2.4 Complex variables

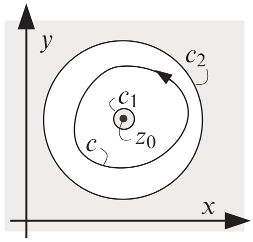
Complex numbers

Cartesian form	$z = x + \mathbf{i}y$	(2.153)	z $\mathbf{i}$ x, y	complex variable $\mathbf{i}^2 = -1$ real variables
Polar form	$z = r\mathrm{e}^{\mathbf{i}\theta} = r(\cos\theta + \mathbf{i}\sin\theta)$	(2.154)	r θ	amplitude (real) phase (real)
Modulus[a]	$\lvert z\rvert = r = (x^2 + y^2)^{1/2}$ $\lvert z_1 \cdot z_2\rvert = \lvert z_1\rvert \cdot \lvert z_2\rvert$	(2.155) (2.156)	$\lvert z\rvert$	modulus of z
Argument	$\theta = \arg z = \arctan \dfrac{y}{x}$ $\arg(z_1 z_2) = \arg z_1 + \arg z_2$	(2.157) (2.158)	$\arg z$	argument of z
Complex conjugate	$z^* = x - \mathbf{i}y = r\mathrm{e}^{-\mathbf{i}\theta}$ $\arg(z^*) = -\arg z$ $z \cdot z^* = \lvert z\rvert^2$	(2.159) (2.160) (2.161)	z^*	complex conjugate of $z = r\mathrm{e}^{\mathbf{i}\theta}$
Logarithm[b]	$\ln z = \ln r + \mathbf{i}(\theta + 2\pi n)$	(2.162)	n	integer

[a] Or "magnitude."
[b] The principal value of $\ln z$ is given by $n = 0$ and $-\pi < \theta \le \pi$.

Complex analysis[a]

Cauchy–Riemann equations[b]	if $f(z) = u(x,y) + \mathbf{i}v(x,y)$ then $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$	(2.163) (2.164)	z $\mathbf{i}$ x, y $f(z)$ u, v	complex variable $\mathbf{i}^2 = -1$ real variables function of z real functions
Cauchy–Goursat theorem[c]	$\oint_c f(z)\,\mathrm{d}z = 0$	(2.165)		
Cauchy integral formula[d]	$f(z_0) = \dfrac{1}{2\pi\mathbf{i}} \oint_c \dfrac{f(z)}{z - z_0}\,\mathrm{d}z$ $f^{(n)}(z_0) = \dfrac{n!}{2\pi\mathbf{i}} \oint_c \dfrac{f(z)}{(z - z_0)^{n+1}}\,\mathrm{d}z$	(2.166) (2.167)	$^{(n)}$ a_n a_{-1} z'	nth derivative Laurent coefficients residue of $f(z)$ at z_0 dummy variable
Laurent expansion[e]	$f(z) = \sum_{n=-\infty}^{\infty} a_n (z - z_0)^n$ where $a_n = \dfrac{1}{2\pi\mathbf{i}} \oint_c \dfrac{f(z')}{(z' - z_0)^{n+1}}\,\mathrm{d}z'$	(2.168) (2.169)		
Residue theorem	$\oint_c f(z)\,\mathrm{d}z = 2\pi\mathbf{i} \sum \text{enclosed residues}$	(2.170)		



[a] Closed contour integrals are taken in the counterclockwise sense, once.
[b] Necessary condition for $f(z)$ to be analytic at a given point.
[c] If $f(z)$ is analytic within and on a simple closed curve c. Sometimes called "Cauchy's theorem."
[d] If $f(z)$ is analytic within and on a simple closed curve c, encircling z_0.
[e] Of $f(z)$, (analytic) in the annular region between concentric circles, c_1 and c_2, centred on z_0. c is any closed curve in this region encircling z_0.