Three Dimensional Geometry

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): $x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$ the equation to the sphere whose centre is at (–2, 3, 4) and radius is 6 units.

Reason (R): Given:

Centre is at (–2, 3, 4) and $r = 6$

$\Rightarrow (x_0, y_0, z_0) = (-2, 3, 4)$ and $r = 6$

We know that general equation of sphere is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

$$\Rightarrow \quad (x - (-2))^2 + (y - 3)^2 + (z - 4)^2 = 6^2$$

$$\Rightarrow \quad (x + 2)^2 + (y - 3)^2 + (z - 4)^2 = 6^2$$

$$\Rightarrow \quad x^2 + 4x + 4 + y^2 - 6y + 9 + z^2 - 8z + 16 = 36$$

$$\Rightarrow \quad x^2 + y^2 + z^2 + 4x - 6y - 8z + 29 = 36$$

$$\Rightarrow \quad x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 2. Assertion (A): If two lines are in the same plane *i.e.*, they are coplanar, they will intersect each other if they are non-parallel. Hence the shortest distance between them is zero.

If the lines are parallel then the shortest distance between them will be the perpendicular distance between the lines *i.e.*, the length of the perpendicular drawn from a point on one line onto the other line.

Reason (R): The angle between the lines with direction ratio $\langle a_1, b_1, c_1 \rangle$ and $\langle a_2, b_2, c_2 \rangle$ is given by:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Ans. Option (B) is correct.

Explanation: Assertion (A) and Reason (R) both are individually correct.

Q. 3. Assertion (A): Direction cosines of a line are the sines of the angles made by the line with the negative directions of the coordinate axes.

Reason (R): The acute angle between the lines $x - 2 = 0$ and $\sqrt{3x - y - 2}$ is 30°.

Ans. Option (D) is correct.

Explanation: Assertion (A) is wrong.

Since, direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.

Reason (R) is correct.

Since, the slope of the line $x - 2 = 0$ is ∞.

The slope of line $\sqrt{3}x - y - 2 = 0$ is $\sqrt{3}$.

Let $m_1 = \infty$, $m_2 = \sqrt{3}$ and the angle between the given lines is θ.

$$\Rightarrow \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 . m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{m_2}{m_1} - 1}{\frac{1}{m_1} + m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30°$$

Q. 4. Assertion (A): P is a point on the line segment joining the points (3, 2, –1) and (6, –4, –2). If x coordinate of P is 5, then its y coordinate is –2.

Reason (R): The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ will be perpendicular, iff $aa' + bb' + cc' = 0$.

Ans. Option (C) is correct.

Explanation: Assertion (A) is correct.

Since $P = (5, y, z)$

Equation of line joining (3, 2, –1) and (6, –4, –2) is

$$\frac{x-3}{6-3} = \frac{y-2}{-4-2} = \frac{z+1}{-2+1} = \frac{x-3}{3} = \frac{y-2}{-6} = \frac{z+1}{-1}$$

so if point P lies on the line then it must satisfy the above equation

$$\frac{5-3}{3} = \frac{y-2}{-6} = \frac{z+1}{-1}$$

$$\frac{5-3}{3} = \frac{y-2}{-6}$$

Hence y co-ordinate of P is –2.

Reason (R) is false.

Since, the two lines $x = ay + b, z = cy + d$ and $= a'y + b', z = c'y + d'$ will be perpendicular, iff $aa' + cc' + 1 = 0$.

Q. 5. Assertion (A): The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} + \frac{y+2}{2} = \frac{z-3}{-3}$ is 90°

Reason (R): Skew lines are lines in different planes which are parallel and intersecting.

Ans. Option (C) is correct.

Explanation: Assertion (A) is correct.

Given : $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$

and $\frac{x-1}{1} + \frac{y+2}{2} = \frac{z-3}{-3}$

Direction ratios of lines are $a_1 = 2, b_1 = 5, c_1 = 4$ and $a_2 - 1, b_2 - 2, c_2 - -3$

As we know, The angle between the lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\left(\sqrt{a_1^2 + b_1^2 + c_1^2}\right).\left(\sqrt{a_2^2 + b_2^2 + c_2^2}\right)}$$

$$\Rightarrow \quad \cos\theta = \frac{2 \times 1 + 5 \times 2 + 4 \times -3}{\left(\sqrt{2^2 + 5^2 + 4^2}\right).\left(\sqrt{1^2 + 2^2 + (-3)^3}\right)}$$

$$= 0$$

$$\therefore \quad \theta = 90°$$

Reason (R) is wrong.

In the space, there are lines neither intersecting nor parallel, such pairs of lines are non-coplanar and are called skew lines.

Q. 6. Assertion (A): The length of the intercepts on the co-ordinate axes made by the plane $5x + 2y + z - 13 = 0$ are $\frac{13}{5}, \frac{13}{2}$, 13 unit

Reason (R): Given:

Equation of plane

$$5x + 2y + z - 13 = 0$$

$$\Rightarrow \quad 5x + 2y + z = 13$$

$$\Rightarrow \quad \frac{5x + 2y + z}{13} = 1$$

$$\Rightarrow \quad \frac{x}{\frac{13}{5}} + \frac{y}{\frac{13}{2}} + \frac{z}{\frac{13}{1}} = 1$$

$\therefore$ Length of intercepts are $\frac{13}{5}, \frac{13}{2}$, 13 units

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).