

SIMPLE HARMONIC MOTION

1. Periodic motion:

The motion of a particle which repeats itself along the same path in equal intervals of time is called periodic motion

- Ex: a. Rotation of earth around the sun.
 b. Oscillations of simple pendulum
 c. Vibrations of prongs of a tuning fork
 d. Motion of hands in a watch.

2. Displacement of a particle in periodic motion can be mathematically expressed in terms of sine (or) cosine function so that periodic motion is also called harmonic motion.

3. Simple harmonic motion:

The to and fro motion of a particle along the straight line such that its acceleration is always directed towards a fixed point in its path and is directly proportional to the displacement from that fixed point is called simple harmonic motion.

$$a \propto -y \Rightarrow a = -\omega^2 y \Rightarrow \frac{d^2 y}{dt^2} + \omega^2 y = 0.$$

Simple harmonic motion is of two types:

i) Linear simple harmonic motion:

If the motion is along the straight line path it is called linear simple harmonic motion

- Ex: a. Vibrations of string
 b. Oscillations of liquid in U. tube
 c. Oscillations of loaded spring
 d. Vibrations of the prongs of a tuning fork.

ii) Angular simple harmonic motion

If vibrations are angular then it is called angular simple harmonic motion

- Ex: a. Oscillations of a torsional pendulum
 b. Oscillations of balance wheel in watches

4. Characteristics of simple harmonic motions:

- The motion is periodic
 - It executes to and fro motion
 - Acceleration is directed towards the mean point
 - The acceleration is always directly proportional to its displacement and which is opposite in direction to the displacement $a \propto -y$
5. Simple harmonic motion is a periodic motion but every periodic motion is need not be a simple harmonic motion.

6. Displacement:

The maximum distance of the particle from the equilibrium position at any instant is called displacement.

7. Amplitude:

The maximum displacement of the vibrating particle from mean position is called amplitude represented by 'A' (or) a

8. Time period:

The time taken for one oscillation is called time period

$$T = \frac{2\pi}{\omega}$$

9. Frequency:

The number of oscillations completed by a particle in one second is called frequency.

$$\text{Frequency } f = \frac{1}{T} \text{ cycles/s.}$$

10. Phase:

It represents the position and direction of vibrating particle at a particular instant at the instant of time $t = 0$ the phase of the particle is called initial phase i.e. ' ϕ ' it is also called 'epoch'

11. Displacement, velocity and acceleration of a particle executing simple harmonic motion.

- a. Displacement of a particle executing simple harmonic motion is given by

$$y = A \sin(\omega t \pm \phi)$$

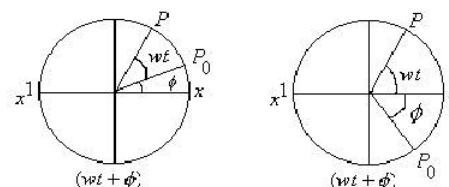
where $\phi \rightarrow$ is initial phase

$\omega \rightarrow$ angular frequency

$A \rightarrow$ amplitude

- b. Velocity of the particle executing simple harmonic motion is given by

$$V = \omega \sqrt{A^2 - y^2}$$



Case (i) When particle at mean position :

$$y = 0$$

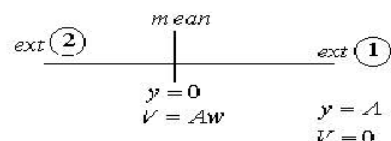
$$\therefore V = \omega \sqrt{A^2 - 0}$$

$$V = A\omega = \text{maximum}$$

Case(ii) When particle at extreme end
 $y = A$

$$\therefore V = \omega \sqrt{A^2 - A^2}$$

$$V = 0 = \text{minimum}$$



- c. Acceleration of a particle executing simple harmonic motion is given by

$$a = -A\omega^2 \sin(\omega t \pm \phi)$$

$$a = -\omega^2 y$$

Case(i) When particle at mean

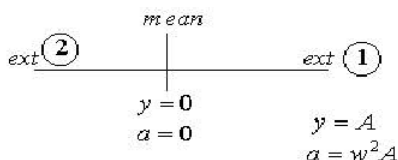
$$y = 0$$

$$a = 0$$

Case(ii) When particle at extreme

$$y = A$$

$$a = \omega^2 A \text{ maximum}$$



12. Time period of a particle executing simple harmonic motion is given by

$$T = 2\pi \sqrt{\frac{y}{a}}$$

Where y = displacement and
 a = acceleration

13. Simple pendulum

Time period of simple pendulum executing simple harmonic motion is given by

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where $l \rightarrow$ length of the simple pendulum
 $g \rightarrow$ acceleration due to gravity

14. Laws of simple pendulum

- a. Time period of a simple pendulum is directly proportional to square root of

its lengths $T \propto \sqrt{l}$

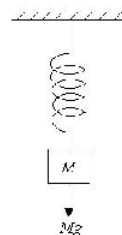
- b. time period of simple pendulum is inversely proportion to square root of the acceleration due to gravity

$$T \propto \frac{1}{\sqrt{g}}$$

$$T^2 \propto \frac{1}{g}$$

- c. time period of simple pendulum is independent of mass and shape of the bob.

15. Loaded spring



- i) If a mass M is suspended vertically from a spring and if the spring elongates ' x ' then spring constant is

$$K = \frac{F}{x}$$

$$K = \frac{Mg}{x}$$

- ii) time period of this loaded spring is

$$T = 2\pi \sqrt{\frac{M}{K}}$$

$$= 2\pi \sqrt{\frac{M}{\frac{Mg}{x}}} \Rightarrow T = 2\pi \sqrt{\frac{x}{g}}$$

- iii) If a mass m is attached to the end of a spring and oscillates time period is

$$T = 2\pi \sqrt{\frac{m}{K}} \dots\dots\dots(1)$$

$$T = 2\pi \sqrt{\frac{m}{\frac{Mg}{x}}}$$

$$T = 2\pi \sqrt{\frac{mx}{Mg}} \dots\dots\dots(2)$$

If m_s is mass of the spring

$$\text{then } T = 2\pi \sqrt{\frac{[m + (m_s / 3)]x}{Mg}} \dots\dots\dots(3)$$

16. Energy of a particle executing simple harmonic motion

Let us consider a particle executing simple harmonic motion it possesses P. E. and K. E.

$$P.E. = \frac{1}{2}mw^2y^2 \dots\dots\dots(1)$$

$$K.E = \frac{1}{2}mw^2(A^2 - y^2) \dots\dots\dots(2)$$

total energy of the particle = PE + KE

$$TE = \frac{1}{2}mw^2A^2 \dots\dots\dots(3)$$