

Chapter 23. Trigonometrical Ratios of Standard Angles [Including Evaluation of an Expression Involving Trigonometric Ratios]

Exercise 23(A)

Solution 1:

$$(i) \sin 30^\circ \cos 30^\circ = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

$$(ii) \tan 30^\circ \tan 60^\circ = \frac{1}{\sqrt{3}} (\sqrt{3}) = 1$$

$$(iii) \cos^2 60^\circ + \sin^2 30^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$(iv) \operatorname{cosec}^2 60^\circ - \tan^2 30^\circ = \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3} - \frac{1}{3} = 1$$

$$(v) \sin^2 30^\circ + \cos^2 30^\circ + \cot^2 45^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1^2 = \frac{1}{4} + \frac{3}{4} + 1 = 2$$

(vi)

$$\begin{aligned} \cos^2 60^\circ + \sec^2 30^\circ + \tan^2 45^\circ &= \left(\frac{1}{2}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 + 1^2 \\ &= \frac{1}{4} + \frac{4}{3} + 1 \\ &= \frac{3+16+12}{12} \\ &= \frac{31}{12} \\ &= 2\frac{7}{12} \end{aligned}$$

Solution 2:

$$(i) \tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 + 1^2 + (\sqrt{3})^2 = \frac{1}{3} + 1 + 3 = \frac{13}{3} = 4\frac{1}{3}$$

$$(ii) \frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ} = \frac{1}{2} + \frac{2}{1} - \frac{5}{2} = \frac{1+4-5}{2} = 0$$

$$(iii) 3 \sin^2 30^\circ + 2 \tan^2 60^\circ - 5 \cos^2 45^\circ$$

$$= 3\left(\frac{1}{2}\right)^2 + 2(\sqrt{3})^2 - 5\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{3}{4} + 6 - \frac{5}{2} = \frac{3+24-10}{4} = 4\frac{1}{4}$$

Solution 3:

$$(i) \text{ LHS} = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1 = \text{RHS}$$

$$(ii) \text{ LHS} = \cos 30^\circ \cdot \cos 60^\circ - \sin 30^\circ \cdot \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0 = \text{RHS}$$

$$(iii) \text{ LHS} = \operatorname{cosec}^2 45^\circ - \cot^2 45^\circ$$

$$= (\sqrt{2})^2 - 1^2 = 2 - 1 = 1 = \text{RHS}$$

$$(iv) \text{ LHS} = \cos^2 30^\circ - \sin^2 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \cos 60^\circ = \text{RHS}$$

$$(v) \text{ LHS} = \left(\frac{\tan 60^\circ + 1}{\tan 60^\circ - 1} \right)^2$$

$$= \left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1} \right)^2 = \frac{4 + 2\sqrt{3}}{4 - 2\sqrt{3}} = \frac{1 + \frac{\sqrt{3}}{2}}{1 - \frac{\sqrt{3}}{2}} = \frac{1 + \cos 30^\circ}{1 - \cos 30^\circ} = \text{RHS}$$

$$(vi) \text{ LHS} = 3 \operatorname{cosec}^2 60^\circ - 2 \cot^2 30^\circ + \sec^2 45^\circ$$

$$= 3 \left(\frac{2}{\sqrt{3}} \right)^2 - 2(\sqrt{3})^2 + (\sqrt{2})^2 = 4 - 6 + 2 = 0 = \text{RHS}$$

Solution 4:

(i)

$$\text{RHS} =$$

$$\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{\sqrt{3}}{2}$$

$$\text{LHS} = \sin (2 \times 30^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \text{LHS} = \text{RHS}$$

(ii)

RHS,

$$\frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{1}{2}$$

LHS,

$$\cos (2 \times 30^\circ) = \cos 60^\circ = \frac{1}{2}$$

LHS = RHS

(iii)

RHS,

$$\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

LHS,

$$\tan (2 \times 30^\circ) = \tan 60^\circ = \sqrt{3}$$

LHS = RHS

Solution 5:

Given that $AB = BC = x$

$$\therefore AC = \sqrt{AB^2 + BC^2} = \sqrt{x^2 + x^2} = x\sqrt{2}$$

$$(i) \sin 45^\circ = \frac{AB}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$(ii) \cos 45^\circ = \frac{BC}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$(iii) \tan 45^\circ = \frac{AB}{BC} = \frac{x}{x} = 1$$

Solution 6:

$$(i) LHS = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$RHS = 2 \sin 60^\circ \cos 60^\circ = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$LHS = RHS$$

$$(ii) LHS = 4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ)$$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 + (1)^4\right]$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right] = \frac{4 \times 2}{16} + 3 \times \frac{1}{2} = 2$$

$$RHS = 2$$

$$LHS = RHS$$

Solution 7:

(i)

The angle, x is acute and hence we have, $0 < x < 90$ degrees

We know that

$$\cos^2 x + \sin^2 x = 1$$

$$\Rightarrow 2\sin^2 x = 1 \quad [\text{since } \cos x = \sin x]$$

$$\Rightarrow \sin x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x = 45^\circ$$

(ii)

$$\sec A = \operatorname{cosec} A$$

$$\cos A = \sin A$$

$$\cos^2 A = \sin^2 A$$

$$\cos^2 A = 1 - \cos^2 A$$

$$2\cos^2 A = 1$$

$$\cos A = \frac{1}{\sqrt{2}}$$

$$A = 45^\circ$$

(iii)

$$\tan \theta = \cot \theta$$

$$\tan \theta = \frac{1}{\tan \theta}$$

$$\tan^2 \theta = 1$$

$$\tan \theta = 1$$

$$\tan \theta = \tan 45^\circ$$

$$\theta = 45^\circ$$

(iv)

$$\sin x = \cos y = \sin (90^\circ - y)$$

If x and y are acute angles,

$$x = 90^\circ - y$$

$$\Rightarrow x + y = 90^\circ$$

Hence x and y are complementary angles

Solution 8:

(i)

$$\sin x = \cos y = \sin\left(\frac{\pi}{2} - y\right)$$

if x and y are acute angles,

$$x = \frac{\pi}{2} - y$$

$$x + y = \frac{\pi}{2}$$

$\therefore x + y = 45^\circ$ is false.

(ii)

$$\sec \theta \cdot \cot \theta = \frac{1}{\cos \theta} \frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta} = \operatorname{cosec} \theta$$

$\sec \theta \cdot \cot \theta = \operatorname{cosec} \theta$ is true

(iii)

$$\sin^2 \theta + \cos^2 \theta = \sin^2 \theta + 1 - \sin^2 \theta = 1$$

Solution 9:

(i) For acute angles, remember what sine means: opposite over hypotenuse. If we increase the angle, then the opposite side gets larger. That means "opposite/hypotenuse" gets larger or increases.

(ii) For acute angles, remember what cosine means: base over hypotenuse. If we increase the angle, then the hypotenuse side gets larger. That means "base/hypotenuse" gets smaller or decreases.

(iii) For acute angles, remember what tangent means: opposite over base. If we decrease the angle, then the opposite side gets smaller. That means "opposite /base" gets decreases.

Solution 10:

$$(i) \sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.87$$

$$(ii) \frac{2}{\tan 30^\circ} = \frac{2}{\frac{1}{\sqrt{3}}} = 2\sqrt{3} = 2 \times 1.732 = 3.46$$

Solution 11:(i) Given that $A = 15^\circ$

$$\begin{aligned}
\frac{\cos 3A - 2 \cos 4A}{\sin 3A + 2 \sin 4A} &= \frac{\cos(3 \times 15^\circ) - 2 \cos(4 \times 15^\circ)}{\sin(3 \times 15^\circ) + 2 \sin(4 \times 15^\circ)} \\
&= \frac{\cos 45^\circ - 2 \cos 60^\circ}{\sin 45^\circ + 2 \sin 60^\circ} \\
&= \frac{\frac{1}{\sqrt{2}} - 2\left(\frac{1}{2}\right)}{\frac{1}{\sqrt{2}} + 2\left(\frac{\sqrt{3}}{2}\right)} \\
&= \frac{\frac{1}{\sqrt{2}} - 1}{\frac{1}{\sqrt{2}} + \sqrt{3}} \\
&= \frac{1 - \sqrt{2}}{1 + \sqrt{6}} \\
&= \frac{1}{5}(\sqrt{6} - 1 - 2\sqrt{3} + \sqrt{2})
\end{aligned}$$

(ii) Given that $B = 20^\circ$

$$\begin{aligned}
\frac{3 \sin 3B + 2 \cos(2B + 5^\circ)}{2 \cos 3B - \sin(2B - 10^\circ)} &= \frac{3 \sin 3 \times 20^\circ + 2 \cos(2 \times 20^\circ + 5^\circ)}{2 \cos 3 \times 20^\circ - \sin(2 \times 20^\circ - 10^\circ)} \\
&= \frac{3 \sin 60^\circ + 2 \cos 45^\circ}{2 \cos 60^\circ - \sin 30^\circ} \\
&= \frac{3\left(\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{\sqrt{2}}\right)}{2\left(\frac{1}{2}\right) - \frac{1}{2}} \\
&= \frac{\frac{3\sqrt{3}}{2} + \sqrt{2}}{\frac{1}{2}} \\
&= 3\sqrt{3} + 2\sqrt{2}
\end{aligned}$$

Exercise 23(B)

Solution 1:

Given $A = 60^\circ$ and $B = 30^\circ$

(i)

$$\begin{aligned} \text{LHS} &= \sin(A + B) \\ &= \sin(60^\circ + 30^\circ) \\ &= \sin 90^\circ \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \sin A \cos B + \cos A \sin B \\ &= \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ \\ &= \frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2} + \frac{1}{2} \frac{1}{2} \\ &= \frac{3}{4} + \frac{1}{4} \\ &= 1 \end{aligned}$$

$$\text{LHS} = \text{RHS}$$

(ii)

$$\begin{aligned} \text{LHS} &= \cos(A + B) \\ &= \cos(60^\circ + 30^\circ) \\ &= \cos 90^\circ \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \cos A \cos B - \sin A \sin B \\ &= \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ \\ &= \frac{1}{2} \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \frac{1}{2} \\ &= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \\ &= 0 \end{aligned}$$

$$\text{LHS} = \text{RHS}$$

(iii)

$$\begin{aligned} \text{LHS} &= \cos(A - B) \\ &= \cos(60^\circ - 30^\circ) \end{aligned}$$

$$= \cos 30^\circ$$

$$= \frac{\sqrt{3}}{2}$$

$$RHS = \cos A \cos B + \sin A \sin B$$

$$= \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

$$= \frac{1}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$LHS = RHS$$

(iv)

$$LHS = \tan(A - B)$$

$$= \tan(60^\circ - 30^\circ)$$

$$= \tan 30^\circ$$

$$= \frac{1}{\sqrt{3}}$$

$$RHS = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

$$= \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \cdot \tan 30^\circ}$$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \left(\frac{1}{\sqrt{3}} \right)}$$

$$= \frac{2}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

$$LHS = RHS$$

Solution 2:

Given $A = 30^\circ$

(i)

$$\sin 2A = \sin 2(30^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} 2\sin A \cos A &= 2\sin 30^\circ \cos 30^\circ \\ &= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \frac{2\tan A}{1+\tan^2 A} &= \frac{2\tan 30^\circ}{1+\tan^2 30^\circ} \\ &= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} \\ &= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} \\ &= \frac{2}{\sqrt{3}} \times \frac{3}{4} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\therefore \sin 2A = 2\sin A \cos A = \frac{2\tan A}{1+\tan^2 A}$$

(ii)

$$\cos 2A = \cos 2(30^\circ) = \cos 60^\circ = \frac{1}{2}$$

$$\begin{aligned}\cos^2 A - \sin^2 A &= \cos^2 30^\circ - \sin^2 30^\circ \\ &= \frac{3}{4} - \frac{1}{4} \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{1 - \tan^2 A}{1 + \tan^2 A} &= \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} \\ &= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} \\ &= \frac{2}{4} \\ &= \frac{1}{2}\end{aligned}$$

$$\therefore \cos 2A = \cos^2 A - \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

(iii)

$$\begin{aligned}2 \cos^2 A - 1 &= 2 \cos^2 30^\circ - 1 \\ &= 2\left(\frac{3}{4}\right) - 1 \\ &= \frac{3}{2} - 1 \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}1 - 2 \sin^2 A &= 1 - 2 \sin^2 30^\circ \\ &= 1 - 2\left(\frac{1}{4}\right) \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}
 1 - 2 \sin^2 A &= 1 - 2 \sin^2 30^\circ \\
 &= 1 - 2 \left(\frac{1}{2} \right) \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\therefore 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

(iv)

$$\begin{aligned}
 \sin 3A &= \sin 3(30^\circ) \\
 &= \sin 90^\circ \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 3 \sin A - 4 \sin^3 A &= 3 \sin 30^\circ - 4 \sin^3 30^\circ \\
 &= 3 \left(\frac{1}{2} \right) - 4 \left(\frac{1}{2} \right)^3 \\
 &= \frac{3}{2} - \frac{1}{2} \\
 &= 1
 \end{aligned}$$

$$\therefore \sin 3A = 3 \sin A - 4 \sin^3 A$$

Solution 3:

Given that $A = B = 45^\circ$

(i)

$$\begin{aligned}LHS &= \sin (A - B) \\&= \sin (45^\circ - 45^\circ) \\&= \sin 0^\circ \\&= 0\end{aligned}$$

$$\begin{aligned}RHS &= \sin A \cos B - \cos A \sin B \\&= \sin 45^\circ \cos 45^\circ - \cos 45^\circ \sin 45^\circ \\&= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\&= 0\end{aligned}$$

$$LHS = RHS$$

(ii)

$$\begin{aligned}LHS &= \cos (A + B) \\&= \cos (45^\circ + 45^\circ) \\&= \cos 90^\circ \\&= 0\end{aligned}$$

$$\begin{aligned}RHS &= \cos A \cos B - \sin A \sin B \\&= \cos 45^\circ \cos 45^\circ - \sin 45^\circ \sin 45^\circ \\&= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\&= 0\end{aligned}$$

$$LHS = RHS$$

Solution 4:

Given that $A = 30^\circ$

(i)

$$LHS = \sin 3A$$

$$= \sin 3(30^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$RHS = 4 \sin A \sin (60^\circ - A) \sin (60^\circ + A)$$

$$= 4 \sin 30^\circ \sin (60^\circ - 30^\circ) \sin (60^\circ + 30^\circ)$$

$$= 4 \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) (1)$$

$$= 1$$

$$LHS = RHS$$

(ii)

$$LHS = (\sin A - \cos A)^2$$

$$= (\sin 30^\circ - \cos 30^\circ)^2$$

$$= \left(\frac{1}{2} - \frac{\sqrt{3}}{2} \right)^2$$

$$= \frac{1}{4} + \frac{3}{4} - \frac{\sqrt{3}}{2}$$

$$= 1 - \frac{\sqrt{3}}{2}$$

$$= \frac{2 - \sqrt{3}}{2}$$

$$\begin{aligned}
RHS &= 1 - \sin 2A \\
&= 1 - \sin 2(30^\circ) \\
&= 1 - \sin 60^\circ \\
&= 1 - \frac{\sqrt{3}}{2} \\
&= \frac{2 - \sqrt{3}}{2}
\end{aligned}$$

$$LHS = RHS$$

(iii)

$$\begin{aligned}
LHS &= \cos 2A \\
&= \cos 2(30^\circ) \\
&= \cos 60^\circ \\
&= \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
RHS &= \cos^4 A - \sin^4 A \\
&= \cos^4 30^\circ - \sin^4 30^\circ \\
&= \left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{2}\right)^4 \\
&= \frac{9}{16} - \frac{1}{16} \\
&= \frac{8}{16} \\
&= \frac{1}{2}
\end{aligned}$$

$$LHS = RHS$$

(iv)

$$\begin{aligned}
LHS &= \frac{1 - \cos 2A}{\sin 2A} \\
&= \frac{1 - \cos 2(30^\circ)}{\sin 2(30^\circ)}
\end{aligned}$$

$$= \frac{1 - \frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}}$$

$$RHS = \tan A$$

$$= \tan 30^\circ$$

$$= \frac{1}{\sqrt{3}}$$

$$LHS = RHS$$

(v)

$$LHS = \frac{1 + \sin 2A + \cos 2A}{\sin A + \cos A}$$

$$= \frac{1 + \sin 2(30^\circ) + \cos 2(30^\circ)}{\sin 30^\circ + \cos 30^\circ}$$

$$= \frac{1 + \frac{\sqrt{3}}{2} + \frac{1}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}}$$

$$= \frac{3 + \sqrt{3}}{\sqrt{3} + 1} \left(\frac{\sqrt{3} - 1}{\sqrt{3} - 1} \right)$$

$$= \frac{3\sqrt{3} - 3 + 3 - \sqrt{3}}{2}$$

$$= \frac{2\sqrt{3}}{2}$$

$$= \sqrt{3}$$

$$RHS = 2 \cos A$$

$$= 2 \cos (30^\circ)$$

$$= 2 \left(\frac{\sqrt{3}}{2} \right)$$

$$= \sqrt{3}$$

(vi)

$$\begin{aligned}LHS &= 4 \cos A \cos (60^\circ - A) \cdot \cos (60^\circ + A) \\&= 4 \cos 30^\circ \cos (60^\circ - 30^\circ) \cdot \cos (60^\circ + 30^\circ) \\&= 4 \cos 30^\circ \cos 30^\circ \cos 90^\circ \\&= 4 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{3}}{2} \right) (0) \\&= 0\end{aligned}$$

$$\begin{aligned}RHS &= \cos 3A \\&= \cos 3(30^\circ) \\&= \cos 90^\circ \\&= 0\end{aligned}$$

$$LHS = RHS$$

(vii)

$$\begin{aligned}LHS &= \frac{\cos^3 A - \cos 3A}{\cos A} + \frac{\sin^3 A + \sin 3A}{\sin A} \\&= \frac{\cos^3 30^\circ - \cos 3(30^\circ)}{\cos 30^\circ} + \frac{\sin^3 30^\circ + \sin 3(30^\circ)}{\sin 30^\circ} \\&= \frac{\left(\frac{\sqrt{3}}{2} \right)^3 - 0}{\frac{\sqrt{3}}{2}} + \frac{\left(\frac{1}{2} \right)^3 + 1}{\frac{1}{2}} \\&= \left(\frac{\sqrt{3}}{2} \right)^2 + \frac{\frac{9}{8}}{\frac{1}{2}} \\&= \frac{3}{4} + \frac{9}{4} \\&= \frac{12}{4} \\&= 3 \\&= RHS\end{aligned}$$

Exercise 23(C)

Solution 1:

(i)

$$2 \sin A = 1$$

$$\sin A = \frac{1}{2}$$

$$\sin A = \sin 30^\circ$$

$$A = 30^\circ$$

(ii)

$$2 \cos 2A = 1$$

$$\cos 2A = \frac{1}{2}$$

$$\cos 2A = \cos 60^\circ$$

$$2A = 60^\circ$$

$$A = 30^\circ$$

(iii)

$$\sin 3A = \frac{\sqrt{3}}{2}$$

$$\sin 3A = \sin 60^\circ$$

$$3A = 60^\circ$$

$$A = 20^\circ$$

(iv)

$$\sec 2A = 2$$

$$\sec 2A = \sec 60^\circ$$

$$2A = 60^\circ$$

$$A = 30^\circ$$

(v)

$$\sqrt{3} \tan A = 1$$

$$\tan A = \frac{1}{\sqrt{3}}$$

$$\tan A = \tan 30^\circ$$

$$A = 30^\circ$$

(vi)

$$\tan 3 A = 1$$

$$\tan 3 A = \tan 45^{\circ}$$

$$3A = 45^{\circ}$$

$$A = 15^{\circ}$$

(vii)

$$2 \sin 3 A = 1$$

$$\sin 3 A = \frac{1}{2}$$

$$\sin 3A = \sin 30^{\circ}$$

$$3A = 30^{\circ}$$

$$A = 10^{\circ}$$

(viii)

$$\sqrt{3} \cot 2 A = 1$$

$$\cot 2 A = \frac{1}{\sqrt{3}}$$

$$\cot 2 A = \cot 60^{\circ}$$

$$2A = 60^{\circ}$$

$$A = 30^{\circ}$$

Solution 2:

(i)

$$(\sin A - 1) (2 \cos A - 1) = 0$$

$$(\sin A - 1) = 0 \text{ and } 2 \cos A - 1 = 0$$

$$\sin A = 1 \quad \text{and} \quad \cos A = \frac{1}{2}$$

$$\sin A = \sin 90^\circ \text{ and } \cos A = \cos 60^\circ$$

$$A = 90^\circ \quad \text{and} \quad A = 60^\circ$$

(ii)

$$(\tan A - 1) (\operatorname{cosec} 3A - 1) = 0$$

$$\tan A - 1 = 0 \text{ and } \operatorname{cosec} 3A - 1 = 0$$

$$\tan A = 1 \text{ and } \operatorname{cosec} 3A = 1$$

$$\tan A = \tan 45^\circ \text{ and } \operatorname{cosec} 3A = \operatorname{cosec} 90^\circ$$

$$A = 45^\circ \quad A = 30^\circ$$

(iii)

$$(\sec 2A - 1) (\operatorname{cosec} 3A - 1) = 0$$

$$\sec 2A - 1 = 0 \text{ and } \operatorname{cosec} 3A - 1 = 0$$

$$\sec 2A = 1 \text{ and } \operatorname{cosec} 3A = 1$$

$$\sec 2A = \sec 0^\circ \text{ and } \operatorname{cosec} 3A = \operatorname{cosec} 90^\circ$$

$$A = 0^\circ \quad A = 30^\circ$$

(iv)

$$\cos 3A. (2 \sin 2A - 1) = 0$$

$$\cos 3A = 0 \quad \text{and} \quad 2 \sin 2A - 1 = 0$$

$$\cos 3A = \cos 90^\circ \text{ and } \sin 2A = \frac{1}{2}$$

$$3A = 90^\circ \text{ and } \sin 2A = \sin 30^\circ$$

$$A = 30^\circ \quad 2A = 30^\circ \Rightarrow A = 15^\circ$$

(v)

$$(\operatorname{cosec} 2A - 2) (\cot 3A - 1) = 0$$

$$\operatorname{cosec} 2A - 2 = 0 \quad \text{and} \quad \cot 3A - 1 = 0$$

$$\operatorname{cosec} 2A = 2 \quad \text{and} \quad \cot 3A = 1$$

$$\operatorname{cosec} 2A = \operatorname{cosec} 30^\circ \text{ and } \cot 3A = \cot 45^\circ$$

$$2A = 30^\circ \text{ and } 3A = 45^\circ$$

$$A = 15^\circ \text{ and } A = 15^\circ$$

Solution 3:

(i)

$$2 \sin x^{\circ} - 1 = 0$$

$$\sin x^{\circ} = \frac{1}{2}$$

(ii)

$$\sin x^{\circ} = \frac{1}{2}$$

$$\sin x^{\circ} = \sin 30^{\circ}$$

$$x^{\circ} = 30^{\circ}$$

(iii)

$$\cos x^{\circ} = \cos 30^{\circ} = \frac{\sqrt{3}}{2}$$

$$\tan x^{\circ} = \tan 30^{\circ} = \frac{1}{\sqrt{3}}$$

Solution 4:

(i)

$$4 \cos^2 x^{\circ} - 1 = 0$$

$$4 \cos^2 x^{\circ} = 1$$

$$\cos^2 x^{\circ} = \left(\frac{1}{2}\right)^2$$

$$\cos x^{\circ} = \frac{1}{2}$$

$$\cos x^{\circ} = \cos 60^{\circ}$$

$$x^{\circ} = 60^{\circ}$$

(ii)

$$\sin^2 x^{\circ} + \cos^2 x^{\circ} = \sin^2 60^{\circ} + \cos^2 60^{\circ}$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

(iii)

$$\frac{1}{\cos^2 x^{\circ}} - \tan^2 x^{\circ} = \frac{1}{\cos^2 60^{\circ}} - \tan^2 60^{\circ}$$

$$= \frac{1}{\left(\frac{1}{2}\right)^2} - (\sqrt{3})^2$$

$$= 4 - 3$$

$$= 1$$

Solution 5:

$$4 \sin^2 \theta - 1 = 0$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \frac{1}{2}$$

$$\sin \theta = \sin 30^\circ$$

$$\theta = 30^\circ$$

$$\cos^2 \theta + \tan^2 \theta = \cos^2 30^\circ + \tan^2 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2} \right)^2 + \left(\frac{1}{\sqrt{3}} \right)^2$$

$$= \frac{3}{4} + \frac{1}{3}$$

$$= \frac{9+4}{12}$$

$$= \frac{13}{12}$$

Solution 6:

$$\sin 3A = 1$$

$$\sin 3A = \sin 90^\circ$$

$$3A = 90^\circ$$

$$A = 30^\circ$$

(i)

$$\sin A = \sin 30^\circ$$

$$\sin A = \frac{1}{2}$$

(ii)

$$\cos 2A = \cos 2(30^\circ)$$

$$= \cos 60^\circ$$

$$= \frac{1}{2}$$

(iii)

$$\tan^2 A - \frac{1}{\cos^2 A} = \tan^2 30^\circ - \frac{1}{\cos^2 30^\circ}$$

$$= \left(\frac{1}{\sqrt{3}} \right)^2 - \frac{1}{\left(\frac{\sqrt{3}}{2} \right)^2}$$

$$= \frac{1}{3} - \frac{4}{3}$$

$$= \frac{-3}{3}$$

$$= -1$$

Solution 7:

(i)

$$2 \cos 2A = \sqrt{3}$$

$$\cos 2A = \frac{\sqrt{3}}{2}$$

$$\cos 2A = \cos 30^\circ$$

$$2A = 30^\circ$$

$$A = 15^\circ$$

(ii)

$$\sin 3A = \sin 3(15^\circ)$$

$$= \sin 45^\circ$$

$$= \frac{1}{\sqrt{2}}$$

(iii)

$$\sin^2(75^\circ - A) + \cos^2(45^\circ + A) = \sin^2(75^\circ - 15^\circ) + \cos^2(45^\circ + 15^\circ)$$

$$= \sin^2 60^\circ + \cos^2 60^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

Solution 8:

(i)

Given that $x = 30^\circ$

$$\sin x + \cos y = 1$$

$$\sin 30^\circ + \cos y = 1$$

$$\cos y = 1 - \sin 30^\circ$$

$$\cos y = 1 - \frac{1}{2}$$

$$\cos y = \frac{1}{2}$$

$$\cos y = \cos 60^\circ$$

$$y = 60^\circ$$

(ii)

Given that $B = 90^\circ$

$$3 \tan A - 5 \cos B = \sqrt{3}$$

$$3 \tan A - 5 \cos 90^\circ = \sqrt{3}$$

$$3 \tan A - 0 = \sqrt{3}$$

$$\tan A = \frac{\sqrt{3}}{3}$$

$$\tan A = \frac{1}{\sqrt{3}}$$

$$\tan A = \tan 30^\circ$$

$$A = 30^\circ$$

Solution 9:

(i)

$$\cos x^\circ = \frac{10}{20}$$

$$\cos x^\circ = \frac{1}{2}$$

(ii)

$$\cos x^\circ = \frac{1}{2}$$

$$\cos x^\circ = \cos 60^\circ$$

$$x^\circ = 60^\circ$$

(iii)

$$\begin{aligned} \frac{1}{\tan^2 x^\circ} - \frac{1}{\sin^2 x^\circ} &= \frac{1}{\tan^2 60^\circ} - \frac{1}{\sin^2 60^\circ} \\ &= \frac{1}{(\sqrt{3})^2} - \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} \\ &= \frac{1}{3} - \frac{4}{3} \\ &= -1 \end{aligned}$$

(iv)

$$\begin{aligned} \tan x^\circ &= \tan 60^\circ \\ &= \sqrt{3} \end{aligned}$$

We know that $\tan x^\circ = \frac{AB}{BC}$

$$\Rightarrow \tan x^\circ = \frac{y}{10}$$

$$\Rightarrow y = 10 \tan x^\circ$$

$$\Rightarrow y = 10 \tan 60^\circ$$

$$\Rightarrow y = 10\sqrt{3}$$

Solution 10:

(i)

$$\tan \theta^{\circ} = \frac{5}{5} = 1$$

(ii)

$$\tan \theta^{\circ} = 1$$

$$\tan \theta^{\circ} = \tan 45^{\circ}$$

$$\theta^{\circ} = 45^{\circ}$$

(iii)

$$\begin{aligned}\sin^2 \theta^{\circ} - \cos^2 \theta^{\circ} &= \sin^2 45^{\circ} - \cos^2 45^{\circ} \\ &= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= 0\end{aligned}$$

(iv)

$$\sin \theta^{\circ} = \frac{5}{x}$$

$$\sin 45^{\circ} = \frac{5}{x}$$

$$x = \frac{5}{\sin 45^{\circ}}$$

$$x = \frac{5}{\frac{1}{\sqrt{2}}}$$

$$x = 5\sqrt{2}$$

Solution 11:

(i)

$$2 \sin A \cos A - \cos A - 2 \sin A + 1 = 0$$

$$2 \sin A \cos A - \cos A = 2 \sin A - 1$$

$$(2 \sin A - 1) \cos A - (2 \sin A - 1) = 0$$

$$(2 \sin A - 1) = 0 \text{ and } \cos A = 1$$

$$\sin A = \frac{1}{2} \text{ and } \cos A = \cos 0^\circ$$

$$A = 30^\circ \text{ and } A = 0^\circ$$

(ii)

$$\tan A - 2 \cos A \tan A + 2 \cos A - 1 = 0$$

$$\tan A - 2 \cos A \tan A = 1 - 2 \cos A$$

$$\tan A(1 - 2 \cos A) - (1 - 2 \cos A) = 0$$

$$(1 - 2 \cos A)(\tan A - 1) = 0$$

$$1 - 2 \cos A = 0 \text{ and } \tan A - 1 = 0$$

$$\cos A = \frac{1}{2} \text{ and } \tan A = 1$$

$$A = 60^\circ \text{ and } A = 45^\circ$$

(iii)

$$2 \cos^2 A - 3 \cos A + 1 = 0$$

$$2 \cos^2 A - \cos A - 2 \cos A + 1 = 0$$

$$\cos A(2 \cos A - 1) - (2 \cos A - 1) = 0$$

$$(2 \cos A - 1)(\cos A - 1) = 0$$

$$2 \cos A - 1 = 0 \text{ and } \cos A - 1 = 0$$

$$\cos A = \frac{1}{2} \text{ and } \cos A = 1$$

$$A = 60^\circ \text{ and } A = 0^\circ$$

(iv)

$$2 \tan 3A \cos 3A - \tan 3A + 1 = 2 \cos 3A$$

$$2 \tan 3A \cos 3A - \tan 3A = 2 \cos 3A - 1$$

$$\tan 3A(2 \cos 3A - 1) = 2 \cos 3A - 1$$

$$(2 \cos 3A - 1)(\tan 3A - 1) = 0$$

$$2 \cos 3A - 1 = 0 \text{ and } \tan 3A - 1 = 0$$

$$\cos 3A = \frac{1}{2} \text{ and } \tan 3A = 1$$

$$3A = 60^\circ \text{ and } 3A = 45^\circ$$

$$A = 20^\circ \text{ and } A = 15^\circ$$

Solution 12:

(i)

$$2 \cos 3x - 1 = 0$$

$$\cos 3x = \frac{1}{2}$$

$$3x = 60^\circ$$

$$x = 20^\circ$$

(ii)

$$\cos \frac{x}{3} - 1 = 0$$

$$\cos \frac{x}{3} = 1$$

$$\frac{x}{3} = 0^\circ$$

$$x = 0^\circ$$

(iii)

$$\sin (x + 10^\circ) = \frac{1}{2}$$

$$\sin (x + 10^\circ) = \sin 30^\circ$$

$$x + 10^\circ = 30^\circ$$

$$x = 20^\circ$$

(iv)

$$\cos (2x - 30^\circ) = 0$$

$$\cos (2x - 30^\circ) = \cos 90^\circ$$

$$2x - 30^\circ = 90^\circ$$

$$2x = 120^\circ$$

$$x = 60^\circ$$

(v)

$$2 \cos (3x - 15^\circ) = 1$$

$$\cos (3x - 15^\circ) = \frac{1}{2}$$

$$\cos (3x - 15^\circ) = \cos 60^\circ$$

$$3x - 15^\circ = 60^\circ$$

$$3x = 75^\circ$$

$$x = 25^\circ$$

(vi)

$$\tan^2 (x - 5^\circ) = 3$$

$$\tan (x - 5^\circ) = \sqrt{3}$$

$$\tan (x - 5^\circ) = \tan 60^\circ$$

$$x - 5^\circ = 60^\circ$$

$$x = 65^\circ$$

(vii)

$$3 \tan^2 (2x - 20^\circ) = 1$$

$$\tan (2x - 20^\circ) = \frac{1}{\sqrt{3}}$$

$$\tan (2x - 20^\circ) = \tan 30^\circ$$

$$2x - 20^\circ = 30^\circ$$

$$2x = 50^\circ$$

$$x = 25^\circ$$

(viii)

$$\cos\left(\frac{x}{2} + 10^\circ\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{x}{2} + 10^\circ\right) = \cos 30^\circ$$

$$\frac{x}{2} + 10^\circ = 30^\circ$$

$$x = 40^\circ$$

(ix)

$$\sin^2 x + \sin^2 30^\circ = 1$$

$$\sin^2 x = 1 - \sin^2 30^\circ$$

$$\sin^2 x = 1 - \frac{1}{4}$$

$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = 60^\circ$$

(x)

$$\cos^2 30^\circ + \cos^2 x = 1$$

$$\cos^2 x = 1 - \cos^2 30^\circ$$

$$\cos^2 x = 1 - \frac{3}{4}$$

$$\cos x = \frac{1}{2}$$

$$x = 60^\circ$$

(xi)

$$\cos^2 30^\circ + \sin^2 2x = 1$$

$$\sin^2 2x = 1 - \cos^2 30^\circ$$

$$\sin^2 2x = 1 - \frac{3}{4}$$

$$\sin 2x = \frac{1}{2}$$

$$2x = 30^\circ$$

$$x = 15^\circ$$

(xii)

$$\sin^2 60^\circ + \cos^2 (3x - 9^\circ) = 1$$

$$\cos^2 (3x - 9^\circ) = 1 - \sin^2 60^\circ$$

$$\cos^2 (3x - 9^\circ) = 1 - \frac{3}{4}$$

$$\cos^2 (3x - 9^\circ) = \frac{1}{4}$$

$$\cos (3x - 9^\circ) = \frac{1}{2}$$

$$3x - 9^\circ = 60^\circ$$

$$3x = 69^\circ$$

$$x = 23^\circ$$

Solution 13:

(i)

$$4 \cos^2 x = 3$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = 30^\circ$$

(ii)

$$\begin{aligned}\cos^2 x + \cot^2 x &= \cos^2 30^\circ + \cot^2 30^\circ \\ &= \frac{3}{4} + 3 \\ &= \frac{15}{4} \\ &= 3\frac{3}{4}\end{aligned}$$

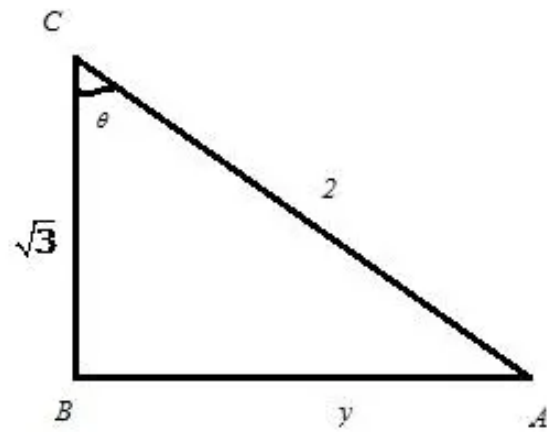
(iii)

$$\cos 3x = \cos 3(30^\circ) = \cos 90^\circ = 0$$

(iv)

$$\sin 2x = \sin 2(30^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

Solution 14:



(i)

From $\triangle ABC$

$$\sin x^\circ = \frac{\sqrt{3}}{2}$$

(ii)

$$\sin x^\circ = \frac{\sqrt{3}}{2}$$

$$\sin x^\circ = \sin 60^\circ$$

$$x^\circ = 60^\circ$$

(iii)

$$\begin{aligned}\tan x^\circ &= \tan 60^\circ \\ &= \sqrt{3}\end{aligned}$$

(iv)

$$\cos x^\circ = \frac{y}{2}$$

$$\cos 60^\circ = \frac{y}{2}$$

$$\frac{1}{2} = \frac{y}{2}$$

$$y = 1$$

Solution 15:

$$2 \cos (A + B) = 1$$

$$\cos (A + B) = \frac{1}{2}$$

$$\cos (A + B) = \cos 60^{\circ}$$

$$A + B = 60^{\circ} \quad \dots\dots\dots(1)$$

$$2 \sin (A - B) = 1$$

$$\sin (A - B) = \frac{1}{2}$$

$$A - B = 30^{\circ} \quad \dots\dots\dots(2)$$

Adding (1) and (2)

$$A + B + A - B = 60^{\circ} + 30^{\circ}$$

$$2A = 90^{\circ}$$

$$A = 45^{\circ}$$

$$A + B = 60^{\circ}$$

$$B = 60^{\circ} - A$$

$$B = 60^{\circ} - 45^{\circ}$$

$$B = 15^{\circ}$$