3.7 Generalised dynamics

Lagrangian dynamics

Action	$$S=\int_{t_1}^{t_2} L(\boldsymbol{q},\dot{\boldsymbol{q}},t)\,\mathrm{d}t$$	(3.213)	S $\boldsymbol{q}$ $\dot{\boldsymbol{q}}$	action ($\delta S=0$ for the motion) generalised coordinates generalised velocities
Euler–Lagrange equation	$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0$$	(3.214)	L t m	Lagrangian time mass
Lagrangian of particle in external field	$$L=\frac{1}{2}mv^2-U(\boldsymbol{r},t)$$ $$=T-U$$	(3.215) (3.216)	$\boldsymbol{v}$ $\boldsymbol{r}$ U T	velocity position vector potential energy kinetic energy
Relativistic Lagrangian of a charged particle	$$L=-\frac{m_0c^2}{\gamma}-e(\phi-\boldsymbol{A}\cdot\boldsymbol{v})$$	(3.217)	m_0 γ $+e$ ϕ $\boldsymbol{A}$	(rest) mass Lorentz factor positive charge electric potential magnetic vector potential
Generalised momenta	$$p_i=\frac{\partial L}{\partial \dot{q}_i}$$	(3.218)	p_i	generalised momenta

Hamiltonian dynamics

Hamiltonian	$$H=\sum_i p_i\dot{q}_i-L$$	(3.219)	L p_i $\dot{q}_i$	Lagrangian generalised momenta generalised velocities
Hamilton's equations	$$\dot{q}_i=\frac{\partial H}{\partial p_i};\qquad \dot{p}_i=-\frac{\partial H}{\partial q_i}$$	(3.220)	H q_i	Hamiltonian generalised coordinates
Hamiltonian of particle in external field	$$H=\frac{1}{2}mv^2+U(\boldsymbol{r},t)$$ $$=T+U$$	(3.221) (3.222)	v $\boldsymbol{r}$ U T	particle speed position vector potential energy kinetic energy
Relativistic Hamiltonian of a charged particle	$$H=(m_0^2c^4+\lvert\boldsymbol{p}-e\boldsymbol{A}\rvert^2c^2)^{1/2}+e\phi$$	(3.223)	m_0 c $+e$ ϕ $\boldsymbol{A}$	(rest) mass speed of light positive charge electric potential vector potential
Poisson brackets	$$[f,g]=\sum_i\left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i}-\frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}\right)$$ $$[q_i,g]=\frac{\partial g}{\partial p_i},\qquad [p_i,g]=-\frac{\partial g}{\partial q_i}$$ $$[H,g]=0\quad\text{if}\quad\frac{\partial g}{\partial t}=0,\quad\frac{\mathrm{d}g}{\mathrm{d}t}=0$$	(3.224) (3.225) (3.226)	$\boldsymbol{p}$ t f,g $[\cdot,\cdot]$	particle momentum time arbitrary functions Poisson bracket (also see *Commutators* on page 26)
Hamilton–Jacobi equation	$$\frac{\partial S}{\partial t}+H\left(q_i,\frac{\partial S}{\partial q_i},t\right)=0$$	(3.227)	S	action