

POLYNOMIAL EXPRESSIONS

A polynomial expression $S(x)$ in one variable x is an algebraic expression in x term as

$$S(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + ax + a_0$$

Where $a_n, a_{n-1}, \dots, a, a_0$ are constant and real numbers and a_n is not equal to zero.

Some Important points to Note:

S.no	Points
1	$a_n, a_{n-1}, a_{n-2}, \dots, a_1, a_0$ are called the coefficients for $x^n, x^{n-1}, \dots, x^1, x^0$
2	n is called the degree of the polynomial
3	when $a_n, a_{n-1}, a_{n-2}, \dots, a_1, a_0$ all are zero, it is called zero polynomial
4	A constant polynomial is the polynomial with zero degree, it is a constant value polynomial
5	A polynomial of one item is called monomial, two items binomial and three items as trinomial
6	A polynomial of one degree is called linear polynomial, two degree as quadratic polynomial and degree three as cubic polynomial

Important Concepts on Polynomial:

Concept	Description
Zero's or roots of the polynomial	It is a solution to the polynomial equation $S(x)=0$ i.e. a number "a" is said to be a zero of a polynomial if $S(a) = 0$. If we draw the graph of $S(x) = 0$, the values where the curve cuts the X-axis are called Zeroes of the polynomial
Remainder Theorem's	If $p(x)$ is an polynomial of degree greater than or equal to 1 and $p(x)$ is divided by the expression $(x-a)$, then the remainder will be $p(a)$
Factor's Theorem's	If $x-a$ is a factor of polynomial $p(x)$ then $p(a)=0$ or if $p(a) = 0, x-a$ is the factor the polynomial $p(x)$

Geometric Meaning of the Zeroes of the Polynomial:

Let's us assume

$Y = p(x)$ where $p(x)$ is the polynomial of any form.

Now we can plot the equation $y = p(x)$ on the Cartesian plane by taking various values of x and y obtained by putting the values. The plot or graph obtained can be of any shapes.

The zeroes of the polynomial are the points where the graph meet x axis in the Cartesian plane. If the graph does not meet x axis, then the polynomial does not have any zero's.

Let us take some useful polynomial and shapes obtained on the Cartesian plane

S.no	$y = p(x)$	Graph obtained	Name of the graph	Name of the equation
1	$y = ax + b$ where a and b can be any values ($a \neq 0$) Example $y = 2x + 3$		Straight line. It intersect the x -axis at $(-b/a, 0)$ Example $(-3/2, 0)$	Linear polynomial
2	$y = ax^2 + bx + c$ where $b^2 - 4ac > 0$ and $a \neq 0$ and $a > 0$ Example $y = x^2 - 7x + 12$		Parabola It intersect the x -axis at two points Example $(3, 0)$ and $(4, 0)$	Quadratic polynomial
3	$y = ax^2 + bx + c$ where $b^2 - 4ac > 0$ and $a \neq 0$ and $a < 0$ Example $y = -x^2 + 2x + 8$		Parabola It intersect the x -axis at two points Example $(-2, 0)$ and $(4, 0)$	Quadratic polynomial
4	$y = ax^2 + bx + c$ where $b^2 - 4ac = 0$ and $a \neq 0$ and $a > 0$ Example $y = (x - 2)^2$		Parabola It intersect the x -axis at one points	Quadratic polynomial

5	$y = ax^2 + bx + c$ where $b^2 - 4ac < 0$ and $a \neq 0$ $a > 0$ Example $y = x^2 - 2x + 6$		Parabola It does not intersect the x-axis It has no zero's	Quadratic polynomial
6	$y = ax^2 + bx + c$ where $b^2 - 4ac < 0$ and $a \neq 0$ $a < 0$ Example $y = -x^2 - 2x - 6$		Parabola It does not intersect the x-axis It has no zero's	Quadratic polynomial
7	$y = ax^3 + bx^2 + cx + d$ where $a \neq 0$	It can be of any shape 	It will cut the x-axis at the most 3 times	Cubic Polynomial
8	$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$ Where $a_n \neq 0$	It can be of any shape 	It will cut the x-axis at the most n times	Polynomial of n degree

Relation between coefficient and zeros of the polynomial:

S.no	Type of Polynomial	General form	Zero's	Relationship between Zero's and coefficients
1	Linear polynomial	$ax+b, a \neq 0$	1	$k = \frac{-\text{constant term}}{\text{Coefficient of } x}$
2	Quadratic	$ax^2+bx+c, a \neq 0$	2	$k_1 + k_2 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$ $k_1 k_2 = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$
3	Cubic	$ax^3+bx^2+cx+d, a \neq 0$	3	$k_1 + k_2 + k_3 = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$ $k_1 k_2 k_3 = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$ $k_1 k_2 + k_2 k_3 + k_1 k_3 = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

Formation of polynomial when the zeroes are given:

Type of polynomial	Zero's	Polynomial Formed
Linear	$k=a$	$(x-a)$
Quadratic	$k_1=a$ and $k_2=b$	$(x-a)(x-b)$ Or $x^2-(a+b)x+ab$ Or $x^2-(\text{Sum of the zero's})x + \text{product of the zero's}$
Cubic	$k_1=a, k_2=b$ and $k_3=c$	$(x-a)(x-b)(x-c)$

Division algorithm for polynomial:

Let's $p(x)$ and $q(x)$ are any two polynomial with $q(x) \neq 0$, then we can find polynomial $s(x)$ and $r(x)$ such that

$$P(x) = s(x) q(x) + r(x)$$

Where $r(x)$ can be zero or degree of $r(x) < \text{degree of } g(x)$

$\text{Dividend} = \text{Quotient} \times \text{Divisor} + \text{Remainder}$
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