

Class 11

Important Formulas

Binomial Theorem

1. (Binomial theorem) If x and a are real numbers, then for all $n \in N$, we have

$$(x+a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + \dots + {}^nC_r x^{n-r} a^r + \dots + {}^nC_{n-1} x^1 a^{n-1} + {}^nC_n x^0 a^n$$

$$\text{i.e., } (x+a)^n = \sum_{r=0}^n {}^nC_r x^{n-r} a^r$$

This expansion has the following properties:

- (i) It has $(n+1)$ terms.
- (ii) The sum of the indices of x and a in each term is n .
- (iii) The coefficients of terms equidistant from the beginning and the end are equal.
- (vi) General term is given by $T_{r+1} = {}^nC_r x^{n-r} a^r$

$$(v) (x+a)^n = \sum_{r=0}^n {}^nC_r x^{n-r} a^r \text{ can also be expressed as } (x+a)^n = \sum_{r+s=n} \frac{n!}{r!s!} x^r a^s$$

(vi) Replacing a by $-a$ in the expansion of $(x+a)^n$, we get

$$(x-a)^n = {}^nC_0 x^n a^0 - {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 - {}^nC_3 x^{n-3} a^3 + \dots + (-1)^r {}^nC_r x^{n-r} a^r + \dots + (-1)^n {}^nC_n x^0 a^n$$

The general term in the expansion of $(x-a)^n$ is $T_{r+1} = (-1)^r {}^nC_r x^{n-r} a^r$

(vii) Putting $x = 1$ and replacing a by x in the expansion of $(x+a)^n$, we get

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n = \sum_{r=0}^n {}^nC_r x^r$$

This is expansion of $(1+x)^n$ in ascending powers of x . In this case, $T_{r+1} = {}^nC_r x^r$

(viii) Putting $a = 1$ in the expansion of $(x+a)^n$, we get

$$(1+x)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} + {}^nC_2 x^{n-2} + \dots + {}^nC_n x^0 = \sum_{r=0}^n {}^nC_r x^{n-r}$$

This is the expansion of $(1+x)^n$ in descending powers of x . In this case,

$$T_{r+1} = {}^nC_r x^{n-r}$$

$$(ix) (x+a)^n + (x-a)^n = 2 \left\{ {}^nC_0 x^n a^0 + {}^nC_2 x^{n-2} a^2 + \dots \right\}$$

$$= 2 \{ \text{Sum of the odd terms in the expansion of } (x+a)^n \}$$

$$(x+a)^n - (x-a)^n = 2 \left\{ {}^nC_1 x^{n-1} a^1 + {}^nC_3 x^{n-3} a^3 + \dots \right\}$$

$$= 2 \{ \text{Sum of the even terms in the expansion of } (x+a)^n \}$$

If n is odd, then $\{(x+a)^n + (x-a)^n\}$ and $\{(x+a)^n - (x-a)^n\}$ both have $\left(\frac{n+1}{2}\right)$ terms.

If n is even, then $\{(x+a)^n + (x-a)^n\}$ has $\left(\frac{n}{2} + 1\right)$ terms whereas $\{(x+a)^n - (x-a)^n\}$ has $\left(\frac{n}{2}\right)$ terms.

(x) If O and E denote respectively the sums of odd terms and even terms in the expansion of $(x+a)^n$, then

$$(a) (x+a)^n = O + E \text{ and } (x-a)^n = O - E \quad (b) (x^2 - a^2)^n = O^2 - E^2$$

$$(c) 4OE = (x-a)^{2n} - (x+a)^{2n} \quad (d) (x+a)^{2n} + (x-a)^{2n} = 2(O^2 + E^2)$$

(xi) If n is even, then $\left(\frac{n}{2} + 1\right)^{\text{th}}$ term is the middle term.

If n is odd, then $\left(\frac{n+1}{2}\right)$ and $\left(\frac{n+3}{2}\right)$ are middle terms.