

PERMUTATION AND COMBINATION

1. **Factorial Notation:** The continued product of the first n natural numbers is defined by the symbol $\underline{n}$ or $n!$ and is read as 'Factorial n '.

$$n! = 1. 2. 3. \dots n$$

$$n! = n (n - 1)! = n (n - 1) (n - 2)!$$

2. **The Sum Rule:** Suppose a work A can occur in m ways and B can occur in n ways and both cannot occur simultaneously. Then A or B (at least one of them) can occur in $(m + n)$ ways. This rule is also applicable for two or more exclusive events.
3. **The Product Rule:** Suppose there are two works A and B. Let A can occur in m ways and B in n ways. Suppose that the ways for A and B are not related in the sense that B

occur in n ways regardless the outcome of A, then both A and B occur in mn ways.

For example, let there are two questions A and B which can be solved by two methods and 3 methods respectively. Then, A or B can be solved in $2 + 3 = 5$ ways and both A and B in $2 \times 3 = 6$ ways.

4. Permutation: Each of the arrangements that can be made by taking some or all of a number of dissimilar things (objects) is called *Permutation*.

5. Combination: Each of the different group or selection which can be made by taking some or all of a number of things (irrespective of order) is called a *combination*.

Derangements: Any change in the given order of the things is called a derangements.

6. The number of permutations of n distinct things taken all at a time $= n!$

7. The number of permutations of n dissimilar things taken r at a time when each thing can be repeated any number of times $= n^r$.

8. Number of permutations of n distinct things taken r at a time, $0 \leq r \leq n$

$$= n(n-1)(n-2) \dots (n-r+1)$$

$$= \frac{n!}{(n-r)!} = {}^n P_r.$$

This is equivalent to filling r places by r objects taking from n distinct objects.

9. The number of permutations of n distinct things taken r at a time when p particulars things always occur $= {}^{(n-p)}C_{(r-p)} \cdot r!$.
10. Number of permutations (arrangements) of n distinct things taken r at a time when p particular things never occur

$$= {}^{(n-p)}C_r \cdot r!$$
11. Number of permutations of n things, taken all at a time when p_1 are alike of one kind, p_2 are alike of second kind, ... p_r of them are alike of the r^{th} kind $p_1 + p_2 + \dots + p_r \leq n$, and remaining things are all different $= \frac{n!}{p_1! p_2! \dots p_r!}$.
12. The number of combinations of n objects taken r at a time, $0 \leq r \leq n$.

$$= {}^nC_r = \frac{n!}{(n-r)!r!}$$

13. The number of combinations of n distinct objects taken r at a time when any object may be repeated any number of times.

$$= \text{coefficient of } x^r \text{ in } (1 + x + x^2 + x^3 + \dots + x^r)^n$$

$$= \text{coefficient of } x^r \text{ in } (1 - x)^{-n} = {}^{n+r-1}C_r$$

14. Number of combinations of n distinct things taken r at a time when p particular things always occur

$$= {}^{(n-p)}C_{(r-p)}$$

15. Number of combinations of n distinct things taken r at a time when p particular things never occur $= {}^{(n-p)}C_r$.

16. The number of combinations if some or all n things be taken at a time

$$= {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n - 1.$$

17. If there are p_1 objects of one kind, p_2 objects of second kind, ..., p_n objects of n^{th} kind, then the number of ways of choosing r objects out of these $(p_1 + p_2 + \dots + p_n)$ objects

$$= \text{coefficient of } x^r \text{ in } (1 + x + \dots + x^{p_1})$$

$$(1 + x + \dots + x^{p_2}) \dots (1 + x + \dots + x^{p_n})$$

If one object of each kind is to be included in such a collection then the number of ways of choosing r objects.

$$= \text{coefficient of } x^r \text{ in the product}$$

$$(x + x^2 + \dots + x^{p_1}) (x + x^2 + \dots + x^{p_2}) \dots$$

$$(x + x^2 + \dots + x^{p_n}).$$

18. Total number of ways to make a selection by taking some or all of $p_1 + p_2 + \dots + p_r$ things where p_1 are alike of one kind, p_2 are alike of second kind, ..., p_r are alike of r^{th} kind is

$$= (p_1 + 1)(p_2 + 1)\dots(p_r + 1) - 1$$

19. The number of ways in which n distinct objects can be split into three groups containing respectively r , s and t objects, r , s and t are distinct and $r + s + t = n$, is given by

$${}^nC_r {}^{n-r}C_s {}^{n-r-s}C_t = \frac{n!}{r!.s!.t!}$$

20. If $3n$ things are to be divided into the three equal groups, then the number of ways

$$= \frac{(3n)!}{n!.n!.n!.3!}$$

21. If $3n$ things are to be divided equally between 3 persons (i.e., division of $3n$ things into 3 equal groups with permutation of groups),

$$\text{then the number of ways} = \frac{(3n)!}{(n!)^3}$$

22. If n things form an arrangement in a row, then the number of ways in which they can

be deranged so that no one of them occupies its original place is

$$n! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \cdot \frac{1}{n!} \right)$$

23. The number of circular permutations of n different things taken all at a time $= (n - 1)!$

24. The number of arrangements of n persons on a round table $= (n - 1)!$

25. The number of arrangement of n flowers to

$$\text{make a garland} = \frac{1}{2} (n - 1)!$$

26. ${}^nC_r = {}^nC_{n-r}$

27. ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

28. ${}^nC_r = {}^nC_s \Rightarrow r = s \text{ or } r + s = n.$

29. Greatest Value of nC_r

nC_r is greatest when $r = n/2$ if n is even.

$r = (n - 1)/2$ or $(n + 1)/2$
if n is odd.

30. Distinction between problems on
“permutation and combination”

(a) In problems on arrangements, ordering, permutations, arranging, line up, we use
the formula for Permutations.

- (b) In problems on selection, subset, committee, groups, choice, we use the formula for Combinations.
 - (c) When to multiply and when to add, Multiply when there is 'AND' in the problem and 'Add' when there is 'OR'.
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