

Square and Square roots

Square Number

if a natural number m can be expressed as n^2 , where n is also a natural number, then m is a square number

Some Important point to Note

S.no	Points
1	All square numbers end with 0, 1, 4, 5, 6 or 9 at unit's place
2	if a number has 1 or 9 in the unit's place, then it's square ends in 1.
3	when a square number ends in 6, the number whose square it is, will have either 4 or 6 in unit's place
4	None of square number with 2, 3, 7 or 8 at unit's place.
5	Even number square is even while odd number square is Odd
6	there are $2n$ non perfect square numbers between the squares of the numbers n and $(n + 1)$
7	if a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square

How to find the square of Number easily

S.no	Method	Working
1	Identity method	We know that $(a+b)^2 = a^2 + 2ab + b^2$

2	Special Cases	Example
		$23^2 = (20+3)^2 = 400+9+120=529$
		$(a5)^2$
		$= a(a+1) \text{ hundred} + 25$
		Example
		$25^2 = 2(3) \text{ hundred} + 25 = 625$

Pythagorean triplets

For any natural number $m > 1$, we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$
 So, $2m$, $m^2 - 1$ and $m^2 + 1$ forms a Pythagorean triplet

Example

6,8,10

$$6^2 + 8^2 = 10^2$$

Square Root

Square root of a number is the number whose square is given number

So we know that

$$m = n^2$$

Square root of m

$$\sqrt{m} = n$$

Square root is denoted by expression $\sqrt{\quad}$

How to Find Square root

Name	Description
Finding square root through repeated subtraction	We know sum of the first n odd natural numbers is n^2. So in this method we subtract the odd number starting from 1 until we get the remainder as zero. The count of odd number will be the square root Consider 36 Then, (i) $36 - 1 = 35$ (ii) $35 - 3 = 32$ (iii) $32 - 5 = 27$ (iv) $27 - 7 = 20$ (v) $20 - 9 = 11$ (vi) $11 - 11 = 0$ So 6 odd number, Square root is 6
Finding square root through prime Factorisation	This method, we find the prime factorization of the number. We will get same prime number occurring in pair for perfect square number. Square root will be given by multiplication of prime factor occurring in pair Consider 81 $81 = (3 \times 3) \times (3 \times 3)$ $\sqrt{81} = 3 \times 3 = 9$
Finding square root by division method	This can be well explained with the example Step 1 Place a bar over every pair of digits starting from the digit at one's place. If the number of digits in it is odd, then the left-most single digit too will have a bar. So in the below example 6 and 25 will have separate bar Step 2 Find the largest number whose square is less than or equal to the number under the extreme left bar. Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend. Divide and get the remainder

In the below example $4 < 6$, So taking 2 as divisor and quotient and dividing, we get 2 as remainder

Step 3 Bring down the number under the next bar to the right of the remainder.

In the below example we bring 25 down with the remainder, so the number is 225

Step 4 Double the quotient and enter it with a blank on its right.

In the below example, it will be 4

Step 5 Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

In this case $45 \times 5 = 225$ so we choose the new digit as 5. Get the remainder.

Step 6 Since the remainder is 0 and no digits are left in the given number, therefore the number on the top is square root

		25
2		<u>625</u>
		4
45		<u>225</u> .
		225
		0

In case of Decimal Number, we count the bar on the integer part in the same manner as we did above, but for the decimal part, we start pairing the digit from first decimal part.