

## Chapter 11. Inequalities

### Exercise 11

#### Solution 1:

In  $\triangle ABC$ ,

$$AB = AC[\text{Given}]$$

$$\therefore \angle ACB = \angle B[\text{angles opposite to equal sides are equal}]$$

$$\angle B = 70^\circ[\text{Given}]$$

$$\Rightarrow \angle ACB = 70^\circ \dots\dots\dots(i)$$

Now,

$$\angle ACB + \angle ACD = 180^\circ[\text{BCD is a straight line}]$$

$$\Rightarrow 70^\circ + \angle ACD = 180^\circ$$

$$\Rightarrow \angle ACD = 110^\circ \dots\dots\dots(ii)$$

In  $\triangle ACD$ ,

$$\angle CAD + \angle ACD + \angle D = 180^\circ$$

$$\Rightarrow \angle CAD + 110^\circ + \angle D = 180^\circ[\text{From (ii)}]$$

$$\Rightarrow \angle CAD + \angle D = 70^\circ$$

$$\text{But } \angle D = 40^\circ[\text{Given}]$$

$$\Rightarrow \angle CAD + 40^\circ = 70^\circ$$

$$\Rightarrow \angle CAD = 30^\circ \dots\dots\dots(iii)$$

In  $\triangle ACD$ ,

$$\angle ACD = 110^\circ[\text{From (ii)}]$$

$$\angle CAD = 30^\circ[\text{From (iii)}]$$

$$\angle D = 40^\circ[\text{Given}]$$

$$\therefore \angle D > \angle CAD$$

$$\Rightarrow AC > CD$$

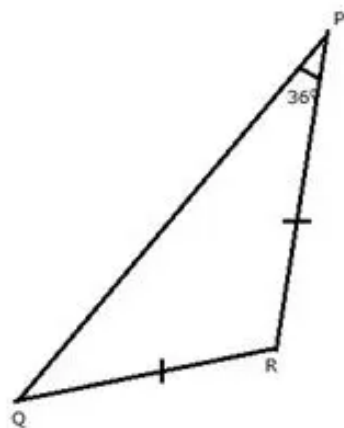
[Greater angle has greater side opposite to it]

Also,

$$AB = AC[\text{Given}]$$

Therefore,  $AB > CD$ .

**Solution 2:**



In  $\triangle PQR$ ,

$$QR = PR[\text{Given}]$$

$$\therefore \angle P = \angle Q[\text{angles opposite to equal sides are equal}]$$

$$\angle P = 36^{\circ}[\text{Given}]$$

$$\Rightarrow \angle Q = 36^{\circ}$$

In  $\triangle PQR$ ,

$$\angle P + \angle Q + \angle R = 180^{\circ}$$

$$\Rightarrow 36^{\circ} + 36^{\circ} + \angle R = 180^{\circ}$$

$$\Rightarrow \angle R + 72^{\circ} = 180^{\circ}$$

$$\Rightarrow \angle R = 108^{\circ}$$

Now,

$$\angle R = 108^{\circ}$$

$$\angle P = 36^{\circ}$$

$$\angle Q = 36^{\circ}$$

Since  $\angle R$  is the greatest, therefore, PQ is the largest side.

**Solution 3:**

The sum of any two sides of the triangle is always greater than third side of the triangle.

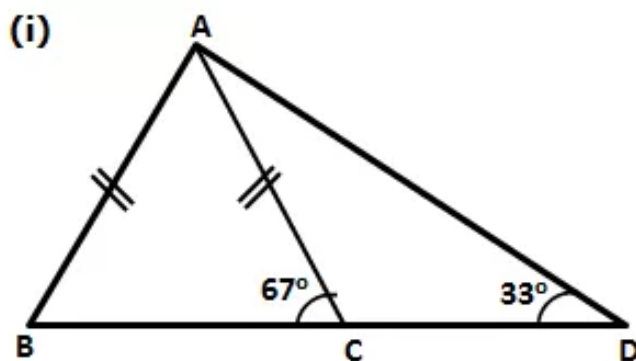
$$\text{Third side} < 13 + 8 = 21 \text{ cm.}$$

The difference between any two sides of the triangle is always less than the third side of the triangle.

$$\text{Third side} > 13 - 8 = 5 \text{ cm.}$$

Therefore, the length of the third side is between 5 cm and 21 cm, respectively.

The value of  $a = 5$  cm and  $b = 21$  cm.

**Solution 4:**

In  $\triangle ABC$ ,

$$AB = AC$$

$$\Rightarrow \angle ABC = \angle ACB \quad (\text{angles opposite to equal sides are equal})$$

$$\Rightarrow \angle ABC = \angle ACB = 67^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - \angle ABC - \angle ACB \quad (\text{angle sum property})$$

$$\Rightarrow \angle BAC = 180^\circ - 67^\circ - 67^\circ = 46^\circ$$

Since  $\angle BAC < \angle ABC$ , we have

$$BC < AC \quad \dots(1)$$

$$\text{Now, } \angle ACD = 180^\circ - \angle ACB \quad (\text{linear pair})$$

$$\Rightarrow \angle ACD = 180^\circ - 67^\circ = 113^\circ$$

Thus, in  $\triangle ACD$ ,

$$\angle CAD = 180^\circ - \angle ACD - \angle ADC$$

$$\Rightarrow \angle CAD = 180^\circ - 113^\circ - 33^\circ = 34^\circ$$

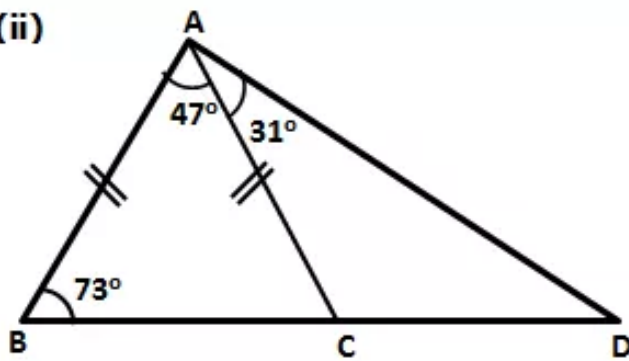
Since  $\angle ADC < \angle CAD$ , we have

$$AC < CD \quad \dots(2)$$

From (1) and (2), we have

$$BC < AC < CD$$

(ii)



In  $\triangle ABC$ ,

$$\angle BAC < \angle ABC$$

$$\Rightarrow BC < AC \quad \dots(1)$$

$$\text{Now, } \angle ACB = 180^\circ - \angle ABC - \angle BAC$$

$$\Rightarrow \angle ACB = 180^\circ - 73^\circ - 47^\circ$$

$$\Rightarrow \angle ACB = 60^\circ$$

$$\text{Now, } \angle ACD = 180^\circ - \angle ACB$$

$$\Rightarrow \angle ACD = 180^\circ - 60^\circ = 120^\circ$$

Now, in  $\triangle ACD$ ,

$$\angle ADC = 180^\circ - \angle ACD - \angle CAD$$

$$\Rightarrow \angle ADC = 180^\circ - 120^\circ - 31^\circ$$

$$\Rightarrow \angle ADC = 29^\circ$$

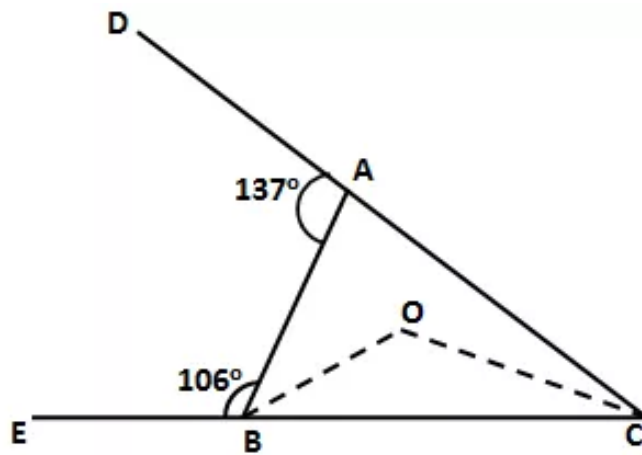
Since  $\angle ADC < \angle CAD$ , we have

$$AC < CD \quad \dots(2)$$

From (1) and (2), we have

$$BC < AC < CD$$

**Solution 5:**



$$\angle BAC = 180^\circ - \angle BAD = 180^\circ - 137^\circ = 43^\circ$$

$$\angle ABC = 180^\circ - \angle ABE = 180^\circ - 106^\circ = 74^\circ$$

Thus, in  $\triangle ABC$ ,

$$\angle ACB = 180^\circ - \angle BAC - \angle ABC$$

$$\Rightarrow \angle ACB = 180^\circ - 43^\circ - 74^\circ = 63^\circ$$

Now,  $\angle ABC = \angle OBC + \angle ABO$

$$\Rightarrow \angle ABC = 2\angle OBC \quad (\text{OB is bisector of } \angle ABC)$$

$$\Rightarrow 74^\circ = 2\angle OBC$$

$$\Rightarrow \angle OBC = 37^\circ$$

Similarly,

$$\angle ACB = \angle OCB + \angle ACO$$

$$\Rightarrow \angle ACB = 2\angle OCB \quad (\text{OC is bisector of } \angle ACB)$$

$$\Rightarrow 63^\circ = 2\angle OCB$$

$$\Rightarrow \angle OCB = 31.5^\circ$$

Now, in  $\triangle BOC$ ,

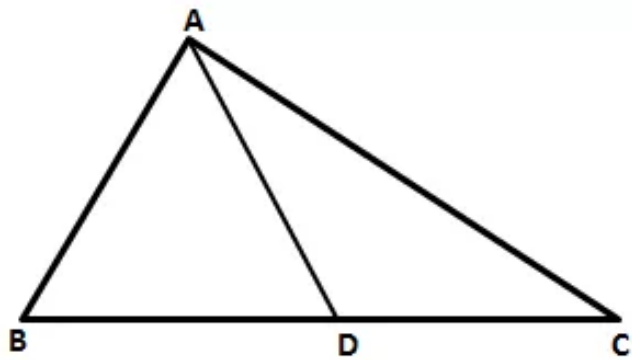
$$\angle BOC = 180^\circ - \angle OBC - \angle OCB$$

$$\Rightarrow \angle BOC = 180^\circ - 37^\circ - 31.5^\circ$$

$$\Rightarrow \angle BOC = 111.5^\circ$$

Since,  $\angle BOC > \angle OBC > \angle OCB$ , we have

$$BC > OC > OB$$

**Solution 6:**

$AD > AC$  (given)

$$\Rightarrow \angle C > \angle ADC \quad \dots(1)$$

Now,  $\angle ADC > \angle B + \angle BAC$  (Exterior Angle Property)

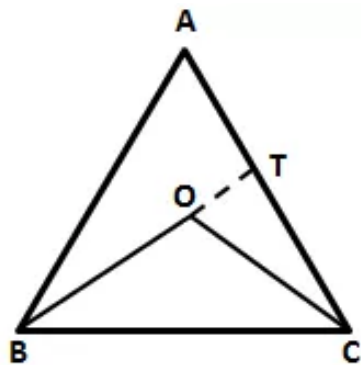
$$\Rightarrow \angle ADC > \angle B \quad \dots(2)$$

From (1) and (2), we have

$$\angle C > \angle ADC > \angle B$$

$$\Rightarrow \angle C > \angle B$$

$$\Rightarrow AB > AC$$

**Solution 7:**

Construction: Produce BO to meet AC at T.

In  $\triangle ABT$ ,

$AB + AT > BT$  (Sum of two sides of a  $\triangle$  is greater than third side)

$$\Rightarrow AB + AT > BO + OT \quad \dots(1)$$

Also, in  $\triangle OCT$ ,

$$OT + TC > OC \quad \dots(2)$$

Adding (1) and (2), we have

$$AB + AT + OT + TC > BO + OT + OC$$

$$\Rightarrow AB + AT + TC > BO + OC$$

$$\Rightarrow AB + AC > OB + OC$$

$$\Rightarrow OB + OC < AB + AC$$

**Solution 8:**

In  $\triangle BEC$ ,

$$\angle B + \angle BEC + \angle BCE = 180^\circ$$

$$\angle B = 65^\circ \text{ [Given]}$$

$$\angle BEC = 90^\circ \text{ [CE is perpendicular to AB]}$$

$$\Rightarrow 65^\circ + 90^\circ + \angle BCE = 180^\circ$$

$$\Rightarrow \angle BCE = 180^\circ - 155^\circ$$

$$\Rightarrow \angle BCE = 25^\circ = \angle DCF \dots\dots\dots(i)$$

In  $\triangle CDF$ ,

$$\angle DCF + \angle FDC + \angle CFD = 180^\circ$$

$$\angle DCF = 25^\circ \text{ [From (i)]}$$

$$\angle FDC = 90^\circ \text{ [AD is perpendicular to BC]}$$

$$\Rightarrow 25^\circ + 90^\circ + \angle CFD = 180^\circ$$

$$\Rightarrow \angle CFD = 180^\circ - 115^\circ$$

$$\Rightarrow \angle CFD = 65^\circ \dots\dots\dots(ii)$$

$$\text{Now, } \angle AFC + \angle CFD = 180^\circ \text{ [AFD is a straight line]}$$

$$\Rightarrow \angle AFC + 65^\circ = 180^\circ$$

$$\Rightarrow \angle AFC = 115^\circ \dots\dots\dots(iii)$$

In  $\triangle ACE$ ,

$$\angle ACE + \angle CEA + \angle BAC = 180^\circ$$

$$\angle BAC = 60^{\circ} [\text{Given}]$$

$$\angle CEA = 90^{\circ} [\text{CE is perpendicular to AB}]$$

$$\Rightarrow \angle ACE + 90^{\circ} + 60^{\circ} = 180^{\circ}$$

$$\Rightarrow \angle ACE = 180^{\circ} - 150^{\circ}$$

$$\Rightarrow \angle ACE = 30^{\circ} \dots\dots\dots(\text{iv})$$

In  $\triangle AFC$ ,

$$\angle AFC + \angle ACF + \angle FAC = 180^{\circ}$$

$$\angle AFC = 115^{\circ} [\text{From (iii)}]$$

$$\angle ACF = 30^{\circ} [\text{From (iv)}]$$

$$\Rightarrow 115^{\circ} + 30^{\circ} + \angle FAC = 180^{\circ}$$

$$\Rightarrow \angle FAC = 180^{\circ} - 145^{\circ}$$

$$\Rightarrow \angle FAC = 35^{\circ} \dots\dots\dots(\text{v})$$

In  $\triangle AFC$ ,

$$\angle FAC = 35^{\circ} [\text{From (v)}]$$

$$\angle ACF = 30^{\circ} [\text{From (iv)}]$$

$$\therefore \angle FAC > \angle ACF$$

$$\Rightarrow CF > AF$$

In  $\triangle CDF$ ,

$$\angle DCF = 25^{\circ} [\text{From (i)}]$$

$$\angle CFD = 65^{\circ} [\text{From (ii)}]$$

$$\therefore \angle CFD > \angle DCF$$

$$\Rightarrow DC > DF$$

**Solution 9:**

$$\angle ACB = 74^{\circ} \dots\dots(i)[\text{Given}]$$

$$\angle ACB + \angle ACD = 180^{\circ}[\text{BCD is a straight line}]$$

$$\Rightarrow 74^{\circ} + \angle ACD = 180^{\circ}$$

$$\Rightarrow \angle ACD = 106^{\circ} \dots\dots(ii)$$

In  $\triangle ACD$ ,

$$\angle ACD + \angle ADC + \angle CAD = 180^{\circ}$$

Given that  $AC = CD$

$$\Rightarrow \angle ADC = \angle CAD$$

$$\Rightarrow 106^{\circ} + \angle CAD + \angle CAD = 180^{\circ}[\text{From (ii)}]$$

$$\Rightarrow 2\angle CAD = 74^{\circ}$$

$$\Rightarrow \angle CAD = 37^{\circ} = \angle ADC \dots\dots(iii)$$

Now,

$$\angle BAD = 110^{\circ}[\text{Given}]$$

$$\angle BAC + \angle CAD = 110^{\circ}$$

$$\angle BAC + 37^{\circ} = 110^{\circ}$$

$$\angle BAC = 73^{\circ} \dots\dots(iv)$$

In  $\triangle ABC$ ,

$$\angle B + \angle BAC + \angle ACB = 180^{\circ}$$

$$\Rightarrow \angle B + 73^{\circ} + 74^{\circ} = 180^{\circ}[\text{From (i) and (iv)}]$$

$$\Rightarrow \angle B + 147^{\circ} = 180^{\circ}$$

$$\Rightarrow \angle B = 33^{\circ} \dots\dots(v)$$

$$\therefore \angle BAC > \angle B \quad [\text{From (iv) and (v)}]$$

$$\Rightarrow BC > AC$$

But,

$$AC = CD \quad [\text{Given}]$$

$$\Rightarrow BC > CD$$

**Solution 10:**

(i)  $\angle ADC + \angle ADB = 180^\circ$  [BDC is a straight line]

$$\angle ADC = 90^\circ \text{ [Given]}$$

$$90^\circ + \angle ADB = 180^\circ$$

$$\angle ADB = 90^\circ \dots\dots\dots(i)$$

In  $\triangle ADB$ ,

$$\angle ADB = 90^\circ \text{ [From (i)]}$$

$$\therefore \angle B + \angle BAD = 90^\circ$$

Therefore,  $\angle B$  and  $\angle BAD$  are both acute, that is less than  $90^\circ$ .

$\therefore AB > BD \dots\dots(ii)$  [Side opposite  $90^\circ$  angle is greater than side opposite acute angle]

(ii) In  $\triangle ADC$ ,

$$\angle ADB = 90^\circ$$

$$\therefore \angle C + \angle DAC = 90^\circ$$

Therefore,  $\angle C$  and  $\angle DAC$  are both acute, that is less than  $90^\circ$ .

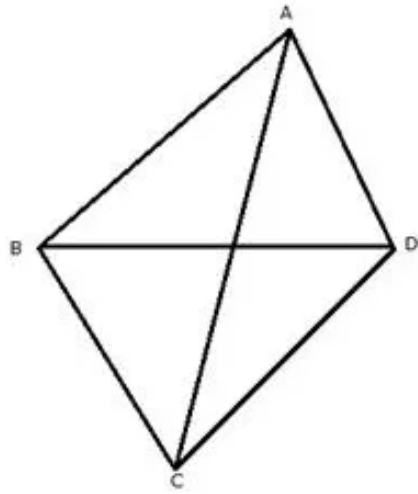
$\therefore AC > CD \dots\dots(iii)$  [Side opposite  $90^\circ$  angle is greater than side opposite acute angle]

Adding (ii) and (iii)

$$AB + AC > BD + CD$$

$$\Rightarrow AB + AC > BC$$

**Solution 11:**



Const: Join AC and BD.

(i) In  $\triangle ABC$ ,

$AB + BC > AC$ ....(i) [Sum of two sides is greater than the third side]

In  $\triangle ACD$ ,

$AC + CD > DA$ ....(ii) [Sum of two sides is greater than the third side]

Adding (i) and (ii)

$$AB + BC + AC + CD > AC + DA$$

$$AB + BC + CD > AC + DA - AC$$

$$AB + BC + CD > DA$$
.....(iii)

(ii) In  $\triangle ACD$ ,

$CD + DA > AC$ ....(iv)[Sum of two sides is greater than the third side]

Adding (i) and (iv)

$$AB + BC + CD + DA > AC + AC$$

$$AB + BC + CD + DA > 2AC$$

(iii) In  $\triangle ABD$ ,

$AB + DA > BD$ ....(v)[Sum of two sides is greater than the third side]

In  $\triangle BCD$ ,

$BC + CD > BD$ ....(vi)[Sum of two sides is greater than the third side]

Adding (v) and (vi)

$$AB + DA + BC + CD > BD + BD$$

$$AB + DA + BC + CD > 2BD$$

**Solution 12:**

(i) In  $\triangle ABC$ ,

$AB = BC = CA$  [ABC is an equilateral triangle]

$$\therefore \angle A = \angle B = \angle C$$

$$\therefore \angle A = \angle B = \angle C = \frac{180^\circ}{3}$$

$$\Rightarrow \angle A = \angle B = \angle C = 60^\circ$$

In  $\triangle ABP$ ,

$$\angle A = 60^\circ$$

$$\angle ABP < 60^\circ$$

$$\therefore \angle A > \angle ABP$$

$$\Rightarrow BP > PA$$

[Side opposite to greater side is greater]

(ii) In  $\triangle BPC$ ,

$$\angle C = 60^\circ$$

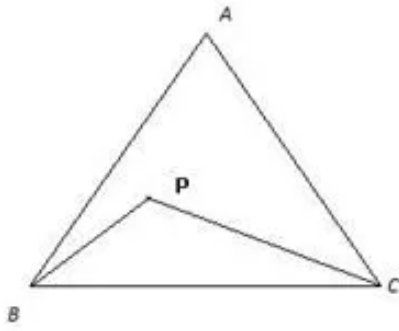
$$\angle CBP < 60^\circ$$

$$\therefore \angle C > \angle CBP$$

$$\Rightarrow BP > PC$$

[Side opposite to greater side is greater]

**Solution 13:**



Let  $\angle PBC = x$  and  $\angle PCB = y$

then,

$$\angle BPC = 180^\circ - (x + y) \dots\dots\dots(i)$$

Let  $\angle ABP = a$  and  $\angle ACP = b$

then,

$$\angle BAC = 180^\circ - (x + a) - (y + b)$$

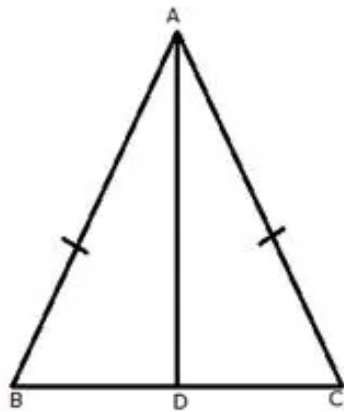
$$\Rightarrow \angle BAC = 180^\circ - (x + y) - (a + b)$$

$$\Rightarrow \angle BAC = \angle BPC - (a + b)$$

$$\Rightarrow \angle BPC = \angle BAC + (a + b)$$

$$\Rightarrow \angle BPC > \angle BAC$$

**Solution 14:**



We know that exterior angle of a triangle is always greater than each of the interior opposite angles.

$\therefore$  In  $\triangle ABD$ ,

$$\angle ADC > \angle B \dots\dots(i)$$

In  $\triangle ABC$ ,

$$AB = AC$$

$$\therefore \angle B = \angle C \dots\dots(ii)$$

From (i) and (ii)

$$\angle ADC > \angle C$$

(i) In  $\triangle ADC$ ,

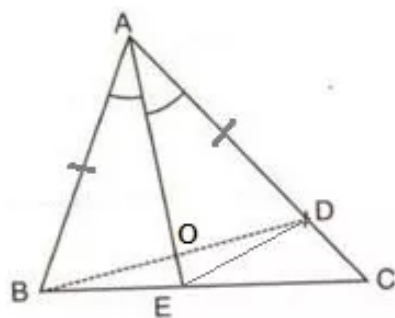
$$\angle ADC > \angle C$$

$$\therefore AC > AD \dots\dots(iii) \text{ [side opposite to greater angle is greater]}$$

(ii) In  $\triangle ABC$ ,

$$AB = AC$$

$$\Rightarrow AB > AD \text{ [From (iii)]}$$

**Solution 15:**

Const: Join ED.

In  $\triangle AOB$  and  $\triangle AOD$ ,

$$AB = AD \text{ [Given]}$$

$$AO = AO \text{ [Common]}$$

$$\angle BAO = \angle DAO \text{ [AO is bisector of } \angle A]$$

$$\therefore \triangle AOB \cong \triangle AOD \text{ [SAS criterion]}$$

Hence,

$$BO = OD \text{ .....(i) [cpct]}$$

$$\angle AOB = \angle AOD \text{ .....(ii) [cpct]}$$

$$\angle ABO = \angle ADO \Rightarrow \angle ABD = \angle ADB \text{ .....(iii) [cpct]}$$

Now,

$$\angle AOB = \angle DOE \text{ [Vertically opposite angles]}$$

$$\angle AOD = \angle BOE \text{ [Vertically opposite angles]}$$

$$\Rightarrow \angle BOE = \angle DOE \text{ .....(iv) [From (ii)]}$$

(i) In  $\triangle BOE$  and  $\triangle DOE$ ,

$$BO = OD \text{ [From (i)]}$$

$$OE = OE \text{ [Common]}$$

$$\angle BOE = \angle DOE \text{ [From (iv)]}$$

$$\therefore \triangle BOE \cong \triangle DOE \text{ [SAS criterion]}$$

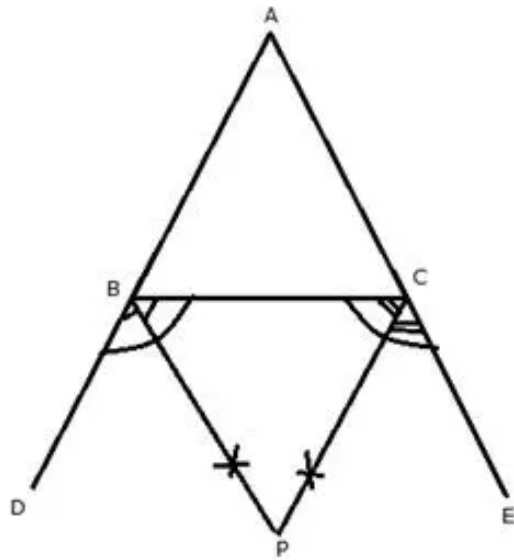
$$\text{Hence, } BE = DE \text{ [cpct]}$$

(ii) In  $\triangle BCD$ ,

$$\angle ADB = \angle C + \angle CBD \text{ [Ext. angle = sum of opp. int. angles]}$$

$$\Rightarrow \angle ADB > \angle C$$

$$\Rightarrow \angle ABD > \angle C \text{ [From (iii)]}$$

**Solution 16:**

In  $\triangle ABC$ ,

$AB > AC$ ,

$$\Rightarrow \angle ABC < \angle ACB$$

$$\therefore 180^\circ - \angle ABC > 180^\circ - \angle ACB$$

$$\Rightarrow \frac{180^\circ - \angle ABC}{2} > \frac{180^\circ - \angle ACB}{2}$$

$$\Rightarrow 90^\circ - \frac{1}{2}\angle ABC > 90^\circ - \frac{1}{2}\angle ACB$$

$$\Rightarrow \angle CBP > \angle BCP \quad \begin{array}{l} \text{[BP is bisector of } \angle CBD \\ \text{and CP is bisector of } \angle BCE] \end{array}$$

$$\Rightarrow PC > PB \quad \text{[side opposite to greater angle is greater]}$$

**Solution 17:**

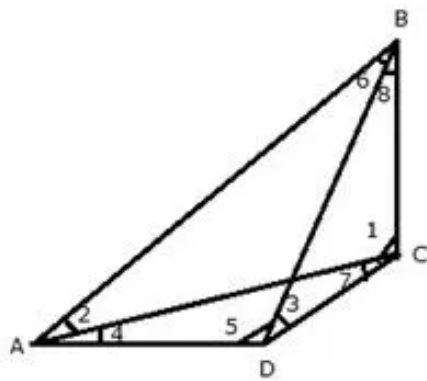
Since AB is the largest side and BC is the smallest side of the triangle ABC

$$AB > AC > BC$$

$$\Rightarrow 180^\circ - z^\circ > 180^\circ - y^\circ > 180^\circ - x^\circ$$

$$\Rightarrow -z^\circ > -y^\circ > -x^\circ$$

$$\Rightarrow z^\circ < y^\circ < x^\circ$$

**Solution 18:**

In the quad. ABCD,

Since AB is the longest side and DC is the shortest side.

$$(i) \angle 1 > \angle 2 [AB > BC]$$

$$\angle 7 > \angle 4 [AD > DC]$$

$$\therefore \angle 1 + \angle 7 > \angle 2 + \angle 4$$

$$\Rightarrow \angle C > \angle A$$

$$(ii) \angle 5 > \angle 6 [AB > AD]$$

$$\angle 3 > \angle 8 [BC > CD]$$

$$\therefore \angle 5 + \angle 3 > \angle 6 + \angle 8$$

$$\Rightarrow \angle D > \angle B$$

**Solution 19:**

(i) Since  $AB > AC$

$$\angle ACB > \angle ABC$$

$$\Rightarrow 180^\circ - z > 180^\circ - y$$

$$\Rightarrow -z > -y$$

$$\Rightarrow z < y \dots \dots (i)$$

Also since  $AC > BC$

$$\angle ABC > \angle BAC$$

$$\Rightarrow 180^\circ - y > 180^\circ - x$$

$$\Rightarrow -y > -x$$

$$\Rightarrow y < x \dots \dots (ii)$$

From (i) and (ii)

$$z < y < x$$

(ii)  $y > x > z$  [Given]

Taking  $y > x$

$$\Rightarrow (180^\circ - \angle ABC) > (180^\circ - \angle BAC)$$

$$\Rightarrow -\angle ABC > -\angle BAC$$

$$\Rightarrow \angle ABC < \angle BAC$$

$$\Rightarrow AC < BC \dots \dots (i)$$

Again taking  $x > z$

$$\Rightarrow (180^\circ - \angle BAC) > (180^\circ - \angle ACB)$$

$$\Rightarrow -\angle BAC > -\angle ACB$$

$$\Rightarrow \angle BAC < \angle ACB$$

$$\Rightarrow BC < AB \dots \dots (ii)$$

From (i) and (ii)

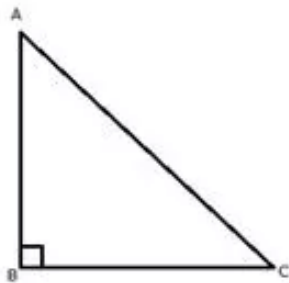
$$AC < BC < AB$$

Writing in descending order

$$AB > BC > AC$$

### Solution 20:

(i)



$$\therefore \angle B = 90^\circ \quad [\text{Given}]$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle C + 90^\circ = 180^\circ$$

$$\Rightarrow \angle A + \angle C = 90^\circ$$

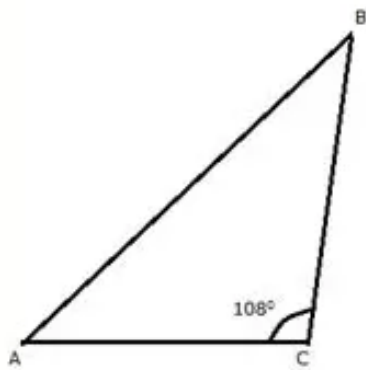
$$\Rightarrow \angle A < 90^\circ \text{ and } \angle C < 90^\circ$$

$$\text{Hence, } \angle B > \angle A \Rightarrow AC > BC$$

$$\text{Similarly, } \angle B > \angle C \Rightarrow AC > AB$$

Hence, hypotenuse is the greatest side.

(ii)



$$\therefore \angle ACB = 108^\circ \quad [\text{Given}]$$

$$\angle A + \angle B + \angle ACB = 180^\circ$$

$$\Rightarrow \angle A + \angle B + 108^\circ = 180^\circ$$

$$\Rightarrow \angle A + \angle B = 72^\circ$$

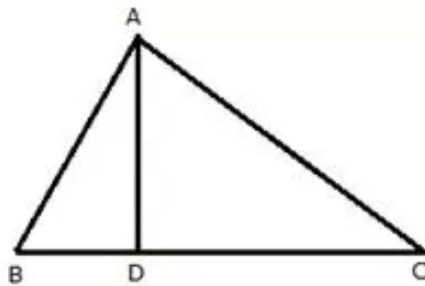
$$\Rightarrow \angle A < 72^\circ \text{ and } \angle B < 72^\circ$$

Hence,  $\angle ACB > \angle A \Rightarrow AB > BC$

Similarly,  $\angle ACB > \angle B \Rightarrow AB > AC$

Therefore, AB is the largest side.

### Solution 21:



In  $\triangle ABD$ ,

$$AB + BD > AD \dots\dots\dots(i)$$

In  $\triangle ACD$ ,

$$AC + DC > AD \dots\dots\dots(ii)$$

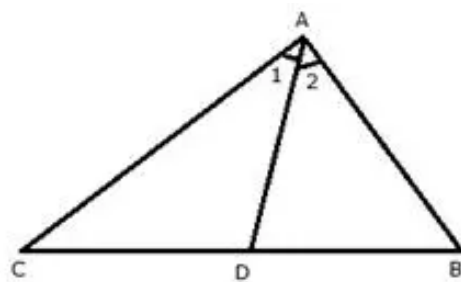
Adding (i) and (ii)

$$AB + BD + AC + DC > 2AD$$

$$AB + BD + DC + AC > 2AD$$

$$AB + BC + AC > 2AD$$

**Solution 22:**



In  $\triangle ADC$ ,

$$\angle ADB = \angle 1 + \angle C \dots\dots\dots(i)$$

In  $\triangle ADB$ ,

$$\angle ADC = \angle 2 + \angle B \dots\dots\dots(ii)$$

But  $AC > AB$ [Given]

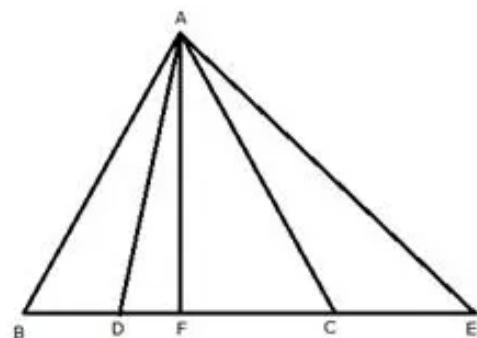
$$\Rightarrow \angle B > \angle C$$

Also given,  $\angle 2 = \angle 1$ [AD is bisector of  $\angle A$ ]

$$\Rightarrow \angle 2 + \angle B > \angle 1 + \angle C \dots\dots(iii)$$

From (i), (ii) and (iii)

$$\Rightarrow \angle ADC > \angle ADB$$

**Solution 23:**

We know that the bisector of the angle at the vertex of an isosceles triangle bisects the base at right angle.

Using Pythagoras theorem in  $\triangle AFB$ ,

$$AB^2 = AF^2 + BF^2 \dots\dots\dots(i)$$

In  $\triangle AFD$ ,

$$AD^2 = AF^2 + DF^2 \dots\dots\dots(ii)$$

We know ABC is isosceles triangle and  $AB = AC$

$$AC^2 = AF^2 + BF^2 \dots\dots(iii) \text{ [ From (i) ]}$$

Subtracting (ii) from (iii)

$$AC^2 - AD^2 = AF^2 + BF^2 - AF^2 - DF^2$$

$$AC^2 - AD^2 = BF^2 - DF^2$$

$$\text{Let } 2DF = BF$$

$$AC^2 - AD^2 = (2DF)^2 - DF^2$$

$$AC^2 - AD^2 = 4DF^2 - DF^2$$

$$AC^2 = AD^2 + 3DF^2$$

$$\Rightarrow AC^2 > AD^2$$

$$\Rightarrow AC > AD$$

Similarly,  $AE > AC$  and  $AE > AD$ .

**Solution 24:**

The sum of any two sides of the triangle is always greater than the third side of the triangle.

In  $\triangle CEB$ ,

$$CE + EB > BC$$

$$\Rightarrow DE + EB > BC \quad [CE = DE]$$

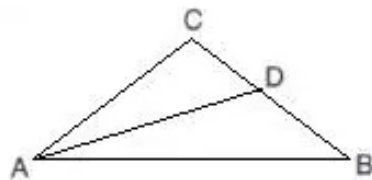
$$\Rightarrow DB > BC \dots\dots(i)$$

In  $\triangle ADB$ ,

$$AD + AB > BD$$

$$\Rightarrow AD + AB > BD > BC \quad [\text{from}(i)]$$

$$\Rightarrow AD + AB > BC$$

**Solution 25:**

Given that,  $AB > AC$

$$\Rightarrow \angle C > \angle B \dots\dots(i)$$

Also in  $\triangle ADC$

$$\angle ADB = \angle DAC + \angle C \quad [\text{Exterior angle}]$$

$$\Rightarrow \angle ADB > \angle C$$

$$\Rightarrow \angle ADB > \angle C > \angle B \quad [\text{From}(i)]$$

$$\Rightarrow \angle ADB > \angle B$$

$$\Rightarrow AB > AD$$