

5. Trigonometric Functions

Exercise 5.1

1. Question

Prove the following identities

$$\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$$

Answer

$$\text{LHS} = \sec^4 x - \sec^2 x$$

$$= (\sec^2 x)^2 - \sec^2 x$$

$$\text{We know } \sec^2 \theta = 1 + \tan^2 \theta.$$

$$= (1 + \tan^2 x)^2 - (1 + \tan^2 x)$$

$$= 1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$$

$$= \tan^4 x + \tan^2 x = \text{RHS}$$

Hence proved.

2. Question

Prove the following identities

$$\sin^6 x + \cos^6 x = 1 - 3 \sin^2 x \cos^2 x$$

Answer

$$\text{LHS} = \sin^6 x + \cos^6 x$$

$$= (\sin^2 x)^3 + (\cos^2 x)^3$$

$$\text{We know that } a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$= (\sin^2 x + \cos^2 x)[(\sin^2 x)^2 + (\cos^2 x)^2 - \sin^2 x \cos^2 x]$$

$$\text{We know that } \sin^2 x + \cos^2 x = 1 \text{ and } a^2 + b^2 = (a + b)^2 - 2ab$$

$$= 1 \times [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x]$$

$$= 1^2 - 3\sin^2 x \cos^2 x$$

$$= 1 - 3\sin^2 x \cos^2 x = \text{RHS}$$

Hence proved.

3. Question

Prove the following identities

$$(\operatorname{cosec} x - \sin x)(\sec x - \cos x)(\tan x + \cot x) = 1$$

Answer

$$\text{LHS} = (\operatorname{cosec} x - \sin x)(\sec x - \cos x)(\tan x + \cot x)$$

$$\text{We know that } \operatorname{cosec} \theta = \frac{1}{\sin \theta}; \sec \theta = \frac{1}{\cos \theta}; \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$= \left(\frac{1}{\sin x} - \sin x \right) \left(\frac{1}{\cos x} - \cos x \right) \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$= \frac{1 - \sin^2 x}{\sin x} \times \frac{1 - \cos^2 x}{\cos x} \times \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$\begin{aligned} &= \frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos x} \times \frac{1}{\sin x \cos x} \\ &= 1 = \text{RHS} \end{aligned}$$

Hence proved.

4. Question

Prove the following identities

$$\operatorname{cosec} x (\sec x - 1) - \cot x (1 - \cos x) = \tan x - \sin x$$

Answer

$$\text{LHS} = \operatorname{cosec} x (\sec x - 1) - \cot x (1 - \cos x)$$

$$\text{We know that } \operatorname{cosec} \theta = \frac{1}{\sin \theta}; \sec \theta = \frac{1}{\cos \theta}; \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\begin{aligned} &= \frac{1}{\sin x} \left(\frac{1}{\cos x} - 1 \right) - \frac{\cos x}{\sin x} (1 - \cos x) \\ &= \frac{1}{\sin x} \left(\frac{1 - \cos x}{\cos x} \right) - \frac{\cos x}{\sin x} (1 - \cos x) \\ &= \left(\frac{1 - \cos x}{\sin x} \right) \left(\frac{1}{\cos x} - \cos x \right) \\ &= \left(\frac{1 - \cos x}{\sin x} \right) \left(\frac{1 - \cos^2 x}{\cos x} \right) \end{aligned}$$

We know that $1 - \cos^2 x = \sin^2 x$.

$$\begin{aligned} &= \left(\frac{1 - \cos x}{\sin x} \right) \left(\frac{\sin^2 x}{\cos x} \right) \\ &= (1 - \cos x) \left(\frac{\sin x}{\cos x} \right) \\ &= \frac{\sin x}{\cos x} - \sin x \\ &= \tan x - \sin x \\ &= \text{RHS} \end{aligned}$$

Hence proved.

5. Question

Prove the following identities

$$\frac{1 - \sin x \cos x}{\cos x (\sec x - \operatorname{cosec} x)} \cdot \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} = \sin x$$

Answer

$$\text{LHS} = \frac{1 - \sin x \cos x}{\cos x (\sec x - \operatorname{cosec} x)} \times \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x}$$

$$\text{We know that } \operatorname{cosec} \theta = \frac{1}{\sin \theta}; \sec \theta = \frac{1}{\cos \theta}$$

$$\begin{aligned} &= \frac{1 - \sin x \cos x}{\cos x \left(\frac{1}{\cos x} - \frac{1}{\sin x} \right)} \times \frac{(\sin x)^2 - (\cos x)^2}{(\sin x)^3 + (\cos x)^3} \end{aligned}$$

We know that $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\begin{aligned}
 &= \frac{1 - \sin x \cos x}{\cos x \left(\frac{\sin x - \cos x}{\cos x \sin x} \right)} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]} \\
 &= \frac{\sin x (1 - \sin x \cos x)}{\sin x - \cos x} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]} \\
 &= \frac{\sin x (1 - \sin x \cos x)}{1} \times \frac{1}{[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}
 \end{aligned}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$= \sin x (1 - \sin x \cos x) \times \frac{1}{(1 - \sin x \cos x)}$$

$$= \sin x$$

$$= \text{RHS}$$

Hence proved.

6. Question

Prove the following identities

$$\frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x} = (\sec x \operatorname{cosec} x + 1)$$

Answer

$$\text{LHS} = \frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x}$$

We know that $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta}$

$$= \frac{\frac{\sin x}{\cos x}}{1 - \frac{\cos x}{\sin x}} + \frac{\frac{\cos x}{\sin x}}{1 - \frac{\sin x}{\cos x}}$$

$$= \frac{\frac{\sin x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}} + \frac{\frac{\cos x}{\sin x}}{\frac{\cos x - \sin x}{\cos x}}$$

$$= \frac{\sin^2 x}{\cos x (\sin x - \cos x)} - \frac{\cos^2 x}{\sin x (\sin x - \cos x)}$$

$$= \frac{\sin^3 x - \cos^3 x}{\sin x \cos x (\sin x - \cos x)}$$

We know that $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$= \frac{(\sin x - \cos x)[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x \cos x (\sin x - \cos x)}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$= \frac{[1 + \sin x \cos x]}{\sin x \cos x}$$

$$= \frac{1}{\sin x \cos x} + \frac{\sin x \cos x}{\sin x \cos x}$$

$$= \frac{1}{\sin x} \times \frac{1}{\cos x} + 1$$

$$\text{We know that } \operatorname{cosec} \theta = \frac{1}{\sin \theta}; \sec \theta = \frac{1}{\cos \theta}$$

$$= \operatorname{cosec} x \times \sec x + 1$$

$$= \sec x \operatorname{cosec} x + 1$$

$$= \text{RHS}$$

Hence proved.

7. Question

Prove the following identities

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 2$$

Answer

$$\text{LHS} = \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$$

$$\text{We know that } a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

$$= \frac{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}{\sin x + \cos x} + \frac{(\sin x - \cos x)[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x - \cos x}$$

$$\text{We know that } \sin^2 x + \cos^2 x = 1.$$

$$= 1 - \sin x \cos x + 1 + \sin x \cos x$$

$$= 2$$

$$= \text{RHS}$$

Hence proved.

8. Question

Prove the following identities

$$(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2 = 1$$

Answer

$$\text{LHS} = (\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$$

$$= [(\sec x \sec y)^2 + (\tan x \tan y)^2 + 2(\sec x \sec y)(\tan x \tan y)] - [(\sec x \tan y)^2 + (\tan x \sec y)^2 + 2(\sec x \tan y)(\tan x \sec y)]$$

$$= [\sec^2 x \sec^2 y + \tan^2 x \tan^2 y + 2(\sec x \sec y)(\tan x \tan y)] - [\sec^2 x \tan^2 y + \tan^2 x \sec^2 y + 2(\sec^2 x \tan^2 y)(\tan x \sec y)]$$

$$= \sec^2 x \sec^2 y - \sec^2 x \tan^2 y + \tan^2 x \tan^2 y - \tan^2 x \sec^2 y$$

$$= \sec^2 x (\sec^2 y - \tan^2 y) + \tan^2 x (\tan^2 y - \sec^2 y)$$

$$= \sec^2 x (\sec^2 y - \tan^2 y) - \tan^2 x (\sec^2 y - \tan^2 y)$$

$$\text{We know that } \sec^2 x - \tan^2 x = 1.$$

$$= \sec^2 x \times 1 - \tan^2 x \times 1$$

$$= \sec^2 x - \tan^2 x$$

$$= 1$$

= RHS

Hence proved.

9. Question

Prove the following identities

$$\frac{\cos x}{1 - \sin x} = \frac{1 + \cos x + \sin x}{1 + \cos x - \sin x}$$

Answer

$$\begin{aligned}\text{RHS} &= \frac{1 + \cos x + \sin x}{1 + \cos x - \sin x} \\&= \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) - (\sin x)} \\&= \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) - (\sin x)} \times \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) + (\sin x)} \\&= \frac{[(1 + \cos x) + (\sin x)]^2}{(1 + \cos x)^2 - (\sin x)^2} \\&= \frac{(1 + \cos x)^2 + (\sin x)^2 + 2(1 + \cos x)(\sin x)}{(1 + \cos^2 x + 2 \cos x) - (\sin^2 x)} \\&= \frac{1 + \cos^2 x + 2 \cos x + \sin^2 x + 2 \sin x + 2 \sin x \cos x}{1 + \cos^2 x + 2 \cos x - \sin^2 x}\end{aligned}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$= \frac{1 + 1 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{(1 - \sin^2 x) + \cos^2 x + 2 \cos x}$$

We know that $1 - \cos^2 x = \sin^2 x$.

$$\begin{aligned}&= \frac{2 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{\cos^2 x + \cos^2 x + 2 \cos x} \\&= \frac{2 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{2 \cos^2 x + 2 \cos x} \\&= \frac{2 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{\cos^2 x + \cos^2 x + 2 \cos x} \\&= \frac{1 + \cos x + \sin x + \sin x \cos x}{\cos x (\cos x + 1)} \\&= \frac{1(1 + \cos x) + \sin x (\cos x + 1)}{\cos x (\cos x + 1)} \\&= \frac{(1 + \sin x)(\cos x + 1)}{\cos x (\cos x + 1)} \\&= \frac{1 + \sin x}{\cos x} \times \frac{\cos x}{\cos x} \\&= \frac{(1 + \sin x) \cos x}{\cos^2 x}\end{aligned}$$

We know that $1 - \sin^2 x = \cos^2 x$.

$$= \frac{(1 + \sin x) \cos x}{1 - \sin^2 x}$$

$$\begin{aligned}
&= \frac{(1 + \sin x) \cos x}{(1 - \sin x)(1 + \sin x)} \\
&= \frac{\cos x}{1 - \sin x} \\
&= \text{LHS}
\end{aligned}$$

Hence proved.

10. Question

Prove the following identities

$$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x} = \frac{1 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

Answer

$$\text{LHS} = \frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x}$$

We know that $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \text{cosec}^2 x$

$$\begin{aligned}
&= \frac{\tan^3 x}{\sec^2 x} + \frac{\cot^3 x}{\text{cosec}^2 x} \\
&= \frac{\frac{\sin^3 x}{\cos^3 x}}{\frac{1}{\cos^2 x}} + \frac{\frac{\cos^3 x}{\sin^3 x}}{\frac{1}{\sin^2 x}} \\
&= \frac{\sin^3 x}{\cos x} + \frac{\cos^3 x}{\sin x} \\
&= \frac{\sin^4 x + \cos^4 x}{\cos x \sin x} \\
&= \frac{(\sin^2 x)^2 + (\cos^2 x)^2}{\cos x \sin x}
\end{aligned}$$

We know that $a^2 + b^2 = (a + b)^2 - 2ab$

$$= \frac{(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$\begin{aligned}
&= \frac{1^2 - 2 \sin^2 x \cos^2 x}{\sin x \cos x} \\
&= \frac{1 - 2 \sin^2 x \cos^2 x}{\sin x \cos x} \\
&= \text{RHS}
\end{aligned}$$

Hence proved.

11. Question

Prove the following identities

$$1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x} = \sin x \cos x$$

Answer

$$\text{LHS} = 1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x}$$

$$\text{We know that } \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$= 1 - \frac{\sin^2 x}{1 + \frac{\cos x}{\sin x}} - \frac{\cos^2 x}{1 + \frac{\sin x}{\cos x}}$$

$$= 1 - \frac{\sin^3 x}{\sin x + \cos x} - \frac{\cos^3 x}{\sin x + \cos x}$$

$$= \frac{\sin x + \cos x - (\sin^3 x + \cos^3 x)}{\sin x + \cos x}$$

$$\text{We know that } a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$= \frac{\sin x + \cos x - ((\sin x + \cos x)(\sin x)^2 + (\cos x)^2 - \sin x \cos x))}{\sin x + \cos x}$$

$$= \frac{(\sin x + \cos x)(1 - \sin^2 x - \cos^2 x + \sin x \cos x)}{\sin x + \cos x}$$

$$= 1 - (\sin^2 x + \cos^2 x) + \sin x \cos x$$

$$\text{We know that } \sin^2 x + \cos^2 x = 1.$$

$$= 1 - 1 + \sin x \cos x$$

$$= \sin x \cos x$$

$$= \text{RHS}$$

Hence proved.

12. Question

Prove the following identities

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{\operatorname{cosec}^2 x - \sin^2 x} \right) \sin^2 x \cos^2 x = \frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$$

Answer

$$\text{LHS} = \left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{\operatorname{cosec}^2 x - \sin^2 x} \right) \sin^2 x \cos^2 x$$

$$\text{We know that } \operatorname{cosec} \theta = \frac{1}{\sin \theta}; \sec \theta = \frac{1}{\cos \theta}$$

$$= \left(\frac{1}{\frac{1}{\cos^2 x} - \cos^2 x} + \frac{1}{\frac{1}{\sin^2 x} - \sin^2 x} \right) \sin^2 x \cos^2 x$$

$$= \left(\frac{\cos^2 x}{1 - \cos^4 x} + \frac{\sin^2 x}{1 - \sin^4 x} \right) \sin^2 x \cos^2 x$$

$$= \left(\frac{\cos^2 x(1 - \sin^4 x) + \sin^2 x(1 - \cos^4 x)}{(1 - \cos^4 x)(1 - \sin^4 x)} \right) \sin^2 x \cos^2 x$$

$$= \left(\frac{\cos^2 x - \cos^2 x \sin^4 x + \sin^2 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x)(1 - \sin^2 x)(1 + \cos^2 x)(1 - \cos^2 x)} \right) \sin^2 x \cos^2 x$$

$$\text{We know that } \sin^2 x + \cos^2 x = 1.$$

$$\begin{aligned}
&= \left(\frac{1 - \cos^2 x \sin^4 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x) \cos^2 x (1 + \cos^2 x) \sin^2 x} \right) \sin^2 x \cos^2 x \\
&= \left(\frac{1 - \cos^2 x \sin^2 x (\sin^2 x + \cos^2 x)}{(1 + \sin^2 x)(1 + \cos^2 x)} \right) \\
&= \left(\frac{1 - \cos^2 x \sin^2 x}{2 + \sin^2 x \cos^2 x} \right) \\
&= \text{RHS}
\end{aligned}$$

Hence proved.

13. Question

Prove the following identities

$$(1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2 = \sec^2 \alpha \sec^2 \beta$$

Answer

$$\begin{aligned}
\text{LHS} &= (1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2 \\
&= 1 + \tan^2 \alpha \tan^2 \beta + 2 \tan \alpha \tan \beta + \tan^2 \alpha + \tan^2 \beta - 2 \tan \alpha \tan \beta \\
&= 1 + \tan^2 \alpha \tan^2 \beta + \tan^2 \alpha + \tan^2 \beta \\
&= \tan^2 \alpha (\tan^2 \beta + 1) + 1 (1 + \tan^2 \beta) \\
&= (1 + \tan^2 \beta) (1 + \tan^2 \alpha)
\end{aligned}$$

$$\text{We know that } 1 + \tan^2 \theta = \sec^2 \theta$$

$$\begin{aligned}
&= \sec^2 \alpha \sec^2 \beta \\
&= \text{RHS}
\end{aligned}$$

Hence proved.

14. Question

Prove the following identities

$$\frac{(1 + \cot x + \tan x)(\sin x - \cos x)}{\sec^3 x - \operatorname{cosec}^3 x} = \sin^2 x \cos^2 x$$

Answer

$$\text{LHS} = \frac{(1 + \cot x + \tan x)(\sin x - \cos x)}{\sec^3 x - \operatorname{cosec}^3 x}$$

$$\text{We know that } \operatorname{cosec} \theta = \frac{1}{\sin \theta}; \sec \theta = \frac{1}{\cos \theta}; \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\begin{aligned}
&= \frac{\left(1 + \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)(\sin x - \cos x)}{\frac{1}{\cos^3 x} - \frac{1}{\sin^3 x}} \\
&= \frac{(\sin x \cos x + \cos^2 x + \sin^2 x)(\sin x - \cos x)(\sin^2 x \cos^2 x)}{\sin^3 x - \cos^3 x}
\end{aligned}$$

$$\text{We know that } a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$$

$$\begin{aligned}
&= \frac{(1 + \sin x \cos x)(\sin x - \cos x)(\sin^2 x \cos^2 x)}{(\sin x - \cos x)(\sin^2 x + \cos^2 x + \sin x \cos x)}
\end{aligned}$$

$$= \frac{(1 + \sin x \cos x)(\sin x - \cos x)(\sin^2 x \cos^2 x)}{(\sin x - \cos x)(1 + \sin x \cos x)}$$

$$= \sin^2 x \cos^2 x$$

$$= \text{RHS}$$

Hence proved.

15. Question

Prove the following identities

$$\frac{2 \sin x \cos x - \cos x}{1 - \sin x + \sin^2 x - \cos^2 x} = \cot x$$

Answer

$$\text{LHS} = \frac{2 \sin x \cos x - \cos x}{1 - \sin x + \sin^2 x - \cos^2 x}$$

$$\text{We know that } 1 - \cos^2 x = \sin^2 x$$

$$= \frac{\cos x (2 \sin x - 1)}{\sin^2 x + \sin^2 x - \sin x}$$

$$= \frac{\cos x (2 \sin x - 1)}{2 \sin^2 x - \sin x}$$

$$= \frac{\cos x (2 \sin x - 1)}{\sin x (2 \sin x - 1)}$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

$$= \text{RHS}$$

Hence proved.

16. Question

Prove the following identities

$$\cos x (\tan x + 2) (2 \tan x + 1) = 2 \sec x + 5 \sin x$$

Answer

$$\text{LHS} = \cos x (\tan x + 2) (2 \tan x + 1)$$

$$= \cos x (2 \tan^2 x + 5 \tan x + 2)$$

$$= \cos x \left(\frac{2 \sin^2 x}{\cos^2 x} + \frac{5 \sin x}{\cos x} + 2 \right)$$

$$\text{We know that } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{2 \sin^2 x + 5 \sin x \cos x + 2 \cos^2 x}{\cos x}$$

$$= \frac{2 + 5 \sin x \cos x}{\cos x}$$

$$= \frac{2}{\cos x} + \frac{5 \sin x \cos x}{\cos x}$$

$$= 2 \sec x + 5 \sin x$$

= RHS

Hence proved.

17. Question

If $a = \frac{2 \sin x}{1 + \cos x + \sin x}$, then prove that $\frac{1 - \cos x + \sin x}{1 + \sin x}$ is also equal to a.

Answer

$$\text{Given } a = \frac{2 \sin x}{1 + \cos x + \sin x}$$

Rationalizing the denominator,

$$= \frac{2 \sin x}{1 + \cos x + \sin x} \times \frac{(1 + \sin x) - \cos x}{(1 + \sin x) - \cos x}$$

$$= \frac{2 \sin x [(1 + \sin x) - \cos x]}{(1 + \sin x)^2 - \cos^2 x}$$

$$= \frac{2 \sin x [(1 + \sin x) - \cos x]}{1 + \sin^2 x + 2 \sin x - \cos^2 x}$$

$$= \frac{2 \sin x [(1 + \sin x) - \cos x]}{2 \sin^2 x + 2 \sin x}$$

$$= \frac{2 \sin x [(1 + \sin x) - \cos x]}{2 \sin x (1 + \sin x)}$$

$$= \frac{(1 + \sin x) - \cos x}{1 + \sin x}$$

$$\therefore a = \frac{1 - \cos x + \sin x}{1 + \sin x}$$

Hence proved.

18. Question

If $\sin x = \frac{a^2 - b^2}{a^2 + b^2}$, find the values of $\tan x$, $\sec x$ and $\operatorname{cosec} x$

Answer

$$\text{Given } \sin x = \frac{a^2 - b^2}{a^2 + b^2}$$

We know that $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{a^2 - b^2}{a^2 + b^2} \right)^2$$

$$= \frac{(a^4 + b^4 + 2a^2b^2) - (a^4 + b^4 - 2a^2b^2)}{(a^2 + b^2)^2}$$

$$= \frac{4a^2b^2}{(a^2 + b^2)^2}$$

$$\Rightarrow \cos x = \frac{2ab}{(a^2 + b^2)}$$

$$\Rightarrow \tan x = \frac{\sin x}{\cos x} = \frac{\frac{a^2 - b^2}{a^2 + b^2}}{\frac{2ab}{(a^2 + b^2)^2}} = \frac{(a^2 - b^2)}{2ab}$$

$$\Rightarrow \sec x = \frac{1}{\cos x} = \frac{1}{\frac{2ab}{(a^2 + b^2)^2}} = \frac{(a^2 + b^2)^2}{2ab}$$

$$\Rightarrow \operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\frac{a^2 - b^2}{a^2 + b^2}} = \frac{a^2 + b^2}{a^2 - b^2}$$

19. Question

If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

Answer

Given $\tan x = b/a$

$$\Rightarrow \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \sqrt{\frac{1 + \frac{b}{a}}{1 - \frac{b}{a}}} + \sqrt{\frac{1 - \frac{b}{a}}{1 + \frac{b}{a}}}$$

$$= \sqrt{\frac{1 + \tan x}{1 - \tan x}} + \sqrt{\frac{1 - \tan x}{1 + \tan x}}$$

$$= \frac{\tan x + 1 + 1 - \tan x}{\sqrt{1 - \tan^2 x}}$$

$$= \frac{2}{\sqrt{1 - \tan^2 x}}$$

$$= \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}}$$

$$\therefore \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}}$$

20. Question

If $\tan x = \frac{a}{b}$, show that $\frac{a \sin x - b \cos x}{a \sin x + b \cos x} = \frac{a^2 - b^2}{a^2 + b^2}$.

Answer

Given $\tan x = a/b$

$$\text{LHS} = \frac{a \sin x - b \cos x}{a \sin x + b \cos x}$$

Dividing by $b \cos x$,

$$= \frac{\frac{a \tan x}{b} - 1}{\frac{a \tan x}{b} + 1}$$

Substituting value of $\tan x$,

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

= RHS

Hence proved.

21. Question

If $\operatorname{cosec} x - \sin x = a^3$, $\sec x - \cos x = b^3$, then prove that $a^2 b^2 (a^2 + b^2) = 1$.

Answer

Given $\operatorname{cosec} x - \sin x = a^3$

We know that $\operatorname{cosec} x = 1/\sin x$.

$$\Rightarrow \frac{1}{\sin x} - \sin x = a^3$$

$$\Rightarrow \frac{1 - \sin^2 x}{\sin x} = a^3$$

We know that $1 - \sin^2 x = \cos^2 x$

$$\therefore a = \left(\frac{\cos^2 x}{\sin x} \right)^{\frac{1}{3}} \dots (1)$$

Also given $\sec x - \cos x = b^3$

We know that $\sec x = 1/\cos x$

$$\Rightarrow \frac{1}{\cos x} - \cos x = b^3$$

$$\Rightarrow \frac{1 - \cos^2 x}{\cos x} = b^3$$

We know that $1 - \cos^2 x = \sin^2 x$

$$\therefore b = \left(\frac{\sin^2 x}{\cos x} \right)^{\frac{1}{3}} \dots (2)$$

Consider LHS = $a^2 b^2 (a^2 + b^2)$

$$= \left(\left(\frac{\cos^2 x}{\sin x} \right)^{\frac{1}{3}} \left(\frac{\sin^2 x}{\cos x} \right)^{\frac{1}{3}} \right) \left(\left(\left(\frac{\cos^2 x}{\sin x} \right)^{\frac{1}{3}} \right)^2 + \left(\left(\frac{\sin^2 x}{\cos x} \right)^{\frac{1}{3}} \right)^2 \right)$$

$$= (\sin x \cos x)^{\frac{2}{3}} \left(\frac{(\cos^2 x)^{\frac{2}{3}}}{(\sin x)^{\frac{2}{3}}} + \frac{(\sin^2 x)^{\frac{2}{3}}}{(\cos x)^{\frac{2}{3}}} \right)$$

$$= (\sin x \cos x)^{\frac{2}{3}} \left(\frac{(\cos^3 x)^{\frac{2}{3}} + (\sin^3 x)^{\frac{2}{3}}}{(\sin x)^{\frac{2}{3}} (\cos x)^{\frac{2}{3}}} \right)$$

$$= (\sin x \cos x)^{\frac{2}{3}} \left(\frac{\cos^2 x + \sin^2 x}{(\sin x \cos x)^{\frac{2}{3}}} \right)$$

We know that $\cos^2 x + \sin^2 x = 1$

= 1

= RHS

Hence proved.

22. Question

If $\cot x(1 + \sin x) = 4m$ and $\cot x(1 - \sin x) = 4n$, prove that $(m^2 - n^2)^2 = mn$.

Answer

Given $4m = \cot x (1 + \sin x)$ and $4n = \cot x (1 - \sin x)$

Multiplying both equations, we get

$$\Rightarrow 16mn = \cot^2 x (1 - \sin^2 x)$$

We know that $1 - \sin^2 x = \cos^2 x$

$$\Rightarrow 16mn = \cot^2 x \cos^2 x$$

$$\Rightarrow mn = \frac{\cos^4 x}{16 \sin^2 x} \dots (1)$$

Squaring the given equations and then subtracting,

$$\Rightarrow 16m^2 = \cot^2 x (1 + \sin x)^2 \text{ and } 16n^2 = \cot^2 x (1 - \sin x)^2$$

$$\Rightarrow 16m^2 - 16n^2 = \cot^2 x (4 \sin x)$$

$$\therefore m^2 - n^2 = \frac{\cot^2 x \sin x}{4}$$

Squaring both sides,

$$\Rightarrow (m^2 - n^2)^2 = \frac{\cot^4 x \sin^2 x}{16}$$

$$\Rightarrow (m^2 - n^2)^2 = \frac{\cos^4 x \sin^2 x}{16 \sin^2 x} \dots (2)$$

From (1) and (2),

$$\Rightarrow (m^2 - n^2)^2 = mn$$

Hence proved.

23. Question

If $\sin x + \cos x = m$, then prove that $\sin^6 x + \cos^6 x = \frac{4 - 3(m^2 - 1)^2}{4}$, where $m^2 \leq 2$

Answer

Given $\sin x + \cos x = m$

We have to prove that $\sin^6 x + \cos^6 x = \frac{4 - 3(m^2 - 1)^2}{4}$

Proof:

$$\text{LHS} = \sin^6 x + \cos^6 x$$

$$= (\sin^2 x)^3 + (\cos^2 x)^3$$

We know that $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)$$

$$= 1 - 3 \sin^2 x \cos^2 x$$

$$\text{RHS} = \frac{4 - 3(m^2 - 1)^2}{4}$$

$$= \frac{4 - 3((\sin x + \cos x)^2 - 1)^2}{4}$$

$$\begin{aligned}
&= \frac{4 - 3(\sin^2 x + \cos^2 x + 2 \sin x \cos x - 1)^2}{4} \\
&= \frac{4 - 3(\sin^2 x - (1 - \cos^2 x) + 2 \sin x \cos x)^2}{4} \\
&= \frac{4 - 3 \times 4 \sin^2 x \cos^2 x}{4} \\
&= 1 - 3 \sin^2 x \cos^2 x
\end{aligned}$$

LHS = RHS

Hence proved.

24. Question

If $a = \sec x - \tan x$ and $b = \operatorname{cosec} x + \cot x$, then show that $ab + a - b + 1 = 0$.

Answer

Given $a = \sec x - \tan x$ and $b = \operatorname{cosec} x + \cot x$

$$a = \frac{1 - \sin x}{\cos x} \text{ and } b = \frac{1 + \cos x}{\sin x}$$

$$\text{LHS} = ab + a - b + 1$$

$$\begin{aligned}
&= \left(\frac{1 - \sin x}{\cos x} \right) \left(\frac{1 + \cos x}{\sin x} \right) + \frac{1 - \sin x}{\cos x} - \frac{1 + \cos x}{\sin x} + 1 \\
&= \frac{1 - \sin x + \cos x - \sin x \cos x + \sin x - \sin^2 x - \cos x - \cos^2 x + \sin x \cos x}{\sin x \cos x} \\
&= \frac{1 - \sin^2 x - \cos^2 x}{\sin x \cos x} \\
&= 0 = \text{RHS}
\end{aligned}$$

Hence proved.

25. Question

Prove that :

$$\left| \sqrt{\frac{1 - \sin x}{1 + \sin x}} + \sqrt{\frac{1 + \sin x}{1 - \sin x}} \right| = -\frac{2}{\cos x}, \text{ where } \frac{\pi}{2} < x < \pi$$

Answer

$$\begin{aligned}
\text{LHS} &= \left| \sqrt{\frac{1 - \sin x}{1 + \sin x}} + \sqrt{\frac{1 + \sin x}{1 - \sin x}} \right| \\
&= \left| \sqrt{\frac{1 - \sin x (1 - \sin x)}{1 + \sin x (1 - \sin x)}} + \sqrt{\frac{1 + \sin x (1 + \sin x)}{1 - \sin x (1 + \sin x)}} \right| \\
&= \left| \sqrt{\frac{(1 - \sin x)^2}{(1 - \sin^2 x)}} + \sqrt{\frac{(1 + \sin x)^2}{1 - \sin^2 x}} \right| \\
&= \left| \frac{1 - \sin x + 1 + \sin x}{\cos x} \right| \\
&= \left| \frac{2}{\cos x} \right|
\end{aligned}$$

$$= -\frac{2}{\cos x} [\because \pi/2 < x < \pi \text{ and in second quadrant, } \cos x \text{ is negative}]$$

$$= \text{RHS}$$

Hence proved.

26 A. Question

If $T_n = \sin^n x + \cos^n x$, prove that

$$\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$$

Answer

Given $T_n = \sin^n x + \cos^n x$

$$\text{LHS} = \frac{T_3 - T_5}{T_1}$$

$$= \frac{(\sin^3 x + \cos^3 x) - (\sin^5 x + \cos^5 x)}{\sin x + \cos x}$$

$$= \frac{\sin^3 x - \sin^5 x + \cos^3 x - \cos^5 x}{\sin x + \cos x}$$

$$= \frac{\sin^3 x (1 - \sin^2 x) + \cos^3 x (1 - \cos^2 x)}{\sin x + \cos x}$$

$$= \frac{\sin^3 x \cos^2 x + \cos^3 x \sin^2 x}{\sin x + \cos x}$$

$$= \frac{\sin^2 x \cos^2 x (\sin x + \cos x)}{\sin x + \cos x}$$

$$= \sin^2 x \cos^2 x$$

$$\text{RHS} = \frac{T_5 - T_7}{T_3}$$

$$= \frac{(\sin^5 x + \cos^5 x) - (\sin^7 x + \cos^7 x)}{\sin^3 x + \cos^3 x}$$

$$= \frac{\sin^5 x - \sin^7 x + \cos^5 x - \cos^7 x}{\sin^3 x + \cos^3 x}$$

$$= \frac{\sin^5 x (1 - \sin^2 x) + \cos^5 x (1 - \cos^2 x)}{\sin x + \cos x}$$

$$= \frac{\sin^5 x \cos^2 x + \cos^5 x \sin^2 x}{\sin x + \cos x}$$

$$= \frac{\sin^2 x \cos^2 x (\sin^3 x + \cos^3 x)}{\sin^3 x + \cos^3 x}$$

$$= \sin^2 x \cos^2 x$$

$$\text{LHS} = \text{RHS}$$

Hence proved.

26 B. Question

If $T_n = \sin^n x + \cos^n x$, prove that

$$2 T_6 - 3 T_4 + 1 = 0$$

Answer

$$\text{Given } T_n = \sin^n x + \cos^n x$$

$$\text{LHS} = 2T_6 - 3T_4 + 1$$

$$= 2 (\sin^6 x + \cos^6 x) - 3 (\sin^4 x + \cos^4 x) + 1$$

$$= 2 (\sin^2 x + \cos^2 x) (\sin^4 x + \cos^4 x - \cos^2 x \sin^2 x) - 3 (\sin^4 x + \cos^4 x) + 1$$

$$\text{We know that } \sin^2 x + \cos^2 x = 1.$$

$$= 2 (1) (\sin^4 x + \cos^4 x - \cos^2 x \sin^2 x) - 3 (\sin^4 x + \cos^4 x) + 1$$

$$= 2\sin^4 x + 2\cos^4 x - 2\sin^2 x \cos^2 x - 3\sin^4 x - 3\cos^4 x + 1$$

$$= -(\sin^4 x + \cos^4 x) - 2\sin^2 x \cos^2 x + 1$$

$$= -(\sin^2 x + \cos^2 x)^2 + 1$$

$$= -1 + 1$$

$$= 0$$

$$= \text{RHS}$$

Hence proved.

26 C. Question

If $T_n = \sin^n x + \cos^n x$, prove that

$$6 T_{10} - 15 T_8 + 10 T_6 - 1 = 0$$

Answer

$$\text{Given } T_n = \sin^n x + \cos^n x$$

$$\text{LHS} = 6T_{10} - 15 T_8 + 10T_6 - 1$$

$$= 6 (\sin^{10} x + \cos^{10} x) - 15 (\sin^8 x + \cos^8 x) + 10 (\sin^6 x + \cos^6 x) - 1$$

$$= 6 (\sin^6 x + \cos^6 x) (\sin^4 x + \cos^4 x) - \cos^4 x \sin^4 x (\sin^2 x + \cos^2 x) - 15 (\sin^6 x + \cos^6 x) (\sin^2 x + \cos^2 x) - \cos^2 x \sin^2 x (\sin^4 x + \cos^4 x) + 10 (\sin^2 x + \cos^2 x) (\sin^4 x + \cos^4 x - \cos^2 x \sin^2 x) - 1$$

$$\text{We know that } \sin^2 x + \cos^2 x = 1.$$

$$= 6 [(1 - 3 \sin^2 x \cos^2 x) (1 - 2 \sin^2 x \cos^2 x) - \sin^4 x \cos^4 x] - 15 [(1 - 3 \sin^2 x \cos^2 x) - \sin^2 x \cos^2 x (1 - 2 \sin^2 x \cos^2 x)] + 10 (1 - 3 \sin^2 x \cos^2 x) - 1$$

$$= 6 (1 - 5 \sin^2 x \cos^2 x + 5 \sin^4 x \cos^4 x) - 15 (1 - 4 \sin^2 x \cos^2 x + 2 \sin^4 x \cos^4 x) + 10 (1 - 3 \sin^2 x \cos^2 x) - 1$$

$$= 6 - 30 \sin^2 x \cos^2 x + 30 \sin^4 x \cos^4 x - 15 + 60 \sin^2 x \cos^2 x - 30 \sin^4 x \cos^4 x + 10 - 30 \sin^2 x \cos^2 x - 1$$

$$= 6 - 15 + 10 - 1$$

$$= 0$$

$$= \text{RHS}$$

Hence proved.

Exercise 5.2**1 A. Question**

Find the values of the other five trigonometric functions in each of the following:

$$\cot x = \frac{12}{5}, x \text{ in quadrant III}$$

Answer

Given $\cot x = 12/5$ and x is in quadrant III

In third quadrant, $\tan x$ and $\cot x$ are positive and $\sin x$, $\cos x$ and $\sec x$ & $\operatorname{cosec} x$ are negative.

$$\text{We know that } \tan x = \frac{1}{\cot x}; \operatorname{cosec} x = \sqrt{1 + \cot^2 x}; \sin x = \frac{1}{\operatorname{cosec} x}; \cos x = -\sqrt{1 - \sin^2 x} \text{ and } \sec x = \frac{1}{\cos x}$$

$$\Rightarrow \tan x = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

$$\Rightarrow \operatorname{cosec} x = -\sqrt{1 + \left(\frac{12}{5}\right)^2}$$

$$= -\sqrt{\frac{25 + 144}{25}}$$

$$= -\sqrt{\frac{169}{25}}$$

$$= -\frac{13}{5}$$

$$\Rightarrow \sin x = \frac{1}{-13/5} = -\frac{5}{13}$$

$$\Rightarrow \cos x = -\sqrt{1 - \left(\frac{-5}{13}\right)^2}$$

$$= -\sqrt{\frac{169 - 25}{169}}$$

$$= -\sqrt{\frac{144}{169}}$$

$$= -\frac{12}{13}$$

$$\Rightarrow \sec x = \frac{1}{-\frac{12}{13}} = -\frac{13}{12}$$

1 B. Question

Find the values of the other five trigonometric functions in each of the following:

$$\cos x = -\frac{1}{2}, x \text{ in quadrant II}$$

Answer

Given $\cot x = -1/2$ and x is in quadrant II

In second quadrant, $\sin x$ and $\operatorname{cosec} x$ are positive and $\tan x$, $\cot x$ and $\cos x$ & $\sec x$ are negative.

We know that $\sin x = \sqrt{1 - \cos^2 x}$; $\tan x = \frac{\sin x}{\cos x}$; $\cot x = \frac{1}{\tan x}$; $\operatorname{cosec} x = \frac{1}{\sin x}$ and $\sec x = \frac{1}{\cos x}$

$$\Rightarrow \sin x = \sqrt{1 - \left(\frac{-1}{2}\right)^2}$$

$$= \sqrt{\frac{4 - 1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

$$\Rightarrow \tan x = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = -\sqrt{3}$$

$$\Rightarrow \cot x = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \operatorname{cosec} x = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sec x = \frac{1}{\frac{-1}{2}} = -2$$

1 C. Question

Find the values of the other five trigonometric functions in each of the following:

$$\tan x = \frac{3}{4}, x \text{ in quadrant III}$$

Answer

Given $\tan x = 3/4$ and x is in quadrant III

In third quadrant, $\tan x$ and $\cot x$ are positive and $\sin x$, $\cos x$, $\sec x$ and $\operatorname{cosec} x$ are negative.

We know that $\sin x = \frac{\tan x}{\sqrt{1 + \tan^2 x}}$; $\tan x = \frac{\sin x}{\cos x}$; $\cot x = \frac{1}{\tan x}$; $\operatorname{cosec} x = \frac{1}{\sin x}$ and $\sec x =$

$$\Rightarrow \cot x = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\Rightarrow \sec x = -\sqrt{1 + \left(\frac{3}{4}\right)^2}$$

$$= -\sqrt{\frac{16 + 9}{16}}$$

$$= -\sqrt{\frac{25}{16}}$$

$$= -\frac{5}{4}$$

$$\Rightarrow \cos x = \frac{1}{-\frac{5}{4}} = -\frac{4}{5}$$

$$\Rightarrow \sin x = -\sqrt{1 - \left(-\frac{4}{5}\right)^2}$$

$$= -\sqrt{\frac{25 - 16}{25}}$$

$$= -\sqrt{\frac{9}{25}}$$

$$= -\frac{3}{5}$$

$$\Rightarrow \operatorname{cosec} x = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}$$

1 D. Question

Find the values of the other five trigonometric functions in each of the following:

$$\sin x = \frac{3}{5}, x \text{ in quadrant I}$$

Answer

Given $\sin x = 3/5$ and x is in first quadrant.

In first quadrant, all trigonometric ratios are positive.

We know that $\tan x = \frac{\sin x}{\cos x}$; $\operatorname{cosec} x = \frac{1}{\sin x}$; $\sin x = \frac{1}{\operatorname{cosec} x}$; $\cos x = \sqrt{1 - \sin^2 x}$ and $\sec x =$

$$\Rightarrow \cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= \sqrt{\frac{25 - 9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$= \frac{4}{5}$$

$$\Rightarrow \tan x = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\Rightarrow \cot x = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\Rightarrow \operatorname{cosec} x = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

$$\Rightarrow \sec x = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

2. Question

If $\sin x = \frac{12}{13}$ and x lies in the second quadrant, find the value of $\sec x + \tan x$.

Answer

Given $\sin x = 12/13$ and x lies in the second quadrant.

In second quadrant, $\sin x$ and $\operatorname{cosec} x$ are positive and all other ratios are negative.

We know that $\cos x = \sqrt{1 - \sin^2 x}$

$$\cos x = -\sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$= -\sqrt{1 - \frac{144}{169}}$$

$$= -\sqrt{\frac{(169 - 144)}{169}}$$

$$= -\sqrt{\frac{25}{169}}$$

$$= -\frac{5}{13}$$

We know that $\tan x = \sin x / \cos x$ and $\sec x = 1/\cos x$

$$\Rightarrow \tan x = \frac{\frac{12}{13}}{\frac{-5}{13}} = -\frac{12}{5}$$

$$\Rightarrow \sec x = \frac{1}{\frac{-5}{13}} = -\frac{13}{5}$$

$$\therefore \sec x + \tan x = -\frac{12}{5} + \left(-\frac{13}{5}\right)$$

$$= \frac{-12 - 13}{5}$$

$$= -\frac{25}{5} = -5$$

3. Question

If $\sin x = \frac{3}{5}$, $\tan y = \frac{1}{2}$ and $\frac{\pi}{2} < x < \pi < y < \frac{3\pi}{2}$, find the value of $8 \tan x - \sqrt{5} \sec y$.

Answer

Given $\sin x = 3/5$, $\tan y = 1/2$ and $\frac{\pi}{2} < x < \pi < y < \frac{3\pi}{2}$

Thus, x is in second quadrant and y is in third quadrant.

In second quadrant, $\cos x$ and $\tan x$ are negative.

In third quadrant, $\sec y$ is negative.

We know that $\cos x = -\sqrt{1 - \sin^2 x}$ and $\tan x = \frac{\sin x}{\cos x}$

$$\Rightarrow \cos x = -\sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= -\sqrt{1 - \frac{9}{25}}$$

$$= -\sqrt{\frac{25 - 9}{25}}$$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

$$\Rightarrow \tan x = \frac{\frac{3}{5}}{-\frac{4}{5}} = \frac{-3}{4}$$

We know that $\sec y = -\sqrt{1 + \tan^2 y}$

$$\Rightarrow \sec y = -\sqrt{1 + \left(\frac{1}{2}\right)^2}$$

$$= -\sqrt{1 + \left(\frac{1}{4}\right)}$$

$$= -\sqrt{\frac{4 + 1}{4}}$$

$$= -\sqrt{\frac{5}{4}}$$

$$= -\frac{\sqrt{5}}{2}$$

$$\therefore 8 \tan x - \sqrt{5} \sec y = 8\left(-\frac{3}{4}\right) - \sqrt{5}\left(-\frac{\sqrt{5}}{2}\right)$$

$$= -6 + \frac{5}{2}$$

$$= -\frac{7}{2}$$

4. Question

If $\sin x + \cos x = 0$ and x lies in the fourth quadrant, find $\sin x$ and $\cos x$.

Answer

Given $\sin x + \cos x = 0$ and x lies in fourth quadrant.

$$\Rightarrow \sin x = -\cos x$$

$$\Rightarrow \frac{\sin x}{\cos x} = -1$$

$$\therefore \tan x = -1$$

In fourth quadrant, $\cos x$ and $\sec x$ are positive and all other ratios are negative.

$$\text{We know that } \sec x = \sqrt{1 + \tan^2 x} ; \cos x = \frac{1}{\sec x} ; \sin x = -\sqrt{1 - \cos^2 x}$$

$$\Rightarrow \sec x = \sqrt{1 + (-1)^2} = \sqrt{2}$$

$$\Rightarrow \cos x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin x = -\sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= -\sqrt{1 - \frac{1}{2}}$$

$$= -\sqrt{\frac{2-1}{2}}$$

$$= -\sqrt{\frac{1}{2}}$$

$$= -\frac{1}{\sqrt{2}}$$

$$\therefore \sin x = -\frac{1}{\sqrt{2}} \text{ and } \cos x = \frac{1}{\sqrt{2}}$$

5. Question

If $\cos x = -\frac{3}{5}$ and $\pi < x < \frac{3\pi}{2}$, find the values of other five trigonometric functions and hence evaluate $\frac{\operatorname{cosec} x + \cot x}{\sec x - \tan x}$.

Answer

Given $\cos x = -3/5$ and $\pi < x < 3\pi/2$

It is in the third quadrant. Here, $\tan x$ and $\cot x$ are positive and all other ratios are negative.

$$\text{We know that } \sin x = -\sqrt{1 - \cos^2 x} ; \tan x = \frac{\sin x}{\cos x} ; \cot x = \frac{1}{\tan x} ; \sec x = \frac{1}{\cos x} \text{ and } \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\Rightarrow \sin x = -\sqrt{1 - \left(\frac{-3}{5}\right)^2}$$

$$= -\sqrt{1 - \frac{9}{25}}$$

$$= -\sqrt{\frac{25-9}{25}}$$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

$$\Rightarrow \tan x = \frac{\frac{-4}{5}}{\frac{-3}{5}} = \frac{4}{3}$$

$$\Rightarrow \cot x = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\Rightarrow \sec x = \frac{1}{\frac{-3}{5}} = -\frac{5}{3}$$

$$\Rightarrow \operatorname{cosec} x = \frac{1}{\frac{-4}{5}} = -\frac{5}{4}$$

$$\therefore \frac{\operatorname{cosec} \theta + \cot \theta}{\sec \theta - \tan \theta} = \frac{\frac{-5}{4} + \frac{3}{4}}{\frac{-5}{3} - \frac{4}{3}}$$

$$= \frac{\frac{-5+3}{4}}{\frac{-5-4}{3}}$$

$$= \frac{\frac{-2}{4}}{\frac{-9}{3}}$$

$$= \frac{\frac{-1}{2}}{-3}$$

$$= \frac{1}{6}$$

Exercise 5.3

1 A. Question

Find the values of the following trigonometric ratios:

$$\sin \frac{5\pi}{3}$$

Answer

$$\text{Given } \sin \frac{5\pi}{3}$$

$$\Rightarrow \frac{5\pi}{3} = \left(\frac{5\pi}{3} \times 180 \right)^\circ = 300^\circ$$

$$= (90^\circ \times 3 + 30^\circ)$$

300° lies in fourth quadrant in which sine function is negative.

$$\therefore \sin \left(\frac{5\pi}{3} \right) = \sin(300^\circ)$$

$$= \sin(90^\circ \times 3 + 30^\circ)$$

$$= -\cos 30^\circ$$

$$= \frac{-\sqrt{3}}{2}$$

1 B. Question

Find the values of the following trigonometric ratios:

$$\sin 17\pi$$

Answer

Given $\sin 17\pi$

$$\Rightarrow \sin 17\pi = \sin 3060^\circ$$

$$\Rightarrow 3060^\circ = 90^\circ \times 34 + 0^\circ$$

3060° is in negative direction of x-axis i.e. on boundary line of II and III quadrants.

$$\therefore \sin(3060^\circ) = \sin(90^\circ \times 34 + 0^\circ)$$

$$= -\sin 0^\circ$$

$$= 0$$

1 C. Question

Find the values of the following trigonometric ratios:

$$\tan \frac{11\pi}{6}$$

Answer

Given $\tan(11\pi/6)$

$$\Rightarrow \frac{11\pi}{6} = \left(\frac{11}{6} \times 180^\circ\right)$$

$$= 330^\circ$$

330° lies in fourth quadrant in which tangent function is negative.

$$\therefore \left(\frac{11\pi}{6}\right) = \tan(330^\circ)$$

$$= \tan(90^\circ \times 3 + 60^\circ)$$

$$= -\cot 60^\circ$$

$$= -\frac{1}{\sqrt{3}}$$

1 D. Question

Find the values of the following trigonometric ratios:

$$\cos\left(-\frac{25\pi}{4}\right)$$

Answer

Given $\cos\left(\frac{-25\pi}{4}\right)$

$$\Rightarrow \cos\left(\frac{-25\pi}{4}\right) = \cos(-1125^\circ)$$

$$\Rightarrow \cos(-1125^\circ) = \cos(1125^\circ)$$

$$= \cos(90^\circ \times 12 + 45^\circ)$$

1125° lies in first quadrant in which cosine function is positive.

$$\therefore \cos(1125^\circ) = \cos(90^\circ \times 12 + 45^\circ)$$

$$= \cos (45^\circ)$$

$$= 1/\sqrt{2}$$

1 E. Question

Find the values of the following trigonometric ratios:

$$\tan \frac{7\pi}{4}$$

Answer

Given $\tan 7\pi/4$

$$\Rightarrow \tan \frac{7\pi}{4} = \tan 315^\circ$$

$$\Rightarrow 315^\circ = (90^\circ \times 3 + 45^\circ)$$

315° lies in fourth quadrant in which tangent function is negative.

$$\therefore \tan (315^\circ) = \tan (90^\circ \times 3 + 45^\circ)$$

$$= -\cot 45^\circ$$

$$= -1$$

1 F. Question

Find the values of the following trigonometric ratios:

$$\sin \frac{17\pi}{6}$$

Answer

Given $\sin \frac{17\pi}{6}$

$$\Rightarrow \sin \frac{17\pi}{6} = \sin 510^\circ$$

$$\Rightarrow 510^\circ = (90^\circ \times 5 + 60^\circ)$$

510° lies in second quadrant in which sine function is positive.

$$\therefore \sin (510^\circ) = \sin (90^\circ \times 5 + 60^\circ)$$

$$= \cos (60^\circ)$$

$$= 1/2$$

1 G. Question

Find the values of the following trigonometric ratios:

$$\cos \frac{19\pi}{6}$$

Answer

Given $\cos \frac{19\pi}{6}$

$$\Rightarrow \cos \frac{19\pi}{6} = \cos 570^\circ$$

$$\Rightarrow 570^\circ = (90^\circ \times 6 + 30^\circ)$$

570° lies in third quadrant in which cosine function is negative.

$$\begin{aligned}
 \therefore \cos (570^{\circ}) &= \cos (90^{\circ} \times 6 + 30^{\circ}) \\
 &= -\cos (30^{\circ}) \\
 &= -\frac{\sqrt{3}}{2}
 \end{aligned}$$

1 H. Question

Find the values of the following trigonometric ratios:

$$\sin\left(-\frac{11\pi}{6}\right)$$

Answer

$$\text{Given } \sin\left(\frac{-11\pi}{6}\right)$$

$$\Rightarrow \sin\frac{-11\pi}{6} = \sin -330^{\circ}$$

$$\Rightarrow -\sin 330^{\circ} = -\sin (90^{\circ} \times 3 + 60^{\circ})$$

330° lies in the fourth quadrant in which the sine function is negative.

$$\therefore \sin (-330)^{\circ} = -\sin (90^{\circ} \times 3 + 60^{\circ})$$

$$= -(-\cos 60^{\circ})$$

$$= -(-1/2)$$

$$= 1/2$$

1 I. Question

Find the values of the following trigonometric ratios:

$$\operatorname{cosec}\left(-\frac{20\pi}{3}\right)$$

Answer

$$\text{Given } \operatorname{cosec}\left(-\frac{20\pi}{3}\right)$$

$$\Rightarrow \operatorname{cosec}\left(-\frac{20\pi}{3}\right) = \operatorname{cosec}(-1200^{\circ})$$

$$\Rightarrow \operatorname{cosec} (-1200^{\circ}) = \operatorname{cosec} (1200^{\circ})$$

$$= \operatorname{cosec} (90^{\circ} \times 13 + 30)$$

1200° lies in second quadrant in which cosec function is positive.

$$\therefore \operatorname{cosec} (-1200^{\circ}) = -\operatorname{cosec} (90^{\circ} \times 13 + 30^{\circ})$$

$$= -\sec 30^{\circ}$$

$$= -\frac{2}{\sqrt{3}}$$

1 J. Question

Find the values of the following trigonometric ratios:

$$\tan\left(-\frac{13\pi}{4}\right)$$

Answer

Given $\tan\left(\frac{-13\pi}{4}\right)$

$$\Rightarrow \tan \frac{-13\pi}{4} = \tan -585^\circ$$

$$\Rightarrow -\tan 585^\circ = -\tan (90^\circ \times 6 + 45^\circ)$$

585° lies in the third quadrant in which the tangent function is positive.

$$\therefore \tan (-585)^\circ = -\tan (90^\circ \times 6 + 45^\circ)$$

$$= -(\tan 45^\circ)$$

$$= -1$$

1 K. Question

Find the values of the following trigonometric ratios:

$$\cos \frac{19\pi}{4}$$

Answer

Given $\cos \frac{19\pi}{4}$

$$\Rightarrow \cos \frac{19\pi}{4} = \cos 855^\circ$$

$$\Rightarrow 855^\circ = 90^\circ \times 9 + 45^\circ$$

855° lies in the second quadrant in which the cosine function is negative.

$$\therefore \cos 855^\circ = \cos (90^\circ \times 9 + 45^\circ)$$

$$= -\sin 45^\circ$$

$$= \frac{-1}{\sqrt{2}}$$

1 L. Question

Find the values of the following trigonometric ratios:

$$\sin \frac{41\pi}{4}$$

Answer

Given $\sin \frac{41\pi}{4}$

$$\Rightarrow \sin \frac{41\pi}{4} = \sin 1845^\circ$$

$$\Rightarrow \sin 1845^\circ = 90^\circ \times 20 + 45^\circ$$

1845° lies in the first quadrant in which the sine function is positive.

$$\therefore \sin 1845^\circ = \sin (90^\circ \times 20 + 45^\circ)$$

$$= \sin 45^\circ$$

$$= \frac{1}{\sqrt{2}}$$

1 M. Question

Find the values of the following trigonometric ratios:

$$\cos \frac{39\pi}{4}$$

Answer

Given $\cos \frac{39\pi}{4}$

$$\Rightarrow \cos \frac{39\pi}{4} = \cos 1755^\circ$$

$$\Rightarrow 1755^\circ = 90^\circ \times 19 + 45^\circ$$

1755° lies in the fourth quadrant in which the cosine function is positive.

$$\therefore \cos 1755^\circ = \cos (90^\circ \times 19 + 45^\circ)$$

$$= \sin 45^\circ$$

$$= \frac{1}{\sqrt{2}}$$

1 N. Question

Find the values of the following trigonometric ratios:

$$\sin \frac{151\pi}{6}$$

Answer

Given $\sin \frac{151\pi}{6}$

$$\Rightarrow \sin \frac{151\pi}{6} = \sin 4530^\circ$$

$$\Rightarrow \sin 4530^\circ = 90^\circ \times 50 + 30^\circ$$

4530° lies in the third quadrant in which the sine function is negative.

$$\therefore \sin 4530^\circ = \sin (90^\circ \times 50 + 30^\circ)$$

$$= -\sin 30^\circ$$

$$= -1/2$$

2 A. Question

prove that :

$$\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ = 0$$

Answer

$$\text{LHS} = \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$$

$$= \tan (90^\circ \times 2 + 45^\circ) \cot (90^\circ \times 4 + 45^\circ) + \tan (90^\circ \times 8 + 45^\circ) \cot (90^\circ \times 7 + 45^\circ)$$

We know that when n is odd, $\cot \rightarrow \tan$.

$$= \tan 45^\circ \cot 45^\circ + \tan 45^\circ [-\tan 45^\circ]$$

$$= \tan 45^\circ \cot 45^\circ - \tan 45^\circ \tan 45^\circ$$

$$= 1 \times 1 - 1 \times 1$$

$$= 1 - 1$$

$$= 0$$

= RHS

Hence proved.

2 B. Question

prove that :

$$\sin \frac{8\pi}{3} \cos \frac{23\pi}{6} + \cos \frac{13\pi}{3} \sin \frac{35\pi}{6} = \frac{1}{2}$$

Answer

$$\text{LHS} = \sin \frac{8\pi}{3} \cos \frac{23\pi}{6} + \cos \frac{13\pi}{3} \sin \frac{35\pi}{6}$$

$$= \sin 480^\circ \cos 690^\circ + \cos 780^\circ \sin 1050^\circ$$

$$= \sin (90^\circ \times 5 + 30^\circ) \cos (90^\circ \times 7 + 60^\circ) + \cos (90^\circ \times 8 + 60^\circ) \sin (90^\circ \times 11 + 60^\circ)$$

We know that when n is odd, $\sin \rightarrow \cos$ and $\cos \rightarrow \sin$.

$$= \cos 30^\circ \sin 60^\circ + \cos 60^\circ [-\cos 60^\circ]$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= 3/4 - 1/4$$

$$= 2/4$$

$$= 1/2$$

$$= \text{RHS}$$

Hence proved.

2 C. Question

prove that :

$$\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ = \frac{1}{2}$$

Answer

$$\text{LHS} = \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$$

$$= \cos 24^\circ + \cos (90^\circ \times 1 - 35^\circ) + \cos (90^\circ \times 1 + 35^\circ) + \cos (90^\circ \times 2 + 24^\circ) + \cos (90^\circ \times 3 + 30^\circ)$$

We know that when n is odd, $\cos \rightarrow \sin$.

$$= \cos 24^\circ + \sin 35^\circ - \sin 35^\circ - \cos 24^\circ + \sin 30^\circ$$

$$= 0 + 0 + 1/2$$

$$= 1/2$$

$$= \text{RHS}$$

Hence proved.

2 D. Question

prove that :

$$\tan (-225^\circ) \cot (-405^\circ) - \tan (-765^\circ) \cot (675^\circ) = 0$$

Answer

$$\text{LHS} = \tan (-225^\circ) \cot (-405^\circ) - \tan (-765^\circ) \cot (675^\circ)$$

We know that $\tan(-x) = -\tan(x)$ and $\cot(-x) = -\cot(x)$.

$$\begin{aligned}
 &= [-\tan(225^\circ)] [-\cot(405^\circ)] - [-\tan(765^\circ)] \cot(675^\circ) \\
 &= \tan(225^\circ) \cot(405^\circ) + \tan(765^\circ) \cot(675^\circ) \\
 &= \tan(90^\circ \times 2 + 45^\circ) \cot(90^\circ \times 4 + 45^\circ) + \tan(90^\circ \times 8 + 45^\circ) \cot(90^\circ \times 7 + 45^\circ) \\
 &= \tan 45^\circ \cot 45^\circ + \tan 45^\circ [-\tan 45^\circ] \\
 &= 1 \times 1 + 1 \times (-1) \\
 &= 1 - 1 \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

Hence proved.

2 E. Question

prove that :

$$\cos 570^\circ \sin 510^\circ + \sin(-330^\circ) \cos(-390^\circ) = 0$$

Answer

$$\text{LHS} = \cos 570^\circ \sin 510^\circ + \sin(-330^\circ) \cos(-390^\circ)$$

We know that $\sin(-x) = -\sin(x)$ and $\cos(-x) = +\cos(x)$.

$$\begin{aligned}
 &= \cos 570^\circ \sin 510^\circ + [-\sin(330^\circ)] \cos(390^\circ) \\
 &= \cos 570^\circ \sin 510^\circ - \sin(330^\circ) \cos(390^\circ) \\
 &= \cos(90^\circ \times 6 + 30^\circ) \sin(90^\circ \times 5 + 60^\circ) - \sin(90^\circ \times 3 + 60^\circ) \cos(90^\circ \times 4 + 30^\circ)
 \end{aligned}$$

We know that cos is negative at $90^\circ + \theta$ i.e. in Q_2 and when n is odd, $\sin \rightarrow \cos$ and $\cos \rightarrow \sin$.

$$\begin{aligned}
 &= -\cos 30^\circ \cos 60^\circ - [-\cos 60^\circ] \cos 30^\circ \\
 &= -\cos 30^\circ \cos 60^\circ + \cos 60^\circ \cos 30^\circ \\
 &= 0
 \end{aligned}$$

= RHS

Hence proved.

2 F. Question

prove that :

$$\tan \frac{11\pi}{3} - 2 \sin \frac{4\pi}{6} - \frac{3}{4} \operatorname{cosec}^2 \frac{\pi}{4} + 4 \cos^2 \frac{17\pi}{6} = \frac{3-4\sqrt{3}}{2}$$

Answer

$$\begin{aligned}
 \text{LHS} &= \tan \frac{11\pi}{3} - 2 \sin \frac{4\pi}{6} - \frac{3}{4} \operatorname{cosec}^2 \frac{\pi}{4} + 4 \cos^2 \frac{17\pi}{6} \\
 &= \tan \frac{11 \times 180^\circ}{3} - 2 \sin \frac{4 \times 180^\circ}{6} - \frac{3}{4} \operatorname{cosec}^2 \frac{180^\circ}{4} + 4 \cos^2 \frac{17 \times 180^\circ}{6} \\
 &= \tan 660^\circ - 2 \sin 120^\circ - \frac{3}{4} [\operatorname{cosec} 45^\circ]^2 + 4 [\cos 510^\circ]^2
 \end{aligned}$$

$$= \tan(90^\circ \times 7 + 30^\circ) - 2 \sin(90^\circ \times 1 + 30^\circ) - \frac{3}{4} [\operatorname{cosec} 45^\circ]^2 + 4 [\cos(90^\circ \times 5 + 60^\circ)]^2$$

We know that tan and cos is negative at $90^\circ + \theta$ i.e. in Q_2 and when n is odd, $\tan \rightarrow \cot$, $\sin \rightarrow \cos$ and $\cos \rightarrow \sin$.

$$\begin{aligned}
&= [-\cot 30^\circ] - 2 \cos 30^\circ - 3/4 [\operatorname{cosec} 45^\circ]^2 + [-\sin 60^\circ]^2 \\
&= -\cot 30^\circ - 2 \cos 30^\circ - 3/4 [\operatorname{cosec} 45^\circ]^2 + [\sin 60^\circ]^2 \\
&= -\sqrt{3} - \frac{2\sqrt{3}}{2} - \frac{3}{4} [\sqrt{2}]^2 + 4 \left[\frac{\sqrt{3}}{2} \right]^2 \\
&= -\sqrt{3} - \sqrt{3} - \frac{6}{4} + \frac{12}{4} \\
&= \frac{3 - 4\sqrt{3}}{2}
\end{aligned}$$

= RHS

Hence proved.

2 G. Question

prove that :

$$3 \sin \frac{\pi}{6} \sec \frac{\pi}{3} - 4 \sin \frac{5\pi}{6} \cot \frac{\pi}{4} = 1$$

Answer

$$\begin{aligned}
\text{LHS} &= 3 \sin \frac{\pi}{6} \sec \frac{\pi}{3} - 4 \sin \frac{5\pi}{6} \cot \frac{\pi}{4} \\
&= 3 \sin \frac{180^\circ}{6} \sec \frac{180^\circ}{3} - 4 \sin \frac{5(180^\circ)}{6} \cot \frac{180^\circ}{4} \\
&= 3 \sin 30^\circ \sec 60^\circ - 4 \sin 150^\circ \cot 45^\circ \\
&= 3 \sin 30^\circ \sec 60^\circ - 4 \sin (90^\circ \times 1 + 60^\circ) \cot 45^\circ
\end{aligned}$$

We know that when n is odd, $\sin \rightarrow \cos$.

$$\begin{aligned}
&= 3 \sin 30^\circ \sec 60^\circ - 4 \cos 60^\circ \cot 45^\circ \\
&= 3 (1/2) (2) - 4 (1/2) (1) \\
&= 3 - 2 \\
&= 1 \\
&= \text{RHS}
\end{aligned}$$

Hence proved.

3 A. Question

Prove that :

$$\frac{\cos(2\pi + x) \operatorname{cosec}(2\pi + x) \tan(\pi/2 + x)}{\sec(\pi/2 + x) \cos x \cot(\pi + x)} = 1$$

Answer

$$\begin{aligned}
\text{LHS} &= \frac{\cos(2\pi + x) \operatorname{cosec}(2\pi + x) \tan(\frac{\pi}{2} + x)}{\sec(\frac{\pi}{2} + x) \cos x \cot(\pi + x)} \\
&= \frac{\cos x \operatorname{cosec} x [-\cot x]}{[-\operatorname{cosec} x] \cos x \cot x} \\
&= \frac{-\cos x \operatorname{cosec} x \cot x}{-\operatorname{cosec} x \cos x \cot x}
\end{aligned}$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

3 B. Question

Prove that :

$$\frac{\operatorname{cosec}(90^\circ + x) + \cot(450^\circ + x)}{\operatorname{cosec}(90^\circ + x) + \tan(180^\circ - x)} + \frac{\tan(180^\circ + x) + \sec(180^\circ - x)}{\tan(360^\circ + x) - \sec(-x)} = 2$$

Answer

$$\begin{aligned} \text{LHS} &= \frac{\operatorname{cosec}(90^\circ + x) + \cot(450^\circ + x)}{\operatorname{cosec}(90^\circ - x) + \tan(180^\circ - x)} + \frac{\tan(180^\circ + x) + \sec(180^\circ - x)}{\tan(360^\circ + x) - \sec(-x)} \\ &= \frac{\operatorname{cosec}(90^\circ + x) + \cot(90^\circ \times 5 + x)}{\operatorname{cosec}(90^\circ - x) + \tan(90^\circ \times 2 - x)} + \frac{\tan(90^\circ \times 2 + x) + \sec(90^\circ \times 2 - x)}{\tan(90^\circ \times 4 + x) - \sec(-x)} \end{aligned}$$

We know that when n is odd, $\operatorname{cosec} \rightarrow \sec$ and also $\sec(-x) = \sec x$.

$$\begin{aligned} &= \frac{\sec x + \cot(90^\circ \times 5 + x)}{\operatorname{cosec}(90^\circ - x) + \tan(90^\circ \times 2 - x)} + \frac{\tan(90^\circ \times 2 + x) + \sec(90^\circ \times 2 - x)}{\tan(90^\circ \times 4 + x) - \sec(x)} \\ &= \frac{\sec x - \tan x}{\sec x - \tan x} + \frac{\tan x - \sec x}{\tan x - \sec x} \\ &= 1 + 1 \end{aligned}$$

$$= 2$$

$$= \text{RHS}$$

Hence proved.

3 C. Question

Prove that :

$$\frac{\sin(\pi - x) \cos\left(\frac{\pi}{2} + x\right) \tan\left(\frac{3\pi}{2} - x\right) \cot(2\pi - x)}{\sin(2\pi - x) \cos(2\pi + x) \operatorname{cosec}(-x) \sin\left(\frac{3\pi}{2} - x\right)} = 1$$

Answer

$$\begin{aligned} \text{LHS} &= \frac{\sin(\pi - x) \cos\left(\frac{\pi}{2} + x\right) \tan\left(\frac{3\pi}{2} - x\right) \cot(2\pi - x)}{\sin(2\pi - x) \cos(2\pi + x) \operatorname{cosec}(-x) \sin\left(\frac{3\pi}{2} - x\right)} \\ &= \frac{\sin(180^\circ - x) \cos(90^\circ + x) \tan(270^\circ - x) \cot(360^\circ - x)}{\sin(360^\circ - x) \cos(360^\circ + x) \operatorname{cosec}(-x) \sin(270^\circ - x)} \end{aligned}$$

We know that $\operatorname{cosec}(-x) = -\operatorname{cosec} x$.

$$= \frac{\sin(90^\circ \times 2 - x) \cos(90^\circ \times 1 + x) \tan(90^\circ \times 3 - x) \cot(90^\circ \times 4 - x)}{\sin(90^\circ \times 4 - x) \cos(90^\circ \times 4 + x) [-\operatorname{cosec}(x)] \sin(90^\circ \times 3 - x)}$$

We know that when n is odd, $\cos \rightarrow \sin$, $\tan \rightarrow \cot$ and $\sin \rightarrow \cos$.

$$= \frac{[-\sin x] [-\sin x] \cot x [-\cot x]}{[-\sin x] \cos x [-\operatorname{cosec} x] [-\cos x]}$$

$$= \frac{\sin^2 x \cot^2 x}{\sin x \operatorname{cosec} x \cos x \cos x}$$

$$= \frac{\sin^2 x \times \frac{\cos^2 x}{\sin^2 x}}{\sin x \times \frac{1}{\sin x} \times \cos^2 x}$$

$$= \frac{\cos^2 x}{\cos^2 x}$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

3 D. Question

Prove that :

$$\left\{ 1 + \cot x - \sec\left(\frac{\pi}{2} + x\right) \right\} \left\{ 1 + \cot x + \sec\left(\frac{\pi}{2} + x\right) \right\} = 2 \cot x$$

Answer

$$\text{LHS} = \left\{ 1 + \cot x - \sec\left(\frac{\pi}{2} + x\right) \right\} \left\{ 1 + \cot x + \sec\left(\frac{\pi}{2} + x\right) \right\}$$

$$= \{1 + \cot x - (-\operatorname{cosec} x)\} \{1 + \cot x + (-\operatorname{cosec} x)\}$$

$$= \{1 + \cot x + \operatorname{cosec} x\} \{1 + \cot x - \operatorname{cosec} x\}$$

$$= \{(1 + \cot x) + (\operatorname{cosec} x)\} \{(1 + \cot x) - (\operatorname{cosec} x)\}$$

$$\text{We know that } (a + b)(a - b) = a^2 - b^2$$

$$= (1 + \cot x)^2 - (\operatorname{cosec} x)^2$$

$$= 1 + \cot^2 x + 2 \cot x - \operatorname{cosec}^2 x$$

$$\text{We know that } 1 + \cot^2 x = \operatorname{cosec}^2 x$$

$$= \operatorname{cosec}^2 x + 2 \cot x - \operatorname{cosec}^2 x$$

$$= 2 \cot x$$

$$= \text{RHS}$$

Hence proved.

3 E. Question

Prove that :

$$\frac{\tan\left(\frac{\pi}{2} - x\right) \sec(\pi - x) \sin(-x)}{\sin(\pi + x) \cot(2\pi - x) \operatorname{cosec}\left(\frac{\pi}{2} - x\right)} = 1$$

Answer

$$\text{LHS} = \frac{\tan\left(\frac{\pi}{2} - x\right) \sec(\pi - x) \sin(-x)}{\sin(\pi + x) \cot(2\pi - x) \operatorname{cosec}\left(\frac{\pi}{2} - x\right)}$$

$$= \frac{\tan(90^\circ - x) \sec(180^\circ - x) \sin(-x)}{\sin(180^\circ + x) \cot(360^\circ - x) \operatorname{cosec}(90^\circ - x)}$$

We know that $\sin(-x) = -\sin x$.

$$= \frac{\tan(90^\circ \times 1 - x) \sec(90^\circ \times 2 - x) [-\sin(x)]}{\sin(90^\circ \times 2 + x) \cot(90^\circ \times 4 - x) \operatorname{cosec}(90^\circ \times 1 - x)}$$

We know that when n is odd, $\tan \rightarrow \cot$ and $\operatorname{cosec} \rightarrow \sec$.

$$= \frac{[\cot x][-\sec x][-\sin x]}{[-\sin x][-\cot x][\sec x]}$$

$$= \frac{\cot x \sec x \sin x}{\cot x \sec x \sin x}$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

4. Question

Prove that :

$$\sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} = 2$$

Answer

$$\text{LHS} = \sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9}$$

$$= \sin^2 \frac{\pi}{18} + \sin^2 \frac{2\pi}{18} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{8\pi}{18}$$

$$= \sin^2 \frac{\pi}{18} + \sin^2 \frac{2\pi}{18} + \sin^2 \left(\frac{\pi}{2} - \frac{2\pi}{18} \right) + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{18} \right)$$

We know that when n is odd, $\sin \rightarrow \cos$.

$$= \sin^2 \frac{\pi}{18} + \sin^2 \frac{2\pi}{18} + \cos^2 \frac{2\pi}{18} + \cos^2 \frac{\pi}{18}$$

$$= \sin^2 \frac{\pi}{18} + \cos^2 \frac{\pi}{18} + \sin^2 \frac{2\pi}{18} + \cos^2 \frac{2\pi}{18}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$= 1 + 1$$

$$= 2$$

$$= \text{RHS}$$

Hence proved.

5. Question

Prove that :

$$\sec\left(\frac{3\pi}{2} - x\right) \sec\left(x - \frac{5\pi}{2}\right) + \tan\left(\frac{5\pi}{2} + x\right) \tan\left(x - \frac{3\pi}{2}\right) = -1.$$

Answer

$$\text{LHS} = \sec\left(\frac{3\pi}{2} - x\right) \sec\left(x - \frac{5\pi}{2}\right) + \tan\left(\frac{5\pi}{2} + x\right) \tan\left(x - \frac{3\pi}{2}\right)$$

$$= \sec\left(\frac{3\pi}{2} - x\right) \left[-\sec\left(\frac{5\pi}{2} - x\right)\right] + \tan\left(\frac{5\pi}{2} + x\right) \left[-\tan\left(\frac{3\pi}{2} - x\right)\right]$$

We know that $\sec(-x) = \sec(x)$ and $\tan(-x) = -\tan(x)$.

$$= \sec\left(\frac{3\pi}{2} - x\right) \left[\sec\left(\frac{5\pi}{2} - x\right) \right] - \tan\left(\frac{5\pi}{2} + x\right) \left[\tan\left(\frac{3\pi}{2} - x\right) \right]$$

$$= \sec\left(\frac{\pi}{2} \times 3 - x\right) \sec\left(\frac{\pi}{2} \times 5 - x\right) - \tan\left(\frac{\pi}{2} \times 5 + x\right) \tan\left(\frac{\pi}{2} \times 3 - x\right)$$

We know that when n is odd, $\sec \rightarrow \operatorname{cosec}$ and $\tan \rightarrow \cot$.

$$= [-\operatorname{cosec} x] [\operatorname{cosec} x] - [-\cot x] [\cot x]$$

$$= -\operatorname{cosec}^2 x + \cot^2 x$$

$$= -[\operatorname{cosec}^2 x - \cot^2 x]$$

We know that $\operatorname{cosec}^2 x - \cot^2 x = 1$

$$= -1$$

$$= \text{RHS}$$

Hence proved.

6. Question

In a ΔABC , prove that :

$$\text{i. } \cos(A + B) + \cos C = 0$$

$$\text{ii. } \cos\left(\frac{A+B}{2}\right) = \sin \frac{C}{2}$$

$$\text{iii. } \tan \frac{A+B}{2} = \cot \frac{C}{2}$$

Answer

We know that in ΔABC , $A + B + C = \pi$

$$\text{(i) Here } A + B = \pi - C$$

$$\text{LHS} = \cos(A + B) + \cos C$$

$$= \cos(\pi - C) + \cos C$$

We know that $\cos(\pi - C) = -\cos C$

$$= -\cos C + \cos C$$

$$= 0$$

$$= \text{RHS}$$

Hence proved.

$$\text{(ii) } \Rightarrow A + B = \pi - C$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi - C}{2}$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\text{LHS} = \cos\left(\frac{A+B}{2}\right)$$

$$= \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

We know that $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$$= \sin\left(\frac{C}{2}\right)$$

= RHS

Hence proved.

(iii)

$$\Rightarrow A + B = \pi - C$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi - C}{2}$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\text{LHS} = \tan\left(\frac{A+B}{2}\right)$$

$$= \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

We know that $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

$$= \cot\left(\frac{C}{2}\right)$$

= RHS

Hence proved

7. Question

If A, B, C, D be the angles of a cyclic quadrilateral taken in order prove that :

$$\cos(180^\circ - A) + \cos(180^\circ + B) + \cos(180^\circ + C) - \sin(90^\circ + D) = 0$$

Answer

Given A, B, C and D are the angles of a cyclic quadrilateral.

$$\therefore A + C = 180^\circ \text{ and } B + D = 180^\circ$$

$$\Rightarrow A = 180^\circ - C \text{ and } B = 180^\circ - D$$

$$\text{Now, LHS} = \cos(180^\circ - A) + \cos(180^\circ + B) + \cos(180^\circ + C) - \sin(90^\circ + D)$$

$$= -\cos A + [-\cos B] + [-\cos C] + [-\cos D]$$

$$= -\cos A - \cos B - \cos C - \cos D$$

$$= -\cos(180^\circ - C) - \cos(180^\circ - D) - \cos C - \cos D$$

$$= -[-\cos C] - [-\cos D] - \cos C - \cos D$$

$$= \cos C + \cos D - \cos C - \cos D$$

$$= 0$$

= RHS

Hence proved.

8 A. Question

Find x from the following equations:

$$\operatorname{cosec}\left(\frac{\pi}{2} + \theta\right) + x \cos \theta \cot\left(\frac{\pi}{2} + \theta\right) = \sin\left(\frac{\pi}{2} + \theta\right)$$

Answer

$$\Rightarrow \operatorname{cosec}\left(\frac{\pi}{2} + \theta\right) + x \cos \theta \cot\left(\frac{\pi}{2} + \theta\right) = \sin\left(\frac{\pi}{2} + \theta\right)$$

$$\Rightarrow \operatorname{cosec}(90^\circ + \theta) + x \cos \theta \cot(90^\circ + \theta) = \cos \theta$$

We know that when n is odd, $\cot \rightarrow \tan$.

$$\Rightarrow \sec \theta + x \cos \theta [-\tan \theta] = \cos \theta$$

$$\Rightarrow \sec \theta - x \cos \theta \tan \theta = \cos \theta$$

$$\Rightarrow \sec \theta - x \cos \theta (\sin \theta / \cos \theta) = \cos \theta$$

$$\Rightarrow \sec \theta - x \sin \theta = \cos \theta$$

$$\Rightarrow \sec \theta - \cos \theta = x \sin \theta$$

$$\Rightarrow \frac{1}{\cos \theta} - \cos \theta = x \sin \theta$$

$$\Rightarrow \frac{1 - \cos^2 \theta}{\cos \theta} = x \sin \theta$$

We know that $1 - \cos^2 \theta = \sin^2 \theta$

$$\Rightarrow \frac{\sin^2 \theta}{\cos \theta} = x \sin \theta$$

$$\Rightarrow \frac{\sin^2 \theta}{\cos \theta \sin \theta} = x$$

$$\Rightarrow \tan \theta = x$$

$$\therefore x = \tan \theta$$

8 B. Question

Find x from the following equations:

$$x \cot\left(\frac{\pi}{2} + \theta\right) + \tan\left(\frac{\pi}{2} + \theta\right) \sin \theta + \operatorname{cosec}\left(\frac{\pi}{2} + \theta\right) = 0$$

Answer

$$\text{Given } x \cot\left(\frac{\pi}{2} + \theta\right) + \tan\left(\frac{\pi}{2} + \theta\right) \sin \theta + \operatorname{cosec}\left(\frac{\pi}{2} + \theta\right) = 0$$

$$\Rightarrow x \cot(90^\circ + \theta) + \tan(90^\circ + \theta) \sin \theta + \operatorname{cosec}(90^\circ + \theta) = 0$$

$$\Rightarrow x [-\tan \theta] + [-\cot \theta] \sin \theta + \sec \theta = 0$$

$$\Rightarrow -x \left[\frac{\sin \theta}{\cos \theta} \right] - \frac{\cos \theta}{\sin \theta} \sin \theta + \frac{1}{\cos \theta} = 0$$

$$\Rightarrow -x \left[\frac{\sin \theta}{\cos \theta} \right] - \cos \theta + \frac{1}{\cos \theta} = 0$$

$$\Rightarrow \frac{-x \sin \theta - \cos^2 \theta + 1}{\cos \theta} = 0$$

$$\Rightarrow -x \sin \theta - \cos^2 \theta + 1 = 0$$

We know that $1 - \cos^2 \theta = \sin^2 \theta$

$$\Rightarrow -x \sin \theta + \sin^2 \theta = 0$$

$$\Rightarrow x \sin \theta = \sin^2 \theta$$

$$\Rightarrow x = \sin^2 \theta / \sin \theta$$

$$\therefore x = \sin \theta$$

9 A. Question

Prove that:

$$\tan 4\pi - \cos \frac{3\pi}{2} - \sin \frac{5\pi}{6} \cos \frac{2\pi}{3} = \frac{1}{4}$$

Answer

$$\text{LHS} = \tan 4\pi - \cos \frac{3\pi}{2} - \sin \frac{5\pi}{6} \cos \frac{2\pi}{3}$$

$$= \tan 720^\circ - \cos 270^\circ - \sin 150^\circ \cos 120^\circ$$

$$= \tan (90^\circ \times 8 + 0^\circ) - \cos (90^\circ \times 3 + 0^\circ) - \sin (90^\circ \times 1 + 60^\circ) \cos (90^\circ \times 1 + 30^\circ)$$

We know that when n is odd, $\cos \rightarrow \sin$ and $\sin \rightarrow \cos$.

$$= \tan 0^\circ - \sin 0^\circ - \cos 60^\circ [-\sin 30^\circ]$$

$$= \tan 0^\circ - \sin 0^\circ + \cos 60^\circ \sin 30^\circ$$

$$= 0 - 0 + 1/2 (1/2)$$

$$= 1/4$$

$$= \text{RHS}$$

Hence proved.

9 B. Question

Prove that:

$$\sin \frac{13\pi}{3} \sin \frac{8\pi}{3} + \cos \frac{2\pi}{3} \sin \frac{5\pi}{6} = \frac{1}{2}$$

Answer

$$\text{LHS} = \sin \frac{13\pi}{3} \sin \frac{8\pi}{3} + \cos \frac{2\pi}{3} \sin \frac{5\pi}{6}$$

$$= \sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 150^\circ$$

$$= \sin (90^\circ \times 8 + 60^\circ) \sin (90^\circ \times 5 + 30^\circ) + \cos (90^\circ \times 1 + 30^\circ) \sin (90^\circ \times 1 + 60^\circ)$$

We know that when n is odd, $\cos \rightarrow \sin$ and $\sin \rightarrow \cos$.

$$= \sin 60^\circ \cos 30^\circ + [-\sin 30^\circ] \cos 60^\circ$$

$$= \sin 60^\circ \cos 30^\circ - \sin 30^\circ \cos 60^\circ$$

We know that $\sin A \cos B - \cos A \sin B = \sin (A - B)$

$$= \sin (60^\circ - 30^\circ)$$

$$= \sin 30^\circ$$

$$= 1/2$$

$$= \text{RHS}$$

Hence proved.

9 C. Question

Prove that:

$$\sin \frac{13\pi}{3} \sin \frac{2\pi}{3} + \cos \frac{4\pi}{3} \sin \frac{13\pi}{6} = \frac{1}{2}$$

Answer

$$\text{LHS} = \sin \frac{13\pi}{3} \sin \frac{2\pi}{3} + \cos \frac{4\pi}{3} \sin \frac{13\pi}{6}$$

$$= \sin 780^\circ \sin 120^\circ + \cos 240^\circ \sin 390^\circ$$

$$= \sin (90^\circ \times 8 + 60^\circ) \sin (90^\circ \times 1 + 30^\circ) + \cos (90^\circ \times 2 + 60^\circ) \sin (90^\circ \times 4 + 30^\circ)$$

We know that when n is odd, $\sin \rightarrow \cos$.

$$= \sin 60^\circ \cos 30^\circ + [-\cos 60^\circ] \sin 30^\circ$$

$$= \sin 60^\circ \cos 30^\circ - \sin 30^\circ \cos 60^\circ$$

We know that $\sin A \cos B - \cos A \sin B = \sin (A - B)$

$$= \sin (60^\circ - 30^\circ)$$

$$= \sin 30^\circ$$

$$= 1/2$$

$$= \text{RHS}$$

Hence proved.

9 D. Question

Prove that:

$$\sin \frac{10\pi}{3} \cos \frac{13\pi}{6} + \cos \frac{8\pi}{3} \sin \frac{5\pi}{6} = -1$$

Answer

$$\text{LHS} = \sin \frac{10\pi}{3} \cos \frac{13\pi}{6} + \cos \frac{8\pi}{3} \sin \frac{5\pi}{6}$$

$$= \sin 600^\circ \cos 390^\circ + \cos 480^\circ \sin 150^\circ$$

$$= \sin (90^\circ \times 6 + 60^\circ) \cos (90^\circ \times 4 + 30^\circ) + \cos (90^\circ \times 5 + 30^\circ) \sin (90^\circ \times 1 + 60^\circ)$$

We know that when n is odd, $\sin \rightarrow \cos$ and $\cos \rightarrow \sin$.

$$= [-\sin 60^\circ] \cos 30^\circ + [-\sin 30^\circ] \cos 60^\circ$$

$$= -\sin 60^\circ \cos 30^\circ - \sin 30^\circ \cos 60^\circ$$

$$= -[\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ]$$

We know that $\sin A \cos B + \cos A \sin B = \sin (A + B)$

$$= -\sin (60^\circ + 30^\circ)$$

$$= -\sin 90^\circ$$

$$= -1$$

$$= \text{RHS}$$

Hence proved.

9 E. Question

Prove that:

$$\tan \frac{5\pi}{4} \cot \frac{9\pi}{4} + \tan \frac{17\pi}{4} \cot \frac{15\pi}{4} = 0$$

Answer

$$\text{LHS} = \tan \frac{5\pi}{4} \cot \frac{9\pi}{4} + \tan \frac{17\pi}{4} \cot \frac{15\pi}{4}$$

$$= \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$$

$$= \tan (90^\circ \times 2 + 45^\circ) \cot (90^\circ \times 4 + 45^\circ) + \tan (90^\circ \times 8 + 45^\circ) \cot (90^\circ \times 7 + 45^\circ)$$

We know that when n is odd, $\cot \rightarrow \tan$.

$$= \tan 45^\circ \cot 45^\circ + \tan 45^\circ [-\tan 45^\circ]$$

$$= \tan 45^\circ \cot 45^\circ - \tan 45^\circ \tan 45^\circ$$

$$= 1 \times 1 - 1 \times 1$$

$$= 1 - 1$$

$$= 0$$

$$= \text{RHS}$$

Hence proved.

Very Short Answer

1. Question

Write the maximum and minimum values of $\cos (\cos x)$.

Answer

Let $\cos x = t$

Range of $t = (-1, 1)$

$\therefore$ Maximum and Minimum value of $\cos x$ is 1 and -1 respectively.

Now,

$$\cos(-x) = \cos x$$

$$\therefore \text{Range of } \cos(\cos x) = [\cos(1), \cos(0)]$$

$$\Rightarrow \cos(\cos x) = [\cos 1, 0]$$

2. Question

Write the maximum and minimum values of $\sin (\sin x)$.

Answer

$\sin(x)$ has maximum value at $x = \pi/2$ and its minimum at

$x = -\pi/2$ which are 1 and -1 respectively.

As $1 < \pi/2$;

so, the argument of the outer $\sin$ always lies within the interval

$$[-\pi/2, \pi/2]$$

So the maximum and minimum of the given function are

$\sin 1$ and $-\sin 1$.

3. Question

Write the maximum value of $\sin (\cos x)$.

Answer

Value of $\cos(x)$ varies from -1 to 1 for all $\mathbb{R}$ and $\sin(x)$ is increasing in $[-\pi/2, \pi/2]$

$\therefore \sin(\cos x)$ has max value of $\sin 1$.

4. Question

If $\sin x = \cos^2 x$, then write the value of $\cos^2 x (1 + \cos^2 x)$.

Answer

Given $\sin x = \cos^2 x$

To find the value of $\cos^2 x (1 + \cos^2 x)$.

$$\Rightarrow \cos^2 x (1 + \cos^2 x).$$

$$\Rightarrow \cos^2 x + \cos^4 x.$$

As $\cos^2 x = 1 - \sin^2 x$ the above equation becomes

$$\Rightarrow 1 - \sin^2 x + \sin^2 x$$

$$\Rightarrow 1.$$

5. Question

If $\sin x = \operatorname{cosec} x = 2$, then write the value of $\sin^n x + \operatorname{cosec}^n x$.

Answer

(Question might be different)

$$\sin x + \operatorname{cosec} x = 2$$

$$\Rightarrow \sin x + \frac{1}{\sin x} = 2$$

$$\Rightarrow \sin^2 x + 1 = 2\sin x$$

$$\Rightarrow \sin^2 x - 2\sin x + 1 = 0$$

$$\Rightarrow (\sin x - 1)^2 = 0$$

$$\Rightarrow \sin x = 1$$

$$\text{As } \sin x = 1$$

$$\sin^n x = 1$$

$$\therefore \sin^n x + \operatorname{cosec}^n x$$

$$\Rightarrow \sin^n x + \frac{1}{\sin^n x} = 1 + 1$$

$$\Rightarrow 2.$$

6. Question

If $\sin x + \sin^2 x = 1$, then write the value of $\cos^{12} x + 3 \cos^{10} x + 3 \cos^8 x + \cos^6 x$.

Answer

$$\text{Given: } \sin x + \sin^2 x = 1$$

To find the value of $\cos^{12} x + 3 \cos^{10} x + 3 \cos^8 x + \cos^6 x$.

$$\Rightarrow \sin x = 1 - \sin^2 x$$

$$\Rightarrow \sin x = \cos^2 x$$

$$\Rightarrow \cos^{12}x = \sin^6x, \cos^{10}x = \sin^5x, \cos^8x = \sin^4x, \cos^6x = \sin^3x.$$

Substituting above values in given equation we get

$$\Rightarrow \sin^6x + 3\sin^5x + 3\sin^8x + \sin^3x [(a+b)^3 = a^3+3a^2b+3ab^2+b^3]$$

$$\Rightarrow (\sin x + \sin^2 x)^3 = (1)^3$$

$$\Rightarrow 1.$$

7. Question

If $\sin x + \sin^2 x = 1$, then write the value of $\cos^8 x + 2 \cos^6 x + \cos^4 x$.

Answer

$$\text{Given: } \sin x + \sin^2 x = 1$$

To find the value of $\cos^8 x + 2 \cos^6 x + \cos^4 x$.

$$\Rightarrow \sin x = 1 - \sin^2 x$$

$$\Rightarrow \sin x = \cos^2 x$$

$$\Rightarrow \cos^8 x = \sin^4 x, \cos^6 x = \sin^3 x, \cos^4 x = \sin^2 x .$$

Substituting above values in given equation we get

$$\Rightarrow \sin^4 x + 2 \sin^3 x + \sin^2 x [(a+b)^2 = a^2+2ab+b^2]$$

$$\Rightarrow (\sin x + \sin^2 x)^2 = (1)^2$$

$$\Rightarrow 1$$

8. Question

If $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$, then write the value of $\cos \theta_1 + \cos \theta_2 + \cos \theta_3$.

Answer

$$\text{Given that } \sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$$

We know that in general the maximum value of $\sin \theta = 1$ when $\theta = \pi/2$

$$\text{As } \sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$$

$$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \pi/2.$$

The above case is the only possible condition for the given condition to satisfy.

$$\therefore \cos \theta_1 + \cos \theta_2 + \cos \theta_3$$

$$\Rightarrow \cos \pi/2 + \cos \pi/2 + \cos \pi/2$$

$$\Rightarrow 0+0+0$$

$$\Rightarrow 0.$$

9. Question

Write the value of $\sin 10^\circ + \sin 20^\circ + \sin 30^\circ + \dots + \sin 360^\circ$.

Answer

$$\text{We know } \sin(180+\theta) = -\sin \theta$$

$$\text{Also, } \sin(360-\theta) = -\sin \theta$$

Given all angles are complementary in nature.

$$\sin 350 = \sin(360-10) = -\sin 10^\circ$$

so finally each of them cancel each other and finally we get
the sum equal to 0.

10. Question

A circular wire of radius 15 cm is cut and bent so as to lie along the circumference of a loop of radius 120 cm. Write the measure of the angle subtended by it at the centre of the loop.

Answer

Let the angle subtended be θ .

For calculating we have the formula $\frac{\text{Radius}}{\text{Circumference}} = \frac{\theta}{360}$

$$\Rightarrow \frac{15}{120} = \frac{\theta}{360}$$

$$\Rightarrow \theta = \frac{15 \times 360}{120}$$

$$\Rightarrow \theta = 45^\circ$$

11. Question

Write the value of $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1$.

Answer

$$\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$$

$$= (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x)$$

$$= 1(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x)$$

Substituting above value in given equation

$$\Rightarrow 2(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) - 3(\sin^4 x + \cos^4 x) + 1$$

$$\Rightarrow 2\sin^4 x + 2\cos^4 x - 2\sin^2 x \cos^2 x - 3\sin^4 x - 3\cos^4 x + 1$$

$$\Rightarrow -\sin^4 x - \cos^4 x - 2\sin^2 x \cos^2 x + 1$$

$$\Rightarrow -[(\sin^2 x)^2 + (\cos^2 x)^2 - 2\sin^2 x \cos^2 x] + 1$$

$$\Rightarrow -[(\sin^2 x + \cos^2 x)^2] + 1$$

$$\Rightarrow -1 + 1$$

$$\Rightarrow 0.$$

12. Question

Write the value of $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 180^\circ$.

Answer

The given expression can be rearranged as:

$$(\cos 1 + \cos 179) + (\cos 2 + \cos 178) + (\cos 3 + \cos 177) + \dots + (\cos 89 + \cos 91) + (\cos 90) + \cos 180$$

We know that: $\cos(180 - x) = -\cos x$.

So all the bracket totals except last 2 terms will be zero.

So given expression is: $0 + (\cos 90) + (\cos 180)$

$$= 0 + 0 + (-1)$$

$$= -1.$$

13. Question

If $\cot(\alpha + \beta) = 0$, then write the value of $\sin(\alpha + 2\beta)$.

Answer

Given: $\cot(\alpha + \beta) = 0$

$$\therefore \frac{\cot\alpha \cdot \cot\beta - 1}{\cot\alpha + \cot\beta} = 0$$

$$\Rightarrow \cot\alpha \cdot \cot\beta = 1$$

$$\Rightarrow \cot\alpha = \frac{1}{\cot\beta}$$

$$\Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\sin\beta}{\cos\beta}$$

Now,

$$\sin(\alpha + 2\beta) = \sin\alpha \cdot \cos 2\beta + \cos\alpha \cdot \sin 2\beta$$

$$= \sin\alpha (2\cos^2\beta - 1) + \cos\alpha \cdot 2\sin\beta \cdot \cos\beta$$

14. Question

If $\tan A + \cot A = 4$, then write the value of $\tan^4 A + \cot^4 A$.

Answer

Given: $\tan A + \cot A = 4$

$$\Rightarrow \tan A + \frac{1}{\tan A} = 4$$

Squaring both sides we get

$$\Rightarrow \left(\tan A + \frac{1}{\tan A}\right)^2 = 4^2$$

$$\Rightarrow \tan^2 A + \frac{1}{\tan^2 A} + 2 \cdot \tan A \cdot \frac{1}{\tan A} = 16$$

$$\Rightarrow \tan^2 A + \frac{1}{\tan^2 A} = 14$$

Squaring both sides we get

$$\Rightarrow \left(\tan^2 A + \frac{1}{\tan^2 A}\right)^2 = 14^2$$

$$\Rightarrow \tan^4 A + \frac{1}{\tan^4 A} + 2 \cdot \tan^2 A \cdot \frac{1}{\tan^2 A} = 196$$

$$\Rightarrow \tan^4 A + \frac{1}{\tan^4 A} = 194$$

15. Question

Write the least value of $\cos^2 x + \sec^2 x$.

Answer

We know that $\cos^2 x$ and $\sec^2 x \geq 0$

$\therefore$ By applying AM and GM we get,

$$\Rightarrow \frac{\cos^2 x + \sec^2 x}{2} \geq \cos^2 x \cdot \sec^2 x$$

$$\Rightarrow \cos^2 x + \sec^2 x \geq 2$$

$\therefore$ Least value of the given function is 2.

16. Question

If $x = \sin^{14}x + \cos^{20}x$, then write the smallest interval in which the value of x lie.

Answer

We know the range of $\sin x$ is

$$-1 \leq \sin x \leq 1$$

$$\therefore 0 \leq \sin^{14}x \leq 1$$

We know the range of $\cos x$ is

$$-1 \leq \cos x \leq 1$$

$$\therefore 0 \leq \cos^{20}x \leq 1$$

$$0 < \sin^{14}x + \cos^{20}x \leq 2$$

which means that the value of x lies in the interval $[0,2]$

But there's a problem, when sine is 0 cosine is 1, they might even be 0 and -1 at particular points (not in this case, since they are even powers), so the minimum we would get should be more than 0. Hence the value of x lies in $(0,1]$

17. Question

If $3 \sin x + 5 \cos x = 5$, then write the value of $5 \sin x - 3 \cos x$.

Answer

$$\Rightarrow 3 \sin x + 5 \cos x = 5$$

$$\Rightarrow 3 \sin x = 5 - 5 \cos x$$

$$\Rightarrow 3 \sin x = 5(1 - \cos x)$$

Squaring both sides we get

$$\Rightarrow 9 \sin^2 x = 25(1 - \cos x)^2$$

$$\Rightarrow 9 \sin^2 x = 25(1 + \cos^2 x - 2 \cos x)$$

$$\Rightarrow 9 \sin^2 x + 9 \cos^2 x = 25 + 25 \cos^2 x - 50 \cos x + 9 \cos^2 x$$

$$\Rightarrow 9(\sin^2 x + \cos^2 x) = 25 + 34 \cos^2 x - 50 \cos x$$

$$\Rightarrow 34 \cos^2 x - 50 \cos x + 16 = 0$$

$$\Rightarrow 17 \cos^2 x - 25 \cos x + 8 = 0$$

$$\Rightarrow 17 \cos^2 x - 17 \cos x - 8 \cos x + 8 = 0$$

$$\Rightarrow 17 \cos x (\cos x - 1) - 8(\cos x - 1) = 0$$

$$\Rightarrow \cos x = \frac{8}{17}, \cos x = 1$$

When $\cos x = 1$

$$3 \sin x + 5 \cos x = 5$$

$$3 \sin x = 0$$

$$\sin x = 0$$

Substituting the value $\cos x = 1$ and $\sin x = 0$

$$5(0) - 3(1) = 0 - 3$$

$$\Rightarrow -3.$$

$$\Rightarrow \cos x = \frac{8}{17}$$

$$\Rightarrow \sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \frac{64}{289}$$

$$\Rightarrow \sin^2 x = \frac{225}{289}$$

$$\Rightarrow \sin x = \frac{15}{17}$$

$$\Rightarrow 5 \sin x - 3 \cos x$$

$$\Rightarrow 5 \times \frac{15}{17} - 3 \times \frac{8}{17}$$

$$\Rightarrow \frac{51}{17} = 3.$$

$\therefore -3$ and 3 .

MCQ

1. Question

Mark the correct alternative in the following:

If $\tan x = x - \frac{1}{4x}$, then $\sec x - \tan x$ is equal to

A. $-2x, \frac{1}{2x}$

B. $-\frac{1}{2x}, 2x$

C. $2x$

D. $2x, \frac{1}{2x}$

Answer

$$\Rightarrow \tan^2 x = x^2 + \frac{1}{16x^2} - 2x \frac{1}{4x}$$

$$\Rightarrow \tan^2 x = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\Rightarrow \sec^2 x - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\Rightarrow \sec^2 x = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \sec^2 x = \left(x + \frac{1}{4x}\right)^2$$

$$\Rightarrow \sec x = x + \frac{1}{4x}, -x - \frac{1}{4x}$$

$$\Rightarrow \sec x - \tan x$$

$$\Rightarrow x + \frac{1}{4x} - \left(x - \frac{1}{4x}\right)$$

$$\Rightarrow \frac{1}{2x}$$

$$\Rightarrow \sec x - \tan x$$

$$\Rightarrow -x - \frac{1}{4x} - x + \frac{1}{4x}$$

$$\Rightarrow -2x$$

$\therefore$ the value of $\sec x - \tan x = -2x, 1/2x$.

2. Question

Mark the correct alternative in the following:

If $\sec x = x + \frac{1}{4x}$, then $\sec x + \tan x =$

A. $x, \frac{1}{x}$

B. $2x, \frac{1}{2x}$

C. $-2x, \frac{1}{2x}$

D. $-\frac{1}{x}, x$

Answer

$$\Rightarrow \sec^2 x = x^2 + \frac{1}{16x^2} + 2x \frac{1}{4x}$$

$$\Rightarrow \sec^2 x = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \tan^2 x + 1 = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \tan^2 x = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\Rightarrow \tan^2 x = \left(x - \frac{1}{4x}\right)^2$$

$$\Rightarrow \tan x = x - \frac{1}{4x}, -x + \frac{1}{4x}$$

$$\Rightarrow \sec x - \tan x$$

$$\Rightarrow x + \frac{1}{4x} - \left(x - \frac{1}{4x}\right)$$

$$\Rightarrow \frac{1}{2x}$$

$$\Rightarrow \sec x - \tan x$$

$$\Rightarrow x + \frac{1}{4x} + x - \frac{1}{4x}$$

$$\Rightarrow 2x$$

$\therefore$ the value of $\sec x - \tan x = 2x, 1/2x$.

3. Question

Mark the correct alternative in the following:

If $\frac{\pi}{2} < x < \frac{3\pi}{2}$, then $\sqrt{\frac{1-\sin x}{1+\sin x}}$ is equal to

- A. $\sec x - \tan x$
- B. $\sec x + \tan x$
- C. $\tan x - \sec x$
- D. none of these

Answer

Given:

$$\Rightarrow \frac{-\pi}{2} < x < \frac{3\pi}{2}$$

Now

$$\Rightarrow \sqrt{\frac{1-\sin x}{1+\sin x}} \text{ (Rationalizing we get)}$$

$$\Rightarrow \sqrt{\frac{1-\sin x}{1+\sin x}} \times \sqrt{\frac{1-\sin x}{1-\sin x}}$$

$$\Rightarrow \sqrt{\frac{(1-\sin x)^2}{1-\sin^2 x}}$$

$$\Rightarrow \sqrt{\frac{(1-\sin x)^2}{\cos^2 x}}$$

$$\Rightarrow \frac{1-\sin x}{\cos x}$$

$$\Rightarrow \sec x - \tan x$$

In the given range $\tan x = -\tan x$ and $\sec x$ is $-\sec x$

$$\therefore -\sec x - (-\tan x)$$

$$\Rightarrow \tan x - \sec x$$

4. Question

Mark the correct alternative in the following:

If $\pi < x < 2\pi$, then $\sqrt{\frac{1+\cos x}{1-\cos x}}$ is equal to

- A. $\operatorname{cosec} x + \cot x$
- B. $\operatorname{cosec} x - \cot x$
- C. $-\operatorname{cosec} x + \cot x$
- D. $-\operatorname{cosec} x - \cot x$

Answer

Given:

$$\Rightarrow \pi < x < 2\pi$$

Now

$$\Rightarrow \sqrt{\frac{1+\cos x}{1-\cos x}} \text{ (Rationalizing we get)}$$

$$\Rightarrow \sqrt{\frac{1+\cos x}{1-\cos x}} \times \sqrt{\frac{1+\cos x}{1+\cos x}}$$

$$\Rightarrow \sqrt{\frac{(1+\cos x)^2}{1-\cos^2 x}}$$

$$\Rightarrow \sqrt{\frac{(1+\cos x)^2}{\sin^2 x}}$$

$$\Rightarrow \frac{1+\cos x}{\sin x}$$

$$\Rightarrow \operatorname{cosec} x + \cot x$$

In the given range $\cot x = -\cot x$ and $\operatorname{cosec} x$ is $-\operatorname{cosec} x$

$$\therefore -\operatorname{cosec} x - (+\cot x)$$

$$\Rightarrow -\operatorname{cosec} x - \cot x$$

5. Question

Mark the correct alternative in the following:

If $0 < x < \frac{\pi}{2}$, and if $\frac{y+1}{1-y} = \sqrt{\frac{1+\sin x}{1-\sin x}}$, then y is equal to

A. $\cot \frac{x}{2}$

B. $\tan \frac{x}{2}$

C. $\cot \frac{x}{2} + \tan \frac{x}{2}$

D. $\cot \frac{x}{2} - \tan \frac{x}{2}$

Answer

$$\frac{y+1}{1-y} = \sqrt{\frac{1+\sin x}{1-\sin x}} \left[\text{Use } 1 = \sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} \right]$$

$$\frac{y+1}{1-y} = \sqrt{\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2}}}$$

$$\frac{y+1}{1-y} = \sqrt{\frac{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2}}$$

If $0 < x < \pi/2$ then we take $\cos\frac{x}{2} - \sin\frac{x}{2}$. So, that square root

is open with positive sign.

$$\frac{y+1}{1-y} = \frac{\sin\frac{x}{2} + \cos\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}$$

Adding 1 on both sides

$$\frac{y+1}{1-y} + 1 = \frac{\sin\frac{x}{2} + \cos\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}} + 1$$

$$\frac{(y+1) + (1-y)}{1-y} = \frac{\sin\frac{x}{2} + \cos\frac{x}{2} + \cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}$$

$$\frac{2}{1-y} = \frac{2\cos\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}$$

$$\frac{1-y}{1} = \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2}}$$

$$1-y = 1 - \tan\frac{x}{2}$$

$$y = \tan\frac{x}{2}$$

6. Question

Mark the correct alternative in the following:

If $\frac{\pi}{2} < x < \pi$, then $\sqrt{\frac{1-\sin x}{1+\sin x}} + \sqrt{\frac{1+\sin x}{1-\sin x}}$ is equal to

- A. $2 \sec x$
- B. $-2 \sec x$
- C. $\sec x$
- D. $-\sec x$

Answer

$$\sqrt{\frac{1-\sin x}{1+\sin x}} + \sqrt{\frac{1+\sin x}{1-\sin x}} \left[\text{Use } 1 = \sin^2\frac{x}{2} + \cos^2\frac{x}{2} \right]$$

$$\Rightarrow \sqrt{\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}} + \sqrt{\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}}$$

$$\Rightarrow \sqrt{\frac{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}} + \sqrt{\frac{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2}}$$

$$\begin{aligned}
&\Rightarrow \frac{\sin \frac{x}{2} - \cos \frac{x}{2}}{\sin \frac{x}{2} + \cos \frac{x}{2}} + \frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\sin \frac{x}{2} - \cos \frac{x}{2}} \\
&\Rightarrow \frac{\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right) + \left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)}{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)} \\
&\Rightarrow \frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2} + \sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)} \\
&\Rightarrow \frac{2}{\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}} \left[\text{Use } \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \cos x \right] \\
&\Rightarrow \frac{2}{-\cos x} \\
&\Rightarrow -2 \sec x
\end{aligned}$$

7. Question

Mark the correct alternative in the following:

If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$ and $z = r \cos \theta$, then $x^2 + y^2 + z^2$ is independent of

- A. θ, ϕ
- B. r, θ
- C. r, ϕ
- D. r

Answer

Given:

$$X = r \sin \theta \cos \phi$$

$$Y = r \sin \theta \sin \phi$$

$$Z = r \cos \theta$$

$$\Rightarrow x^2 + y^2 + z^2$$

$$\Rightarrow (r \sin \theta \cos \phi)^2 + (r \sin \theta \sin \phi)^2 + (r \cos \theta)^2$$

$$\Rightarrow r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \sin^2 \phi + r^2 \cos^2 \theta$$

$$\Rightarrow r^2 \sin^2 \theta (\cos^2 \phi + \sin^2 \phi) + r^2 \cos^2 \theta$$

$$\Rightarrow r^2 \sin^2 \theta + r^2 \cos^2 \theta$$

$$\Rightarrow r^2 (\sin^2 \theta + \cos^2 \theta)$$

$$\Rightarrow r^2.$$

$\therefore$ It is independent of θ and ϕ .

8. Question

Mark the correct alternative in the following:

If $\tan x + \sec x = \sqrt{3}$, $0 < x < \pi$, then x is equal to

- A. $\frac{5\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer

Given: $\tan x + \sec x = \sqrt{3}$

squaring on both sides

$$(\tan x + \sec x)^2 = \sqrt{3}^2$$

$$\tan^2 x + \sec^2 x + 2 \tan x \sec x = 3$$

Also, $\sec^2 x - \tan^2 x = 1$

$$\tan^2 x + 1 + \tan^2 x + 2 \tan x \sec x = 3$$

$$2 \tan^2 x + 2 \tan x \sec x = 3 - 1$$

$$\tan^2 x + \tan x \sec x = 2/2$$

$$\tan^2 x + \tan x \sec x = 1$$

$$\tan x \sec x = 1 - \tan^2 x$$

again, squaring on both sides

$$\tan^2 x \sec^2 x = 1 + \tan^4 x - 2 \tan^2 x$$

$$(1 + \tan^2 x) \tan^2 x = 1 + \tan^4 x - 2 \tan^2 x$$

$$\tan^4 x + \tan^2 x = 1 + \tan^4 x - 2 \tan^2 x$$

$$3 \tan^2 x = 1$$

$$\tan x = 1/\sqrt{3}$$

$$x = \pi/6.$$

9. Question

Mark the correct alternative in the following:

If $\tan x = -\frac{1}{\sqrt{5}}$ and x lies in the IV quadrant, then the value of $\cos x$ is

A. $\frac{\sqrt{5}}{\sqrt{6}}$

B. $\frac{2}{\sqrt{6}}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{6}}$

Answer

In IV quadrant $\cos x$ is positive

We know

$$\tan^2 x + 1 = \sec^2 x$$

$$\Rightarrow \left(-\frac{1}{\sqrt{5}}\right)^2 + 1 = \sec^2 x$$

$$\Rightarrow \frac{1}{5} + 1 = \sec^2 x$$

$$\Rightarrow \sec^2 x = \frac{6}{5}$$

$$\Rightarrow \cos^2 x = \frac{5}{6}$$

$$\therefore \cos x = \frac{\sqrt{5}}{\sqrt{6}}$$

10. Question

Mark the correct alternative in the following:

If $\frac{3\pi}{4} < \alpha < \pi$, then $\sqrt{2 \cot \alpha + \frac{1}{\sin^2 \alpha}}$ is equal to

A. $1 - \cot \alpha$

B. $1 + \cot \alpha$

C. $-1 + \cot \alpha$

D. $-1 - \cot \alpha$

Answer

Given:

$$\Rightarrow \frac{3\pi}{4} < \alpha < \pi$$

$$\Rightarrow \sqrt{2 \cot \alpha + \frac{1}{\sin^2 \alpha}}$$

$$\Rightarrow \sqrt{2 \cot \alpha + \csc^2 \alpha}$$

We know $\csc^2 \alpha = \cot^2 \alpha + 1$

$$\Rightarrow \sqrt{2 \cot \alpha + 1 + \cot^2 \alpha}$$

$$\Rightarrow \sqrt{(\cot \alpha + 1)^2}$$

$$\Rightarrow \cot \alpha + 1$$

In the given range $\cot$ is negative

$$\therefore -1 - \cot \alpha$$

11. Question

Mark the correct alternative in the following:

$$\sin^6 A + \cos^6 A + 3 \sin^2 A \cos^2 A =$$

- A. 0
- B. 1
- C. 2
- D. 3

Answer

$$\begin{aligned}
 \sin^6 A + \cos^6 A &= (\sin^2 A)^3 + (\cos^2 A)^3 \\
 &= (\sin^2 A + \cos^2 A)(\sin^4 A + \cos^4 A - \sin^2 A \cos^2 A) \\
 &= 1 (\sin^4 A + \cos^4 A - \sin^2 A \cos^2 A) \\
 \therefore \sin^4 A + \cos^4 A - \sin^2 A \cos^2 A + 3 \sin^2 A \cos^2 A \\
 &\Rightarrow \sin^4 A + \cos^4 A + 2 \sin^2 A \cos^2 A \\
 &\Rightarrow (\sin^2 A + \cos^2 A)^2 \\
 &= 1^2 \\
 &= 1
 \end{aligned}$$

12. Question

Mark the correct alternative in the following:

If $\operatorname{cosec} x - \cot x = \frac{1}{2}$, $0 < x < \frac{\pi}{2}$, then $\cos x$ is equal to

- A. $\frac{5}{3}$
- B. $\frac{3}{5}$
- C. $-\frac{3}{5}$
- D. $-\frac{5}{3}$

Answer

Given:

Let $\operatorname{cosec} x = a$, $\cot x = b$

$\therefore$ According to the question

$$\Rightarrow a - b = \frac{1}{2}$$

But, $\operatorname{cosec}^2 x - \cot^2 x = 1$

$$\Rightarrow a^2 - b^2 = 1$$

$$\Rightarrow (a-b)(a+b) = 1$$

$$\Rightarrow \frac{1}{2}(a+b) = 1$$

$$\Rightarrow a + b = 2$$

$$a - b = 1/2 \dots (1)$$

$$a + b = 2 \dots (2)$$

Adding (1) and (2)

$$2a = \frac{1}{2} + 2$$

$$\Rightarrow a = \frac{5}{4}$$

$$\therefore \csc x = \frac{5}{4}$$

$$\Rightarrow \sin x = \frac{4}{5}$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \frac{16}{25}$$

$$\Rightarrow \cos^2 x = \frac{9}{25}$$

$$\Rightarrow \cos x = \frac{3}{5}$$

13. Question

Mark the correct alternative in the following:

If $\operatorname{cosec} x + \cot x = \frac{11}{2}$, then $\tan x =$

A. $\frac{21}{22}$

B. $\frac{15}{16}$

C. $\frac{44}{117}$

D. $\frac{117}{44}$

Answer

Let $\operatorname{cosec} x = a$, $\cot x = b$

$\therefore$ According to the question

$$\Rightarrow a - b = \frac{11}{2}$$

But, $\operatorname{cosec}^2 x - \cot^2 x = 1$

$$\Rightarrow a^2 - b^2 = 1$$

$$\Rightarrow (a-b)(a+b) = 1$$

$$\Rightarrow \frac{11}{2}(a+b) = 1$$

$$\Rightarrow a+b = \frac{2}{11}$$

$$a-b = \frac{11}{2} \dots (1)$$

$$a + b = \frac{2}{11} \dots (2)$$

Adding (1) and (2)

$$\Rightarrow 2a = \frac{11}{2} + \frac{2}{11}$$

$$\Rightarrow a = \frac{125}{44}$$

$$\Rightarrow \frac{125}{44} - \frac{11}{2} = b$$

$$\Rightarrow b = \frac{-117}{44}$$

$$\therefore \cot x = \frac{-117}{44}$$

$$\Rightarrow \tan x = \frac{44}{117}$$

14. Question

Mark the correct alternative in the following:

$$\sec^2 x = \frac{4xy}{(x+y)^2} \text{ is true if and only if}$$

- A. $x + y \neq 0$
- B. $x + y, x \neq 0$
- C. $x = y$
- D. $x \neq 0, y \neq 0$

Answer

First of all we need to check the condition on x

If $x = 0$ then $\sec^2 x$ attains to infinity, so that condition must be true i.e x should not be zero

Again if $x+y = 0$ then the RHS part becomes infinity so that condition must be true i.e. $x+y$ should not be zero.

$\therefore$ Option B is the correct answer.

15. Question

Mark the correct alternative in the following:

$$\text{If } x \text{ is an acute angle and } \tan x = \frac{1}{\sqrt{7}}, \text{ then the value of } \frac{\operatorname{cosec}^2 x - \sec^2 x}{\operatorname{cosec}^2 x + \sec^2 x} \text{ is}$$

- A. $3/4$
- B. $1/2$
- C. 2
- D. $5/4$

Answer

Given x is an acute angle and value of $\tan x = 1/\sqrt{7}$.

$$\Rightarrow \text{We know } \tan^2 x + 1 = \sec^2 x$$

$$\Rightarrow \text{Also, } \cot^2 x + 1 = \operatorname{cosec}^2 x$$

$$\therefore \tan^2 x = \frac{1}{7}$$

$$\therefore \tan^2 x + 1 = \frac{1}{7} + 1 = \frac{8}{7}$$

$$\therefore \sec^2 x = \frac{8}{7}$$

$$\Rightarrow \cot^2 x = 7$$

$$\Rightarrow \cot^2 x + 1 = 7 + 1$$

$$= 8$$

$$\Rightarrow \operatorname{cosec}^2 x = 8$$

$$\therefore \frac{\csc^2 x - \sec^2 x}{\csc^2 x + \sec^2 x} = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$\Rightarrow \frac{\frac{48}{7}}{\frac{64}{7}}$$

$$\Rightarrow \frac{48}{64} = \frac{3}{4}$$

$$\Rightarrow \frac{3}{4}$$

16. Question

Mark the correct alternative in the following:

The value of $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 85^\circ + \sin^2 90^\circ$ is

- A. 7
- B. 8
- C. 9.5
- D. 10

Answer

$$= \sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 45^\circ + \dots + \sin^2 75^\circ + \sin^2 80^\circ + \sin^2 85^\circ + \sin^2 90^\circ$$

We know that $\sin(90-x) = \cos x$

$$\text{So } \sin^2(90-x) = \cos^2 x$$

$$= \sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 45^\circ + \dots + \cos^2 15^\circ + \cos^2 10^\circ + \cos^2 5^\circ + \sin^2 90^\circ$$

$$\text{And } \sin^2 x + \cos^2 x = 1$$

So, in given series on rearranging terms we get 8 cases where $\sin^2 x + \cos^2 x = 1$

So, given changes to

$$8 + \sin^2 45^\circ + \sin^2 90^\circ$$

$$= 8 + \frac{1}{2} + 1$$

$$= 9 + \frac{1}{2}$$

$$= 9.5$$

17. Question

Mark the correct alternative in the following:

$$\sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} =$$

- A. 1
- B. 4
- C. 2
- D. 0

Answer

We know that $\sin(90-x) = \cos x$

So $\sin^2(90-x) = \cos^2 x$

$$\Rightarrow \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{9} \right)$$

$$\Rightarrow \cos^2 \frac{7\pi}{18}$$

$$\Rightarrow \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{18} \right)$$

$$\Rightarrow \cos^2 \frac{4\pi}{9}$$

And $\sin^2 x + \cos^2 x = 1$

Rearranging we get,

$$\Rightarrow \sin^2 \frac{7\pi}{18} + \cos^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} + \cos^2 \frac{4\pi}{9}$$

$$= 1 + 1$$

$$= 2$$

18. Question

Mark the correct alternative in the following:

If $\tan A + \cot A = 4$, then $\tan^4 A + \cot^4 A$ is equal to

- A. 110
- B. 191
- C. 80
- D. 194

Answer

Given: $\tan A + \cot A = 4$

$$\Rightarrow \tan A + \frac{1}{\tan A} = 4$$

Squaring both sides we get

$$\Rightarrow \left(\tan A + \frac{1}{\tan A} \right)^2 = 4^2$$

$$\Rightarrow \tan^2 A + \frac{1}{\tan^2 A} + 2 \cdot \tan A \cdot \frac{1}{\tan A} = 16$$

$$\Rightarrow \tan^2 A + \frac{1}{\tan^2 A} = 14$$

Squaring both sides we get

$$\Rightarrow \left(\tan^2 A + \frac{1}{\tan^2 A} \right)^2 = 14^2$$

$$\Rightarrow \tan^4 A + \frac{1}{\tan^4 A} + 2 \cdot \tan^2 A \cdot \frac{1}{\tan^2 A} = 196$$

$$\Rightarrow \tan^4 A + \frac{1}{\tan^4 A}$$

$$= 194$$

19. Question

Mark the correct alternative in the following:

$$\text{If } x \sin 45^\circ \cos^2 60^\circ = \frac{\tan^2 60^\circ \operatorname{cosec} 30^\circ}{\sec 45^\circ \cot^2 30^\circ}, \text{ then } x =$$

A. 2

B. 4

C. 8

D. 16

Answer

According to the given question:

$$\Rightarrow x = \frac{\tan^2 60 \csc 30}{\sec 45 \cot^2 30 \sin 45 \cos^2 60}$$

$$\Rightarrow x = \frac{(\sqrt{3})^2 \cdot 2}{\sqrt{2} \cdot (\sqrt{3})^2 \cdot \frac{1}{\sqrt{2}} \cdot \left(\frac{1}{2}\right)^2}$$

$$\Rightarrow x = 8.$$

20. Question

Mark the correct alternative in the following:

If A lies in second quadrant and $3 \tan A + 4 = 0$, then the value of $2 \cot A - 5 \cos A + \sin A$ is equal to

A. $-53/10$

B. $23/10$

C. $37/10$

D. $7/10$

Answer

Given:

$$3 \tan A + 4 = 0$$

$$\Rightarrow \tan A = \frac{-4}{3}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\Rightarrow \left(\frac{-4}{3}\right)^2 + 1 = \sec^2 x$$

$$\Rightarrow \sec^2 A = \frac{16}{9} + 1$$

$$\Rightarrow \sec^2 A = \frac{25}{9}$$

$$\Rightarrow \sec A = \frac{5}{3}$$

$$\Rightarrow \cos A = -\frac{3}{5}$$

Because in second quadrant cos is negative.

$$\Rightarrow \cot A = \frac{-3}{4}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 A = 1 - \left(\frac{9}{25}\right)$$

$$\Rightarrow \sin^2 A = \frac{16}{25}$$

$$\Rightarrow \sin A = \frac{4}{5}$$

$\therefore$ The value of $2 \cot A - 5 \cos A + \sin A =$

$$\Rightarrow 2\left(\frac{-3}{4}\right) - 5\left(\frac{-3}{5}\right) + \frac{4}{5}$$

$$\Rightarrow \frac{-6}{4} + \frac{15}{5} + \frac{4}{5}$$

$$\Rightarrow \frac{-6}{4} + \frac{19}{5}$$

$$\Rightarrow \frac{-30 + 76}{20}$$

$$\Rightarrow \frac{23}{10}$$

21. Question

Mark the correct alternative in the following:

If $\operatorname{cosec} x + \cot x = \frac{11}{2}$, then $\tan x =$

A. 21/22

B. 15/16

C. 44/117

D. 117/43

Answer

Let $\operatorname{cosec} x = a$, $\cot x = b$

∴ According to the question

$$\Rightarrow a + b = \frac{11}{2}$$

But, $\operatorname{cosec}^2 x - \cot^2 x = 1$

$$\Rightarrow a^2 - b^2 = 1$$

$$\Rightarrow (a-b)(a+b) = 1$$

$$\Rightarrow \frac{11}{2}(a-b) = 1$$

$$\Rightarrow a - b = \frac{2}{11}$$

$$a + b = 11/2 \dots(1)$$

$$a - b = 2/11 \dots(2)$$

Adding (1) and (2)

$$\Rightarrow 2a = \frac{11}{2} + \frac{2}{11}$$

$$\Rightarrow a = \frac{125}{44}$$

$$\Rightarrow \frac{125}{44} - \frac{2}{11} = b$$

$$\Rightarrow b = \frac{117}{44}$$

$$\therefore \cot x = \frac{117}{44}$$

$$\therefore \tan x = \frac{44}{117}$$

22. Question

Mark the correct alternative in the following:

If $\tan \theta + \sec \theta = e^x$, then $\cos \theta$ equals

A. $\frac{e^x + e^{-x}}{2}$

B. $\frac{2}{e^x + e^{-x}}$

C. $\frac{e^x - e^{-x}}{2}$

D. $\frac{e^x - e^{-x}}{e^x + e^{-x}}$

Answer

We know $\tan^2 x + 1 = \sec^2 x \dots(1)$

Let $\tan \theta$ be a and $\sec \theta$ be b

According to question

$$a + b = e^x$$

Manipulating and Substituting in 1 we get

$$\Rightarrow a^2 - b^2 = -1$$

$$\Rightarrow (a-b)(a+b) = -1$$

$$\Rightarrow (a-b).e^x = -1$$

$$\Rightarrow a-b = -e^{-x}$$

$$a + b = e^x$$

$$a-b = -e^{-x}$$

subtracting above equations we get

$$2b = e^x + e^{-x}$$

$$\Rightarrow b = \frac{e^x + e^{-x}}{2}$$

$$\Rightarrow \sec \theta = \frac{e^x + e^{-x}}{2}$$

$$\Rightarrow \cos \theta = \frac{2}{e^x + e^{-x}}$$

23. Question

Mark the correct alternative in the following:

If $\sec x + \tan x = k$, $\cos x =$

A. $\frac{k^2 + 1}{2k}$

B. $\frac{2k}{k^2 + 1}$

C. $\frac{k}{k^2 + 1}$

D. $\frac{k}{k^2 - 1}$

Answer

We know $\tan^2 x + 1 = \sec^2 x \dots (1)$

Let $\tan x$ be a and $\sec x$ be b

According to question

$$a + b = k$$

Manipulating and Substituting in 1 we get

$$\Rightarrow a^2 - b^2 = -1$$

$$\Rightarrow (a-b)(a+b) = -1$$

$$\Rightarrow (a-b).k = -1$$

$$\Rightarrow a-b = -k^{-1}$$

$$a + b = k$$

$$a - b = -k^{-1}$$

subtracting above equations we get

$$2b = k + k^{-1}$$

$$\Rightarrow b = \frac{k^2 + 1}{2k}$$

$$\Rightarrow \sec \theta = \frac{k^2 + 1}{2k}$$

$$\Rightarrow \cos \theta = \frac{2k}{k^2 + 1}$$

24. Question

Mark the correct alternative in the following:

If $f(x) = \cos^2 x + \sec^2 x$, the

- A. $f(x) < 1$
- B. $f(x) = 1$
- C. $2 < f(x) < 1$
- D. $f(x) \geq 2$

Answer

$$\tan^2 x + 1 = \sec^2 x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\therefore \cos^2 x = 1 - \sin^2 x$$

Substituting in $f(x)$ we get

$$1 - \sin^2 x + \tan^2 x + 1$$

$$\Rightarrow 2 - \sin^2 x + \frac{\sin^2 x}{\cos^2 x}$$

$$\Rightarrow 2 - \frac{\sin^2 x \cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\Rightarrow 2 + \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$$

$$\Rightarrow 2 + \frac{\sin^4 x}{\cos^2 x}$$

Minimum value of $\frac{\sin^4 x}{\cos^2 x}$ is 0.

$$\therefore f(x) \geq 2.$$

25. Question

Mark the correct alternative in the following:

Which of the following is incorrect?

- A. $\sin x = -1/5$
- B. $\cos x = 1$

C. $\sec x = 1/2$

D. $\tan x = 20$

Answer

$\sec x = 1/2$ is incorrect because for no real value of x $\sec x$ attains $1/2$.

26. Question

Mark the correct alternative in the following:

The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ$ is

A. $1/\sqrt{2}$

B. 0

C. 1

D. -1

Answer

$$\cos 1^\circ \times \cos 2^\circ \times \cos 3^\circ \times \dots \times \cos 179^\circ$$

$$= \cos 1^\circ \times \cos 2^\circ \times \cos 3^\circ \times \dots \times \cos 90^\circ \times \dots \times \cos 179^\circ$$

$$= \cos 1^\circ \times \cos 2^\circ \times \cos 3^\circ \times \dots \times 0 \times \dots \times \cos 179^\circ$$

$$= 0 \times \cos 1^\circ \times \cos 2^\circ \times \cos 3^\circ \times \dots \times \cos 179^\circ$$

$$= 0$$

27. Question

Mark the correct alternative in the following:

The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$ is

A. 0

B. 1

C. $1/2$

D. not defined

Answer

$$\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ = \tan (90^\circ - 89^\circ) \tan (90^\circ - 88^\circ) \tan (90^\circ - 87^\circ) \dots \tan (90^\circ - 46^\circ) \tan 45^\circ \tan 46^\circ \dots \tan 89^\circ$$

$$= \cot 89^\circ \cot 88^\circ \cot 87^\circ \dots \cot 46^\circ \tan 45^\circ \tan 46^\circ \dots \tan 89^\circ$$

$$(\because \tan (90^\circ - \theta) = \cot \theta)$$

$$= \frac{1}{\tan 89^\circ} \times \frac{1}{\tan 88^\circ} \times \frac{1}{\tan 87^\circ} \times \dots \times \frac{1}{\tan 46^\circ} \times \tan 45^\circ \tan 46^\circ \dots \tan 89^\circ$$

$$= \tan 45^\circ$$

$$= 1$$

28. Question

Mark the correct alternative in the following:

Which of the following is correct?

A. $\sin 1^\circ > \sin 1$

B. $\sin 1^\circ < \sin 1$

C. $\sin 1^\circ = \sin 1$

D. $\sin 1^\circ = \frac{\pi}{180} \sin 1$

Answer

$$\Rightarrow 1^\circ = \frac{\pi}{180} \text{rad}$$

$$\Rightarrow 1 \text{rad} = \frac{180^\circ}{\pi}$$

$$\therefore 1 \text{rad} = 57.32^\circ$$

In Range 0 to $\pi/2$ $\sin x$ is an increasing function

$\therefore \sin 1$ will always be greater than $\sin 1^\circ$

Because $\sin 1 = \sin 57.32^\circ$