PROBABILITY

Def: A mathematically measure of uncertainty is known as probability.

Experiment: An experiment in any action which we do or intend to do. An experiment is performed for achieving some objective. For example, coin is tossed in order to choose batting or fielding first in cricket match.

Random Experiment: Word "random experiment" signifies that the experiment is performed in a haphazard way. It is performed in an unplanned manner. The word random signifies that the possibility of a particular outcome as a result of performing an experiment is not a certainty.

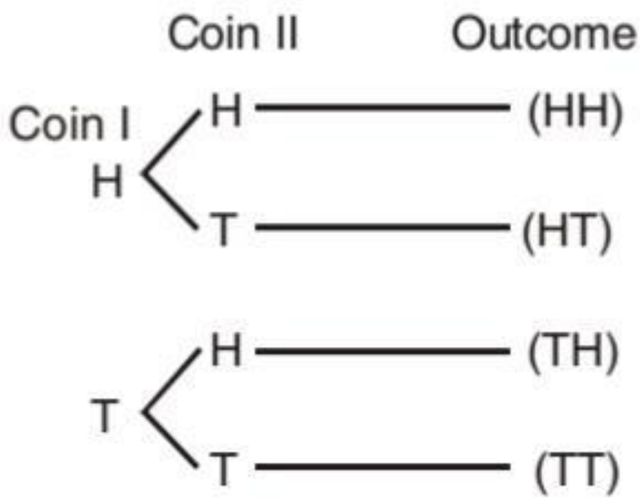
Tossing of a coin and throwing a die are examples of random experiment.

Outcome: The result of a random experiment is called an outcome.

Sample space: The set of all possible outcomes of a random experiment is known as sample space. For example

(*i*) The sample space in a throw of a die is the set {1, 2, 3, 4, 5, 6}

(*ii*) If we toss coins, the sample space in this consists of four outcomes.



The sample space is the set {(H, H), (H, T), (T, H), (T, T)}

Trial: The performance of a random experimental is called a trial.

Events: An event is a set of experimental outcomes, or in other words it is a subset of sample space S.

Note:

(*i*) an event may contain any number of the elements of sample space S.

(*ii*) Elements of a sample space are called elementary events.

(*iii*) As yet S is a subset of S therefore S is also an event.

(*iv*) We know ϕ is subset of every set so, ϕ is also an example of an event and sometimes is called impossible event.

For example, let us perform the experiment of tossing a die and A denote the event that an even number appears on top

Obviously A = {2, 4, 6}

which is a subset of sample space.

S = {1, 2, 3, 4, 5, 6}.

Occurrence of an event: If on performing an experiment, the results of the experiment belongs to the subset defining an event then the event is said to have occurred. Event is said to have not occurred if the result of the experiment does not belong to the subset defining the event.

Simple event: An event consisting of only one sample point of sample space is called a simple event.

An event consisting of more than one sample point is called a compound event.

Mutually exclusive events: The events are said to be mutually exclusive or incomplete if no two or more of them can happen simultaneously in the same trial. For example, appearing of head and tail in tossing of a coin are mutually exclusive as there is only one possibility – either head or tail. Both head and tail cannot occur.

Equally likely events: The events are said to be equally likely if there is no reason except any one in preference to any other. For example, in tossing of a coin, appearing a head or tail are equally likely.

Exhaustive events: All possible outcomes of an event are known as exhaustive events. For example, in a throw of a single dice the exhaustive events are six i.e. 1, 2, 3, 4, 5, 6. If two dice are thrown the exhaustive events would be $6 \times 6 = 36$.

Favourable events: The cases which ensures the occurrence of the event are callled favourable. For example, when two dice are thrown the number of cases favourable for getting a sum 6 is 5 i.e. (1, 5) , (5, 1), (2, 4) (4, 2) and (3, 3).

Independent events: Two or more events are said to be independent if the happening or non-happening or any other event. For example the result of first toss of a coin does not influence the result of second toss of a coin.

Dependent events: If the happening of an event influences the happening of the other event, the events are said to be dependent events.

For example, if a card is drawn from a pack of shuffled cards and not replaced before drawing the second card, then the second draw is dependent on the first one.

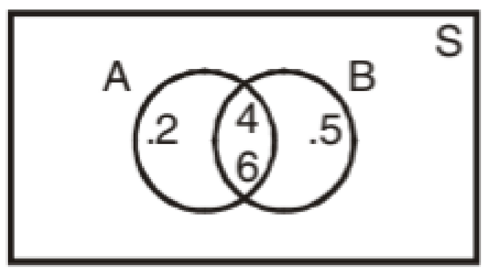
Algebra of events: In a single throw of a dice, let A denote the event "The appearance of the even number" and B denotes the event "The appearance of the number of greater than 3".

$\therefore$ Set A = {2, 4, 6}

Set B = {4, 5, 6} and S = {1, 2, 3, 4, 5, 6}

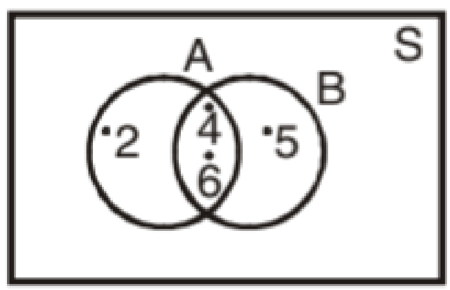
(*i*) Consider a new event "A or B" which occurs when A or B or both occur. Clearly the event "A or B" will be represented by

$A \cup B = \{2, 4, 5, 6\}$



(*ii*) Consider another event "A and B" which can occur only when both occurs. Clearly "A and B" will be represented by

$A \cap B = \{4, 6\}$



Complementary events: Event which is complementary to the event E is set of those elements of sample space which do not belong to E. Event complement to E is denoted by E^c or $\overline{E}$. It is also called negation of E.

Probability of an event: If a trial results in 'n' exhaustive, mutually exclusive and equally likely cases, out of which 'm' are favourable in the occurrence of an event A, then the probability of occurrence of event A, usually denoted P (A) is given by

$$P(A) = \frac{\text{Number of favourable cases}}{\text{Exhaustive number of cases}} = \frac{m}{n}$$

Note:

1. If the probability of an event A is zero then we say that the event A is impossible, and if the probability of an event is one then we say that the event A is sure or certain.
2. If the probability of an event A is 'p', $0 \leq p \leq 1$ *i.e.*, $P(A) = p$, then we say
   (*i*) The chance that event A happen is p
   (*ii*) The chance that event A fails to happen is $1 - p$
   (*iii*) Odds in favour of event A are $p : (1 - p)$
   (iv) Odds against the event A are $(1 - p) : p$

3. If an event A can happen in 'm' ways and fail in 'n' ways, then the following three statement are equivalent.

(*i*) $P(A) = \dfrac{m}{m+n}$

(*ii*) The odds in favour of the event A are

$$\frac{m}{m+n};\left(1-\frac{m}{m+n}\right) i.e., m:n$$

(*iii*) The odds against the event A are

$$\left(1-\frac{m}{m+n}\right);\frac{m}{m+n} \; i.e., n:m$$

(*iv*) Probability of non-happening of event A is denoted by

$$P(\overline{A}) = \frac{\text{Number of favourable cases (}i.e.\text{ unfavourable)}}{\text{Exhaustive number of outcomes}}$$

$$= \frac{n-m}{m} = 1-\frac{m}{n} = 1-P(A)$$

$$\Rightarrow P(A) + P(\overline{A}) = 1$$

Theorems on Probability

If A and B are two mutually exclusive events of a random experiment, the probability of occurrence of the event "A or B" is the sum of the probabilities of the events, A and B, or

$$P\ (A \text{ or } B) = P\ (A) + P(B)$$

if A and B are mutually exclusive events or

In the language of subsets of the sample space S, theorem is written as follows:

If $A \cap B = \phi$ then $P\ (A \cup B) = P\ (A) + P\ (B)$

Cor I. If $A_1, A_2, A_3 \ldots A_k$ are mutually exclusive events, then

$P(A_1 \cup A_2 \cup A_3 \ldots \cup A_k) = P\ (A_1) + (A_2) + \ldots + P(A_k)$

II. If A and B are two events associated with a random experiment then,

$P(\ A \text{ or } B) = P(A) + P(B) - P\ (A \text{ and } B)$ or

In the language of the theory of sets it can be written as

$P\ (A \cup B) = P(A) + P(B) - P\ (A \cap B)$

III. For every event A associated with random experiment, we have

$P\ (\text{not } A) = 1 - P\ (A)$

or $\quad P(\overline{A}) = 1 - P\ (A)$

IV. If the event A implies the event, B, then

$$P\ (A) \leq P\ (B)$$

V. For any two events A and B

$$P(\overline{A} \cap B) = P\ (B) - P\ (\ A \cap B)$$

VI. If $B \subset A$, then

$$P(A \cap \overline{B}) = P(A) - P(B)$$

VII. If A and B are any two events (subsets of sample space S) and are disjoint, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability: Let A and B be two events associated with sample space S. Then the probability of occurrence of the event A when B has occurred is called the conditional probability of A and is denoted by P (A/B)

$\therefore$

$$P(A / B) = \frac{\text{Number of cases favourable to both A and B}}{\text{Numbe of cases in which B happens}}$$

$$= = \frac{n(A \cap B)}{n(B)}$$

Multiplicative Rule of Probability or Theorem of Compound Probability: The probability of the simultaneous occurrences of two dependent events is given by the product of the unconditional probability of one of them multiplied by the conditional probability of the other asssuming that the first has already occurred. Symbolically for two events

$$P(A \cap B) = P(B/A).\ P(A),\ P(A) \neq 0$$
$$P(A \cap B) = P(A/B).\ P(B),\ P(B) \neq 0$$

Multiplicative Rule for Independent Events: The probabilityof simultaneous occurrences of two independent events is equal to the product of their individual probabilities *i.e.*, for two independent events A and B,

$P(A \cap B) = P(A).\ P(B)$; it can also be written as

$P(AB) = P(A)\ .\ P(B)$

Baye's Theorem: If $E_1, E_2, E_3 \dots E_n$ form a set of mutually exclusive and exhaustive events and A is any other event, then

$$P(E_i / A) = \frac{P(E_i)P(A / E_i)}{P(E_1)P(A / E_1) + P(E_2)P(A / E_2)\dots + P(E_n)P(A / E_n)}$$

Random Variable: A random variable is a real valued function defined over the sample space of an experiment i.e., a variable whose value is a number determined by the sample point (outcome of the experiment) of a sample space is called a random variable. A random variable is denoted by X, Y, Z ... etc.

For example, let X be a random variable which is the number of heads obtained in two independent tosses of an unbiased coin.

$S = \{HH, HT, TH, TT\}$

Then $X(HH) = 2$, $X(HT) = 1$, $X(TH) = 1$, $X(TT) = 0$

$\therefore$ X can take values 0, 1, 2

Mean of Random Variable:

Mean $\mu = \frac{\Sigma p_i x_i}{\Sigma p_i}$

Variance of Random Variable:

$$\sigma^2 = \Sigma p_i x_i^2 - (p_i x_i)^2$$

Discrete and Continuous Random Variables: If the random variable X assumes only a finite or countably infinite set of values it is called *discrete random variable*. For example, marks obtained by students in an examination, the number of students in a school, the number of effective bulbs etc., are all discrete random variable.

But if the random variable X can assume infinite and uncountable set of values it is said to be *a continuous random variable*. For example, the age, height or weight of a group of students are all continuous random variable.

Binomial Distribution: Binomial distribution is a probability distribution which is obtained when the probability p of the happening of an event is same in all the trials. For example the probability of getting head, when the coin is tossed a number of times must remain same in each toss *i.e.* $\frac{1}{2}$.

$$P(r) = {}^nC_r p^r q^{n-r}$$

Here, n is the trials, p is the probability of success and q be the probability of failure.

Mean of binomial distribution. $(\mu) = np$

Variance of binomial distribution $= (\sigma^2) = npq$

Properties of binomial distribution:

1. The binomial distribution has its

   Mean $= np$, Variance $= npq$ or S.D. $= \sqrt{npq}$

   $\mu_1' = np, \mu_2' = n^2p^2 - np^2 + np, \mu_2 = \mu_2' - (\mu_1')^2$.

2. Binomial distribution is symmetrical if

   $p = q = \frac{1}{2}$

3. The shape and location of a Binomial distribution changes as p changes for a given n or as n changes for a given p.
4. Binomial distribution can be represented graphically.