounded by the curve

= Area of region $PACQP$ + Area of region $QDRBQ$.

$$= \int_a^c |f(x) - g(x)|\,dx + \int_c^b |g(x) - f(x)|\,dx.$$

Reason (R): Let the two curves by $y = f(x)$ and $y = g(x)$, as shown in the figure. Suppose these curves intersect at $f(x)$ with width dx.



$$\text{Area} = \int_a^b [f(x) - g(x)]\,dx$$
$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

= Area bounded by the curve $\{y = f(x)\}$
–Area bounded by the curve $\{y = g(x)\}$,

where $f(x) > g(x)$.

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are individually correct.