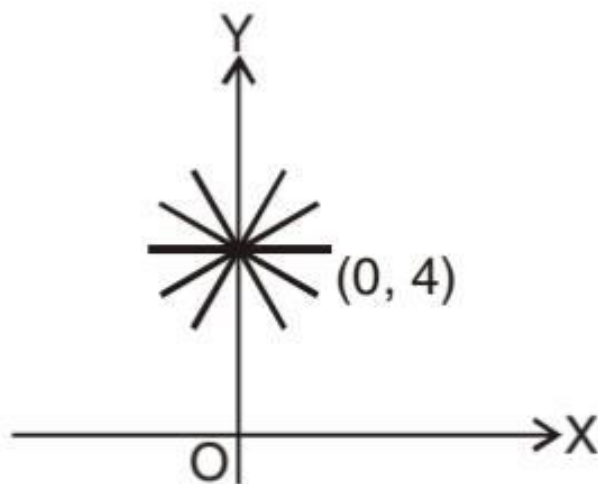


FAMILY OF LINES

We know that the eqn. $y = mx + 4$ represents a straight line having slope m and making an intercept 4 on y -axis.



For different value of m represents different straight lines each making an intercept 4 on y -axis. All these lines (as shown in figure) taken together constitute a family of lines having the common property that each makes an intercept of 4 units on y -axis.

The arbitrary constant m , which is same for one line but is different for different lines, is called the parameter of the family.

Let $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

and $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$.

Then, $S = 0$ represents.

- (i) a pair of straight lines, if $\Delta = 0$ and $h^2 \geq ab$
- (ii) a pair of intersecting lines, if $\Delta = 0$ and $h^2 > ab$
- (iii) a pair of parallel lines, if $\Delta = 0$ and $h^2 = ab$ or if $h^2 = ab$ and $bg^2 = af^2$

The equation $ax^2 + 2hxy + by^2 = 0$ represents

- (i) two real and different lines if $h^2 > ab$
- (iii) two coincident lines if $h^2 = ab$

Consider the equation of pair of lines through the origin, i.e., $S = ax^2 + 2hxy + by^2 = 0$, then

- (i) if $y = m_1 x$ and $y = m_2 x$ are two lines represented by $S = 0$, then $m_1 + m_2 = -2h/b$ and $m_1 m_2 = a/b$
- (ii) The angle θ between the lines represented

$$\text{by } S = 0, \text{ is given by } \theta = \tan^{-1} \left| \frac{2\sqrt{(h^2 - ab)}}{a + b} \right|$$

- (iii) The lines given by $S = 0$ are parallel if $h^2 = ab$.
- (iv) The line given by $S = 0$ are perpendicular if $a + b = 0$, *i.e.*, if coeff. of x^2 + coeff. of $y^2 = 0$
- (v) The equation of the pair of bisectors of the angles between the lines $S = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}.$$
