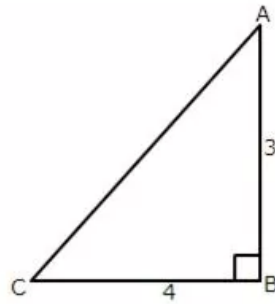


Chapter 22. Trigonometrical Ratios [Sine, Consine, Tangent of an Angle and their Reciprocals]

Exercise 22(A)

Solution 1:

Given angle $\angle ABC = 90^\circ$



$$\Rightarrow AC^2 = AB^2 + BC^2 \text{ (AC is hypotenuse)}$$

$$\Rightarrow AC^2 = 3^2 + 4^2$$

$$\therefore AC^2 = 9 + 16 = 25 \text{ and } AC = 5$$

(i)

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{4}{5}$$

(ii)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{3}{5}$$

(iii)

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \frac{3}{4}$$

(iv)

$$\sec C = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{BC} = \frac{5}{4}$$

(v)

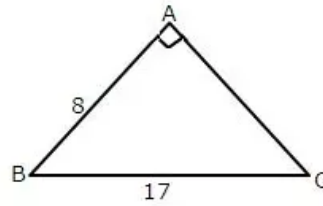
$$\operatorname{cosec} C = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{AB} = \frac{5}{3}$$

(vi)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{3}{4}$$

Solution 2:

Given angle $BAC = 90^\circ$



$$\Rightarrow BC^2 = AB^2 + AC^2 \text{ (BC is hypotenuse)}$$

$$\Rightarrow 17^2 = 8^2 + AC^2$$

$$\therefore AC^2 = 289 - 64 = 225 \text{ and } AC = 15$$

(i)

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

(ii)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{AC} = \frac{8}{15}$$

(iii)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

$$\begin{aligned} \sin^2 B + \cos^2 B &= \left(\frac{15}{17}\right)^2 + \left(\frac{8}{17}\right)^2 \\ &= \frac{225 + 64}{289} \\ &= \frac{289}{289} \\ &= 1 \end{aligned}$$

(iv)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

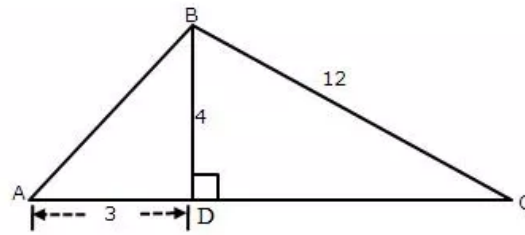
$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{8}{17}$$

$$\cos C = \frac{\text{base}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{15}{17}$$

$$\begin{aligned} \sin B \cdot \cos C + \cos B \cdot \sin C &= \frac{15}{17} \cdot \frac{15}{17} + \frac{8}{17} \cdot \frac{8}{17} \\ &= \frac{225 + 64}{289} \\ &= \frac{289}{289} \\ &= 1 \end{aligned}$$

Solution 3:

Consider the diagram as



Given angle $ADB = 90^\circ$ and $BDC = 90^\circ$

$$\Rightarrow AB^2 = AD^2 + BD^2 \text{ (} AB \text{ is hypotenuse in } \triangle ABD \text{)}$$

$$\Rightarrow AB^2 = 3^2 + 4^2$$

$$\therefore AB^2 = 9 + 16 = 25 \text{ and } AB = 5$$

$$\Rightarrow BC^2 = BD^2 + DC^2 \text{ (} BC \text{ is hypotenuse in } \triangle BDC \text{)}$$

$$\Rightarrow DC^2 = 12^2 - 4^2$$

$$\therefore DC^2 = 144 - 16 = 128 \text{ and } DC = 8\sqrt{2}$$

(i)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{3}{5}$$

(ii)

$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AB}{BD} = \frac{5}{4}$$

(iii)

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{BD}{AD} = \frac{4}{3}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AB}{AD} = \frac{5}{3}$$

$$\begin{aligned} \tan^2 A - \sec^2 A &= \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2 \\ &= \frac{16}{9} - \frac{25}{9} \\ &= \frac{-9}{9} \\ &= -1 \end{aligned}$$

(iv)

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BD}{BC} = \frac{4}{12} = \frac{1}{3}$$

(v)

$$\sec C = \frac{\text{hypotenuse}}{\text{base}} = \frac{BC}{DC} = \frac{12}{8\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

(vi)

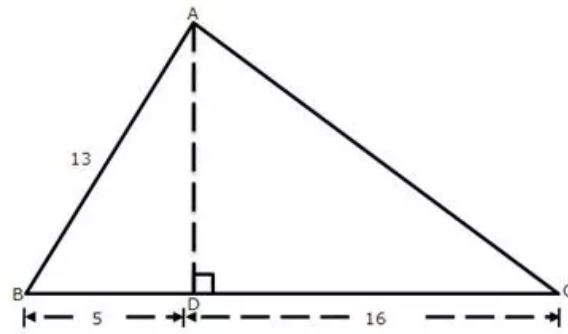
$$\cot C = \frac{\text{base}}{\text{perpendicular}} = \frac{DC}{BD} = \frac{8\sqrt{2}}{4} = 2\sqrt{2}$$

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BD}{BC} = \frac{4}{12} = \frac{1}{3}$$

$$\begin{aligned} \cot^2 C - \frac{1}{\sin^2 C} &= (2\sqrt{2})^2 - \frac{1}{\left(\frac{1}{3}\right)^2} \\ &= 8 - 9 \\ &= -1 \end{aligned}$$

Solution 4:

Given angle $\angle ADB = 90^\circ$ and $\angle ADC = 90^\circ$



$$\Rightarrow AB^2 = AD^2 + BD^2 \text{ (} AB \text{ is hypotenuse in } \triangle ABD \text{)}$$

$$\Rightarrow 13^2 = AD^2 + 5^2$$

$$\therefore AD^2 = 169 - 25 = 144 \text{ and } AD = 12$$

$$\Rightarrow AC^2 = AD^2 + DC^2 \text{ (} AC \text{ is hypotenuse in } \triangle ADC \text{)}$$

$$\Rightarrow AC^2 = 12^2 + 16^2$$

$$\therefore AC^2 = 144 + 256 = 400 \text{ and } AC = 20$$

(i)

$$\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{12}{13}$$

(ii)

$$\tan C = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{DC} = \frac{12}{16} = \frac{3}{4}$$

(iii)

$$\sec B = \frac{\text{hypotenuse}}{\text{base}} = \frac{AB}{BD} = \frac{13}{5}$$

$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{12}{5}$$

$$\begin{aligned} \sec^2 B - \tan^2 B &= \left(\frac{13}{5}\right)^2 - \left(\frac{12}{5}\right)^2 \\ &= \frac{169 - 144}{25} \\ &= \frac{25}{25} \\ &= 1 \end{aligned}$$

(iv)

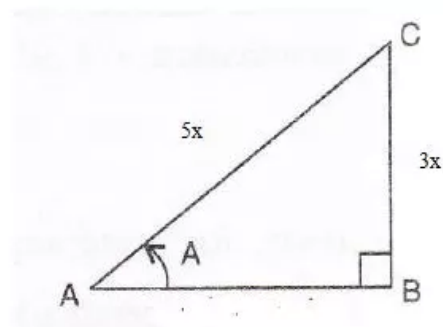
$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AC} = \frac{12}{20} = \frac{3}{5}$$

$$\cos C = \frac{\text{base}}{\text{hypotenuse}} = \frac{DC}{AC} = \frac{16}{20} = \frac{4}{5}$$

$$\begin{aligned} \sin^2 C + \cos^2 C &= \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 \\ &= \frac{9 + 16}{25} \\ &= \frac{25}{25} \\ &= 1 \end{aligned}$$

Solution 5:

Consider the diagram below:



$$\sin A = \frac{3}{5}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3}{5} \Rightarrow \frac{BC}{AC} = \frac{3}{5}$$

Therefore if length of $BC = 3x$, length of $AC = 5x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$AB^2 + (3x)^2 = (5x)^2$$

$$AB^2 = 25x^2 - 9x^2 = 16x^2$$

$$\therefore AB = 4x \text{ (base)}$$

Now

(i)

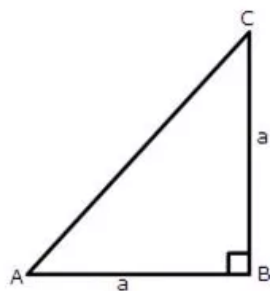
$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{3x}{4x} = \frac{3}{4}$$

(ii)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$$

Solution 6:

Given angle $\angle ABC = 90^\circ$ in the figure



$$\Rightarrow AC^2 = AB^2 + BC^2 \text{ (} AC \text{ is hypotenuse in } \triangle ABC \text{)}$$

$$\Rightarrow AC^2 = a^2 + a^2$$

$$\therefore AC^2 = 2a^2 \text{ and } AC = \sqrt{2}a$$

Now

$$(i) \sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$(ii) \sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{\sqrt{2}a}{a} = \sqrt{2}$$

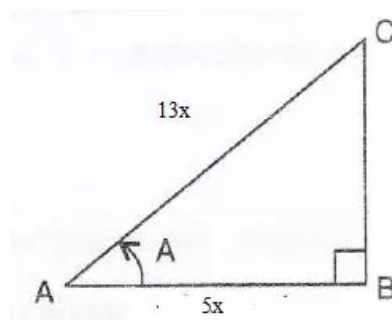
$$(iii) \sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\begin{aligned} \cos^2 A + \sin^2 A &= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= \frac{1}{2} + \frac{1}{2} \\ &= 1 \end{aligned}$$

Solution 7:

Consider the diagram below:



$$\cos A = \frac{5}{13}$$

$$\text{i.e. } \frac{\text{base}}{\text{hypotenuse}} = \frac{5}{13} \Rightarrow \frac{AB}{AC} = \frac{5}{13}$$

Therefore if length of $AB = 5x$, length of $AC = 13x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(5x)^2 + BC^2 = (13x)^2$$

$$BC^2 = 169x^2 - 25x^2 = 144x^2$$

$$\therefore BC = 12x \text{ (perpendicular)}$$

Now

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{12x}{5x} = \frac{12}{5}$$

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{12x}{13x} = \frac{12}{13}$$

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{5x}{12x} = \frac{5}{12}$$

(i)

$$\frac{\sin A - \cot A}{2 \tan A}$$

$$= \frac{\frac{12}{13} - \frac{5}{12}}{2 \left(\frac{12}{5} \right)}$$

$$= \frac{79}{156} \cdot \frac{5}{24}$$

$$= \frac{395}{3744}$$

(ii)

$$\cot A + \frac{1}{\cos A}$$

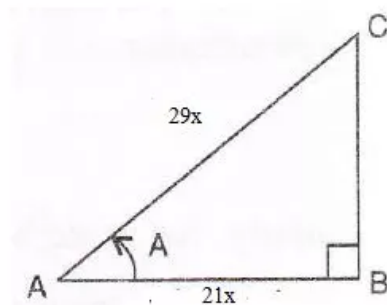
$$= \frac{5}{12} + \frac{1}{\frac{13}{13}}$$

$$= \frac{5}{12} + \frac{13}{5}$$

$$= \frac{181}{60}$$

Solution 8:

Consider the diagram below:



$$\sec A = \frac{29}{21}$$

$$\text{i.e. } \frac{\text{hypotenuse}}{\text{base}} = \frac{29}{21} \Rightarrow \frac{AC}{AB} = \frac{29}{21}$$

Therefore if length of $AB = 21x$, length of $AC = 29x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(21x)^2 + BC^2 = (29x)^2$$

$$BC^2 = 841x^2 - 441x^2 = 400x^2$$

$$\therefore BC = 20x \text{ (perpendicular)}$$

Now

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{20x}{29x} = \frac{20}{29}$$

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{20x}{21x} = \frac{20}{21}$$

Therefore

$$\sin A - \frac{1}{\tan A}$$

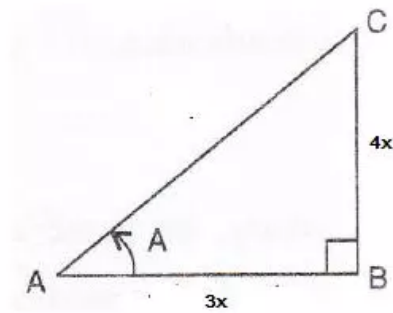
$$= \frac{20}{29} - \frac{1}{\frac{20}{21}}$$

$$= \frac{20}{29} - \frac{21}{20}$$

$$= -\frac{209}{580}$$

Solution 9:

Consider the diagram below:



$$\tan A = \frac{4}{3}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{base}} = \frac{4}{3} \Rightarrow \frac{BC}{AB} = \frac{4}{3}$$

Therefore if length of $AB = 3x$, length of $BC = 4x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (4x)^2 = AC^2$$

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

$$\therefore AC = 5x (\text{hypotenuse})$$

Now

$$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$$

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC} = \frac{3x}{4x} = \frac{3}{4}$$

$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$$

Therefore

$$\frac{\operatorname{cosec} A}{\cot A - \sec A}$$

$$= \frac{\frac{5}{4}}{\frac{3}{4} - \frac{5}{3}}$$

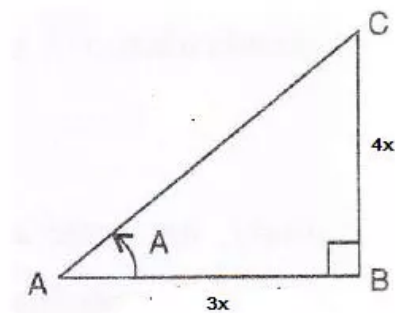
$$= \frac{\frac{5}{4}}{-\frac{11}{12}}$$

$$= -\frac{60}{44}$$

$$= -\frac{15}{11}$$

Solution 10:

Consider the diagram below:



$$4 \cot A = 3$$

$$\cot A = \frac{3}{4}$$

$$\text{i.e. } \frac{\text{base}}{\text{perpendicular}} = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$$

Therefore if length of $AB = 3x$, length of $BC = 4x$
Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (4x)^2 = AC^2$$

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

$$\therefore AC = 5x (\text{hypotenuse})$$

(i)

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$$

(ii)

$$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$$

(iii)

$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$$

$$\cot A = \frac{3}{4}$$

$$\operatorname{cosec}^2 A - \cot^2 A$$

$$= \left(\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2$$

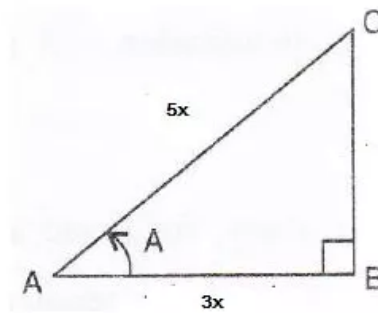
$$= \frac{25-9}{16}$$

$$= \frac{16}{16}$$

$$= 1$$

Solution 11:

Consider the diagram below:



$$\cos A = 0.6$$

$$\cos A = \frac{6}{10} = \frac{3}{5}$$

$$\text{i.e. } \frac{\text{base}}{\text{hypotenuse}} = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$$

Therefore if length of $AB = 3x$, length of $AC = 5x$
Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + BC^2 = (5x)^2$$

$$BC^2 = 25x^2 - 9x^2 = 16x^2$$

$$\therefore BC = 4x (\text{perpendicular})$$

Now all other trigonometric ratios are

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$$

$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC} = \frac{5x}{4x} = \frac{5}{4}$$

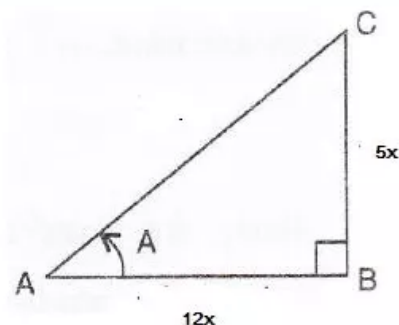
$$\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5x}{3x} = \frac{5}{3}$$

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{4x}{3x} = \frac{4}{3}$$

$$\cot A = \frac{\text{base}}{\text{perpendicular}} = \frac{3x}{4x} = \frac{3}{4}$$

Solution 12:

Consider the diagram below:



$$\tan A = \frac{5}{12}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{base}} = \frac{5}{12} \Rightarrow \frac{BC}{AB} = \frac{5}{12}$$

Therefore if length of $AB = 12x$, length of $BC = 5x$
Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(12x)^2 + (5x)^2 = AC^2$$

$$AC^2 = 144x^2 + 25x^2 = 169x^2$$

$$\therefore AC = 13x (\text{hypotenuse})$$

(i)

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12x}{13x} = \frac{12}{13}$$

(ii)

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{5x}{13x} = \frac{5}{13}$$

(iii)

$$\frac{\cos A + \sin A}{\cos A - \sin A}$$

$$\frac{\frac{12}{13} + \frac{5}{13}}{\frac{12}{13} - \frac{5}{13}}$$

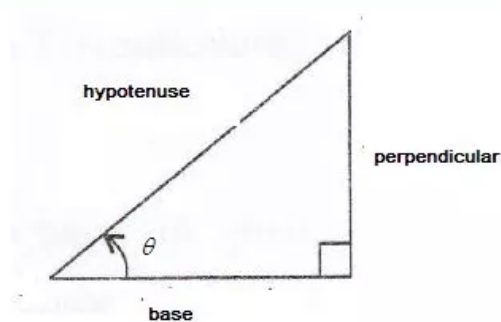
$$= \frac{17}{7}$$

$$= \frac{17}{7}$$

$$= 2\frac{3}{7}$$

Solution 13:

Consider the diagram below:



$$\sin \theta = \frac{p}{q}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{p}{q}$$

Therefore if length of perpendicular = px , length of hypotenuse = qx

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \text{ [Using Pythagoras Theorem]}$$

$$\text{base}^2 + (px)^2 = (qx)^2$$

$$\text{base}^2 = q^2x^2 - p^2x^2 = (q^2 - p^2)x^2$$

$$\therefore \text{base} = \sqrt{q^2 - p^2}x$$

Now

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{q^2 - p^2}}{q}$$

Therefore

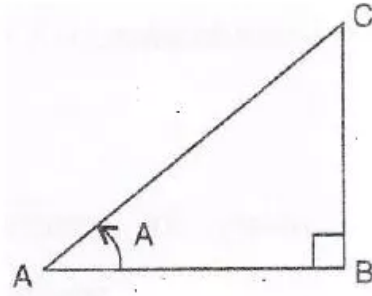
$$\cos \theta + \sin \theta$$

$$= \frac{\sqrt{q^2 - p^2}}{q} + \frac{p}{q}$$

$$= \frac{p + \sqrt{q^2 - p^2}}{q}$$

Solution 14:

Consider the diagram below:



$$\cos A = \frac{1}{2}$$

$$\text{i.e. } \frac{\text{base}}{\text{hypotenuse}} = \frac{1}{2} \Rightarrow \frac{AB}{AC} = \frac{1}{2}$$

Therefore if length of $AB = x$, length of $AC = 2x$

Since

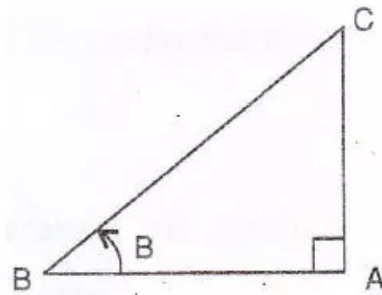
$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(x)^2 + BC^2 = (2x)^2$$

$$BC^2 = 4x^2 - x^2 = 3x^2$$

$$\therefore BC = \sqrt{3}x \text{ (perpendicular)}$$

Consider the diagram below:



$$\sin B = \frac{1}{\sqrt{2}}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{AC}{BC} = \frac{1}{\sqrt{2}}$$

Therefore if length of $AC = x$, length of $BC = \sqrt{2}x$

Since

$$AB^2 + AC^2 = BC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$AB^2 + x^2 = (\sqrt{2}x)^2$$

$$AB^2 = 2x^2 - x^2 = x^2$$

$$\therefore AB = x \text{ (base)}$$

Now

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{x} = 1$$

Therefore

$$\frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$

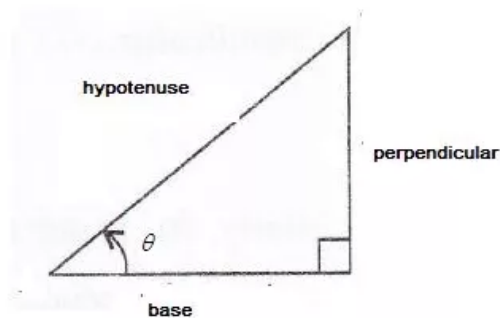
$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

Solution 15:

Consider the diagram below:



$$5 \cot \theta = 12$$

$$\cot \theta = \frac{12}{5}$$

$$\text{i.e. } \frac{\text{base}}{\text{perpendicular}} = \frac{12}{5}$$

Therefore if length of base = $12x$, length of perpendicular = $5x$

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \text{ [Using Pythagoras Theorem]}$$

$$(12x)^2 + (5x)^2 = \text{hypotenuse}^2$$

$$\text{hypotenuse}^2 = 144x^2 + 25x^2 = 169x^2$$

$$\therefore \text{hypotenuse} = 13x$$

Now

$$\operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{13x}{5x} = \frac{13}{5}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{13x}{12x} = \frac{13}{12}$$

Therefore

$$\operatorname{cosec} \theta + \sec \theta$$

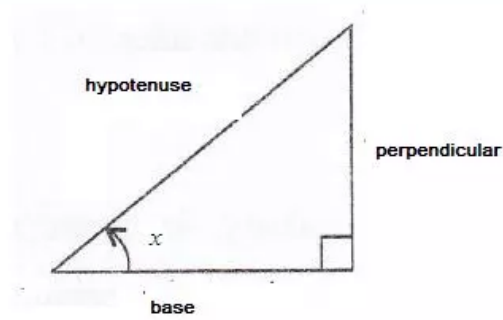
$$= \frac{13}{5} + \frac{13}{12}$$

$$= \frac{221}{60}$$

$$= 3\frac{41}{60}$$

Solution 16:

Consider the diagram below:



$$\tan x = 1\frac{1}{3}$$

$$\tan x = \frac{4}{3}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{base}} = \frac{4}{3}$$

Therefore if length of base = $3x$, length of perpendicular = $4x$

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (4x)^2 = \text{hypotenuse}^2$$

$$\text{hypotenuse}^2 = 9x^2 + 16x^2 = 25x^2$$

$$\therefore \text{hypotenuse} = 5x$$

Now

$$\sin x = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{4x}{5x} = \frac{4}{5}$$

$$\cos x = \frac{\text{base}}{\text{hypotenuse}} = \frac{3x}{5x} = \frac{3}{5}$$

Therefore

$$4 \sin^2 x - 3 \cos^2 x + 2$$

$$= 4 \left(\frac{4}{5} \right)^2 - 3 \left(\frac{3}{5} \right)^2 + 2$$

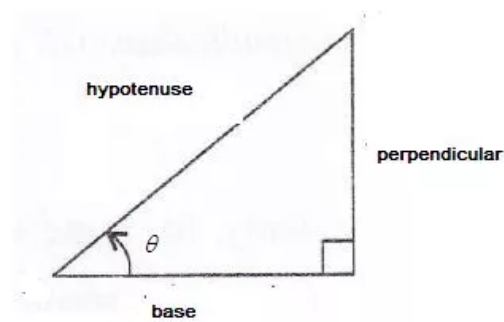
$$= \frac{64}{25} - \frac{27}{25} + 2$$

$$= \frac{87}{25}$$

$$= 3\frac{12}{25}$$

Solution 17:

Consider the diagram below:



$$\operatorname{cosec} \theta = \sqrt{5}$$

$$\text{i.e. } \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{\sqrt{5}}{1}$$

Therefore if length of hypotenuse $= \sqrt{5}x$, length of perpendicular $= x$

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \text{ [Using Pythagoras Theorem]}$$

$$\text{base}^2 + (x)^2 = (\sqrt{5}x)^2$$

$$\text{base}^2 = 5x^2 - x^2 = 4x^2$$

$$\therefore \text{base} = 2x$$

Now

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{x}{\sqrt{5}x} = \frac{1}{\sqrt{5}}$$

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{2x}{\sqrt{5}x} = \frac{2}{\sqrt{5}}$$

(i)

$$2 - \sin^2 \theta - \cos^2 \theta$$

$$= 2 - \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2$$

$$= 2 - \frac{1}{5} - \frac{4}{5}$$

$$= \frac{5}{5}$$

$$= 1$$

(ii)

$$2 + \frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta}$$

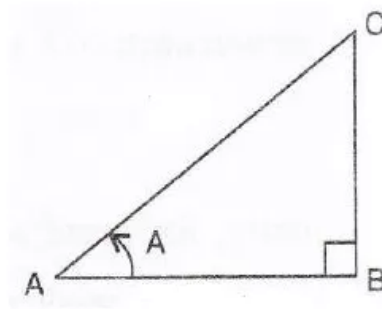
$$= 2 + \frac{1}{\left(\frac{1}{\sqrt{5}}\right)^2} - \frac{\left(\frac{2}{\sqrt{5}}\right)^2}{\left(\frac{1}{\sqrt{5}}\right)^2}$$

$$= 2 + 5 - 4$$

$$= 3$$

Solution 18:

Consider the diagram below:



$$\sec A = \sqrt{2}$$

$$\text{i.e. } \frac{\text{hypotenuse}}{\text{base}} = \frac{\sqrt{2}}{1} \Rightarrow \frac{AC}{AB} = \frac{\sqrt{2}}{1}$$

Therefore if length of $AB = x$, length of $AC = \sqrt{2}x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(x)^2 + BC^2 = (\sqrt{2}x)^2$$

$$BC^2 = 2x^2 - x^2 = x^2$$

$$\therefore BC = x \text{ (perpendicular)}$$

Now

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{x} = 1$$

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

Therefore

$$\frac{3\cos^2 A + 5\tan^2 A}{4\tan^2 A - \sin^2 A}$$

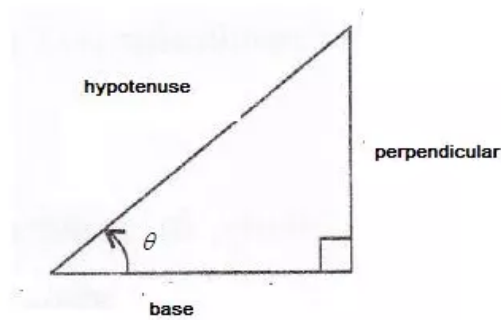
$$= \frac{3\left(\frac{1}{\sqrt{2}}\right)^2 + 5(1)^2}{4(1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{13}{2}$$

$$= 1\frac{6}{7}$$

Solution 19:

Consider the diagram below:



$$\cot \theta = 1$$

$$\text{i.e. } \frac{\text{base}}{\text{perpendicular}} = \frac{1}{1}$$

Therefore if length of base = x , length of perpendicular = x

Since

$$\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2 \text{ [Using Pythagoras Theorem]}$$

$$(x)^2 + (x)^2 = \text{hypotenuse}^2$$

$$\text{hypotenuse}^2 = x^2 + x^2 = 2x^2$$

$$\therefore \text{hypotenuse} = \sqrt{2}x$$

Now

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{x} = 1$$

Therefore

$$5 \tan^2 \theta + 2 \sin^2 \theta - 3$$

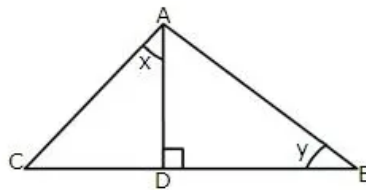
$$= 5(1)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 - 3$$

$$= 5 + 1 - 3$$

$$= 3$$

Solution 20:

Given angle $\angle DAC = 90^\circ$ and $\angle ADB = 90^\circ$ in the figure



$$\Rightarrow AC^2 = AD^2 + DC^2 \text{ (AC is hypotenuse in } \triangle ADC \text{)}$$

$$\Rightarrow AD^2 = 26^2 - 10^2$$

$$\therefore AD^2 = 576 \text{ and } AD = 24$$

Again

$$\Rightarrow AB^2 = AD^2 + BD^2 \text{ (AB is hypotenuse in } \triangle ABD \text{)}$$

$$\Rightarrow AB^2 = 24^2 + 32^2$$

$$\therefore AB^2 = 1600 \text{ and } AB = 40$$

Now

$$(i) \quad \cot x = \frac{\text{base}}{\text{perpendicular}} = \frac{AD}{CD} = \frac{24}{10} = 2.4$$

$$(ii) \quad \sin y = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{24}{40} = \frac{3}{5}$$

$$\tan y = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{24}{32} = \frac{3}{4}$$

Therefore

$$\begin{aligned}& \frac{1}{\sin^2 y} - \frac{1}{\tan^2 y} \\&= \frac{1}{\left(\frac{3}{5}\right)^2} - \frac{1}{\left(\frac{3}{4}\right)^2} \\&= \frac{25}{9} - \frac{16}{9} \\&= \frac{9}{9} \\&= 1\end{aligned}$$

(iii)

$$\begin{aligned}\tan y &= \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{24}{32} = \frac{3}{4} \\ \cos x &= \frac{\text{base}}{\text{hypotenuse}} = \frac{AD}{AC} = \frac{24}{26} = \frac{12}{13} \\ \cos y &= \frac{\text{base}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{32}{40} = \frac{4}{5}\end{aligned}$$

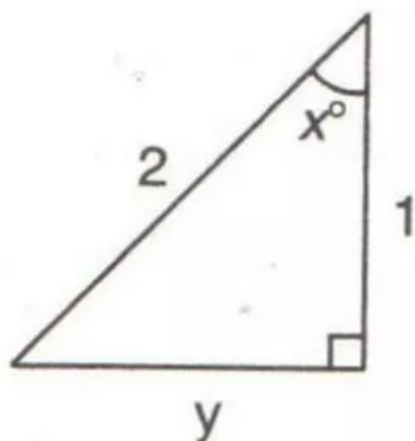
Therefore

$$\begin{aligned}& \frac{6}{\cos x} - \frac{5}{\cos y} + 8 \tan y \\&= \frac{6}{\frac{12}{13}} - \frac{5}{\frac{4}{5}} + 8 \left(\frac{3}{4} \right) \\&= \frac{13}{2} - \frac{25}{4} + 6 \\&= \frac{26 - 25 + 24}{4} \\&= \frac{25}{4} \\&= 6\frac{1}{4}\end{aligned}$$

Exercise 22(B)

Solution 1:

Consider the given figure



(i)

Since the triangle is a right angled triangle, so using Pythagorean Theorem

$$2^2 = y^2 + 1^2$$

$$y^2 = 4 - 1 = 3$$

$$y = \sqrt{3}$$

(ii)

$$\sin x^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$

(iii)

$$\tan x^\circ = \frac{\text{perpendicular}}{\text{base}} = \sqrt{3}$$

$$\sec x^\circ = \frac{\text{hypotenuse}}{\text{base}} = 2$$

Therefore

$$(\sec x^\circ - \tan x^\circ)(\sec x^\circ + \tan x^\circ)$$

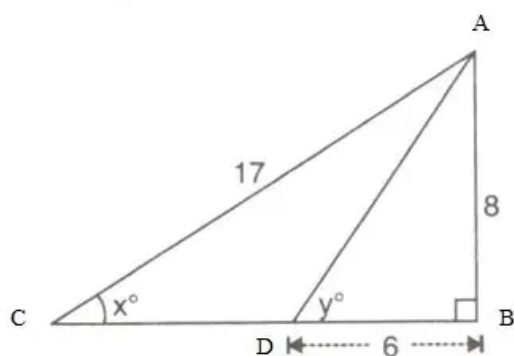
$$= (2 - \sqrt{3})(2 + \sqrt{3})$$

$$= 4 - 3$$

$$= 1$$

Solution 2:

Consider the given figure



Since the triangle is a right angled triangle, so using Pythagorean Theorem

$$AD^2 = 8^2 + 6^2$$

$$AD^2 = 64 + 36 = 100$$

$$AD = 10$$

Also

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 17^2 - 8^2 = 225$$

$$BC = 15$$

(i)

$$\sin x^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{8}{17}$$

(ii)

$$\cos y^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$

(iii)

$$\sin y^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AD} = \frac{8}{10} = \frac{4}{5}$$

$$\cos y^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$

$$\tan x^\circ = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{8}{15}$$

Therefore

$$3 \tan x^\circ - 2 \sin y^\circ + 4 \cos y^\circ$$

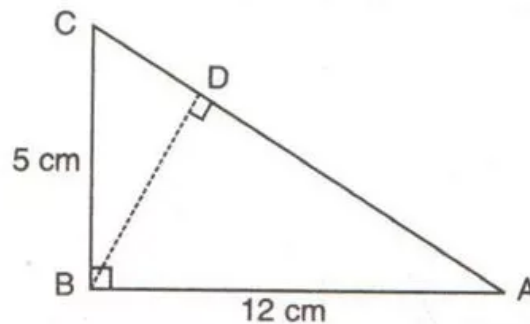
$$= 3 \left(\frac{8}{15} \right) - 2 \left(\frac{4}{5} \right) + 4 \left(\frac{3}{5} \right)$$

$$= \frac{8}{5} - \frac{8}{5} + \frac{12}{5}$$

$$= 2 \frac{2}{5}$$

Solution 3:

Consider the given figure



Since the triangle is a right angled triangle, so using Pythagorean Theorem

$$AC^2 = 5^2 + 12^2$$

$$AC^2 = 25 + 144 = 169$$

$$AC = 13$$

In $\triangle CBD$ and $\triangle CBA$, the $\angle C$ is common to both the triangles, $\angle CDB = \angle CBA = 90^\circ$ so therefore $\angle CBD = \angle CAB$.

Therefore $\triangle CBD$ and $\triangle CBA$ are similar triangles according to AAA Rule

So

$$\frac{AC}{BC} = \frac{AB}{BD}$$

$$\frac{13}{5} = \frac{12}{BD}$$

$$BD = \frac{60}{13}$$

(i)

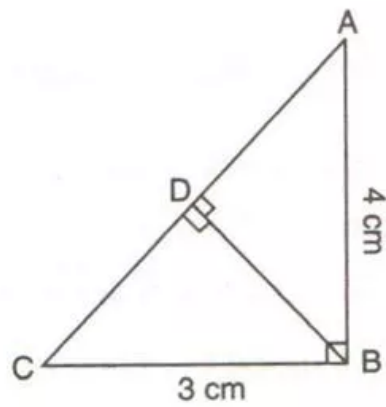
$$\cos \angle DBC = \frac{\text{base}}{\text{hypotenuse}} = \frac{BD}{BC} = \frac{\frac{60}{13}}{5} = \frac{12}{13}$$

(ii)

$$\cot \angle DBA = \frac{\text{base}}{\text{perpendicular}} = \frac{BD}{AB} = \frac{\frac{60}{13}}{12} = \frac{5}{13}$$

Solution 4:

Consider the given figure



Since the triangle is a right angled triangle, so using Pythagorean Theorem

$$AC^2 = 4^2 + 3^2$$

$$AC^2 = 16 + 9 = 25$$

$$AC = 5$$

In $\triangle CBD$ and $\triangle CBA$, the $\angle C$ is common to both the triangles, $\angle CDB = \angle CBA = 90^\circ$ so therefore $\angle CBD = \angle CAB$.

Therefore $\triangle CBD$ and $\triangle CBA$ are similar triangles according to AAA Rule

So

$$\frac{AC}{BC} = \frac{AB}{BD}$$

$$\frac{5}{3} = \frac{4}{BD}$$

$$BD = \frac{12}{5}$$

Now using Pythagorean Theorem

$$DC^2 = 3^2 - \left(\frac{12}{5}\right)^2$$

$$DC^2 = 9 - \frac{144}{25} = \frac{81}{25}$$

$$DC = \frac{9}{5}$$

Therefore

$$AD = AC - DC$$

$$= 5 - \frac{9}{5}$$

$$= \frac{16}{5}$$

(i)

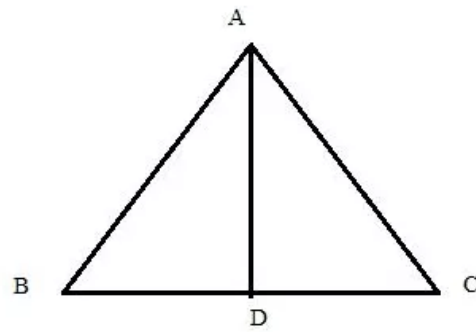
$$\tan \angle DBC = \frac{\text{perpendicular}}{\text{base}} = \frac{DC}{BD} = \frac{\frac{9}{5}}{\frac{12}{5}} = \frac{3}{4}$$

(ii)

$$\sin \angle DBA = \frac{AD}{AB} = \frac{\frac{16}{5}}{4} = \frac{4}{5}$$

Solution 5:

Consider the figure below

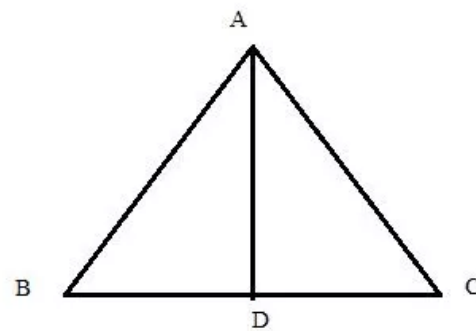


In the isosceles $\triangle ABC$, $AB = AC = 15\text{ cm}$ and $BC = 18\text{ cm}$ the perpendicular drawn from angle A to the side BC divides the side BC into two equal parts $BD = DC = 9\text{ cm}$

$$\cos \angle ABC = \frac{\text{base}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{9}{15} = \frac{3}{5}$$

Solution 6:

Consider the figure below



In the isosceles $\triangle ABC$, $AB = AC = 5\text{ cm}$ and $BC = 8\text{ cm}$ the perpendicular drawn from angle A to the side BC divides the side BC into two equal parts $BD = DC = 4\text{ cm}$

Since $\angle ADB = 90^\circ$

$$\Rightarrow AB^2 = AD^2 + BD^2 \text{ (} AB \text{ is hypotenuse in } \triangle ABD \text{)}$$

$$\Rightarrow AD^2 = 5^2 - 4^2$$

$$\therefore AD^2 = 9 \text{ and } AD = 3$$

(i)

$$\sin B = \frac{AD}{AB} = \frac{3}{5}$$

(ii)

$$\tan C = \frac{AD}{DC} = \frac{3}{4}$$

(iii)

$$\sin B = \frac{AD}{AB} = \frac{3}{5}$$

$$\cos B = \frac{BD}{AB} = \frac{4}{5}$$

Therefore

$$\sin^2 B + \cos^2 B$$

$$= \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$$

$$= \frac{25}{25}$$

$$= 1$$

(iv)

$$\tan C = \frac{AD}{DC} = \frac{3}{4}$$

$$\cot B = \frac{BD}{AD} = \frac{4}{3}$$

Therefore

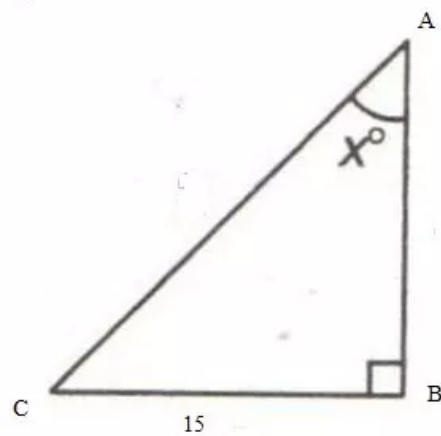
$$\tan C - \cot B$$

$$= \frac{3}{4} - \frac{4}{3}$$

$$= -\frac{7}{12}$$

Solution 7:

Consider the figure



$$\tan x^\circ = \frac{3}{4}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB} = \frac{3}{4}$$

Therefore if length of base = $4x$, length of perpendicular = $3x$

Since

$$BC^2 + AB^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (4x)^2 = AC^2$$

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

$$\therefore AC = 5x$$

Now

$$BC = 15$$

$$3x = 15$$

$$x = 5$$

Therefore

$$AB = 4x$$

$$= 4 \times 5$$

$$= 20 \text{ cm}$$

And

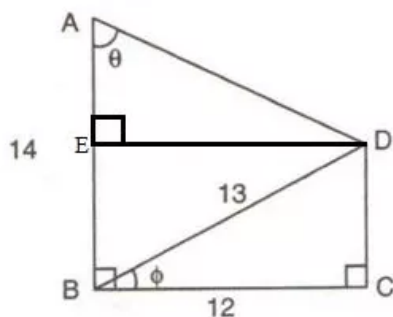
$$AC = 5x$$

$$= 5 \times 5$$

$$= 25 \text{ cm}$$

Solution 8:

Consider the figure



A perpendicular is drawn from D to the side AB at point E which makes BCDE is a rectangle.

Now in right angled triangle BCD using Pythagorean Theorem

$$\Rightarrow BD^2 = BC^2 + CD^2 \quad (BD \text{ is hypotenuse in } \triangle BCD)$$

$$\Rightarrow CD^2 = 13^2 - 12^2 = 25$$

$$\therefore CD = 5$$

Since BCDE is rectangle so ED = 12 cm, EB = 5 and AE = 14 - 5 = 9

(i)

$$\sin \phi = \frac{CD}{BD} = \frac{5}{13}$$

$$\tan \theta = \frac{ED}{AE} = \frac{12}{9} = \frac{4}{3}$$

(ii)

$$\sec \theta = \frac{AD}{AE}$$

$$\sec \theta = \frac{AD}{9}$$

$$AD = 9 \sec \theta$$

Or

$$\operatorname{cosec} \theta = \frac{AD}{ED}$$

$$\operatorname{cosec} \theta = \frac{AD}{12}$$

$$AD = 12 \operatorname{cosec} \theta$$

Solution 9:

Given

$$\sin B = \frac{4}{5}$$

$$i.e. \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{4}{5}$$

Therefore if length of perpendicular = 4x, length of hypotenuse = 5x

Since

$$BC^2 + AC^2 = AB^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(5x)^2 - (4x)^2 = BC^2$$

$$BC^2 = 9x^2$$

$$\therefore BC = 3x$$

Now

$$BC = 15$$

$$3x = 15$$

$$x = 5$$

(i)

$$AC = 4x$$

$$= 4 \times 5$$

$$= 20 \text{ cm}$$

And

$$AB = 5x$$

$$= 5 \times 5$$

$$= 25 \text{ cm}$$

(ii)

Given

$$\tan \angle ADC = \frac{1}{1}$$

$$i.e. \frac{\text{perpendicular}}{\text{base}} = \frac{AC}{CD} = \frac{1}{1}$$

Therefore if length of perpendicular = x, length of hypotenuse = x

Since

$$AC^2 + CD^2 = AD^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(x)^2 + (x)^2 = AD^2$$

$$AD^2 = 2x^2$$

$$\therefore AD = \sqrt{2}x$$

Now

$$AC = 20$$

$$x = 20$$

So

$$AD = \sqrt{2}x$$

$$= \sqrt{2} \times 20$$

$$= 20\sqrt{2} \text{ cm}$$

And

$$CD = 20 \text{ cm}$$

Now

$$\tan B = \frac{AC}{BC} = \frac{20}{15} = \frac{4}{3}$$

$$\cos B = \frac{BC}{AB} = \frac{15}{25} = \frac{3}{5}$$

So

$$\tan^2 B - \frac{1}{\cos^2 B}$$

$$= \left(\frac{4}{3}\right)^2 - \frac{1}{\left(\frac{3}{5}\right)^2}$$

$$= \frac{16}{9} - \frac{25}{9}$$

$$= -\frac{9}{9}$$

$$= -1$$

Solution 10:

$$\sin A + \operatorname{cosec} A = 2$$

Squaring both sides

$$(\sin A + \operatorname{cosec} A)^2 = 2^2$$

$$\sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \cdot \operatorname{cosec} A = 4$$

$$\sin^2 A + \operatorname{cosec}^2 A + 2 \cancel{\sin A} \cdot \frac{1}{\cancel{\sin A}} = 4$$

$$\sin^2 A + \operatorname{cosec}^2 A = 2$$

Solution 11:

$$\tan A + \cot A = 5$$

Squaring both sides

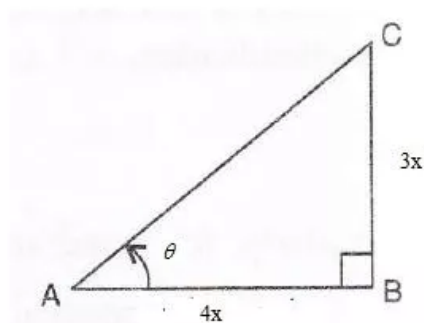
$$(\tan A + \cot A)^2 = 5^2$$

$$\tan^2 A + \cot^2 A + 2 \tan A \cdot \cot A = 25$$

$$\tan^2 A + \cot^2 A + 2 \cancel{\tan A} \cdot \frac{1}{\cancel{\tan A}} = 25$$

$$\tan^2 A + \cot^2 A = 23$$

Consider the diagram below:



$$4 \sin \theta = 3 \cos \theta$$

$$\tan \theta = \frac{3}{4}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{base}} = \frac{3}{4} \Rightarrow \frac{BC}{AB} = \frac{3}{4}$$

Therefore if length of $BC = 3x$, length of $AB = 4x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(4x)^2 + (3x)^2 = AC^2$$

$$AC^2 = 25x^2$$

$$\therefore AC = 5x \text{ (hypotenuse)}$$

(i)

$$\sin \theta = \frac{BC}{AC} = \frac{3}{5}$$

(ii)

$$\cos \theta = \frac{AB}{AC} = \frac{4}{5}$$

Solution 12:

(iii)

$$\cot \theta = \frac{AB}{BC} = \frac{4}{3}$$

$$\operatorname{cosec} \theta = \frac{AC}{BC} = \frac{5}{3}$$

Therefore

$$\cot^2 \theta - \operatorname{cosec}^2 \theta$$

$$= \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2$$

$$= \frac{16 - 25}{9}$$

$$= -\frac{9}{9}$$

$$= -1$$

(iv)

$$4 \cos^2 \theta - 3 \sin^2 \theta + 2$$

$$= 4 \left(\frac{4}{5}\right)^2 - 3 \left(\frac{3}{5}\right)^2 + 2$$

$$= \frac{64}{25} - \frac{27}{25} + 2$$

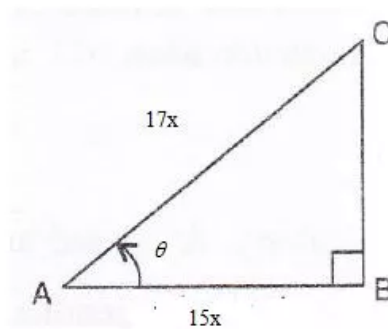
$$= \frac{64 - 27 + 50}{25}$$

$$= \frac{87}{25}$$

$$= 3\frac{12}{25}$$

Solution 13:

Consider the diagram below:



$$17 \cos \theta = 15$$

$$\cos \theta = \frac{15}{17}$$

$$\text{i.e. } \frac{\text{base}}{\text{hypotenuse}} = \frac{15}{17} \Rightarrow \frac{AB}{AC} = \frac{15}{17}$$

Therefore if length of $AB = 15x$, length of $AC = 17x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(17x)^2 - (15x)^2 = BC^2$$

$$BC^2 = 64x^2$$

$$\therefore BC = 8x (\text{perpendicular})$$

Now

$$\sec \theta = \frac{AC}{AB} = \frac{17}{15}$$

$$\tan \theta = \frac{BC}{AB} = \frac{8}{15}$$

Therefore

$$\tan \theta + 2 \sec \theta$$

$$= \frac{8}{15} + 2 \cdot \frac{17}{15}$$

$$= \frac{42}{15}$$

$$= \frac{14}{5}$$

$$= 2\frac{4}{5}$$

Solution 14:

$$5 \cos A - 12 \sin A = 0$$

$$5 \cos A = 12 \sin A$$

$$\frac{\sin A}{\cos A} = \frac{5}{12}$$

$$\tan A = \frac{5}{12}$$

Now

$$\frac{\sin A + \cos A}{2 \cos A - \sin A} = \frac{\frac{\sin A}{\cos A} + \frac{\cos A}{\cos A}}{\frac{2 \cos A}{\cos A} - \frac{\sin A}{\cos A}}$$

$$= \frac{\tan A + 1}{2 - \tan A}$$

$$= \frac{\frac{5}{12} + 1}{2 - \frac{5}{12}}$$

$$= \frac{\frac{17}{12}}{\frac{19}{12}}$$

$$= \frac{17}{19}$$

Solution 15:

Since D is mid-point of AC so $AC = 2DC$

(i)

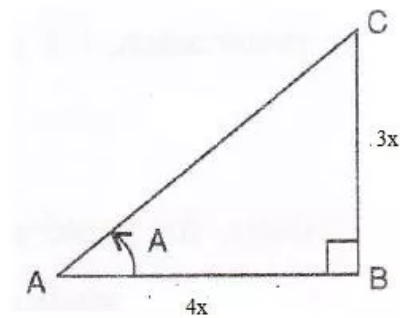
$$\begin{aligned} & \frac{\tan \angle CAB}{\tan \angle CDB} \\ &= \frac{\frac{BC}{AC}}{\frac{BC}{DC}} \\ &= \frac{BC}{2DC} \cdot \frac{DC}{BC} \\ &= \frac{1}{2} \end{aligned}$$

(ii)

$$\begin{aligned} & \frac{\tan \angle ABC}{\tan \angle DBC} \\ &= \frac{\frac{AC}{BC}}{\frac{DC}{BC}} \\ &= \frac{2DC}{BC} \cdot \frac{BC}{DC} \\ &= 2 \end{aligned}$$

Solution 16:

Consider the diagram below:



$$3 \cos A = 4 \sin A$$

$$\cot A = \frac{4}{3}$$

$$\text{i.e. } \frac{\text{base}}{\text{perpendicular}} = \frac{4}{3} \Rightarrow \frac{AB}{BC} = \frac{4}{3}$$

Therefore if length of $AB = 4x$, length of $BC = 3x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(4x)^2 + (3x)^2 = AC^2$$

$$AC^2 = 25x^2$$

$$\therefore AC = 5x (\text{hypotenuse})$$

(i)

$$\cos A = \frac{AB}{AC} = \frac{4}{5}$$

(ii)

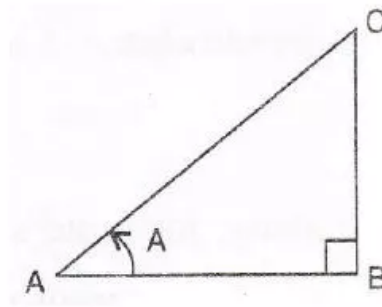
$$\operatorname{cosec} A = \frac{AC}{BC} = \frac{5}{3}$$

Therefore

$$\begin{aligned} & 3 - \cot^2 A + \operatorname{cosec}^2 A \\ &= 3 - \left(\frac{4}{3}\right)^2 + \left(\frac{5}{3}\right)^2 \\ &= \frac{27 - 16 + 25}{9} \\ &= \frac{36}{9} \\ &= 4 \end{aligned}$$

Solution 17:

Consider the figure



$$\tan A = \frac{75}{100} = \frac{3}{4}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB} = \frac{3}{4}$$

Therefore if length of base = $4x$, length of perpendicular = $3x$

Since

$$BC^2 + AB^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (4x)^2 = AC^2$$

$$AC^2 = 9x^2 + 16x^2 = 25x^2$$

$$\therefore AC = 5x$$

Now

$$AC = 30$$

$$5x = 30$$

$$x = 6$$

Therefore

$$AB = 4x$$

$$= 4 \times 6$$

$$= 24 \text{ cm}$$

And

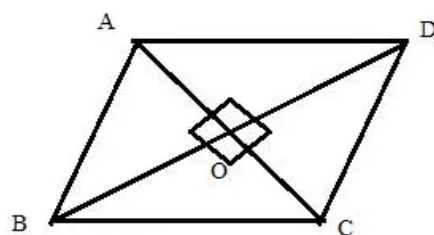
$$BC = 3x$$

$$= 3 \times 6$$

$$= 18 \text{ cm}$$

Solution 18:

Consider the figure



The diagonals of a rhombus bisect each other perpendicularly

$$\cos \angle CAB = \frac{6}{10} = \frac{3}{5}$$

$$\text{i.e. } \frac{\text{base}}{\text{hypotenuse}} = \frac{OA}{AB} = \frac{3}{5}$$

Therefore if length of base = $3x$, length of hypotenuse = $5x$

Since

$$OB^2 + OA^2 = AB^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(5x)^2 - (3x)^2 = OB^2$$

$$OB^2 = 16x^2$$

$$\therefore OB = 4x$$

Now

$$OB = 8$$

$$4x = 8$$

$$x = 2$$

Therefore

$$AB = 5x$$

$$= 5 \times 2$$

$$= 10 \text{ cm}$$

And

$$OA = 3x$$

$$= 3 \times 2$$

$$= 6 \text{ cm}$$

Since the sides of a rhombus are equal so the length of the side of the rhombus = 10 cm

The diagonals are

$$BD = 8 \times 2$$

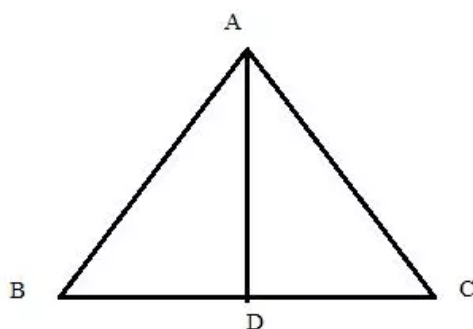
$$= 16 \text{ cm}$$

$$AC = 6 \times 2$$

$$= 12 \text{ cm}$$

Solution 19:

Consider the figure below



In the isosceles $\triangle ABC$, the perpendicular drawn from angle A to the side BC divides the side BC into two equal parts $BD = DC = 9 \text{ cm}$

Since $\angle ADB = 90^\circ$

$$\Rightarrow AB^2 = AD^2 + BD^2 \quad (AB \text{ is hypotenuse in } \triangle ABD)$$

$$\Rightarrow AD^2 = 15^2 - 9^2$$

$$\therefore AD^2 = 144 \text{ and } AD = 12$$

(i)

$$\cos B = \frac{\text{base}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{9}{15} = \frac{3}{5}$$

(ii)

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{12}{15} = \frac{4}{5}$$

(iii)

$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{12}{9} = \frac{4}{3}$$

$$\sec B = \frac{\text{hypotenuse}}{\text{base}} = \frac{AB}{BD} = \frac{15}{9} = \frac{5}{3}$$

Therefore

$$\tan^2 B - \sec^2 B + 2$$

$$= \left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2 + 2$$

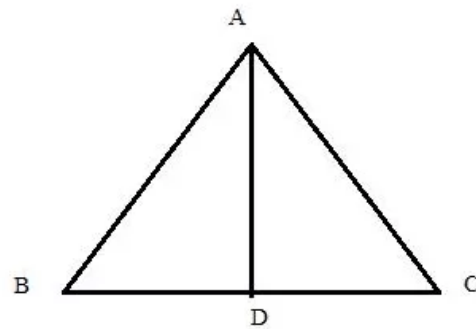
$$= \frac{16 - 25 + 18}{9}$$

$$= \frac{9}{9}$$

$$= 1$$

Solution 20:

Consider the figure below



$$\sin B = \frac{8}{10} = \frac{4}{5}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{4}{5}$$

Therefore if length of perpendicular = $4x$, length of hypotenuse = $5x$

Since

$$AD^2 + BD^2 = AB^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(5x)^2 - (4x)^2 = BD^2$$

$$BD^2 = 9x^2$$

$$\therefore BD = 3x$$

Now

$$BD = 9$$

$$3x = 9$$

$$x = 3$$

Therefore

$$\begin{aligned}AB &= 5x \\&= 5 \times 3 \\&= 15 \text{ cm}\end{aligned}$$

And

$$\begin{aligned}AD &= 4x \\&= 4 \times 3 \\&= 12 \text{ cm}\end{aligned}$$

Again

$$\begin{aligned}\tan C &= \frac{1}{1} \\i.e. \frac{\text{perpendicular}}{\text{base}} &= \frac{AD}{DC} = \frac{1}{1}\end{aligned}$$

Therefore if length of perpendicular = x, length of base = x

Since

$$AD^2 + DC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(x)^2 + (x)^2 = AC^2$$

$$AC^2 = 2x^2$$

$$\therefore AC = \sqrt{2}x$$

Now

$$\begin{aligned}AD &= 12 \\x &= 12\end{aligned}$$

Therefore

$$\begin{aligned}DC &= x \\&= 12 \text{ cm}\end{aligned}$$

And

$$\begin{aligned}AC &= \sqrt{2}x \\&= \sqrt{2} \times 12 \\&= 12\sqrt{2} \text{ cm}\end{aligned}$$

Solution 21:

$$q \tan A = p$$

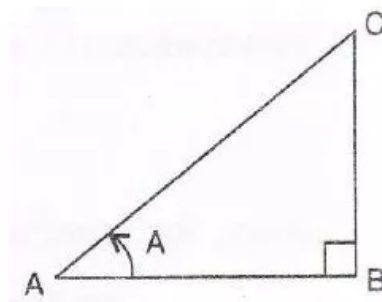
$$\tan A = \frac{p}{q}$$

Now

$$\begin{aligned}\frac{p \sin A - q \cos A}{p \sin A + q \cos A} &= \frac{\frac{p \sin A}{\cos A} - \frac{q \cos A}{\cos A}}{\frac{p \sin A}{\cos A} + \frac{q \cos A}{\cos A}} \\&= \frac{p \tan A - q}{p \tan A + q} \\&= \frac{p \left(\frac{p}{q} \right) - q}{p \left(\frac{p}{q} \right) + q} \\&= \frac{p^2 - q^2}{p^2 + q^2} \\&= \frac{q}{q}\end{aligned}$$

Solution 22:

Consider the figure



$$\sin A = \cos A$$

$$\tan A = \frac{1}{1}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB} = \frac{1}{1}$$

Therefore if length of perpendicular = x , length of base = x

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(x)^2 + (x)^2 = AC^2$$

$$AC^2 = 2x^2$$

$$\therefore AC = \sqrt{2}x$$

Now

$$\sec A = \frac{AC}{AB} = \sqrt{2}$$

Therefore

$$2 \tan^2 A - 2 \sec^2 A + 5$$

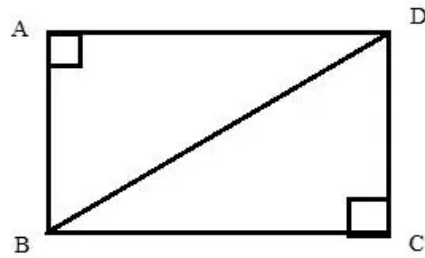
$$= 2(1)^2 - 2(\sqrt{2})^2 + 5$$

$$= 2 - 4 + 5$$

$$= 3$$

Solution 23:

Consider the diagram



$$\cot \angle ABD = \frac{15}{10} = \frac{3}{2}$$

$$\text{i.e. } \frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BD} = \frac{3}{2}$$

Therefore if length of base = $3x$, length of perpendicular = $2x$

Since

$$AB^2 + AD^2 = BD^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(3x)^2 + (2x)^2 = BD^2$$

$$BD^2 = 13x^2$$

$$\therefore BD = \sqrt{13}x$$

Now

$$BD = 26$$

$$\sqrt{13}x = 26$$

$$x = \frac{26}{\sqrt{13}}$$

Therefore

$$AD = 2x$$

$$= 2 \times \frac{26}{\sqrt{13}}$$

$$= \frac{52}{\sqrt{13}} \text{ cm}$$

$$AB = 3x$$

$$= 3 \times \frac{26}{\sqrt{13}}$$

$$= \frac{78}{\sqrt{13}} \text{ cm}$$

Now

$$\text{Area of rectangle } ABCD = AB \times AD$$

$$= \frac{78}{\sqrt{13}} \times \frac{52}{\sqrt{13}}$$

$$= 312 \text{ cm}^2$$

$$\text{Perimeter of rectangle } ABCD = 2(AB + AD)$$

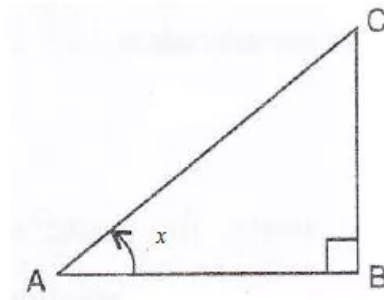
$$= 2 \left(\frac{78}{\sqrt{13}} + \frac{52}{\sqrt{13}} \right)$$

$$= \frac{260}{\sqrt{13}}$$

$$= 20\sqrt{13} \text{ cm}$$

Solution 24:

Consider the figure



$$2 \sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$$

Therefore if length of perpendicular = $\sqrt{3}x$, length of hypotenuse = $2x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(2x)^2 - (\sqrt{3}x)^2 = AB^2$$

$$AB^2 = x^2$$

$$\therefore AB = x$$

Now

$$\cos x = \frac{AB}{AC} = \frac{1}{2}$$

(i)

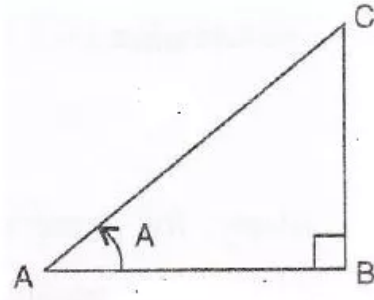
$$\begin{aligned} 4 \sin^3 x - 3 \sin x &= 4 \left(\frac{\sqrt{3}}{2} \right)^3 - 3 \left(\frac{\sqrt{3}}{2} \right) \\ &= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} \\ &= 0 \end{aligned}$$

(ii)

$$\begin{aligned} 3 \cos x - 4 \cos^3 x &= 3 \cdot \frac{1}{2} - 4 \cdot \left(\frac{1}{2} \right)^3 \\ &= \frac{3}{2} - \frac{1}{2} \\ &= 1 \end{aligned}$$

Solution 25:

Consider the diagram below:



$$\sin A = \frac{\sqrt{3}}{2}$$

$$\text{i.e. } \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{BC}{AC} = \frac{\sqrt{3}}{2}$$

Therefore if length of $BC = \sqrt{3}x$, length of $AC = 2x$

Since

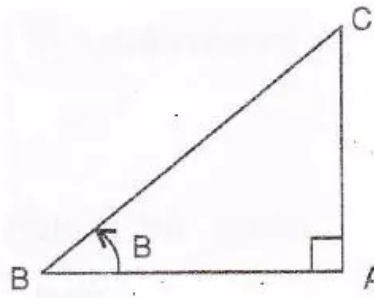
$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(\sqrt{3}x)^2 + AB^2 = (2x)^2$$

$$AB^2 = x^2$$

$$\therefore AB = x (\text{base})$$

Consider the diagram below:



$$\cos B = \frac{\sqrt{3}}{2}$$

$$\text{i.e. } \frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{AB}{BC} = \frac{\sqrt{3}}{2}$$

Therefore if length of $AB = \sqrt{3}x$, length of $BC = 2x$

Since

$$AB^2 + AC^2 = BC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$AC^2 + (\sqrt{3}x)^2 = (2x)^2$$

$$AC^2 = x^2$$

$$\therefore AC = x (\text{perpendicular})$$

Now

$$\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

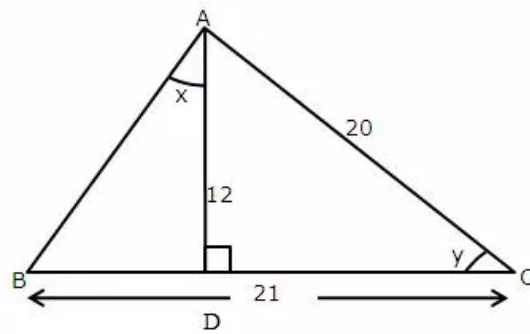
$$\tan B = \frac{\text{perpendicular}}{\text{base}} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}}$$

Therefore

$$\begin{aligned} \frac{\tan A - \tan B}{1 + \tan A \tan B} &= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}} \\ &= \frac{\frac{2}{\sqrt{3}}}{2} \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

Solution 26:

Consider the given diagram as



Using Pythagorean Theorem

$$AD^2 + DC^2 = AC^2$$

$$DC^2 = 20^2 - 12^2 = 256$$

$$DC = 16$$

Now

$$BC = BD + DC$$

$$21 = BD + 16$$

$$BD = 5$$

Again using Pythagorean Theorem

$$AD^2 + BD^2 = AB^2$$

$$12^2 + 5^2 = AB^2$$

$$AB^2 = 169$$

$$AB = 13$$

Now

$$\sin x = \frac{BD}{AB} = \frac{5}{13}$$

$$\sin y = \frac{AD}{AC} = \frac{12}{20} = \frac{3}{5}$$

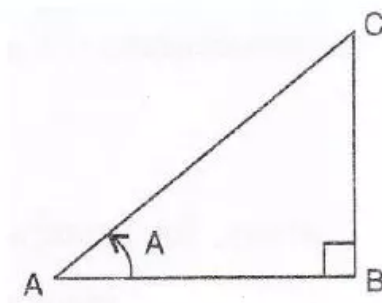
$$\cot y = \frac{DC}{AD} = \frac{16}{12} = \frac{4}{3}$$

Therefore

$$\begin{aligned} \frac{10}{\sin x} + \frac{6}{\sin y} - 6 \cot y &= \frac{10}{\frac{5}{13}} + \frac{6}{\frac{3}{5}} - 6 \left(\frac{4}{3} \right) \\ &= \frac{130}{5} + \frac{30}{3} - \frac{24}{3} \\ &= 26 + 10 - 8 \\ &= 28 \end{aligned}$$

Solution 27:

Consider the figure



$$\sec A = \sqrt{2}$$

$$\text{i.e. } \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \sqrt{2}$$

Therefore if length of base = x , length of hypotenuse = $\sqrt{2}x$

Since

$$AB^2 + BC^2 = AC^2 \quad [\text{Using Pythagoras Theorem}]$$

$$(\sqrt{2}x)^2 - (x)^2 = BC^2$$

$$BC^2 = x^2$$

$$\therefore BC = x$$

Now

$$\cos A = \frac{1}{\sec A} = \frac{1}{\sqrt{2}}$$

$$\sin A = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$$

$$\tan A = \frac{BC}{AB} = 1$$

$$\cot A = \frac{1}{\tan A} = 1$$

Therefore

$$\begin{aligned} \frac{3\cot^2 A + 2\sin^2 A}{\tan^2 A - \cos^2 A} &= \frac{3(1)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2}{1^2 - \left(\frac{1}{\sqrt{2}}\right)^2} \\ &= \frac{3+1}{1-\frac{1}{2}} \\ &= \frac{4}{\frac{1}{2}} \\ &= 8 \end{aligned}$$

Solution 28:

$$\cos \theta = \frac{3}{5}$$

Now

$$\begin{aligned} \frac{\operatorname{cosec} \theta - \cot \theta}{\operatorname{cosec} \theta + \cot \theta} &= \frac{\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta}} \\ &= \frac{1 - \cos \theta}{1 + \cos \theta} \\ &= \frac{1 - \frac{3}{5}}{1 + \frac{3}{5}} \\ &= \frac{\frac{2}{5}}{\frac{8}{5}} \\ &= \frac{2}{8} \\ &= \frac{1}{4} \end{aligned}$$

Solution 29:

$$\operatorname{cosec} A + \sin A = 5\frac{1}{5}$$

Squaring both sides

$$\begin{aligned} (\operatorname{cosec} A + \sin A)^2 &= \left(5\frac{1}{5}\right)^2 \\ \operatorname{cosec}^2 A + \sin^2 A + 2 \cancel{\operatorname{cosec} A} \cdot \frac{1}{\cancel{\operatorname{cosec} A}} &= \frac{26}{5} \\ \operatorname{cosec}^2 A + \sin^2 A &= \frac{626}{25} \\ \operatorname{cosec}^2 A + \sin^2 A &= 25\frac{1}{25} \end{aligned}$$

Solution 30:

$$5 \cos \theta = 6 \sin \theta$$

$$\tan \theta = \frac{5}{6}$$

Now

(i)

$$\tan \theta = \frac{5}{6}$$

(ii)

$$\begin{aligned} \frac{12 \sin \theta - 3 \cos \theta}{12 \sin \theta + 3 \cos \theta} &= \frac{\frac{12 \sin \theta}{\cos \theta} - \frac{3 \cos \theta}{\cos \theta}}{\frac{12 \sin \theta}{\cos \theta} + \frac{3 \cos \theta}{\cos \theta}} \\ &= \frac{12 \tan \theta - 3}{12 \tan \theta + 3} \\ &= \frac{12 \left(\frac{5}{6}\right) - 3}{12 \left(\frac{5}{6}\right) + 3} \end{aligned}$$

$$\begin{aligned} &= \frac{\frac{42}{6}}{\frac{78}{6}} \\ &= \frac{42}{78} \\ &= \frac{7}{13} \end{aligned}$$