

Chapter 25. Complementary Angles

Exercise 25(A)

Solution 1(i):

$$\frac{\cos 22^\circ}{\sin 68^\circ} = \frac{\cos(90^\circ - 68^\circ)}{\sin 68^\circ} = \frac{\sin 68^\circ}{\sin 68^\circ} = 1$$

Solution 1(ii):

$$\frac{\tan 47^\circ}{\cot 43^\circ} = \frac{\tan(90^\circ - 43^\circ)}{\cot 43^\circ} = \frac{\cot 43^\circ}{\cot 43^\circ} = 1$$

Solution 1(iii):

$$\frac{\sec 75^\circ}{\operatorname{cosec} 15^\circ} = \frac{\sec(90^\circ - 15^\circ)}{\operatorname{cosec} 15^\circ} = \frac{\operatorname{cosec} 15^\circ}{\operatorname{cosec} 15^\circ} = 1$$

Solution 1(iv):

$$\begin{aligned} & \frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ} \\ &= \frac{\cos(90^\circ - 35^\circ)}{\sin 35^\circ} + \frac{\cot(90^\circ - 55^\circ)}{\tan 55^\circ} \\ &= \frac{\sin 35^\circ}{\sin 35^\circ} + \frac{\tan 55^\circ}{\tan 55^\circ} \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

Solution 1(v):

$$\begin{aligned} & \sin^2 40^\circ - \cos^2 50^\circ \\ &= \sin^2(90^\circ - 50^\circ) - \cos^2 50^\circ \\ &= \cos^2 50^\circ - \cos^2 50^\circ \\ &= 0 \end{aligned}$$

Solution 1(vi):

$$\begin{aligned} & \sec^2 18^\circ - \operatorname{cosec}^2 72^\circ \\ &= [\sec(90^\circ - 72^\circ)]^2 - \operatorname{cosec}^2 72^\circ \\ &= \operatorname{cosec}^2 72^\circ - \operatorname{cosec}^2 72^\circ \\ &= 0 \end{aligned}$$

Solution 1(vii):

$$\begin{aligned} & \sin 15^\circ \cos 15^\circ - \cos 75^\circ \sin 75^\circ \\ &= \sin(90^\circ - 75^\circ) \cos 15^\circ - \cos 75^\circ \sin(90^\circ - 15^\circ) \\ &= \cos 75^\circ \cos 15^\circ - \cos 75^\circ \cos 15^\circ \\ &= 0 \end{aligned}$$

Solution 1(viii):

$$\begin{aligned}
& \sin 42^\circ \sin 48^\circ - \cos 42^\circ \cos 48^\circ \\
&= \sin(90^\circ - 48^\circ) \sin 48^\circ - \cos(90^\circ - 48^\circ) \cos 48^\circ \\
&= \cos 48^\circ \sin 48^\circ - \sin 48^\circ \cos 48^\circ \\
&= \cos 48^\circ \sin 48^\circ - \cos 48^\circ \sin 48^\circ \\
&= 0
\end{aligned}$$

Solution 2(i):

$$\begin{aligned}
& \sin(90^\circ - A) \sin A - \cos(90^\circ - A) \cos A \\
&= \cos A \sin A - \sin A \cos A \\
&= 0
\end{aligned}$$

Solution 2(ii):

$$\begin{aligned}
& \sin^2 35^\circ - \cos^2 55^\circ \\
&= \sin^2 35^\circ - [\cos(90^\circ - 35^\circ)]^2 \\
&= \sin^2 35^\circ - \sin^2 35^\circ \\
&= 0
\end{aligned}$$

Solution 2(iii):

$$\begin{aligned}
& \frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2 \\
&= \frac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} - 2 \\
&= \frac{\tan 36^\circ}{\tan 36^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2 \\
&= 1 + 1 - 2 \\
&= 2 - 2 \\
&= 0
\end{aligned}$$

Solution 2(iv):

$$\begin{aligned}
& \frac{2 \tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ} \\
&= \frac{2 \tan(90^\circ - 37^\circ)}{\cot 37^\circ} - \frac{\cot(90^\circ - 10^\circ)}{\tan 10^\circ} \\
&= \frac{2 \cot 37^\circ}{\cot 37^\circ} - \frac{\tan 10^\circ}{\tan 10^\circ} \\
&= 2 - 1 \\
&= 1
\end{aligned}$$

Solution 2(v):

$$\begin{aligned}
& \cos^2 25^\circ - \sin^2 65^\circ - \tan^2 45^\circ \\
&= [\cos(90^\circ - 65^\circ)]^2 - \sin^2 65^\circ - (\tan 45^\circ)^2 \\
&= \sin^2 65^\circ - \sin^2 65^\circ - (1)^2 \\
&= 0 - 1 \\
&= -1
\end{aligned}$$

Solution 2(vi):

$$\begin{aligned}
& \left(\frac{\sin 77^\circ}{\cos 13^\circ} \right)^2 + \left(\frac{\cos 77^\circ}{\sin 13^\circ} \right)^2 - 2 \cos^2 45^\circ \\
&= \left(\frac{\sin (90^\circ - 13^\circ)}{\cos 13^\circ} \right)^2 + \left(\frac{\cos (90^\circ - 13^\circ)}{\sin 13^\circ} \right)^2 - 2 (\cos 45^\circ)^2 \\
&= \left(\frac{\cos 13^\circ}{\cos 13^\circ} \right)^2 + \left(\frac{\sin 13^\circ}{\sin 13^\circ} \right)^2 - 2 \left(\frac{1}{\sqrt{2}} \right)^2 \\
&= (1)^2 + (1)^2 - 2 \times \frac{1}{2} \\
&= 1 + 1 - 1 \\
&= 1
\end{aligned}$$

Solution 3(i):

L.H.S.

$$\begin{aligned}
&= \tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ \\
&= \tan (90^\circ - 80^\circ) \tan (90^\circ - 75^\circ) \tan 75^\circ \tan 80^\circ \\
&= \cot 80^\circ \cot 75^\circ \tan 75^\circ \tan 80^\circ \\
&= (\cot 80^\circ \tan 80^\circ)(\cot 75^\circ \tan 75^\circ) \\
&= (1)(1) \\
&= 1 \\
&= \text{R.H.S.}
\end{aligned}$$

Solution 3(ii):

L.H.S.

$$\begin{aligned}
&= \sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ \\
&= \sin(90^\circ - 48^\circ) \times \frac{1}{\cos 48^\circ} + \cos(90^\circ - 48^\circ) \times \frac{1}{\sin 48^\circ} \\
&= \cos 48^\circ \times \frac{1}{\cos 48^\circ} + \sin 48^\circ \times \frac{1}{\sin 48^\circ} \\
&= 1 + 1 \\
&= 2 \\
&= \text{R.H.S.}
\end{aligned}$$

Solution 4:

$$\begin{aligned} & \text{(i) } \sin 59^\circ + \tan 63^\circ \\ &= \sin(90 - 31)^\circ + \tan(90 - 27)^\circ \\ &= \cos 31^\circ + \cot 27^\circ \\ & \text{(ii) } \operatorname{cosec} 68^\circ + \cot 72^\circ \\ &= \operatorname{cosec} (90 - 22)^\circ + \cot(90 - 18)^\circ \\ &= \sec 22^\circ + \tan 18^\circ \\ & \text{(iii) } \cos 74^\circ + \sec 67^\circ \\ &= \cos(90 - 16)^\circ + \sec(90 - 23)^\circ \\ &= \sin 16^\circ + \operatorname{cosec} 23^\circ \end{aligned}$$

Solution 5:

(i) We know that for a triangle ΔABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\frac{\angle B + \angle A}{2} = 90^\circ - \frac{\angle C}{2}$$

$$\begin{aligned} \sin\left(\frac{A+B}{2}\right) &= \sin\left(90^\circ - \frac{C}{2}\right) \\ &= \cos\left(\frac{C}{2}\right) \end{aligned}$$

(ii) We know that for a triangle ΔABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle C = 180^\circ - \angle A$$

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$$\begin{aligned} \tan\left(\frac{B+C}{2}\right) &= \tan\left(90^\circ - \frac{A}{2}\right) \\ &= \cot\left(\frac{A}{2}\right) \end{aligned}$$

Solution 6:

(i)

$$\begin{aligned}
& 3 \frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ} \\
&= 3 \frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ} \\
&= 3 \frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} = 3 - 1 = 2
\end{aligned}$$

(ii) $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$

$$\begin{aligned}
&= 3 \cos(90^\circ - 10^\circ) \operatorname{cosec} 10^\circ + 2 \cos(90^\circ - 31^\circ) \operatorname{cosec} 31^\circ \\
&= 3 \sin 10^\circ \operatorname{cosec} 10^\circ + 2 \sin 31^\circ \operatorname{cosec} 31^\circ \\
&= 3 + 2 = 5
\end{aligned}$$

(iii) $\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$

$$\begin{aligned}
&= \frac{\sin(90^\circ - 10^\circ)}{\cos 10^\circ} + \sin(90^\circ - 31^\circ) \sec 31^\circ \\
&= \frac{\cos 10^\circ}{\cos 10^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ} \\
&= 1 + 1 = 2
\end{aligned}$$

(iv) $\tan(55^\circ - A) - \cot(35^\circ + A)$

$$\begin{aligned}
&= \tan[90^\circ - (35^\circ + A)] - \cot(35^\circ + A) \\
&= \cot(35^\circ + A) - \cot(35^\circ + A) \\
&= 0
\end{aligned}$$

(v) $\operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A)$

$$\begin{aligned}
&= \operatorname{cosec}[90^\circ - (25^\circ - A)] - \sec(25^\circ - A) \\
&= \sec(25^\circ - A) - \sec(25^\circ - A) \\
&= 0
\end{aligned}$$

(vi) $2 \frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2} \cos 45^\circ$

$$\begin{aligned}
&= 2 \frac{\tan(90^\circ - 33^\circ)}{\cot 33^\circ} - \frac{\cot(90^\circ - 20^\circ)}{\tan 20^\circ} - \sqrt{2} \left(\frac{1}{\sqrt{2}} \right) \\
&= 2 \frac{\cot 33^\circ}{\cot 33^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 1 \\
&= 2 - 1 - 1 \\
&= 0
\end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad & \frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2 \frac{\sin^2 75^\circ}{\cos^2 15^\circ} \\
 &= \frac{[\cot(90^\circ - 49^\circ)]^2}{\tan^2 49^\circ} - 2 \frac{[\sin(90^\circ - 15^\circ)]^2}{\cos^2 15^\circ} \\
 &= \frac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2 \frac{\cos^2 15^\circ}{\cos^2 15^\circ} \\
 &= 1 - 2 = -1
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad & \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ \\
 &= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8 \left(\frac{1}{2}\right)^2 \\
 &= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 2 \\
 &= 1 + 1 - 2 = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(ix)} \quad & 14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ \\
 &= 14 \left(\frac{1}{2}\right) + 6 \left(\frac{1}{2}\right) - 5(1) \\
 &= 7 + 3 - 5 = 5
 \end{aligned}$$

Solution 7:

Since $\triangle ABC$ is a right-angled triangle, right-angled at B,

$$A + C = 90^\circ$$

$$\begin{aligned}
 \therefore & \frac{\sec A \cdot \sin C - \tan A \cdot \tan C}{\sin B} \\
 &= \frac{\sec A (90^\circ - C) \sin C - \tan (90^\circ - C) \tan C}{\sin 90^\circ} \\
 &= \frac{\operatorname{cosec} C \sin C - \cot C \tan C}{1} \\
 &= \frac{1}{\sin C} \times \sin C - \frac{1}{\tan C} \times \tan C \\
 &= 1 - 1 \\
 &= 0
 \end{aligned}$$

Solution 8(i):

$$\begin{aligned}
 & \sin(90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1 \\
 \Rightarrow & \sin(90^\circ - 3A) = \frac{1}{\operatorname{cosec} 42^\circ} \\
 \Rightarrow & \cos 3A = \frac{1}{\operatorname{cosec}(90^\circ - 48^\circ)} \\
 \Rightarrow & \cos 3A = \frac{1}{\sec 48^\circ} \\
 \Rightarrow & \cos 3A = \cos 48^\circ \\
 \Rightarrow & 3A = 48^\circ \\
 \Rightarrow & A = 16^\circ
 \end{aligned}$$

Solution 8(ii):

$$\cos (90^{\circ}-3 A) . \sec 77^{\circ}=1$$

$$\Rightarrow \cos \left(90^{\circ}-3 A\right)=\frac{1}{\sec 77^{\circ}}$$

$$\Rightarrow \sin 3 A=\frac{1}{\sec \left(90^{\circ}-12^{\circ}\right)}$$

$$\Rightarrow \sin 3 A=\frac{1}{\operatorname{cosec} 12^{\circ}}$$

$$\Rightarrow \sin 3 A=\sin 12^{\circ}$$

$$\Rightarrow 3 A=12^{\circ}$$

$$\Rightarrow A=3^{\circ}$$