

CIRCLES AND FAMILY OF CIRCLES

Circle: A circle is a locus of a point which moves in a plane so that it is always at a constant distance from a fixed point in the plane. The fixed point is called the centre and the constant distance is called the radius of the circle.

Equation of a Circle: The equation of a circle whose centre is C (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

Particular Case: If a centre of the circle is origin (0, 0), then the above equation reduces to

$$(x - 0)^2 + (y - 0)^2 = r^2$$

or $x^2 + y^2 = r^2$

General Equation of a Circle: $x^2 + y^2 + 2gx + 2fy + c = 0$ is the equation of a circle whose centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$

(A) To find the centre and radius of circle

(i) make the coefficient of x^2 and y^2 each

equal to 1 by dividing throughout (if necessary) by the coefficients of x^2 and y^2 .

(ii) The coordinates of the centre are

$$\left(-\frac{1}{2} \text{coeff. of } x, -\frac{1}{2} \text{coeff. of } y \right)$$

(iii) Radius

$$= \sqrt{\left(-\frac{1}{2} \text{coeff. of } x \right)^2 + \left(-\frac{1}{2} \text{coeff. of } y \right)^2 - \text{constant term}}$$

(B) Nature of the circle

- (i) If $g^2 + f^2 - c = 0$, the radius is zero and the circle becomes a point, which is known as a point circle.
- (ii) If $g^2 + f^2 - c > 0$, the radius is real and hence, the circle is a real circle.
- (iii) If $g^2 + f^2 - c < 0$, the radius is imaginary and therefore, the circle is an imaginary circle and is called a virtual circle.

Condition of a Circle: Any equation satisfying the following conditions must be a circle.

- (i) The coeff. of $x^2 =$ coeff. of y^2
- (ii) It contains no term involving the product xy .
- (iii) It is an equation of second degree in x and y .

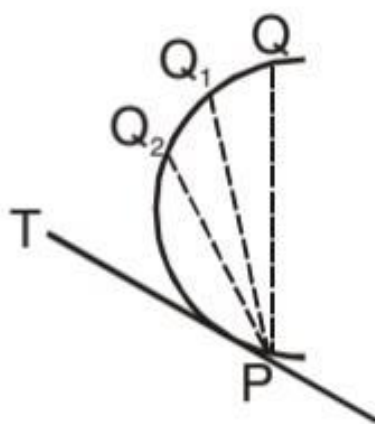
Diameter form of a Circle: The equation of a circle whose end diameter are (x_1, y_1) and (x_2, y_2) is given by $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Intersection of a line and a circle:

- (i) The line $y = mx + c$ will intersect the circle $x^2 + y^2 = r^2$ in two distinct and real points if $c^2 < r^2 (1 + m^2)$
- (ii) The line $y = mx + c$ will intersect the circle $x^2 + y^2 = r^2$ at one point if $c^2 = r^2 (1 + m^2)$
- (iii) The line $y = mx + c$ will intersect the circle $x^2 + y^2 = r^2$ at two imaginary points if $c^2 > r^2 (1 + m^2)$

Tangent to a Circle:

Let P be any point on a curve and Q be any other point on the curve, join PQ and that PQ is called a chord of the Circle.



Let the point Q moves along the curve towards P taking the position $Q_1, Q_2 \dots$ and finally coincide with P such that the chord PQ takes the position PT. The limiting position PT of the line PQ as Q tends to P is called the tangent to the curve at P.

Equation of a tangent

- (i) The equation of the tangent at point (x_1, y_1) on the circle is $xx_1 + yy_1 = r^2$
- (ii) Equation of the tangent at the point (x_1, y_1) on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

Working Rule:

- (i) Change x^2 into xx_1 and y^2 into yy_1
- (ii) Change x into $\frac{x + x_1}{2}$ and y to $\frac{y + y_1}{2}$
and keep constant term same.

Normal: The normal at a point of a curve is the straight line through the point and perpendicular to the tangent at the point.

Equation of a Normal

- (i) Equation of a normal at the point (x_1, y_1) to the circle $x^2 + y^2 = r^2$ is
$$xy_1 - yx_1 = 0$$

Note: (a) The normal at any point (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$x(y_1 + f) - y(x_1 + g) + (gy_1 - fx_1) = 0$$

A Circle and a line: A line $y = mx + c$ will be a tangent to the circle $x^2 + y^2 = a^2$ is $c = \pm a\sqrt{1 + m^2}$

Rule to find the equation of radical axis:

- (i) Subtract the equations of two circles, after making coefficients of x^2 and y^2 unity in both the equations.
- (ii) If the two circles intersect, then radical axis is the common chord of the two circles.
- (iii) If the two circles touch each other, then radical axis is the common tangent.

Radical Centre of three circle: The point of intersection of radical axes of three circles taken in pairs is called the radical centre of the three circles.

Family of Circles: A collection of circles is called a family of circles.

The general equation of a circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ where g, f, c are three arbitrary constants (called parameters). We also know that three constants g, f, c can have fixed value only when three independent conditions satisfied by a circle are given.
