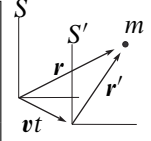


3.2 Frames of reference

Galilean transformations

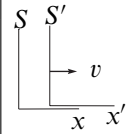
Time and position ^a	$\mathbf{r} = \mathbf{r}' + \mathbf{v}t$	(3.1)	$\mathbf{r}, \mathbf{r}'$	position in frames S and S'
	$t = t'$	(3.2)	$\mathbf{v}$ t, t'	velocity of S' in S time in S and S'
Velocity	$\mathbf{u} = \mathbf{u}' + \mathbf{v}$	(3.3)	$\mathbf{u}, \mathbf{u}'$	velocity in frames S and S'
Momentum	$\mathbf{p} = \mathbf{p}' + m\mathbf{v}$	(3.4)	$\mathbf{p}, \mathbf{p}'$ m	particle momentum in frames S and S' particle mass
Angular momentum	$\mathbf{J} = \mathbf{J}' + m\mathbf{r}' \times \mathbf{v} + \mathbf{v} \times \mathbf{p}'t$	(3.5)	$\mathbf{J}, \mathbf{J}'$	angular momentum in frames S and S'
Kinetic energy	$T = T' + m\mathbf{u}' \cdot \mathbf{v} + \frac{1}{2}mv^2$	(3.6)	T, T'	kinetic energy in frames S and S'



^aFrames coincide at $t=0$.

Lorentz (spacetime) transformations^a

Lorentz factor	$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$	(3.7)	γ v c	Lorentz factor velocity of S' in S speed of light
Time and position	$x = \gamma(x' + vt')$; $x' = \gamma(x - vt)$	(3.8)	x, x' t, t' X	x-position in frames S and S' (similarly for y and z) time in frames S and S' spacetime four-vector
	$y = y'$; $y' = y$	(3.9)		
	$z = z'$; $z' = z$	(3.10)		
	$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$; $t' = \gamma\left(t - \frac{v}{c^2}x\right)$	(3.11)		
Differential four-vector ^b	$dX = (cdt, -dx, -dy, -dz)$	(3.12)		

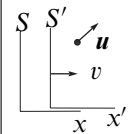


^aFor frames S and S' coincident at $t=0$ in relative motion along x . See page 141 for the transformations of electromagnetic quantities.

^bCovariant components, using the $(1, -1, -1, -1)$ signature.

Velocity transformations^a

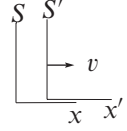
Velocity	$u_x = \frac{u'_x + v}{1 + u'_x v/c^2}$; $u'_x = \frac{u_x - v}{1 - u_x v/c^2}$	(3.13)	γ v c u_i, u'_i	Lorentz factor $= [1 - (v/c)^2]^{-1/2}$ velocity of S' in S speed of light particle velocity components in frames S and S'
	$u_y = \frac{u'_y}{\gamma(1 + u'_x v/c^2)}$; $u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)}$	(3.14)		
	$u_z = \frac{u'_z}{\gamma(1 + u'_x v/c^2)}$; $u'_z = \frac{u_z}{\gamma(1 - u_x v/c^2)}$	(3.15)		



^aFor frames S and S' coincident at $t=0$ in relative motion along x .

Momentum and energy transformations^a

Momentum and energy		γ	Lorentz factor $= [1 - (v/c)^2]^{-1/2}$
$p_x = \gamma(p'_x + vE'/c^2);$	$p'_x = \gamma(p_x - vE/c^2)$	v	velocity of S' in S
$p_y = p'_y;$	$p'_y = p_y$	c	speed of light
$p_z = p'_z;$	$p'_z = p_z$	p_x, p'_x	x components of momentum in S and S' (sim. for y and z)
$E = \gamma(E' + vp'_x);$	$E' = \gamma(E - vp_x)$	E, E'	energy in S and S'
$E^2 - p^2c^2 = E'^2 - p'^2c^2 = m_0^2c^4$		m_0	(rest) mass
		p	total momentum in S
Four-vector ^b	$\mathbf{P} = (E/c, -p_x, -p_y, -p_z)$	$\mathbf{P}$	momentum four-vector

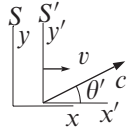
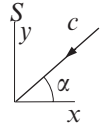


^aFor frames S and S' coincident at $t=0$ in relative motion along x .

^bCovariant components, using the $(1, -1, -1, -1)$ signature.

Propagation of light^a

Doppler effect	$\frac{v'}{v} = \gamma \left(1 + \frac{v}{c} \cos \alpha \right)$	v	frequency received in S
		v'	frequency emitted in S'
		α	arrival angle in S
Aberration ^b	$\cos \theta = \frac{\cos \theta' + v/c}{1 + (v/c) \cos \theta'}$	γ	Lorentz factor $= [1 - (v/c)^2]^{-1/2}$
	$\cos \theta' = \frac{\cos \theta - v/c}{1 - (v/c) \cos \theta}$	v	velocity of S' in S
		c	speed of light
		θ, θ'	emission angle of light in S and S'
Relativistic beaming ^c	$P(\theta) = \frac{\sin \theta}{2\gamma^2 [1 - (v/c) \cos \theta]^2}$	$P(\theta)$	angular distribution of photons in S



^aFor frames S and S' coincident at $t=0$ in relative motion along x .

^bLight travelling in the opposite sense has a propagation angle of $\pi + \theta$ radians.

^cAngular distribution of photons from a source, isotropic and stationary in S' . $\int_0^\pi P(\theta) d\theta = 1$.

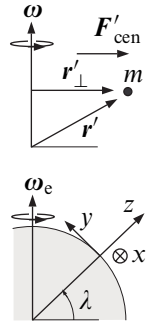
Four-vectors^a

Covariant and contravariant components	$x_0 = x^0$ $x_1 = -x^1$ $x_2 = -x^2$ $x_3 = -x^3$	x_i	covariant vector components
		x^i	contravariant components
Scalar product	$x^i y_i = x^0 y_0 + x^1 y_1 + x^2 y_2 + x^3 y_3$		
Lorentz transformations		x^i, x'^i	four-vector components in frames S and S'
$x^0 = \gamma[x'^0 + (v/c)x'^1];$	$x'^0 = \gamma[x^0 - (v/c)x^1]$	γ	Lorentz factor $= [1 - (v/c)^2]^{-1/2}$
$x^1 = \gamma[x'^1 + (v/c)x'^0];$	$x'^1 = \gamma[x^1 - (v/c)x^0]$	v	velocity of S' in S
$x^2 = x'^2;$	$x'^3 = x^3$	c	speed of light

^aFor frames S and S' , coincident at $t=0$ in relative motion along the (1) direction. Note that the $(1, -1, -1, -1)$ signature used here is common in special relativity, whereas $(-1, 1, 1, 1)$ is often used in connection with general relativity (page 67).

Rotating frames

Vector transformation	$\left[\frac{d\mathbf{A}}{dt} \right]_S = \left[\frac{d\mathbf{A}}{dt} \right]_{S'} + \boldsymbol{\omega} \times \mathbf{A} \quad (3.31)$	$\mathbf{A}$ any vector S stationary frame S' rotating frame $\boldsymbol{\omega}$ angular velocity of S' in S
Acceleration	$\dot{\mathbf{v}} = \dot{\mathbf{v}}' + 2\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \quad (3.32)$	$\dot{\mathbf{v}}, \dot{\mathbf{v}}'$ accelerations in S and S' $\mathbf{v}'$ velocity in S' $\mathbf{r}'$ position in S'
Coriolis force	$\mathbf{F}'_{\text{cor}} = -2m\boldsymbol{\omega} \times \mathbf{v}' \quad (3.33)$	$\mathbf{F}'_{\text{cor}}$ coriolis force m particle mass
Centrifugal force	$\mathbf{F}'_{\text{cen}} = -m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \quad (3.34)$	$\mathbf{F}'_{\text{cen}}$ centrifugal force
	$= +m\omega^2 \mathbf{r}'_{\perp} \quad (3.35)$	$\mathbf{r}'_{\perp}$ perpendicular to particle from rotation axis
Motion relative to Earth	$m\ddot{\mathbf{x}} = F_x + 2m\omega_e(\dot{y} \sin \lambda - \dot{z} \cos \lambda) \quad (3.36)$	F_i nongravitational force
	$m\ddot{y} = F_y - 2m\omega_e \dot{x} \sin \lambda \quad (3.37)$	λ latitude
	$m\ddot{z} = F_z - mg + 2m\omega_e \dot{x} \cos \lambda \quad (3.38)$	z local vertical axis y northerly axis x easterly axis
Foucault's pendulum ^a	$\Omega_f = -\omega_e \sin \lambda \quad (3.39)$	Ω_f pendulum's rate of turn ω_e Earth's spin rate



^aThe sign is such as to make the rotation clockwise in the northern hemisphere.