

Chapter 10. Isosceles Triangles

Exercise 10(A)

Solution 1:

In $\triangle ABC$,

$$\angle BAC + \angle ACB + \angle ABC = 180^\circ$$

$$48^\circ + \angle ACB + \angle ABC = 180^\circ$$

$$\text{But } \angle ACB = \angle ABC \text{ [AB = AC]}$$

$$2\angle ABC = 180^\circ - 48^\circ$$

$$2\angle ABC = 132^\circ$$

$$\angle ABC = 66^\circ = \angle ACB \dots\dots(i)$$

$$\angle ACB = 66^\circ$$

$$\angle ACD + \angle DCB = 66^\circ$$

$$18^\circ + \angle DCB = 66^\circ$$

$$\angle DCB = 48^\circ \dots\dots(ii)$$

Now, In $\triangle DCB$,

$$\angle DBC = 66^\circ \text{ [From (i), Since } \angle ABC = \angle DBC]$$

$$\angle DCB = 48^\circ \text{ [From (ii)]}$$

$$\angle BDC = 180^\circ - 48^\circ - 66^\circ$$

$$\angle BDC = 66^\circ$$

$$\text{Since } \angle BDC = \angle DBC$$

Therefore, $BC = CD$

Equal angles have equal sides opposite to them.

Solution 2:

Given: $\angle ACE = 130^\circ$; $AD = BD = CD$

Proof:

(i)

$$\angle ACD + \angle ACE = 180^\circ \quad [\text{DCE is a st. line}]$$

$$\Rightarrow \angle ACD = 180^\circ - 130^\circ$$

$$\Rightarrow \angle ACD = 50^\circ$$

$$\text{Now, } CD = AD$$

$$\Rightarrow \angle ACD = \angle DAC = 50^\circ \dots (i)$$

[Since angles opposite to equal sides are equal]

In $\triangle ADC$,

$$\angle ACD = \angle DAC = 50^\circ$$

$$\angle ACD + \angle DAC + \angle ADC = 180^\circ$$

$$50^\circ + 50^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 180^\circ - 100^\circ$$

$$\angle ADC = 80^\circ$$

(ii)

$$\angle ADC = \angle ABD + \angle DAB \quad [\text{Exterior angle is equal to sum of opp. interior angles}]$$

$$\text{But } AD = BD$$

$$\therefore \angle DAB = \angle ABD$$

$$\Rightarrow 80^\circ = \angle ABD + \angle ABD$$

$$\Rightarrow 2\angle BD = 80^\circ$$

$$\Rightarrow \angle ABD = 40^\circ = \angle DAB \dots (ii)$$

(iii)

$$\angle BAC = \angle DAB + \angle DAC$$

substituting the values from (i) and (ii)

$$\angle BAC = 40^\circ + 50^\circ$$

$$\Rightarrow \angle BAC = 90^\circ$$

Solution 3:

$$\angle FAB = 128^\circ \quad [\text{Given}]$$

$$\angle BAC + \angle FAB = 180^\circ \quad [\text{FAC is a st. line}]$$

$$\Rightarrow \angle BAC = 180^\circ - 128^\circ$$

$$\Rightarrow \angle BAC = 52^\circ$$

In $\triangle ABC$,

$$\angle A = 52^\circ$$

$$\angle B = \angle C \quad [\text{Given } AB = AC \text{ and angles opposite to equal sides are equal}]$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 52^\circ + 2\angle B = 180^\circ$$

$$\Rightarrow 2\angle B = 128^\circ$$

$$\Rightarrow \angle B = 64^\circ = \angle C \dots\dots\dots(i)$$

$$\angle B = \angle ADE \quad [\text{Given } DE \parallel BC]$$

(i)

Now,

$$\angle ADE + \angle CDE + \angle B = 180^\circ \quad [ADB \text{ is a st. line}]$$

$$\Rightarrow 64^\circ + \angle CDE + 64^\circ = 180^\circ$$

$$\Rightarrow \angle CDE = 180^\circ - 128^\circ$$

$$\Rightarrow \angle CDE = 52^\circ$$

(ii)

Given $DE \parallel BC$ and DC is the transversal.

$$\Rightarrow \angle CDE = \angle DCB = 52^\circ \dots\dots(ii)$$

$$\text{Also, } \angle ECB = 64^\circ \dots\dots[\text{From (i)}]$$

But,

$$\angle ECB = \angle DCE + \angle DCB$$

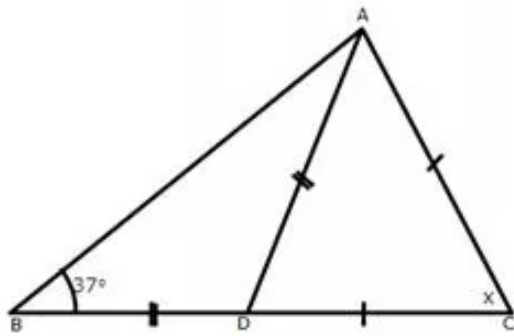
$$\Rightarrow 64^\circ = \angle DCE + 52^\circ$$

$$\Rightarrow \angle DCE = 64^\circ - 52^\circ$$

$$\Rightarrow \angle DCE = 12^\circ$$

Solution 4:

(i) Let the triangle be ABC and the altitude be AD.



In $\triangle ABD$,

$$\angle DBA = \angle DAB = 37^\circ \quad \begin{array}{l} \text{[Given } BD = AD \text{ and} \\ \text{angles opposite to equal sides are equal]} \end{array}$$

Now,

$$\angle CDA = \angle DBA + \angle DAB \quad \begin{array}{l} \text{[Exterior angle is equal to the sum of} \\ \text{opp. interior angles]} \end{array}$$

$$\therefore \angle CDA = 37^\circ + 37^\circ$$

$$\Rightarrow \angle CDA = 74^\circ$$

Now in $\triangle ADC$,

$$\angle CDA = \angle CAD = 74^\circ \quad \begin{array}{l} \text{[Given } CD = AC \text{ and} \\ \text{angles opposite to equal sides are equal]} \end{array}$$

Now,

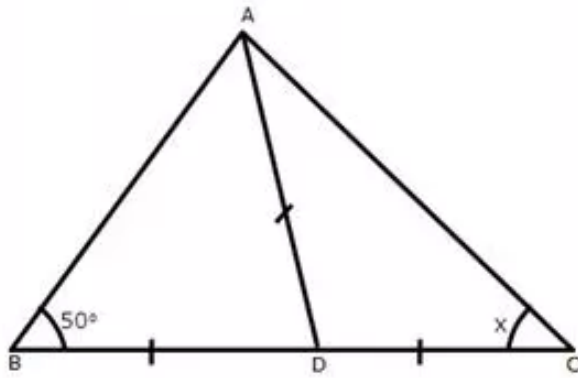
$$\angle CAD + \angle CDA + \angle ACD = 180^\circ$$

$$\Rightarrow 74^\circ + 74^\circ + x = 180^\circ$$

$$\Rightarrow x = 180^\circ - 148^\circ$$

$$\Rightarrow x = 32^\circ$$

(ii) Let triangle be ABC and altitude be AD.



In $\triangle ABD$,

$$\angle DBA = \angle DAB = 50^\circ \quad \begin{array}{l} \text{[Given } BD = AD \text{ and} \\ \text{angles opposite to equal sides are equal]} \end{array}$$

Now,

$$\angle CDA = \angle DBA + \angle DAB \quad \begin{array}{l} \text{[Exterior angle is equal to the sum of} \\ \text{opp. interior angles]} \end{array}$$

$$\therefore \angle CDA = 50^\circ + 50^\circ$$

$$\Rightarrow \angle CDA = 100^\circ$$

In $\triangle ADC$,

$$\angle DAC = \angle DCA = x \quad \begin{array}{l} \text{[Given } AD = DC \text{ and} \\ \text{angles opposite to equal sides are equal]} \end{array}$$

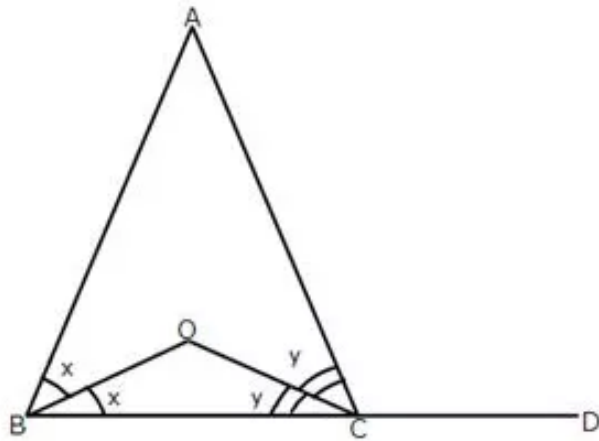
$$\therefore \angle DAC + \angle DCA + \angle ADC = 180^\circ$$

$$\Rightarrow x + x + 100^\circ = 180^\circ$$

$$\Rightarrow 2x = 80^\circ$$

$$\Rightarrow x = 40^\circ$$

Solution 5:



Let $\angle ABO = \angle OBC = x$ and $\angle ACO = \angle OCB = y$

In $\triangle ABC$,

$$\angle BAC = 180^\circ - 2x - 2y \dots\dots\dots (i)$$

Since $\angle B = \angle C$ [$AB = AC$]

$$\frac{1}{2}B = \frac{1}{2}C$$

$$\Rightarrow x = y$$

Now,

$$\angle ACD = 2x + \angle BAC \quad \text{[Exterior angle is equal to sum of opp. interior angles]}$$

$$= 2x + 180^\circ - 2x - 2y \quad \text{[From (i)]}$$

$$\angle ACD = 180^\circ - 2y \dots\dots\dots (ii)$$

In $\triangle OBC$,

$$\angle BOC = 180^\circ - x - y$$

$$\Rightarrow \angle BOC = 180^\circ - y - y \quad \text{[Already proved]}$$

$$\Rightarrow \angle BOC = 180^\circ - 2y \dots\dots (iii)$$

From (i) and (ii)

$$\angle BOC = \angle ACD$$

Solution 6:

Given: $\angle PLN = 110^\circ$

(i) We know that the sum of the measure of all the angles of a quadrilateral is 360° .

In quad. PQNL,

$$\angle QPL + \angle PLN + \angle LNQ + \angle NQP = 360^\circ$$

$$\Rightarrow 90^\circ + 110^\circ + \angle LNQ + 90^\circ = 360^\circ$$

$$\Rightarrow \angle LNQ = 360^\circ - 290^\circ$$

$$\Rightarrow \angle LNQ = 70^\circ$$

$$\Rightarrow \angle LNM = 70^\circ \dots\dots\dots(i)$$

In $\triangle LMN$,

$$LM = LN \quad \quad \quad [\text{Given}]$$

$$\therefore \angle LNM = \angle LMN \quad \quad \quad [\text{angles opp. to equal sides are equal}]$$

$$\Rightarrow \angle LMN = 70^\circ \dots\dots\dots(ii) \quad [\text{From (i)}]$$

(ii)

In $\triangle LMN$,

$$\angle LMN + \angle LNM + \angle MLN = 180^\circ$$

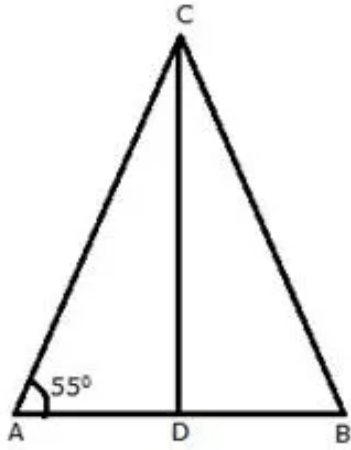
$$\text{But, } \angle LNM = \angle LMN = 70^\circ \quad \quad \quad [\text{From (i) and (ii)}]$$

$$\therefore 70^\circ + 70^\circ + \angle MLN = 180^\circ$$

$$\Rightarrow \angle MLN = 180^\circ - 140^\circ$$

$$\Rightarrow \angle MLN = 40^\circ$$

Solution 7:



In $\triangle ABC$,

$$AC = BC \quad [\text{Given}]$$

$$\therefore \angle CAB = \angle CBD \quad [\text{angles opp. to equal sides are equal}]$$

$$\Rightarrow \angle CBD = 55^\circ$$

In $\triangle ABC$,

$$\angle CBA + \angle CAB + \angle ACB = 180^\circ$$

$$\text{but, } \angle CAB = \angle CBA = 55^\circ$$

$$\Rightarrow 55^\circ + 55^\circ + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ACB = 180^\circ - 110^\circ$$

$$\Rightarrow \angle ACB = 70^\circ$$

Now,

In $\triangle ACD$ and $\triangle BCD$,

$$AC = BC \quad [\text{Given}]$$

$$CD = CD \quad [\text{Common}]$$

$$AD = BD \quad [\text{Given : } CD \text{ bisects } AB]$$

$$\therefore \triangle ACD \cong \triangle BCD$$

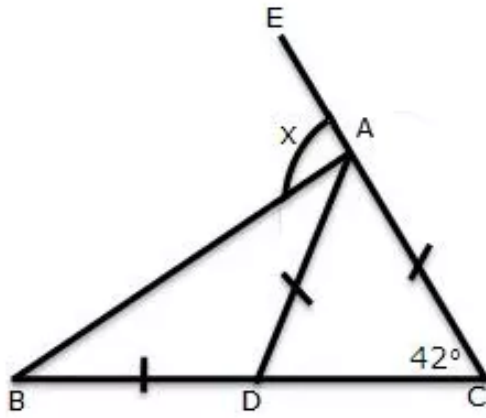
$$\Rightarrow \angle DCA = \angle DCB$$

$$\Rightarrow \angle DCB = \frac{\angle ACB}{2} = \frac{70^\circ}{2}$$

$$\Rightarrow \angle DCB = 35^\circ$$

Solution 8:

Let us name the figure as following:



In $\triangle ABC$,

$$AD = AC \quad [\text{Given}]$$

$$\therefore \angle ADC = \angle ACD \quad [\text{angles opp. to equal sides are equal}]$$

$$\Rightarrow \angle ADC = 42^\circ$$

Now,

$$\angle ADC = \angle DAB + \angle DBA \quad [\text{Exterior angle is equal to the sum of opp. interior angles}]$$

But,

$$\angle DAB = \angle DBA \quad [\text{Given : } BD = DA]$$

$$\therefore \angle ADC = 2\angle DBA$$

$$\Rightarrow 2\angle DBA = 42^\circ$$

$$\Rightarrow \angle DBA = 21^\circ$$

For x:

$$x = \angle CBA + \angle BCA \quad [\text{Exterior angle is equal to the sum of opp. interior angles}]$$

We know that,

$$\angle CBA = 21^\circ$$

$$\angle BCA = 42^\circ$$

$$\therefore x = 21^\circ + 42^\circ$$

$$\Rightarrow x = 63^\circ$$

Solution 9:

In $\triangle ABD$ and $\triangle DBC$,

$$BD = BD \quad [\text{Common}]$$

$$\angle BDA = \angle BDC \quad [\text{each equal to } 90^\circ]$$

$$\angle ABD = \angle DBC \quad [BD \text{ bisects } \angle ABC]$$

$$\therefore \triangle ABD \cong \triangle DBC \quad [\text{ASA criterion}]$$

Therefore,

$$AD = DC$$

$$x + 1 = y + 2$$

$$\Rightarrow x = y + 1 \dots (i)$$

$$\text{and } AB = BC$$

$$3x + 1 = 5y - 2$$

Substituting the value of x from (i)

$$3(y + 1) + 1 = 5y - 2$$

$$\Rightarrow 3y + 3 + 1 = 5y - 2$$

$$\Rightarrow 3y + 4 = 5y - 2$$

$$\Rightarrow 2y = 6$$

$$\Rightarrow y = 3$$

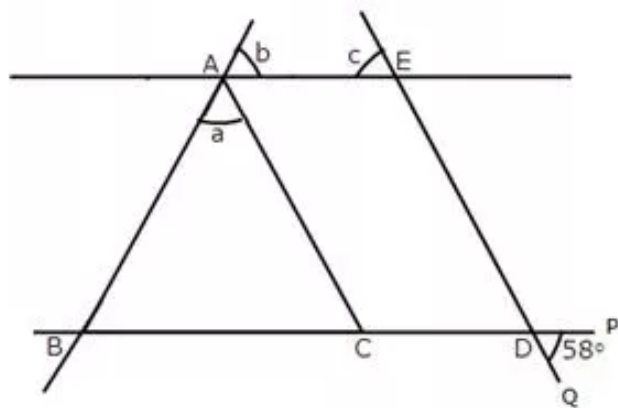
Putting $y = 3$ in (i)

$$x = 3 + 1$$

$$\therefore x = 4$$

Solution 10:

Let P and Q be the points as shown below:



Given: $\angle PDQ = 58^\circ$

$$\angle PDQ = \angle EDC = 58^\circ \quad [\text{Vertically opp. angles}]$$

$$\angle EDC = \angle ACB = 58^\circ \quad [\text{Corresponding angles } \therefore AC \parallel ED]$$

In $\triangle ABC$,

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle ACB = \angle ABC = 58^\circ \quad [\text{angles opp. to equal sides are equal}]$$

Now,

$$\angle ACB + \angle ABC + \angle BAC = 180^\circ$$

$$\Rightarrow 58^\circ + 58^\circ + a = 180^\circ$$

$$\Rightarrow a = 180^\circ - 116^\circ$$

$$\Rightarrow a = 64^\circ$$

Since $AE \parallel BD$ and AC is the transversal

$$\angle ABC = b \quad [\text{Corresponding angles}]$$

$$\therefore b = 58^\circ$$

Also since $AE \parallel BD$ and ED is the transversal

$$\angle EDC = c \quad [\text{Corresponding angles}]$$

$$\therefore c = 58^\circ$$

Solution 11:

In $\triangle ACD$,

$$AC = CD \quad [\text{Given}]$$

$$\therefore \angle CAD = \angle CDA$$

$$\angle ACD = 58^\circ \quad [\text{Given}]$$

$$\angle ACD + \angle CDA + \angle CAD = 180^\circ$$

$$\Rightarrow 58^\circ + 2\angle CAD = 180^\circ$$

$$\Rightarrow 2\angle CAD = 122^\circ$$

$$\Rightarrow \angle CAD = \angle CDA = 61^\circ \dots\dots\dots (i)$$

Now,

$$\angle CDA = \angle DAB + \angle DBA \quad [\text{Ext. angle is equal to sum of opp. int. angles}]$$

But,

$$\angle DAB = \angle DBA \quad [\text{Given : } AD = DB]$$

$$\therefore \angle DAB + \angle DAB = \angle CDA$$

$$\Rightarrow 2\angle DAB = 61^\circ$$

$$\Rightarrow \angle DAB = 30.5^\circ \dots\dots\dots (ii)$$

In $\triangle ABC$,

$$\angle CAB = \angle CAD + \angle DAB$$

$$\therefore \angle CAB = 61^\circ + 30.5^\circ$$

$$\Rightarrow \angle AB = 91.5^\circ$$

Solution 12:

In $\triangle ACD$,

$$AC = AD = CD \quad [\text{Given}]$$

Hence, $\triangle ACD$ is an equilateral triangle

$$\therefore \angle ACD = \angle CDA = \angle CAD = 60^\circ$$

$$\angle CDA = \angle DAB + \angle ABD \quad [\text{Ext. angle is equal to sum of opp. int. angles}]$$

But,

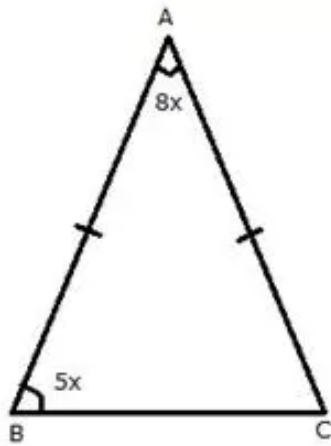
$$\angle DAB = \angle ABD \quad [\text{Given : } AD = DB]$$

$$\therefore \angle ABD + \angle ABD = \angle CDA$$

$$\Rightarrow 2\angle ABD = 60^\circ$$

$$\Rightarrow \angle ABD = \angle ABC = 30^\circ$$

Solution 13:



Let $\angle A = 8x$ and $\angle B = 5x$

Given: $AB = AC$

$\Rightarrow \angle B = \angle C = 5x$ [Angles opp. to equal sides are equal]

Now,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 8x + 5x + 5x = 180^\circ$$

$$\Rightarrow 18x = 180^\circ$$

$$\Rightarrow x = 10^\circ$$

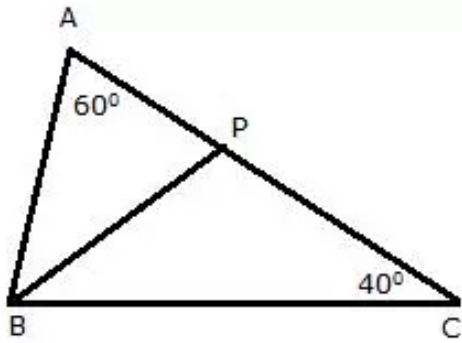
Given that :

$$\angle A = 8x$$

$$\Rightarrow \angle A = 8 \times 10^\circ$$

$$\Rightarrow \angle A = 80^\circ$$

Solution 14:



In $\triangle ABC$,

$$\angle A = 60^\circ$$

$$\angle C = 40^\circ$$

$$\therefore \angle B = 180^\circ - 60^\circ - 40^\circ$$

$$\Rightarrow \angle B = 80^\circ$$

Now,

BP is the bisector of $\angle ABC$

$$\therefore \angle PBC = \frac{\angle ABC}{2}$$

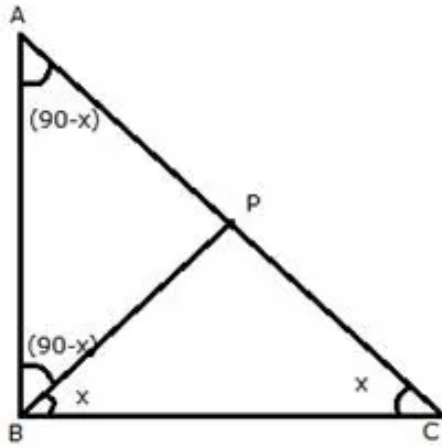
$$\Rightarrow \angle PBC = 40^\circ$$

In $\triangle PBC$

$$\angle PBC = \angle PCB = 40^\circ$$

$$\therefore BP = CP \quad [\text{Sides opp. to equal angles are equal}]$$

Solution 15:



Let $\angle PBC = \angle PCB = x$

In the right angled triangle ABC,

$$\angle ABC = 90^\circ$$

$$\angle ACB = x$$

$$\Rightarrow \angle BAC = 180^\circ - (90^\circ + x)$$

$$\Rightarrow \angle BAC = (90^\circ - x) \dots \dots \dots (i)$$

and

$$\angle ABP = \angle ABC - \angle PBC$$

$$\Rightarrow \angle ABP = 90^\circ - x \dots \dots \dots (ii)$$

Therefore in the triangle ABP;

$$\angle BAP = \angle ABP$$

Hence,

$$PA = PB \text{ [sides opp. to equal angles are equal]}$$

Solution 16:

$\triangle ABC$ is an equilateral triangle

$$\Rightarrow \text{Side } AB = \text{Side } AC$$

$$\Rightarrow \angle ABC = \angle ACB \quad \left[\begin{array}{l} \text{If two sides of a triangle are equal, then angles} \\ \text{opposite to them are equal} \end{array} \right]$$

Similarly, Side $AC = \text{Side } BC$

$$\Rightarrow \angle CAB = \angle ABC \quad \left[\begin{array}{l} \text{If two sides of a triangle are equal, then angles} \\ \text{opposite to them are equal} \end{array} \right]$$

Hence, $\angle ABC = \angle CAB = \angle ACB = y$ (say)

As the sum of all the angles of the triangle is 180°

$$\angle ABC + \angle CAB + \angle ACB = 180^\circ$$

$$\Rightarrow 3y = 180^\circ$$

$$\Rightarrow y = 60^\circ$$

$$\angle ABC = \angle CAB = \angle ACB = 60^\circ$$

Sum of two non-adjacent interior angles of a triangle is equal to the exterior angle.

$$\Rightarrow \angle CAB + \angle CBA = \angle ACE$$

$$\Rightarrow 60^\circ + 60^\circ = \angle ACE$$

$$\Rightarrow \angle ACE = 120^\circ$$

Now $\triangle ACE$ is an isosceles triangle with $AC = CE$

$$\Rightarrow \angle EAC = \angle AEC$$

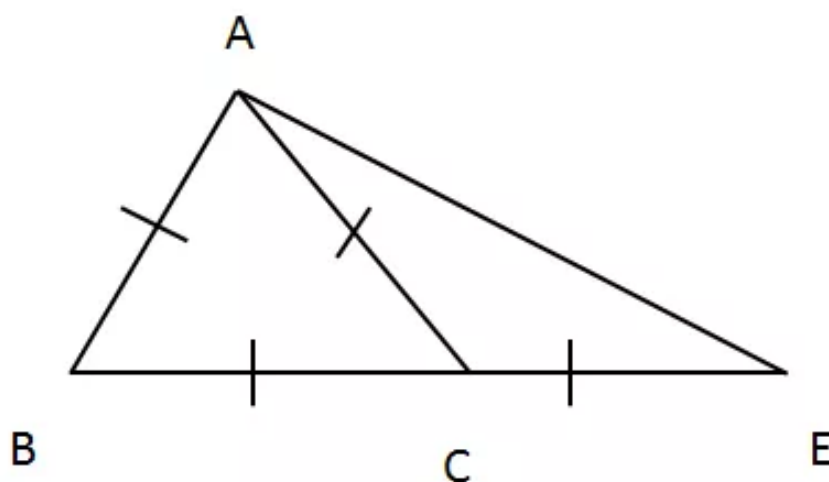
Sum of all the angles of a triangle is 180°

$$\angle EAC + \angle AEC + \angle ACE = 180^\circ$$

$$\Rightarrow 2\angle AEC + 120^\circ = 180^\circ$$

$$\Rightarrow 2\angle AEC = 180^\circ - 120^\circ$$

$$\Rightarrow \angle AEC = 30^\circ$$



Solution 17:

$\triangle DBC$ is an isosceles triangle

As, Side $CD =$ Side DB

$$\Rightarrow \angle DBC = \angle DCB \quad \left[\begin{array}{l} \text{If two sides of a triangle are equal, then angles} \\ \text{opposite to them are equal} \end{array} \right]$$

$$\text{And } \angle B = \angle DBC = \angle DCB = 28^\circ$$

As the sum of all the angles of the triangle is 180°

$$\angle DCB + \angle DBC + \angle BCD = 180^\circ$$

$$\Rightarrow 28^\circ + 28^\circ + \angle BCD = 180^\circ$$

$$\Rightarrow \angle BCD = 180^\circ - 56^\circ$$

$$\Rightarrow \angle BCD = 124^\circ$$

Sum of two non-adjacent interior angles of a triangle is equal to the exterior angle.

$$\Rightarrow \angle DBC + \angle DCB = \angle DAC$$

$$\Rightarrow 28^\circ + 28^\circ = 56^\circ$$

$$\Rightarrow \angle DAC = 56^\circ$$

Now $\triangle ACD$ is an isosceles triangle with $AC = DC$

$$\Rightarrow \angle ADC = \angle DAC = 56^\circ$$

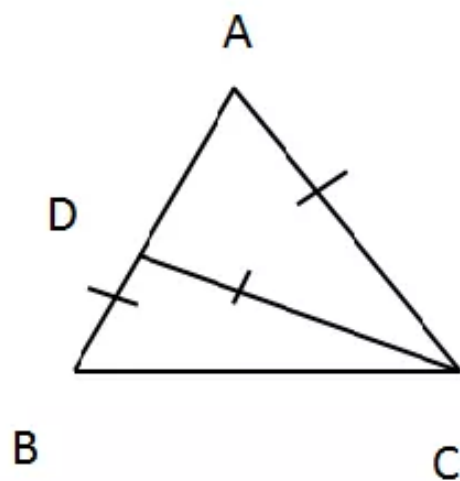
Sum of all the angles of a triangle is 180°

$$\angle ADC + \angle DAC + \angle DCA = 180^\circ$$

$$\Rightarrow 56^\circ + 56^\circ + \angle DCA = 180^\circ$$

$$\Rightarrow \angle DCA = 180^\circ - 112^\circ$$

$$\Rightarrow \angle DCA = 64^\circ = \angle ACD$$



Solution 18:

We can see that the $\triangle ABC$ is an isosceles triangle with Side $AB =$ Side AC .

$$\Rightarrow \angle ACB = \angle ABC$$

$$\text{As } \angle ACB = 65^\circ$$

$$\text{hence } \angle ABC = 65^\circ$$

Sum of all the angles of a triangle is 180°

$$\angle ACB + \angle CAB + \angle ABC = 180^\circ$$

$$65^\circ + 65^\circ + \angle CAB = 180^\circ$$

$$\angle CAB = 180^\circ - 130^\circ$$

$$\angle CAB = 50^\circ$$

As BD is parallel to CA

Therefore, $\angle CAB = \angle DBA$ since they are alternate angles.

$$\angle CAB = \angle DBA = 50^\circ$$

We see that $\triangle ADB$ is an isosceles triangle with Side $AD =$ Side AB .

$$\Rightarrow \angle ADB = \angle DBA = 50^\circ$$

Sum of all the angles of a triangle is 180°

$$\angle ADB + \angle DAB + \angle DBA = 180^\circ$$

$$50^\circ + \angle DAB + 50^\circ = 180^\circ$$

$$\angle DAB = 180^\circ - 100^\circ = 80^\circ$$

$$\angle DAB = 80^\circ$$

The angle DAC is sum of angle DAB and CAB .

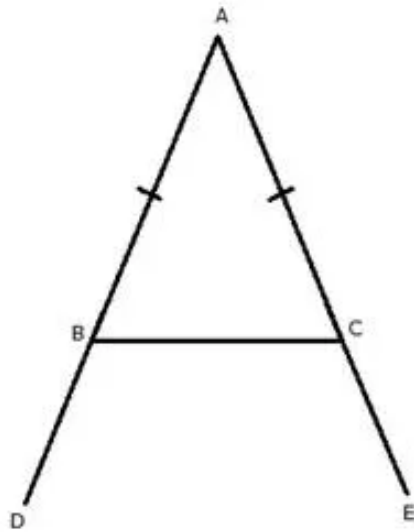
$$\angle DAC = \angle CAB + \angle DAB$$

$$\angle DAC = 50^\circ + 80^\circ$$

$$\angle DAC = 130^\circ$$

Exercise 10(B)

Solution 1:



Const: AB is produced to D and AC is produced to E so that exterior angles $\angle DBC$ and $\angle ECB$ is formed.

In $\triangle ABC$,

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle C = \angle B \dots\dots (i) \quad [\text{angles opp. to equal sides are equal}]$$

Since angle B and angle C are acute they cannot be right angles or obtuse angles.

$$\angle ABC + \angle DBC = 180^\circ \quad [\text{ABD is a st. line}]$$

$$\Rightarrow \angle DBC = 180^\circ - \angle ABC$$

$$\Rightarrow \angle DBC = 180^\circ - \angle B \dots\dots (ii)$$

Similarly,

$$\angle ACB + \angle ECB = 180^\circ \quad [\text{ACE is a st. line}]$$

$$\Rightarrow \angle ECB = 180^\circ - \angle ACB$$

$$\Rightarrow \angle ECB = 180^\circ - \angle C \dots\dots (iii)$$

$$\Rightarrow \angle ECB = 180^\circ - \angle B \dots\dots (iv) \quad [\text{from (i) and (iii)}]$$

$$\Rightarrow \angle DBC = \angle ECB \quad [\text{from (ii) and (iv)}]$$

Now,

$$\angle DBC = 180^\circ - \angle B$$

But $\angle B = \text{Acute angle}$

$$\therefore \angle DBC = 180^\circ - \text{Acute angle} = \text{obtuse angle}$$

Similarly,

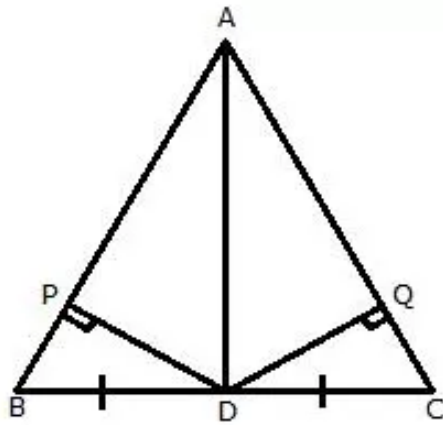
$$\angle ECB = 180^\circ - \angle C.$$

But $\angle C = \text{Acute angle}$

$$\therefore \angle ECB = 180^\circ - \text{Acute angle} = \text{obtuse angle}$$

Therefore, exterior angles formed are obtuse and equal.

Solution 2:



Const: Join AD.

In $\triangle ABC$,

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle C = \angle B, \dots (i) \quad [\text{angles opp. to equal sides are equal}]$$

(i)

In $\triangle BPD$ and $\triangle CQD$,

$$\angle BPD = \angle CQD \quad [\text{Each} = 90^\circ]$$

$$\angle B = \angle C \quad [\text{proved}]$$

$$BD = DC \quad [\text{Given}]$$

$$\therefore \triangle BPD \cong \triangle CQD \quad [\text{AAS criterion}]$$

$$\therefore DP = DQ \quad [\text{cpct}]$$

(ii) We have already proved that $\triangle BPD \cong \triangle CQD$

Therefore, $BP = CQ$ [cpct]

Now,

$$AB = AC [\text{Given}]$$

$$\Rightarrow AB - BP = AC - CQ$$

$$\Rightarrow AP = AQ$$

(iii)

In $\triangle APD$ and $\triangle AQD$,

$$DP = DQ \quad [\text{proved}]$$

$$AD = AD \quad [\text{common}]$$

$$AP = AQ \quad [\text{Proved}]$$

$$\therefore \triangle APD \cong \triangle AQD \quad [\text{SSS}]$$

$$\Rightarrow \angle PAD = \angle QAD \quad [\text{cpct}]$$

Hence, AD bisects angle A.

Solution 3:

(i)

In $\triangle AEB$ and $\triangle AFC$,

$$\angle A = \angle A \quad [\text{Common}]$$

$$\angle AEB = \angle AFC = 90^\circ \quad [\text{Given: } BE \perp AC]$$

$$[\text{Given: } CF \perp AB]$$

$$AB = AC \quad [\text{Given}]$$

$$\Rightarrow \triangle AEB \cong \triangle AFC \quad [\text{AAS}]$$

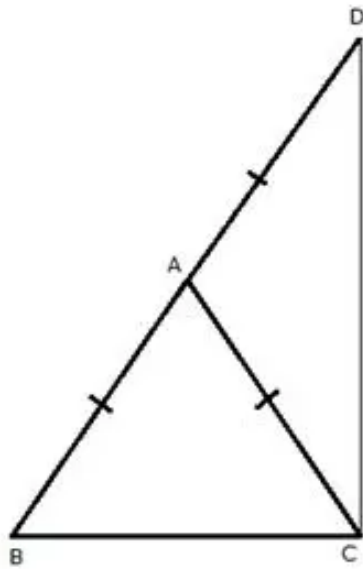
$$\therefore BE = CF \quad [\text{cpct}]$$

(ii) Since $\triangle AEB \cong \triangle AFC$

$$\angle ABE = \angle AFC$$

$$\therefore AF = AE \quad [\text{congruent angles of congruent triangles}]$$

Solution 4:



Const: Join CD.

In $\triangle ABC$,

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle C = \angle B \dots\dots(i) \quad [\text{angles opp. to equal sides are equal}]$$

In $\triangle ACD$,

$$AC = AD \quad [\text{Given}]$$

$$\therefore \angle ADC = \angle ACD \dots\dots\dots(ii)$$

Adding (i) and (ii)

$$\angle B + \angle ADC = \angle C + \angle ACD$$

$$\angle B + \angle ADC = \angle BCD \dots\dots\dots(iii)$$

In $\triangle BCD$,

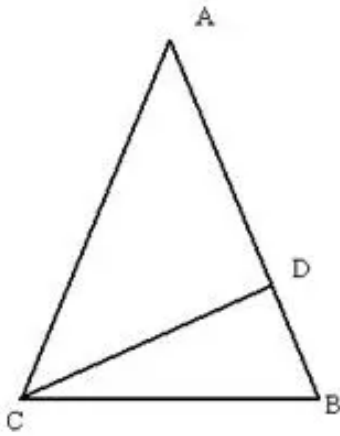
$$\angle B + \angle ADC + \angle BCD = 180^\circ$$

$$\angle BCD + \angle BCD = 180^\circ \quad [\text{From (iii)}]$$

$$2\angle BCD = 180^\circ$$

$$\angle BCD = 90^\circ$$

Solution 5:



$$AB = AC$$

$\triangle ABC$ is an isosceles triangle.

$$\angle A = 36^\circ$$

$$\angle B = \angle C = \frac{180^\circ - 36^\circ}{2} = 72^\circ$$

$$\angle ACD = \angle BCD = 36^\circ \quad [\because CD \text{ is the angle bisector of } \angle C]$$

$\triangle ADC$ is an isosceles triangle since $\angle DAC = \angle DCA = 36^\circ$

$$\therefore AD = CD \dots\dots (i)$$

In $\triangle DCB$,

$$\begin{aligned} \angle CDB &= 180^\circ - (\angle DCB + \angle DBC) \\ &= 180^\circ - (36^\circ + 72^\circ) \\ &= 180^\circ - 108^\circ \\ &= 72^\circ \end{aligned}$$

$\triangle DCB$ is an isosceles triangle since $\angle CDB = \angle CBD = 72^\circ$

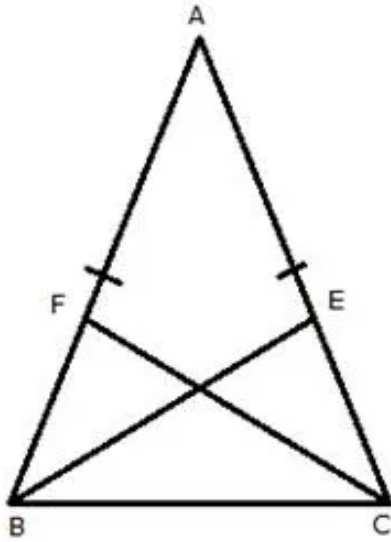
$$\therefore DC = BC \dots\dots (ii)$$

From (i) and (ii), we get

$$AD = BC$$

Hence proved

Solution 6:



In $\triangle ABC$,

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle C = \angle B \dots (i) \quad [\text{angles opp. to equal sides are equal}]$$

$$\Rightarrow \frac{1}{2} \angle C = \frac{1}{2} \angle B$$

$$\Rightarrow \angle BCF = \angle CBE \dots (ii)$$

In $\triangle BCE$ and $\triangle CBF$,

$$\angle C = \angle B \quad [\text{From (i)}]$$

$$\angle BCF = \angle CBE \quad [\text{From (ii)}]$$

$$BC = BC \quad [\text{Common}]$$

$$\therefore \triangle BCE \cong \triangle CBF \quad [\text{AAS}]$$

$$\Rightarrow BE = CF \quad [\text{cpct}]$$

Solution 7:

In $\triangle ABC$,

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle ACB = \angle ABC \quad [\text{angles opp. to equal sides are equal}]$$

$$\Rightarrow \angle ABC = \angle ACB \dots\dots(i)$$

$$\angle DBC = \angle ECB = 90^\circ [\text{Given}]$$

$$\Rightarrow \angle DBC = \angle ECB \dots\dots(ii)$$

Subtracting (i) from (ii)

$$\angle DCB - \angle ABC = \angle ECB - \angle ACB$$

$$\Rightarrow \angle DBA = \angle ECA \dots\dots(iii)$$

In $\triangle DBA$ and $\triangle ECA$,

$$\angle DBA = \angle ECA \quad [\text{From (iii)}]$$

$$\angle DAB = \angle EAC \quad [\text{Vertically opposite angles}]$$

$$AB = AC \quad [\text{Given}]$$

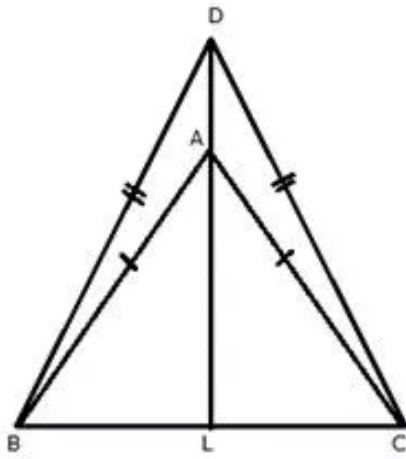
$$\therefore \triangle DBA \cong \triangle ECA \quad [\text{ASA}]$$

$$\Rightarrow BD = CE \quad [\text{cpct}]$$

Also,

$$AD = AE \quad [\text{cpct}]$$

Solution 8:



DA is produced to meet BC in L.

In $\triangle ABC$,

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle ACB = \angle ABC \dots\dots (i) \quad [\text{angles opposite to equal sides are equal}]$$

In $\triangle DBC$,

$$DB = DC \quad [\text{Given}]$$

$$\therefore \angle DCB = \angle DBC \dots\dots (ii) \quad [\text{angles opposite to equal sides are equal}]$$

Subtracting (i) from (ii)

$$\angle DCB - \angle ACB = \angle DBC - \angle ABC$$

$$\Rightarrow \angle DCA = \angle DBA \dots\dots (iii)$$

In $\triangle DBA$ and $\triangle DCA$,

$$DB = DC \quad [\text{Given}]$$

$$\angle DBA = \angle DCA \quad [\text{From (iii)}]$$

$$AB = AC \quad [\text{Given}]$$

$$\therefore \triangle DBA \cong \triangle DCA \quad [\text{SAS}]$$

$$\Rightarrow \angle BDA = \angle CDA \dots\dots (iv) \quad [\text{cpct}]$$

In $\triangle DBA$,

$$\angle BAL = \angle DBA + \angle BDA \dots\dots\dots (v)$$

[Ext. angle = sum of opp. int. angles]

From (iii), (iv) and (v)

$$\angle BAL = \angle DCA + \angle CDA \dots\dots\dots (vi)$$

In $\triangle DCA$,

$$\angle CAL = \angle DCA + \angle CDA \dots\dots\dots (vii)$$

[Ext. angle = sum of opp. int. angles]

From (vi) and (vii)

$$\angle BAL = \angle CAL \dots\dots\dots (viii)$$

In $\triangle BAL$ and $\triangle CAL$,

$$\angle BAL = \angle CAL \quad \text{[From (viii)]}$$

$$\angle ABL = \angle ACL \quad \text{[From (i)]}$$

$$AB = AC \quad \text{[Given]}$$

$$\therefore \triangle BAL \cong \triangle CAL \quad \text{[ASA]}$$

$$\Rightarrow \angle ALB = \angle ALC \quad \text{[cpct]}$$

$$\text{and } BL = LC \dots\dots\dots (ix) \quad \text{[cpct]}$$

Now,

$$\angle ALB + \angle ALC = 180^\circ$$

$$\Rightarrow \angle ALB + \angle ALB = 180^\circ$$

$$\Rightarrow 2\angle ALB = 180^\circ$$

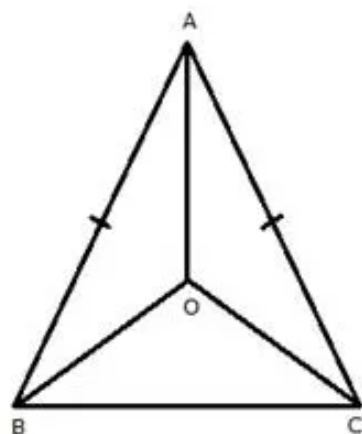
$$\Rightarrow \angle ALB = 90^\circ$$

$$\therefore AL \perp BC$$

$$\text{or } DL \perp BC \text{ and } BL = LC$$

$$\therefore DA \text{ produced bisects } BC \text{ at right angle.}$$

Solution 9:



In $\triangle ABC$, we have $AB = AC$

$\Rightarrow \angle B = \angle C$ [angles opposite to equal sides are equal]

$$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle C$$

$$\Rightarrow \angle OBC = \angle OCB \dots\dots\dots(i)$$

$$\Rightarrow OB = OC \dots\dots\dots(ii)$$

[angles opposite to equal sides are equal]

Now,

In $\triangle ABO$ and $\triangle ACO$,

$$AB = AC \text{ [Given]}$$

$$\angle OBC = \angle OCB \text{ [From (i)]}$$

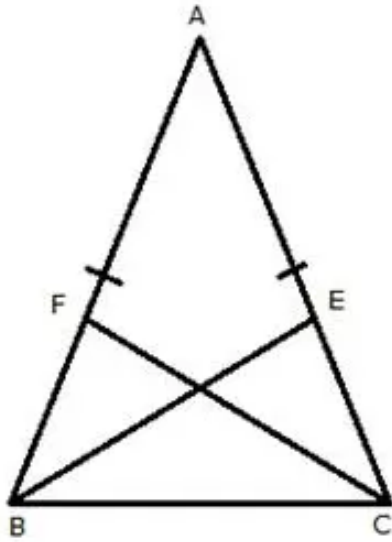
$$OB = OC \text{ [From (ii)]}$$

$$\triangle ABO \cong \triangle ACO \quad \text{[SAS criterion]}$$

$$\Rightarrow \angle BAO = \angle CAO \quad \text{[cpct]}$$

Therefore, AO bisects $\angle BAC$.

Solution 10:



In $\triangle ABC$,

$$AB = AC \quad [\text{Given}]$$

$$\therefore \angle C = \angle B \dots\dots (i) \quad [\text{angles opp. to equal sides are equal}]$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} AC$$

$$\Rightarrow BF = CE \dots\dots (ii)$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} AC$$

$$\Rightarrow BF = CE \dots\dots (ii)$$

In $\triangle BCE$ and $\triangle CBF$,

$$\angle C = \angle B \quad [\text{From (i)}]$$

$$BF = CE \quad [\text{From (ii)}]$$

$$BC = BC \quad [\text{Common}]$$

$$\therefore \triangle BCE \cong \triangle CBF \quad [\text{SAS}]$$

$$\Rightarrow BE = CF \quad [\text{cpct}]$$

Solution 11:

In $\triangle APQ$,

$$AP = AQ \quad [\text{Given}]$$

$$\therefore \angle APQ = \angle AQP \dots\dots (i)$$

[angles opposite to equal sides are equal]

In $\triangle ABP$,

$$\angle APQ = \angle BAP + \angle ABP \dots\dots (ii)$$

[Ext. angle is equal to sum of opp. int. angles]

In $\triangle AQC$,

$$\angle AQP = \angle CAQ + \angle ACQ \dots\dots (iii)$$

[Ext. angle is equal to sum of opp. int. angles]

From (i), (ii) and (iii)

$$\angle BAP + \angle ABP = \angle CAQ + \angle ACQ$$

$$\text{But, } \angle BAP = \angle CAQ \quad [\text{Given}]$$

$$\Rightarrow \angle CAQ + \angle ABP = \angle CAQ + \angle ACQ$$

$$\Rightarrow \angle ABP = \angle CAQ + \angle ACQ - \angle CAQ$$

$$\Rightarrow \angle ABP = \angle ACQ$$

$$\Rightarrow \angle B = \angle C \dots\dots (iv)$$

In $\triangle ABC$,

$$\angle B = \angle C$$

$$\Rightarrow AB = AC \quad [\text{Sides opposite to equal angles are equal}]$$

Solution 12:

Since $AE \parallel BC$ and DAB is the transversal

$$\therefore \angle DAE = \angle ABC = \angle B \quad [\text{Corresponding angles}]$$

Since $AE \parallel BC$ and AC is the transversal

$$\angle CAE = \angle ACB = \angle C \quad [\text{Alternate Angles}]$$

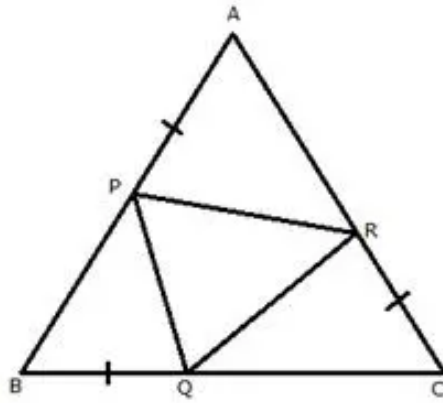
But AE bisects $\angle CAD$

$$\therefore \angle DAE = \angle CAE$$

$$\Rightarrow \angle B = \angle C$$

$$\Rightarrow AB = AC [\text{Sides opposite to equal angles are equal}]$$

Solution 13:



$$AB = BC = CA \dots\dots (i) \text{ [Given]}$$

$$AP = BQ = CR \dots\dots (ii) \text{ [Given]}$$

Subtracting (ii) from (i)

$$AB - AP = BC - BQ = CA - CR$$

$$BP = CQ = AR \dots\dots\dots (iii)$$

$$\therefore \angle A = \angle B = \angle C \dots\dots (iv) \text{ [angles opp. to equal sides are equal]}$$

In $\triangle BPQ$ and $\triangle CQR$,

$$BP = CQ \quad \text{[From (iii)]}$$

$$\angle B = \angle C \quad \text{[From (iv)]}$$

$$BQ = CR \quad \text{[Given]}$$

$$\therefore \triangle BPQ \cong \triangle CQR \quad \text{[SAS criterion]}$$

$$\Rightarrow PQ = QR \dots\dots\dots (v)$$

In $\triangle CQR$ and $\triangle APR$,

$$CQ = AR \quad \text{[From (iii)]}$$

$$\angle C = \angle A \quad \text{[From (iv)]}$$

$$CR = AP \quad \text{[Given]}$$

$$\therefore \triangle CQR \cong \triangle APR \quad \text{[SAS criterion]}$$

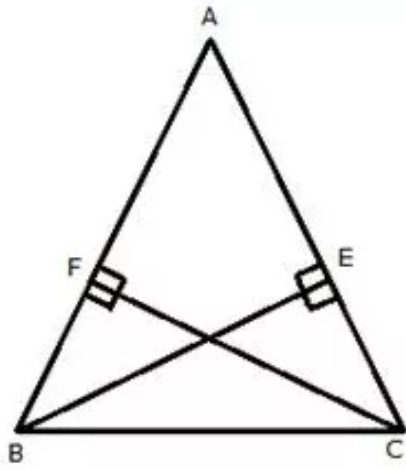
$$\Rightarrow QR = PR \dots\dots\dots (vi)$$

From (v) and (vi)

$$PQ = QR = PR$$

Therefore, PQR is an equilateral triangle.

Solution 14:



In $\triangle ABE$ and $\triangle ACF$,

$$\angle A = \angle A [\text{Common}]$$

$$\angle AEB = \angle AFC = 90^\circ [\text{Given: } BE \perp AC; CF \perp AB]$$

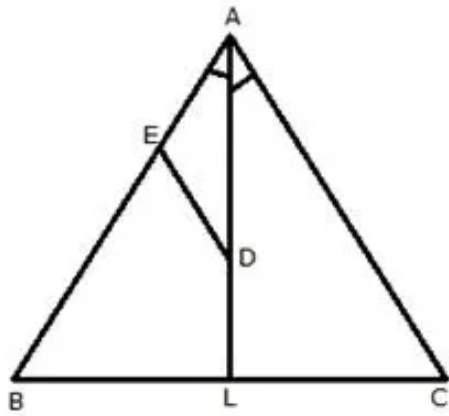
$$BE = CF [\text{Given}]$$

$$\therefore \triangle ABE \cong \triangle ACF \quad [\text{AAS criterion}]$$

$$\Rightarrow AB = AC$$

Therefore, ABC is an isosceles triangle.

Solution 15:



AL is bisector of angle A. Let D is any point on AL. From D, a straight line DE is drawn parallel to AC.

$DE \parallel AC$ [Given]

$\therefore \angle ADE = \angle DAC \dots (i)$ [Alternate angles]

$\angle DAC = \angle DAE \dots (ii)$ [AL is bisector of $\angle A$]

From (i) and (ii)

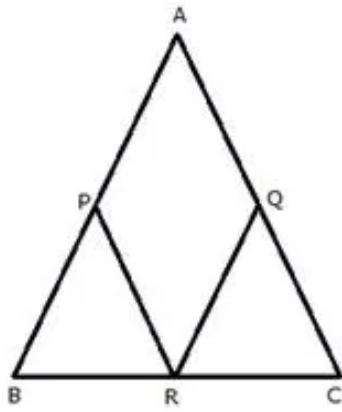
$$\angle ADE = \angle DAE$$

$\therefore AE = ED$ [Sides opposite to equal angles are equal]

Therefore, AED is an isosceles triangle.

Solution 16:

(i)

In $\triangle ABC$,

$$AB = AC$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} AC$$

$$\Rightarrow AP = AQ \text{(i) [Since P and Q are mid - points]}$$

In $\triangle BCA$,

$$PR = \frac{1}{2} AC \text{ [PR is line joining the mid - points of AB and BC]}$$

$$\Rightarrow PR = AQ \text{(ii)}$$

In $\triangle CAB$,

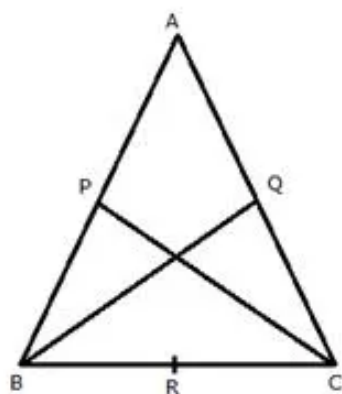
$$QR = \frac{1}{2} AB \text{ [QR is line joining the mid - points of AC and BC]}$$

$$\Rightarrow QR = AP \text{(iii)}$$

From (i), (ii) and (iii)

$$PR = QR$$

(ii)



$$AB = AC$$

$$\Rightarrow \angle B = \angle C$$

Also,

$$\frac{1}{2}AB = \frac{1}{2}AC$$

$$\Rightarrow BP = CQ \quad [P \text{ and } Q \text{ are mid-points of } AB \text{ and } AC]$$

In $\triangle BPC$ and $\triangle CQB$,

$$BP = CQ$$

$$\angle B = \angle C$$

$$BC = BC$$

Therefore, $\triangle BPC \cong \triangle CQB$ [SAS]

$$BP = CP$$

Solution 17:

(i) In $\triangle ACB$,

$$AC = AC \text{ [Given]}$$

$$\therefore \angle ABC = \angle ACB \text{(i) [angles opposite to equal sides are equal]}$$

$$\angle ACD + \angle ACB = 180^\circ \text{(ii) [DCB is a straight line]}$$

$$\angle ABC + \angle CBE = 180^\circ \text{(iii) [ABE is a straight line]}$$

Equating (ii) and (iii)

$$\angle ACD + \angle ACB = \angle ABC + \angle CBE$$

$$\Rightarrow \angle ACD + \angle ACB = \angle ACB + \angle CBE \text{ [From (i)]}$$

$$\Rightarrow \angle ACD = \angle CBE$$

(ii)

In $\triangle ACD$ and $\triangle CBE$,

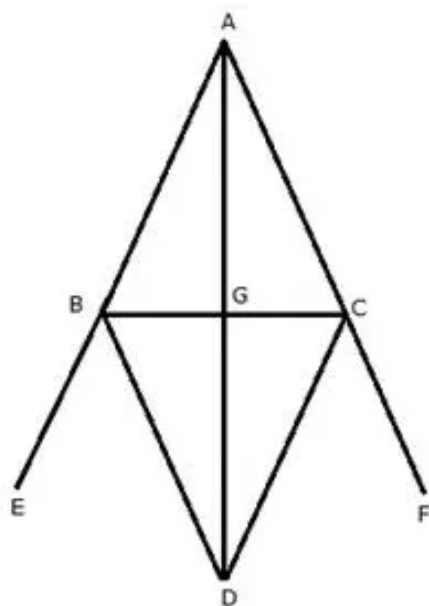
$$DC = CB \quad \text{[Given]}$$

$$AC = BE \quad \text{[Given]}$$

$$\angle ACD = \angle CBE \quad \text{[Proved Earlier]}$$

$$\therefore \triangle ACD \cong \triangle CBE \quad \text{[SAS criterion]}$$

$$\Rightarrow AD = CE \quad \text{[cpct]}$$

Solution 18:

AB is produced to E and AC is produced to F. BD is bisector of angle CBE and CD is bisector of angle BCF. BD and CD meet at D.

In $\triangle ABC$,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$ [angles opposite to equal sides are equal]

$\angle CBE = 180^\circ - \angle B$ [ABE is a straight line]

$$\Rightarrow \angle CBD = \frac{180^\circ - \angle B}{2} \text{ [BD is bisector of } \angle CBE]$$

$$\Rightarrow \angle CBD = 90^\circ - \frac{\angle B}{2} \dots\dots\dots(i)$$

Similarly,

$\angle BCF = 180^\circ - \angle C$ [ACF is a straight line]

$$\Rightarrow \angle BCD = \frac{180^\circ - \angle C}{2} \text{ [CD is bisector of } \angle BCF]$$

$$\Rightarrow \angle BCD = 90^\circ - \frac{\angle C}{2} \dots\dots\dots(ii)$$

Now,

$$\Rightarrow \angle CBD = 90^\circ - \frac{\angle C}{2} \quad [\because \angle B = \angle C]$$

$$\Rightarrow \angle CBD = \angle BCD$$

In $\triangle BCD$,

$$\angle CBD = \angle BCD$$

$$\therefore BD = CD$$

In $\triangle ABD$ and $\triangle ACD$,

$AB = AC$ [Given]

$AD = AD$ [Common]

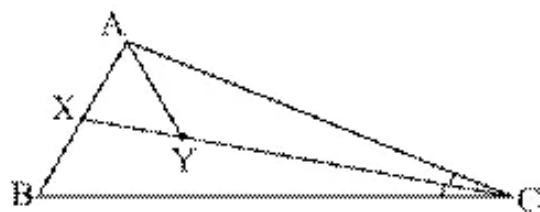
$BD = CD$ [Proved]

$\therefore \triangle ABD \cong \triangle ACD$ [SSS criterion]

$$\Rightarrow \angle BAD = \angle CAD \quad [\text{cpct}]$$

Therefore, AD bisects $\angle A$.

Solution 19:



In $\triangle ABC$,

CX is the angle bisector of $\angle C$

$$\Rightarrow \angle ACY = \angle BCX \dots\dots (i)$$

In $\triangle AXY$,

$$AX = AY \text{ [Given]}$$

$$\angle AXY = \angle AYX \dots\dots(ii) \text{ [angles opposite to equal sides are equal]}$$

$$\text{Now } \angle XYC = \angle AXB = 180^\circ \text{ [straight line]}$$

$$\Rightarrow \angle AYX + \angle AYC = \angle AXY + \angle BXY$$

$$\Rightarrow \angle AYC = \angle BXY \dots\dots (iii) \text{ [From (ii)]}$$

In $\triangle AYC$ and $\triangle BXC$

$$\angle AYC + \angle ACY + \angle CAY = \angle BXC + \angle BCX + \angle XBC = 180^\circ$$

$$\Rightarrow \angle CAY = \angle XBC \text{ [From (i) and (iii)]}$$

$$\Rightarrow \angle CAY = \angle ABC$$

Solution 20:

Since $IA \parallel CP$ and CA is a transversal

$$\therefore \angle CAI = \angle PCA \text{ [Alternate angles]}$$

Also, $IA \parallel CP$ and AP is a transversal

$$\therefore \angle IAB = \angle APC \text{ [Corresponding angles]}$$

But $\therefore \angle CAI = \angle IAB$ [Given]

$$\therefore \angle PCA = \angle APC$$

$$\Rightarrow AC = AP$$

Similarly,

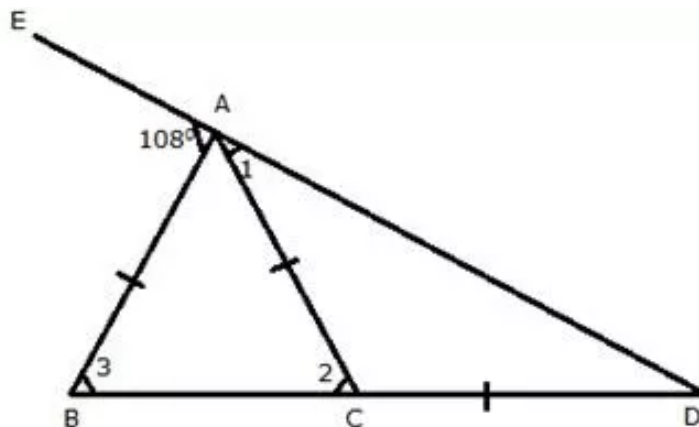
$$BC = BQ$$

Now,

$$PQ = AP + AB + BQ$$

$$= AC + AB + BC$$

$$= \text{Perimeter of } \triangle ABC$$

Solution 21:

In $\triangle ABD$,

$$\angle BAE = \angle 3 + \angle ADB$$

$$\Rightarrow 108^\circ = \angle 3 + \angle ADB$$

But $AB = AC$

$$\Rightarrow \angle 3 = \angle 2$$

$$\Rightarrow 108^\circ = \angle 2 + \angle ADB \dots\dots(i)$$

Now,

In $\triangle ACD$,

$$\angle 2 = \angle 1 + \angle ADB$$

But $AC = CD$

$$\Rightarrow \angle 1 = \angle ADB$$

$$\Rightarrow \angle 2 = \angle ADB + \angle ADB$$

$$\Rightarrow \angle 2 = 2\angle ADB$$

Putting this value in (i)

$$\Rightarrow 108^\circ = 2\angle ADB + \angle ADB$$

$$\Rightarrow 3\angle ADB = 108^\circ$$

$$\Rightarrow \angle ADB = 36^\circ$$

Solution 22:

ABC is an equilateral triangle.

Therefore, $AB = BC = AC = 15 \text{ cm}$

$$\angle A = \angle B = \angle C = 60^\circ$$

In $\triangle ADE$, $DE \parallel BC$ [Given]

$$\angle AED = 60^\circ [\because \angle ACB = 60^\circ]$$

$$\angle ADE = 60^\circ [\because \angle ABC = 60^\circ]$$

$$\angle DAE = 180^\circ - (60^\circ + 60^\circ) = 60^\circ$$

Similarly, $\triangle BDF$ & $\triangle GEC$ are equilateral triangles.

$$= 60^\circ [\because \angle C = 60^\circ]$$

Let $AD = x$, $AE = x$, $DE = x$ [$\because \triangle ADE$ is an equilateral triangle]

Let $BD = y$, $FD = y$, $FB = y$ [$\because \triangle BDF$ is an equilateral triangle]

Let $EC = z$, $GC = z$, $GE = z$ [$\because \triangle GEC$ is an equilateral triangle]

$$\text{Now, } AD + DB = 15 \Rightarrow x + y = 15 \dots\dots(i)$$

$$AE + EC = 15 \Rightarrow x + z = 15 \dots\dots(ii)$$

$$\text{Given, } DE + DF + EG = 20$$

$$\Rightarrow x + y + z = 20$$

$$\Rightarrow 15 + z = 20 \text{ [from (i)]}$$

$$\Rightarrow z = 5$$

$$\text{From (ii), we get } x = 10$$

$$\therefore y = 5$$

$$\text{Also, } BC = 15$$

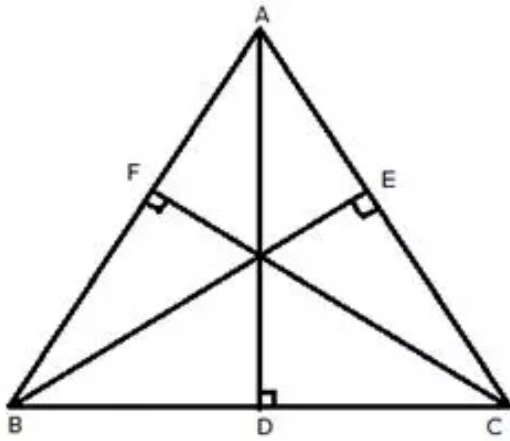
$$BF + FG + GC = 15$$

$$\Rightarrow y + FG + z = 15$$

$$\Rightarrow 5 + FG + 5 = 15$$

$$\Rightarrow FG = 5$$

Solution 23:



In right $\triangle BEC$ and $\triangle BFC$,

$$BE = CF[\text{Given}]$$

$$BC = BC[\text{Common}]$$

$$\angle BEC = \angle BFC[\text{each} = 90^\circ]$$

$$\therefore \triangle BEC \cong \triangle BFC [\text{RHS}]$$

$$\Rightarrow \angle B = \angle C$$

Similarly,

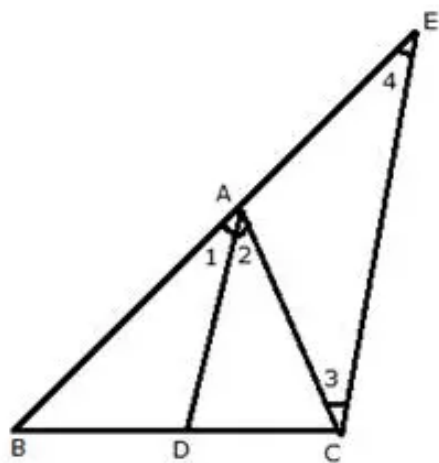
$$\angle A = \angle B$$

$$\text{Hence, } \angle A = \angle B = \angle C$$

$$\Rightarrow AB = BC = AC$$

Therefore, ABC is an equilateral triangle.

Solution 24:



$DA \parallel CE$ [Given]

$\Rightarrow \angle 1 = \angle 4$ (i) [Corresponding angles]

$\angle 2 = \angle 3$ (ii) [Alternate angles]

But $\angle 1 = \angle 2$ (iii) [AD is the bisector of $\angle A$]

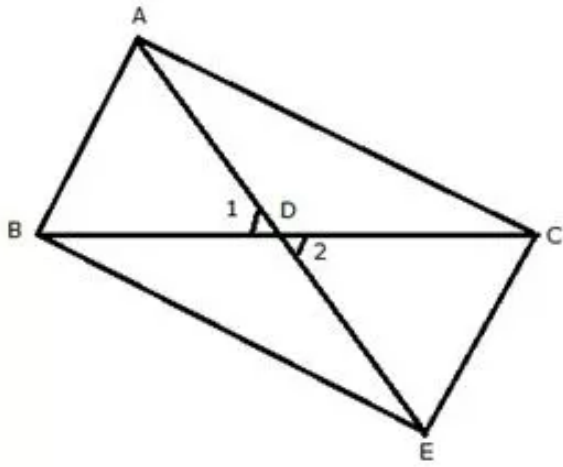
From (i), (ii) and (iii)

$$\angle 3 = \angle 4$$

$$\Rightarrow AC = AE$$

$\Rightarrow \triangle ACE$ is an isosceles triangle.

Solution 25:



Produce AD upto E such that $AD = DE$.

In $\triangle ABD$ and $\triangle EDC$,

$AD = DE$ [by construction]

$BD = CD$ [Given]

$\angle 1 = \angle 2$ [vertically opposite angles]

$\therefore \triangle ABD \cong \triangle EDC$ [SAS]

$\Rightarrow AB = CE$(i)

and $\angle BAD = \angle CED$

But, $\angle BAD = \angle CAD$ [AD is bi sector of $\angle BAC$]

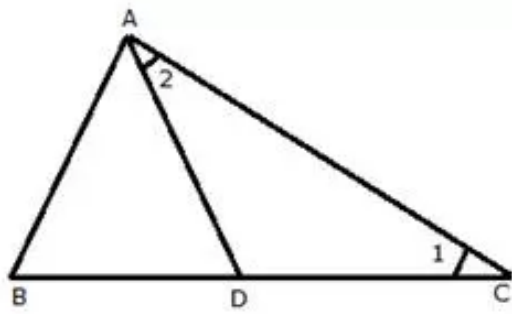
$\therefore \angle CED = \angle CAD$

$\Rightarrow AC = CE$(ii)

From (i) and (ii)

$AB = AC$

Hence, ABC is an isosceles triangle.

Solution 26:

Since $AB = AD = BD$

$\therefore \triangle ABD$ is an equilateral triangle.

$$\therefore \angle ADB = 60^\circ$$

$$\begin{aligned}\Rightarrow \angle ADC &= 180^\circ - \angle ADB \\ &= 180^\circ - 60^\circ \\ &= 120^\circ\end{aligned}$$

Again in $\triangle ADC$,

$$AD = DC$$

$$\therefore \angle 1 = \angle 2$$

But,

$$\angle 1 + \angle 2 + \angle ADC = 180^\circ$$

$$\Rightarrow 2\angle 1 + 120^\circ = 180^\circ$$

$$\Rightarrow 2\angle 1 = 60^\circ$$

$$\Rightarrow \angle 1 = 30^\circ$$

$$\Rightarrow \angle C = 30^\circ$$

$$\therefore \angle ADC : \angle C = 120^\circ : 30^\circ$$

$$\Rightarrow \angle ADC : \angle C = 4 : 1$$

Solution 27:

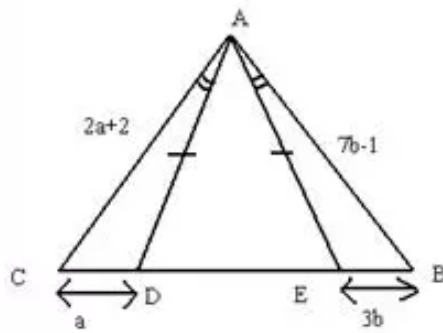
(i)

$$\text{In } \triangle CAE, \angle CAE = \angle AEC = \frac{180^\circ - 68^\circ}{2} = 56^\circ [\because CE = AC]$$

$$\text{In } \angle BEA, a = 180^\circ - 56^\circ = 124^\circ$$

$$\begin{aligned}\text{In } \triangle ABE, \angle ABE &= 180^\circ - (a + \angle BAE) \\ &= 180^\circ - (124^\circ + 14^\circ) \\ &= 180^\circ - 138^\circ = 42^\circ\end{aligned}$$

(ii)



In $\triangle AEB$ & $\triangle CAD$,

$\angle EAB = \angle CAD$ [Given]

$\angle ADC = \angle AEB$ [$\because \angle ADE = \angle AED$ { $AE = AD$ }

$$180^\circ - \angle ADE = 180^\circ - \angle AED$$

$$\angle ADC = \angle AEB$$

$AE = AD$ [Given]

$\therefore \triangle AEB \cong \triangle CAD$ [ASA]

$AC = AB$ [By C.P.C.T.]

$$2a + 2 = 7b - 1$$

$$\Rightarrow 2a - 7b = -3 \dots (i)$$

$$CD = EB$$

$$\Rightarrow a = 3b \dots (ii)$$

Solving (i) & (ii), we get

$$a = 9, b = 3$$