Class 11

Important Formulas

Limits and Derivatives

Limits:

1. $\lim_{x \to a} f(x)$ exists $\Leftrightarrow \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

2. For a function $f(x)$ and a real number a, $\lim_{x \to a} f(x)$ and $f(a)$ may not be same. In fact:
   (i) $\lim_{x \to a} f(x)$ exists but $f(a)$ (the value of $f(x)$ at $x = a$) may not exists
   (ii) The value $f(a)$ exists but $\lim_{x \to a} f(x)$ does not exist
   (iii) $\lim_{x \to a} f(x)$ and $f(a)$ both exist but are unequal
   (iv) $\lim_{x \to a} f(x)$ and $f(a)$ both exist and are equal.

3. Let $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$. If l and m both exist, then
   (i) $\lim_{x \to a} k\, f(x) = k \lim_{x \to a} f(x)$
   (ii) $\lim_{x \to a} (f \pm g)(x) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = l + m$
   (iii) $\lim_{x \to a} (fg)(x) = \lim_{x \to a} f(x) \lim_{x \to a} g(x) = lm$
   (iv) $\lim_{x \to a} \left(\dfrac{f}{g}\right)(x) = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \dfrac{l}{m}$
   (v) $\lim_{x \to a} \{f(a)\}^{g(x)} = l^m$

4. Following are some standard limits:
   (i) $\lim_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$
   (ii) $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$
   (iii) $\lim_{x \to 0} \dfrac{\tan x}{x} = 1$
   (iv) $\lim_{x \to a} \dfrac{\sin(x - a)}{x - a} = 1$
   (v) $\lim_{x \to a} \dfrac{\tan(x - a)}{x - a} = 1$
   (vi) $\lim_{x \to 0} \dfrac{\log(1 + x)}{x} = 1$
   (vii) $\lim_{x \to 0} \dfrac{a^x - 1}{x} = \log_e a,\ a \neq 0,\ a > 1$
   (viii) $\lim_{x \to 0} \dfrac{e^x - 1}{x} = 1$

Derivatives:

1. A function $f(x)$ is differentiable at $x = c$ iff $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists finitely.

   This limit is called the derivative or differentiation of $f(x)$ at $x = c$ and is denoted by $f'(c)$.

2. Geometrically the derivative of a function $f(x)$ at a point $x = c$ is the slope of the tangent to the curve $y = f(x)$ at the point $(c, f(c))$.

3. If $f(x)$ is a differentiable function, then $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ is called the differentiation of $f(x)$ or differentiation of $f(x)$ with respect to x.

4. Mechanically, $\frac{d}{dx}(f(x))$ measures the rate of change of $f(x)$ with respect to x.

5. Following are some standard derivatives:

   (i) $\frac{d}{dx}(x^n) = n\,x^{n-1}$

   (ii) $\frac{d}{dx}(a^x) = a^x \log_e a,\ a > 0,\ a \neq 1$

   (iii) $\frac{d}{dx}(e^x) = e^x$

   (iv) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

   (v) $\frac{d}{dx}(\sin x) = \cos x$

   (vi) $\frac{d}{dx}(\cos x) = -\sin x$

   (vii) $\frac{d}{dx}(\tan x) = \sec^2 x$

   (viii) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

   (ix) $\frac{d}{dx}(\sec x) = \sec x \tan x$

   (x) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

6. Following are the fundamental rules for differentiation:

   (i) Differentiation of a constant function is zero i.e., $\frac{d}{dx}(c) = 0$

   (ii) Differentiation of a constant and a function is equal to constant times the differentiation of the function.

   (iii) If $f(x)$ and $g(x)$ are differentiable functions, then

   (a) $\frac{d}{dx}\{f(x) \pm g(x)\} = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

   (b) $\frac{d}{dx}\{f(x) \times g(x)\} = g(x) \times \frac{d}{dx}(f(x)) + f(x) \times \frac{d}{dx}(g(x))$

   (c) $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x) \times \frac{d}{dx}(f(x)) - f(x) \times \frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$