CONIC SECTION

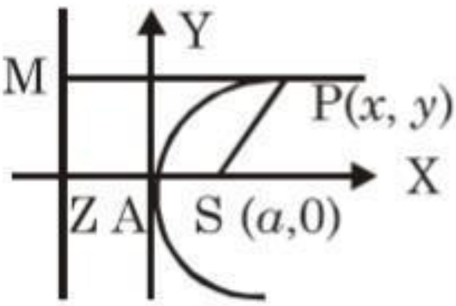
A Conic Section or conic is the locus of a point P which moves so that its distance from a fixed point S is in a constant ratio to its perpendicular distance from a fixed straight line, all being in the same plane.

The fixed point is called the focus and is generally denoted by S. The fixed straight line is called the directrix and the constant ratio is called the ecentricity and is denoted by the letter e.

The straight line passing through the focus and perpendicular to the directrix is called the axis of the conic, and the point where the axis meets the conic is called the vertex of the conic.

The conic is called a parabola if $e = 1$

The conic is called a ellipse if $e < 1$



The conic is called a hyperbola if $e > 1$.

Parabola: A parabola is the locus of a point which moves in a plane so that its distance from a fixed point is equal to its distance from a fixed straight line.

The fixed point is called the focus and fixed straight line is called the directrix of the parabola.

Equation of the parabola: It is $y^2 = 4ax$

Note:

(*i*) The coordinates of the vertex A are (0,0).

(ii) The coordinates of the focus S are $(a, 0)$.

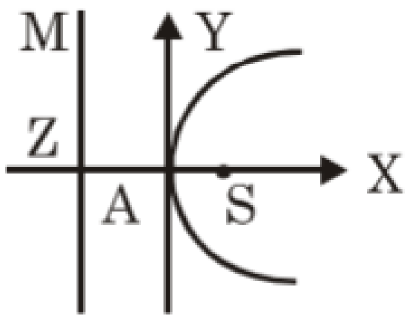
(iii) The equation of the axis of the parabola is $y = 0$

(iv) The equation of the directrix is $x = -a$.

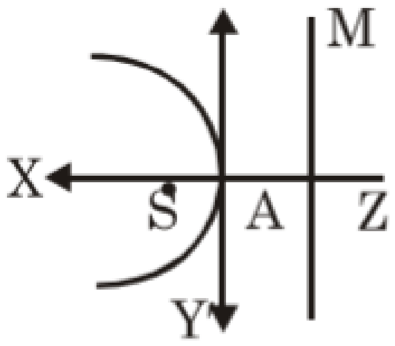
Definitions:

1. **Focal distance:** The distance of a point on the parabola from the focus is called the focal distance of the point.
2. **Focal chord:** A chord of the parabola which passes through the focus is called a focal chord.
3. **Latus rectum:** The latus rectum of a parabola is a chord passing through the focus and perpendicular to the axis.

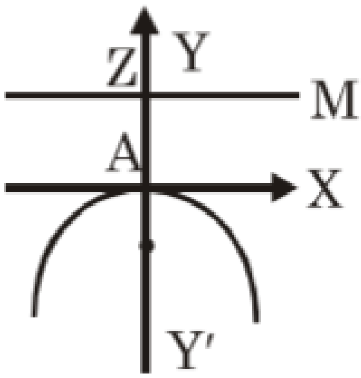
Four standard forms of the parabola



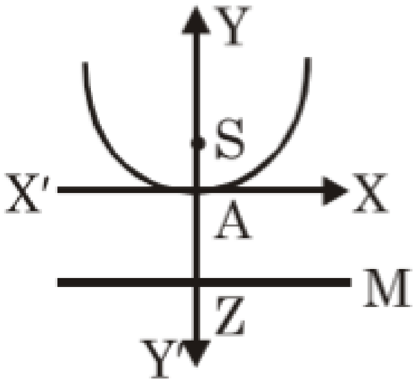
(i) $y^2 = 4ax$
[Right handed parabola]



(ii) $y^2 - 4ax$
[Left handed parabola]



(i) $x^2 - -4ay$
[Down parabola]

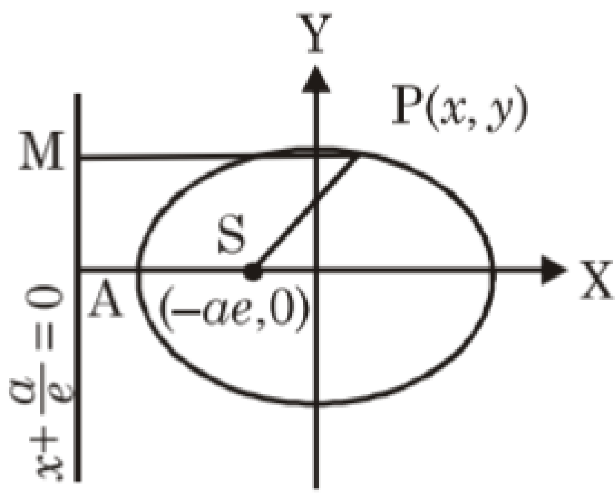


(i) $x^2 = 4ay$
[Upward parabola]

Important facts relating to parabola

Form	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Axis	$y = a$	$y = 0$	$x = 0$	$x = 0$
Vertex	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
Tangent at vertex	$x = 0$	$x = 0$	$y = 0$	$y = 0$
Rectum	$4a$	$4a$	$4a$	$4a$

Ellipse: An ellipse is the locus of a point which moves in a plane so that its distance from a fixed point bears a constant ratio, less than one, to its distance from a fixed straight line.



If P is any point on the ellipse whose focus is S, and ecentricity is e and PM ⊥ on the directrix, then

$$\frac{SP}{PM} = e \text{ or } SP = e\,PM\,(e < 1)$$

Equation of the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Definitions:

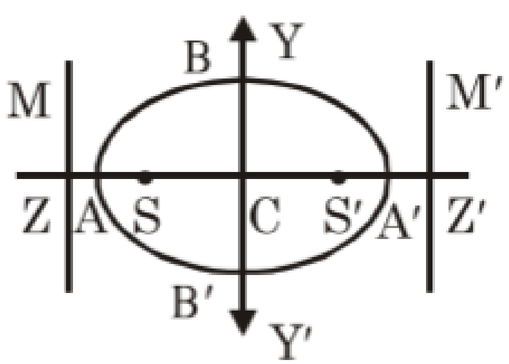
(*i*) The lines AA′ and BB′ are called the major and minor axis. Both together are called principal axis.

(*ii*) The points A, A′, the extremities of the major axis are called the vertices of the ellipse.

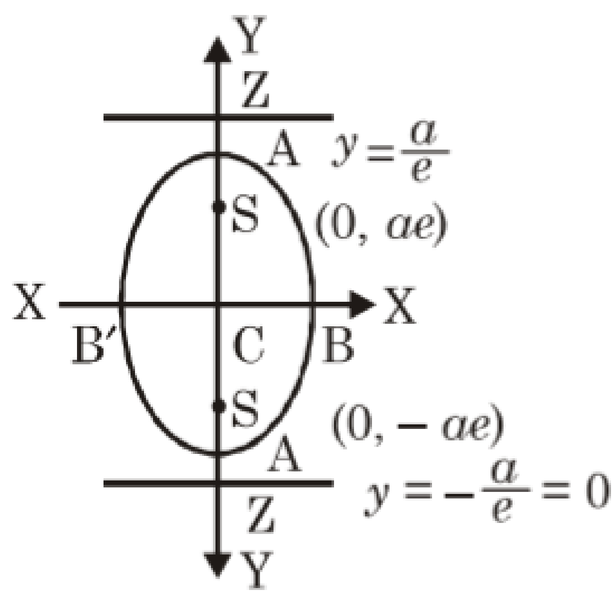
(*iii*) C is centre of the ellipse. The centre C bisects every chord of the ellipse which passes through it.

Important facts relating to an ellipse:

Ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



Ellipse: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$



1.	Centre	(0, 0)	(0, 0)
2.	Foci	$(\pm ae, 0)$	$(0 \pm ae)$
3.	Length of major axis	$2a$	$2a$

4. Length of minor axis	$2b$	$2b$
5. Equation of major axis	$y = 0$	$x = 0$
6. Equation of minor axis	$x = 0$	$y = 0$
7. Equation of directrices	$x = \pm\frac{a}{e}$	$y = \pm\frac{a}{e}$
8. Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
9. Equation of latera-recta	$x = \pm\, ae$	$y = \pm\, ae$
10. Vertices	$(\pm\, a, 0), (0, \pm\, b)$	$(\pm b, 0), (0, \pm\, a)$

Hyperbola: A hyperbola is the locus of a point which moves in a plane so that its distance from a fixed point bears a constant ratio greater than one, to its distance, from a fixed straight line.

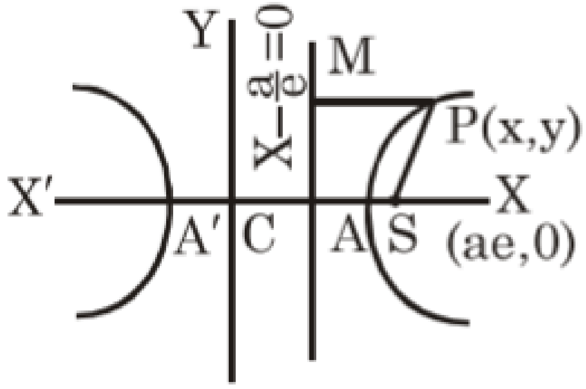
The fixed point is called the focus and is generally denoted by S. The fixed straight line is called the directrix of the hyperbola and the constant ratio is usually denoted by e, is called the ecentricity of the hyperbola.

Equation of hyperbola in standard form: is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Note:

The y-axis *i.e.*, $x = 0$ meets the hyperbola



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

where $y^2 = -b^2$ or $y = \pm ib$

$\therefore$ The points of intersection are imaginary.

Note:

(*i*) **Vertices:** The point A and A' are called vertices of the hyperbola.

(*ii*) **Transverse axis:** AA' is called the transverse axis of the hyperbola.

(*iii*) **Conjugate axis:** Y is called conjugate of the hyperbola.

(*iv*) **Principal axis:** The transverse and conjugate axes together are called the principal axes of the hyperbola.

(*v*) **Centre:** C is called the centre of the hyperbola. It is the point of intersection of the transverse and conjugate axes. It bisects every chord of the hyperbola that passes through it.

Important facts relating to hyperbola

Form of the hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
1. Co-ordinates of centre are	(0, 0)	(0, 0)
2. Coordinates of foci are	$(\pm ae, 0)$	$(0, \pm ae)$
3. Coordinates or vertices are	$(\pm a, 0)$	$(0, \pm a)$
4. Equation of transverse axis	$y = 0$	$x = 0$
5. Equation of conjugate axis	$x = 0$	$y = 0$
6. Equation of the directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{a}{e}$

7. Ecentricity is given by $b^2 = a^2 (e^2 - 1)$ or

$$e^2 = \frac{a^2 + b^2}{a^2}$$

8. Length of latus rectum $= \frac{2b^2}{a}$
9. Length of transverse axis $= 2a$
10. Length of conjugate axis $= 2b$
11. For a rectangular hyperbola $b = a$ and $e = \sqrt{2}$
12. Equation of hyperbola with centre, (h,k) is

$$\frac{(x-h)^2}{a^2} = \frac{(y-h)^2}{b^2} = 1$$

13. The coordinates of the ends of latera recta are:

$$\left(ae, \frac{b^2}{a}\right); \left(ae, \frac{-b^2}{a}\right); \left(-ae, \frac{b^2}{a^2}\right); \left(-ae, \frac{-b^2}{a^2}\right)$$

14. The equation of latera recta are $x = \pm\, ae$
15. Focal distances of any point $P(x, y)$ on the hyperbola are $ex \pm a$

Condition of Tangency of the line $y = mx + c$

(*a*) **Parabola:** Condition that line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$ is $c = \dfrac{a}{m}$.

and the point of contact is $\left(\dfrac{a}{m^2}, \dfrac{2a}{m}\right)$.

Parametric coordinates: Any point on the parabola $y^2 = 4ax$ is $(at^2, 2at)$ and is generally referred to as the point 't'.

(**b**) **Ellipse:** Condition that line $y = mx + c$ is tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is

$$c = \pm\sqrt{a^2m^2 + b^2}$$

Point of contact: The point of contact of this tangent with the ellipse is

$$\left(-\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \frac{b^2}{\sqrt{a^2m^2+b^2}}\right) \text{ if } c = \sqrt{a^2m^2+b^2}$$

and $\left(\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$ if $c = \sqrt{a^2m^2+b^2}$

Point 'θ': Any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is (a cos θ, b sin θ) and this point is referred as the point'θ'.

The parametric equation of the ellipse is

$x = a \cos\theta, y = b \sin\theta$

θ is called the ecentric angle.

(c) Hyperbola: Condition that line $y = mx + c$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$$c = \pm\sqrt{a^2m^2 - b^2}$$

Point of contact is $\left(\frac{ma^2}{\sqrt{a^2m^2-b^2}}, \frac{b^2}{\sqrt{a^2m^2-b^2}}\right)$

Point 'θ': (a sec θ, b tan θ) are the coordinates of any point on the hyperbola.

The parametric equation of hyperbola can be written as $x = a \sec\theta$, $y = b \tan\theta$