Differential Equations

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): The order of the differential equation given by $\frac{dy}{dx} + 4y = \sin x$ is 1.

Reason (R): Since the order of a differential equation is defined as the order of the highest derivative occurring in the differential equation, *i.e.*, for nth derivative $\frac{d^n y}{d^x n}$ if $n = 1$. then it's order = 1.

Given differential equation contains only $\frac{dy}{dx}$ derivative with variables and constants.

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 2. Assertion (A): The degree of the differential equation given by $\frac{dy}{dx} = \frac{x^4 - y^4}{(x^2 + y^2)xy}$ is 1.

Reason (R): The degree of a differential equation is the degree of the highest order derivative when differential coefficients are free from radicals and fraction.

The given differential equation has first order derivative which is free from radical and fraction with power = 1, thus it has a degree of 1.

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 3. Assertion (A): Solution of the differential equation

$\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$ is $\frac{e^{2y}}{3} = \frac{e^{3x}}{3} + \frac{x^2}{2} + C$

Reason (R):

$$\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$$

$$\frac{dy}{dx} = e^{-2y}(e^{3x} + x^2)$$

separating the variables

$$e^{2y}dy = (e^{3x} + x^2)dx \qquad \text{[integrating]}$$

$$\int e^{2y}dy = \int (e^{3x} + x^2)dx$$

$$\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + C.$$

Ans. Option (D) is correct.

> ***Explanation:*** Assertion (A) is wrong. The correct solution is given in Reason (R).

Q. 4. Assertion (A): The solution of differential equation

$\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ is $\cos\left(\frac{y}{x}\right) = xc$

Reason (R): $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ we can clearly see that it is an homogeneous equation substituting

$$y = vx$$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v + \tan v$$

separating the variables and integrating we get

$$\int \frac{1}{\tan v}dv = \int \frac{1}{x}dx$$

$$\log(\sin v) = \log x + \log C$$

$$\sin(v) = xC$$

$$\Rightarrow \quad \sin\left(\frac{y}{x}\right) = xC$$

is the solution where, C is constant.

Ans. Option (D) is correct.

> ***Explanation:*** Assertion (A) is wrong. The correct solution is given in Reason (R).

Q. 5. Assertion (A): The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx} + 5}$ are 2 and 1 respectively

Reason (R): The differential equation

$$\left(\frac{dx}{dy}\right)^3 + 2y^{1/2} = x$$

is of order 1 and degree 3.

Ans. Option (B) is correct.

> ***Explanation:*** Squaring both sides of the given differential equation,
>
> $$\left(\sqrt{\frac{d^2y}{dx^2}}\right)^2 = \left(\sqrt{\frac{dy}{dx} + 5}\right)^2$$
>
> $$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} + 5$$
>
> The highest order is 2 and its power is 1
> ∴ Order is 2, degree is 1
> Hence, Assertion (A) is true.
> The equation given in reason (R) is,
>
> $$\left(\frac{1}{\frac{dy}{dx}}\right)^3 + 2\sqrt{y} = x$$
>
> $$\Rightarrow \quad \frac{1 + 2\sqrt{y}\left(\frac{dy}{dx}\right)^3}{\left(\frac{dy}{dx}\right)^3} = x$$
>
> $$\Rightarrow \quad 1 + 2\sqrt{y}\left(\frac{dy}{dx}\right)^3 = x\left(\frac{dy}{dx}\right)^3$$
>
> Highest order is 1 and its power is 3
> ∴ Order is 1 and degree is 3.
> Hence, reason (R) is also true.

Q. 6. Assertion (A): The differential equation formed by eliminating a and b from $y = ae^x + be^{-x}$ is $\frac{d^2y}{dx^2} - y = 0$

Reason (R):

$$y = ae^x + be^{-x} \qquad \ldots(i)$$

Differentiating w.r.t. 'x'

$$\frac{dy}{dx} = ae^x - be^{-x}$$

Differentiating again w.r.t. 'x'

$$\frac{d^2y}{dx^2} = ae^x + be^{-x} \qquad \ldots(ii)$$

Subtracting eqn. (i) from eqn. (ii)

$$\frac{d^2y}{dx^2} - y = ae^x + be^{-x} - ae^x - be^{-x}$$

$$= 0$$

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).