

Chapter – 6

Lines and Angles

Introduction to Lines and Angles

Introduction

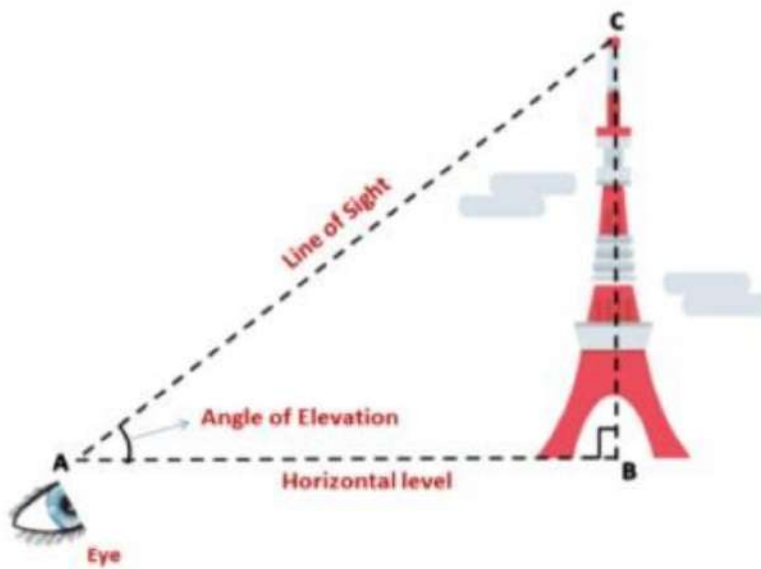
We see lines and angles all around us in different objects. In our daily life, we come across different types of angles formed between the edges of different surfaces.



An architect applies the knowledge of lines and angles for drawing a plan of a multi-storied building.

In science, lines are extensively used to represent the properties of light using the ray diagrams.

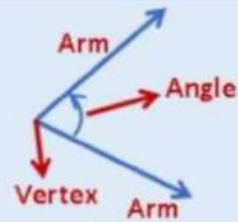
We can easily find the height of a tower or a tall building if we know the angle formed between the horizontal line and the line of sight.



Basic Terms and Definitions

Line – It is a collection of points which has only length, no breadth or thickness and is endless in both directions. Line AB is denoted by $\overleftrightarrow{AB}$.	
Line segment – A portion of a line with two endpoints is called a line segment. Line segment AB is denoted by $\overline{AB}$.	
Ray – A part of a line with one endpoint is called a ray. Ray AB is represented by $\overrightarrow{AB}$.	
Collinear Points - If three or more points lie on the same line, they are called collinear points otherwise they are called non-collinear points.	

Angle – When two rays originate from the same endpoint, they form an angle.
The rays making an angle are called the arms and the endpoint is called the vertex of the angle.

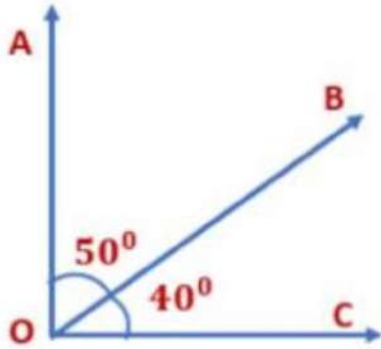


Types of Angles

Acute Angle is an angle whose measure is more than 0° but less than 90° $0^\circ < x < 90^\circ$	A diagram showing an acute angle labeled 'x' in red. The angle is formed by two rays meeting at a vertex. A dashed line indicates the 90° reference angle.
A right angle is an angle whose measure is 90°. $y = 90^\circ$	A diagram showing a right angle labeled 'y' in red. The angle is formed by two rays meeting at a vertex, forming a square corner. A dashed line indicates the 90° reference angle.
An obtuse angle is an angle whose measure is greater than 90° but less than 180°. $90^\circ < z < 180^\circ$	A diagram showing an obtuse angle labeled 'z' in red. The angle is formed by two rays meeting at a vertex. A dashed line indicates the 90° reference angle, and another dashed line indicates the 180° reference angle.
A straight angle is an angle whose measure is 180°. Thus, a straight angle looks like a straight line. $u = 180^\circ$	A diagram showing a straight angle labeled 'u' in red. The angle is formed by two rays meeting at a vertex, forming a straight line. A dashed line indicates the 180° reference angle.
A reflex angle is an angle whose measure is more than 180° but less than 360° $180^\circ < v < 360^\circ$	A diagram showing a reflex angle labeled 'v' in red. The angle is formed by two rays meeting at a vertex, forming an angle greater than a straight line. A dashed line indicates the 180° reference angle.

Complementary and Supplementary Angles

A pair of angles are said to be complementary if the sum of the angles is equal to 90°

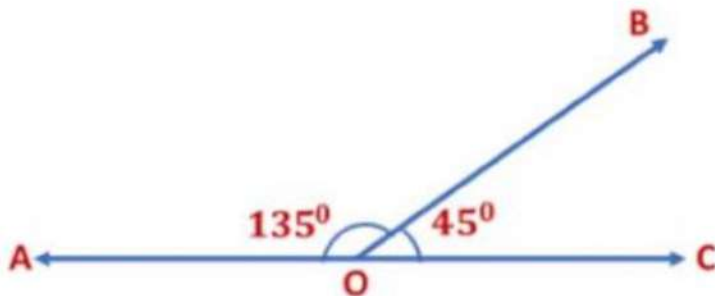


$$\angle AOB + \angle BOC = 50^\circ + 40^\circ = 90^\circ$$

$$\angle AOB + \angle BOC = 90^\circ$$

$\therefore \angle AOB$ and $\angle BOC$ are complementary angles.

A pair of angles are said to be supplementary if the sum of the angles is equal to 180°



$$\angle AOB + \angle BOC = 135^\circ + 45^\circ = 180^\circ$$

$$\angle AOB + \angle BOC = 180^\circ$$

$\therefore \angle AOB$ and $\angle BOC$ are supplementary angles.

Example: If $(2x - 20^\circ)$ and $(x + 5^\circ)$ are complementary angles, find the angles.

A pair of angles is said to be complementary if the sum of the angles is equal to 90° .

If $(2x - 20^\circ)$ and $(x + 5^\circ)$ are complementary angles then their sum will be equal to 90° .

$$(2x - 20^\circ) + (x + 5^\circ) = 90^\circ$$

$$2x - 20^\circ + x + 5^\circ = 90^\circ$$

$$2x + x - 20^\circ + 5^\circ = 90$$

$$3x - 15^\circ = 90^\circ \Rightarrow 3x = 15^\circ + 90^\circ$$

$$3x = 105^\circ \Rightarrow x = \frac{105}{3}^\circ = 35^\circ$$

$$(2x - 20^\circ) = (2 \times 35^\circ - 20^\circ) \Rightarrow 70^\circ - 20^\circ = 50^\circ$$

$$(2x - 20^\circ) = 50^\circ$$

$$(x + 5^\circ) = 35^\circ + 5^\circ = 40^\circ$$

$$(x + 5^\circ) = 40^\circ$$

Example: Two supplementary are in the ratio 3: 6, find the angles.

Let the two supplementary angles be $3x$ and $6x$,

$$3x + 6x = 180^\circ$$

A pair of angles are said to be supplementary if the sum of the angles is equal to 180°

$$9x = 180^\circ \Rightarrow x = \frac{180}{9} = 20^\circ$$

$$x = 20^\circ$$

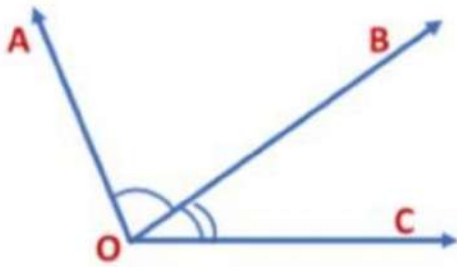
$$3x = 3 \times 20 = 60^\circ$$

$$6x = 6 \times 20^\circ = 120^\circ$$

Adjacent Angles

Two angles are adjacent if

- i) they have a common vertex
- ii) they have a common arm
- iii) their non-common arms are on different sides of the common arm.



Here, $\angle AOB$ and $\angle BOC$ are adjacent angles because these angles have a common vertex O . Ray OB is the common arm. Rays AO and CO are the non-common arms.

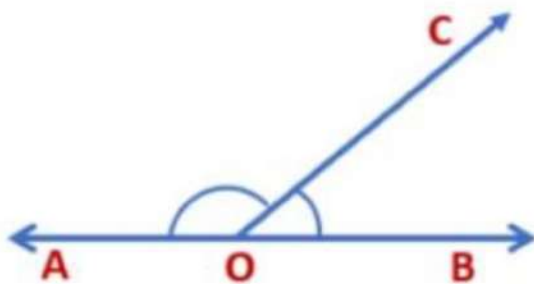
When two angles are adjacent, then their sum is always equal to the angle formed by the two non-common arms.

Therefore, $\angle AOC = \angle AOB + \angle BOC$

We see that $\angle AOC$ and $\angle AOB$ are not adjacent as their non-common arms OB and OC are on the same side of the common arm AO .

Linear pair of Angles

If the non-common arms of two adjacent angles are two opposite rays, then these angles are called linear pairs of angles.

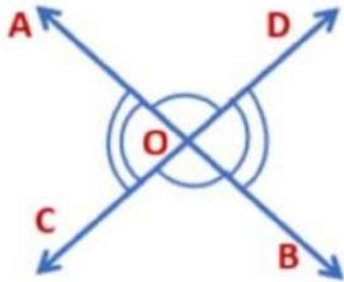


Here, OA and OB are two opposite rays and $\angle AOC$, $\angle BOC$ are adjacent angles.

Therefore, $\angle AOC$ and $\angle BOC$ form a linear pair.

Vertically Opposite Angles

When two lines AB and CD intersect each other at point O, then there are two pairs of vertically opposite angles.



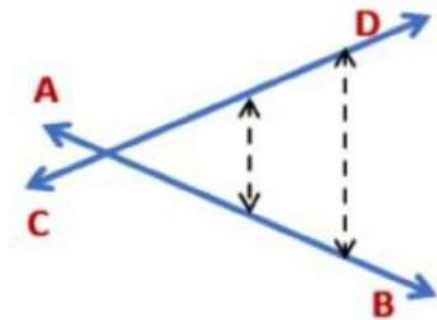
Here,

- 1) $\angle AOC$ and $\angle BOD$ are vertically opposite angles.
- 2) $\angle AOD$ and $\angle BOC$ are vertically opposite angles

Intersecting and non-intersecting lines

Intersecting Lines and Non-intersecting Lines

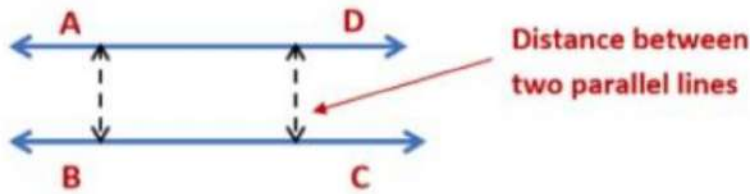
Intersecting Lines – Two lines are said to be intersecting when the perpendicular distance between the two lines is not the same everywhere and they intersect at one point.



Here, lines AB and CD are intersecting lines.

Non-Intersecting Lines

Two lines are said to be non-intersecting when the perpendicular distance between them is the same everywhere and they do not meet.



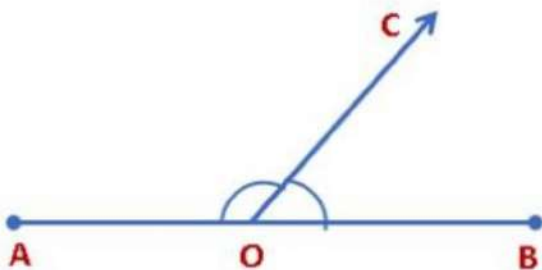
The lengths of the common perpendiculars at different points on these lines are the same and so these lines are parallel. This equal length is called the distance between two parallel lines.

Two lines in a plane will either intersect at one point or do not intersect at all, that is they are parallel.

Pairs of Angles

Pairs of Angles

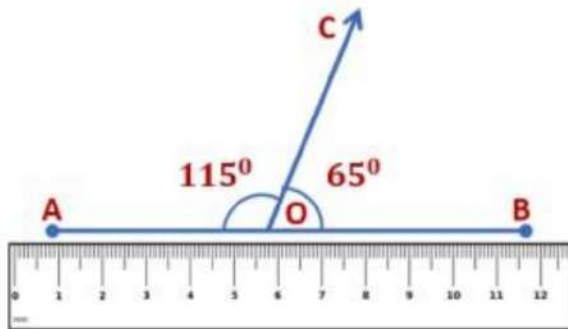
Axiom 1: If a ray stands on a line, then the sum of two adjacent angles so formed is 180° .



Here, ray OC stands on line AB, then $\angle AOC$ and $\angle COB$ are adjacent angles. Therefore, $\angle AOC + \angle COB = 180^\circ$

In Axiom 1, it is given that a ray stands on a line and we concluded that the sum of two adjacent angles so formed is 180° . Now, if we do the reverse and take the 'conclusion' of Axiom 1 as 'given' and 'given' as 'conclusion' then it becomes:

Statement A: If the sum of two adjacent angles is 180° then a ray stands on a line.



We see that Axiom 1 and Statement A are converse of each other.

If we place a ruler along with one of the non-common arms we see that the other non-common arm also lies along the ruler.

Therefore, points A, O and B lie on the same line and ray OC stands on it.

$$\angle AOC + \angle COB = 115^\circ + 65^\circ = 180^\circ$$

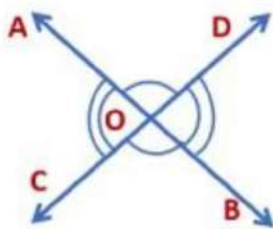
Therefore, statement A is true.

The given statement can be stated in the form of an axiom as follows:

Axiom 2: If the sum of two adjacent angles is 180° , then the non-common arms of the angles form a line.

Axiom 1 and 2, together are called the linear pair axiom.

Theorem 1: If two lines intersect each other, then the vertically opposite angles are equal.



Given: Lines AB and CD intersect each other at point O. So, two pairs of vertically opposite angles are formed,

i) $\angle AOC$ and $\angle BOD$

ii) $\angle AOD$ and $\angle COB$

To prove: $\angle AOC = \angle BOD$ and $\angle AOD = \angle COB$

Proof: Here, ray AO stands on line CD.

$$\therefore \angle AOC + \angle AOD = 180^\circ$$

(by Linear Pair axiom) (1)

Similarly, ray DO stands on line AB.

$$\therefore \angle AOD + \angle BOD = 180^\circ \quad (\text{by Linear Pair axiom}) \dots\dots (2)$$

Using Eq 1 and 2 we get,

$$\angle AOC + \angle AOD = \angle AOD + \angle BOD$$

Subtracting $\angle AOD$ from both sides we get,

$$\angle AOC + \angle AOD - \angle AOD = \angle AOD + \angle BOD - \angle AOD$$

(Euclid's axiom 3 states that if equals are subtracted from equals, the remainders are equal)

$$\angle AOC + \angle AOD - \angle AOD = \angle BOD + \angle AOD - \angle AOD$$

$$\therefore \angle AOC = \angle BOD$$

Now, we know ray AO stands on line CD.

$$\therefore \angle AOC + \angle AOD = 180^\circ$$

(by Linear Pair axiom) (1)

Similarly, ray CO stands on line AB.

$$\therefore \angle AOC + \angle BOC = 180^\circ$$

(by Linear Pair axiom) (2)

Using Eq 1 and 2 we get,

$$\angle AOC + \angle AOD = \angle AOC + \angle BOC$$

Subtracting $\angle AOC$ from both sides we get,

$$\angle AOC + \angle AOD - \angle AOC = \angle AOC + \angle BOC - \angle AOC$$

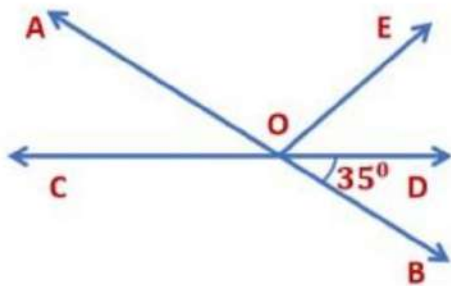
(Using Euclid's axiom 3 states which that if equals are subtracted from equals, the remainders are equal)

$$\angle AOD + \angle AOC - \angle AOC = \angle BOC + \angle AOC - \angle AOC$$

$$\therefore \angle AOD = \angle BOC$$

We see that two pairs of vertically opposite angles are equal.

Example: In the given figure AB and CD intersect at O. If $\angle AOC + \angle DOE = 60^\circ$ and $\angle BOD = 35^\circ$, find $\angle DOE$ and reflex $\angle AOE$.



We know,

i) $\angle AOC + \angle DOE = 60^\circ$

ii) $\angle BOD = 35^\circ$

Here, lines AB and CD intersect each other at O.

So, $\angle AOC = \angle BOD = 35^\circ$

(If two lines intersect each other, then the vertically opposite angles are equal)

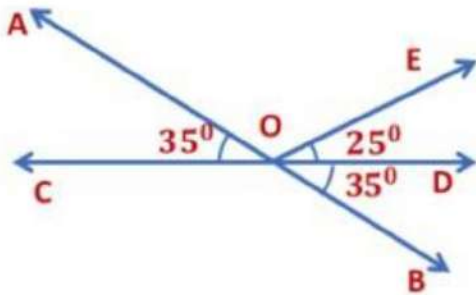
$$\angle AOC + \angle DOE = 60^\circ$$

$$35^\circ + \angle DOE = 60^\circ$$

$$(\because \angle AOC = 35^\circ)$$

$$\angle DOE = 60^\circ - 35^\circ = 25^\circ$$

$$\angle DOE = 25^\circ$$



We see that COD is a straight line, that is the measure of the $\angle COD$ is equal to 180° .

$$\angle AOC + \angle AOE + \angle EOD = 180^\circ$$

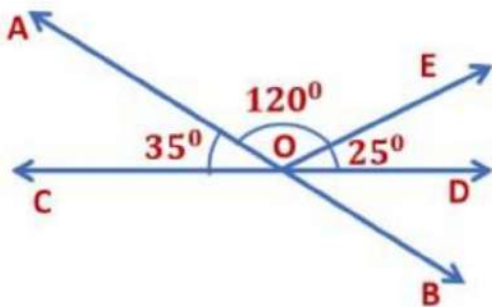
$$\angle AOE + (\angle AOC + \angle EOD) = 180^\circ$$

$$\angle AOE + 60^\circ = 180^\circ$$

$$(\because \angle AOC + \angle DOE = 60^\circ)$$

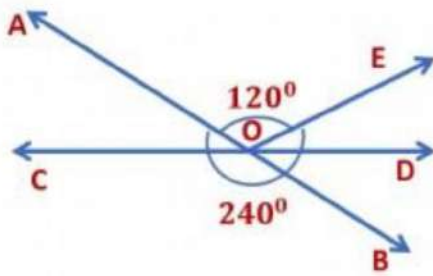
$$\angle AOE = 180^\circ - 60^\circ$$

$$\angle AOE = 120^\circ$$

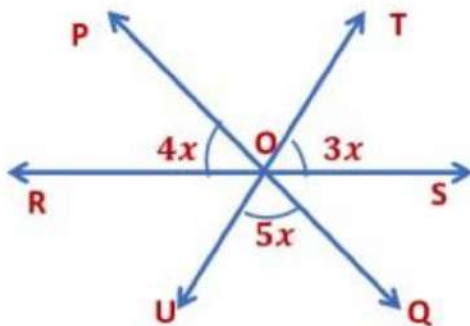


$$\text{Reflex } \angle AOE = 360^\circ - 120^\circ = 240^\circ$$

(reflex angle is an angle whose measure is greater than 180° but less than 360°)

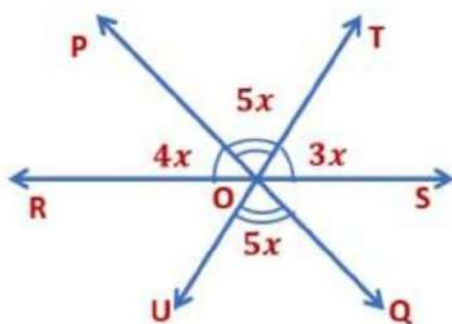


Example: In the figure given below, lines PQ, RS and TU meet at point O. Find the value of x, hence find all the three indicated angles.



Here, lines PQ and TU intersect each other at O.

So, $\angle POT = \angle UOQ = 5x$ (If two lines intersect each other, then the vertically opposite angles are equal)



We see that ROS is a straight line, that is the measure of the $\angle ROS$ is equal to 180° .

$$\angle ROP + \angle POT + \angle TOS = 180^\circ$$

$$4x + 5x + 3x = 180^\circ$$

$$12x = 180^\circ$$

$$x = \frac{180}{12}^\circ$$

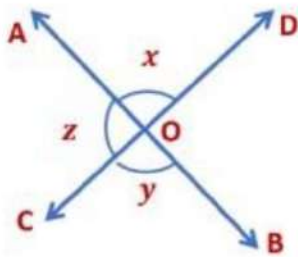
$$x = 15^\circ$$

$$\angle POR = 4x = 4 \times 15^\circ = 60^\circ$$

$$\angle UOQ = 5x = 5 \times 15^\circ = 75^\circ$$

$$\angle TOS = 3x = 3 \times 15^\circ = 45^\circ$$

Example: Lines AB and CD intersect each other at O. If $\angle AOD : \angle AOC = 2 : 4$, find all the angles x, y and z .



We know,

$$\angle AOD : \angle AOC = 2 : 4$$

$$\text{Let, } \angle AOD = 2a \text{ and } \angle AOC = 4a$$

Ray AO stands on line CD.

$$\angle AOD + \angle AOC = 180^\circ$$

(If a ray stands on a line, then the sum of two adjacent angles so formed is 180°)

$$2a + 4a = 180^\circ$$

$$6a = 180^\circ$$

$$a = \frac{180}{6} = 30^\circ$$

$$\angle AOD = x = 2a = 2 \times 30^\circ = 60^\circ$$

$$\angle AOC = z = 4a = 4 \times 30^\circ = 120^\circ$$

$$x = y = 60^\circ \text{ (Vertically opposite angles are equal)}$$

$$\text{So, } x = 60^\circ, y = 60^\circ \text{ and } z = 120^\circ$$

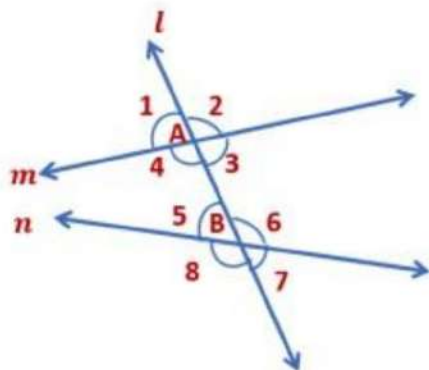
Parallel Lines and Transversal Lines

If two lines in the same plane do not intersect, when produced on either side, then such lines are said to be parallel to each other.



Here, lines l and m are parallel to each other.

A line that intersects two or more straight lines at distinct points is called a transversal line.

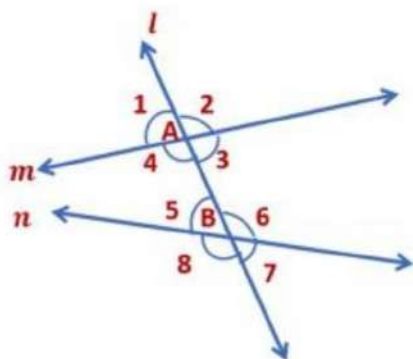


Here, line l intersects lines m and n at points A and B respectively.

We see that four angles are formed at each point A and B , namely $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7$ and $\angle 8$.

$\angle 3, \angle 4, \angle 5$ and $\angle 6$	Interior Angles
$\angle 1, \angle 2, \angle 7$ and $\angle 8$	Exterior Angles

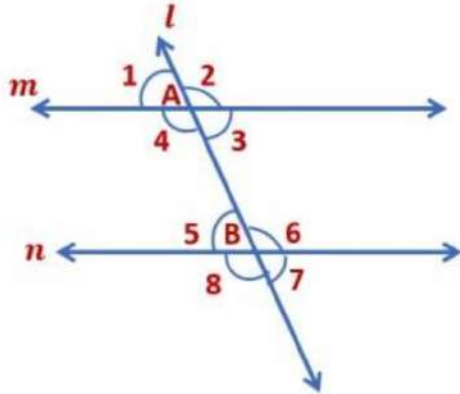
The pairs of angles formed, when a transversal intersects two lines are as follows:



Corresponding Angles – The angles on the same side of a transversal are known as corresponding angles if both lie either above or below the lines.	$\angle 1$ & $\angle 5$, $\angle 2$ & $\angle 6$, $\angle 4$ & $\angle 8$, $\angle 3$ & $\angle 7$
Alternate Interior Angles – The pairs of interior angles on opposite sides of the transversal are called alternate interior angles.	$\angle 4$ & $\angle 6$, $\angle 3$ & $\angle 5$
Alternate Exterior Angles – The pairs of exterior angles on opposite sides of the transversal are called alternate exterior angles.	$\angle 1$ & $\angle 7$, $\angle 2$ & $\angle 8$
Interior angles on the same side of the transversal – They are also referred to as consecutive interior angles or allied angles or co-interior angles.	$\angle 4$ & $\angle 5$, $\angle 3$ & $\angle 6$

Here, we see that the lines m and n are not parallel. Can you tell what will happen if the line l intersects two parallel lines m and n ?

When a transversal l intersects two parallel lines m and n , then the relation between angles formed are obtained as axioms and theorems.



Axiom 1: If a transversal intersects two parallel lines, then each pair of corresponding angles is equal. This axiom is known as a corresponding angle axiom.

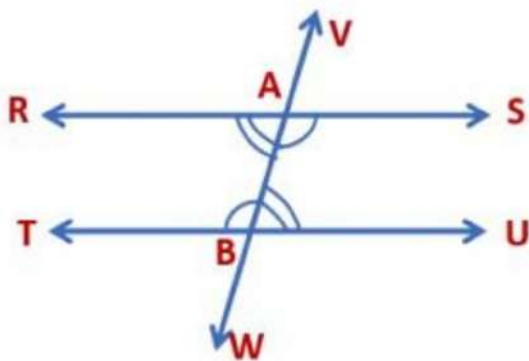
When line l intersects two parallel lines m and n , then we see that each pair of corresponding angles is equal.

$$\therefore \angle 1 = \angle 5, \angle 2 = \angle 6, \angle 4 = \angle 8 \text{ and } \angle 3 = \angle 7$$

The converse of this axiom is as follows:

Axiom 2 (Converse of axiom 1): If a transversal intersects two lines such that a pair of corresponding angles are equal, then the two lines are parallel to each other.

Theorem 1: If a transversal intersects two parallel lines, then each pair of alternate angles is equal.



Here, line VW intersects two parallel lines RS and TU at points A and B respectively.

The two pairs of alternate interior angles are $\angle SAB$ and $\angle TBA$, $\angle RAB$ and $\angle ABU$

To prove:

- i) $\angle SAB = \angle TBA$
- ii) $\angle RAB = \angle ABU$

We know,

$$\angle VAR = \angle SAB \text{ (Vertically opposite angles)} \rightarrow \text{Eq 1}$$

$$\angle VAR = \angle TBA \text{ (Corresponding angles axiom) Eq } \rightarrow 2$$

Using Eq 1 and 2 we see that,

$$\angle VAR = \angle SAB \text{ and } \angle VAR = \angle TBA. \text{ Therefore, } \angle SAB = \angle TBA$$

Similarly,

$$\angle SAV = \angle RAB \text{ (Vertically opposite angles) Eq } \rightarrow 3$$

$$\angle SAV = \angle ABU \text{ (Corresponding angles axiom) Eq } \rightarrow 4$$

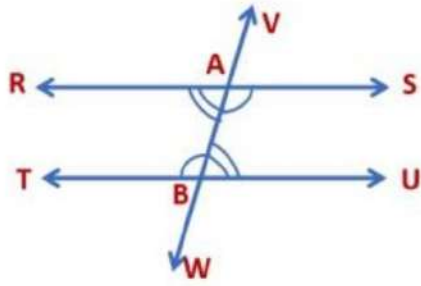
Using Eq 3 and 4 we get,

$$\angle SAV = \angle RAB \text{ and } \angle SAV = \angle ABU. \text{ Therefore, } \angle RAB = \angle ABU$$

$$\text{So, } \angle SAB = \angle TBA \text{ and } \angle RAB = \angle ABU$$

Therefore, the pairs of alternate interior angles are equal.

Theorem 2 (Converse of theorem 1): If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel.



Here, line VW intersects two parallel lines RS and TU at points A and B respectively in such a way that, $\angle SAB = \angle TBA$ and $\angle RAB = \angle ABU$.

To prove: RS is parallel to TU

We know,

$\angle SAB = \angle TBA$ (alternate interior angles) Eq $\rightarrow 1$

$\angle SAB = \angle RAV$ (vertically opposite angles) Eq $\rightarrow 2$

Using Eq 1 and 2 we see that,

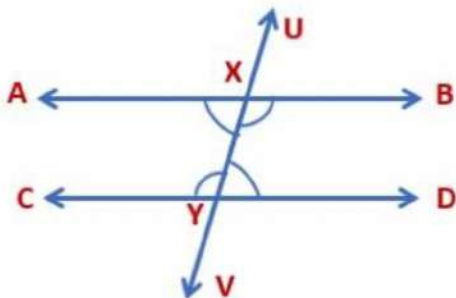
$\angle SAB = \angle TBA$ and $\angle SAB = \angle RAV$. Therefore, $\angle TBA = \angle RAV$.

We know that $\angle TBA$ and $\angle RAV$ are corresponding angles.

According to the converse of corresponding angles axiom, if a transversal intersects two lines such that a pair of corresponding angles are equal, then the two lines are parallel to each other.

So, RS and TU are parallel lines, that are $RS \parallel TU$.

Theorem 3: If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.



Here, line UV intersects two parallel lines AB and CD at points X and Y respectively. So, two pairs of interior angles are formed; $\angle AXY$ and $\angle CYX$, $\angle BXY$ and $\angle XYD$. To prove: $\angle AXY + \angle CYX = 180^\circ$ and $\angle BXY + \angle XYD = 180^\circ$

(Pair of interior angles on the same side of the transversal is supplementary)

Here, ray XY stands on line AB. Therefore, $\angle AXY$ and $\angle BXY$ are adjacent angles.

So, $\angle AXY + \angle BXY = 180^\circ \rightarrow \text{Eq 1}$

(Linear pair axiom states that if a ray stands on a line, then the sum of two adjacent angles so formed is 180°)

Now, $\angle BXY = \angle CYX$ (alternate interior angles) $\rightarrow \text{Eq 2}$

On putting $\angle BXY = \angle CYX$ in Eq 1 we get,

$$\angle AXY + \angle CYX = 180^\circ$$

Similarly, ray XY stands on line CD. Therefore, $\angle CYX$ and $\angle XYD$ are adjacent angles.

So, $\angle CYX + \angle XYD = 180^\circ$ (linear pair axiom) Eq $\rightarrow 3$

But, $\angle CYX = \angle BXY$ (alternate interior angles) Eq $\rightarrow 4$

On putting $\angle CYX = \angle BXY$ in Eq 1 we get,

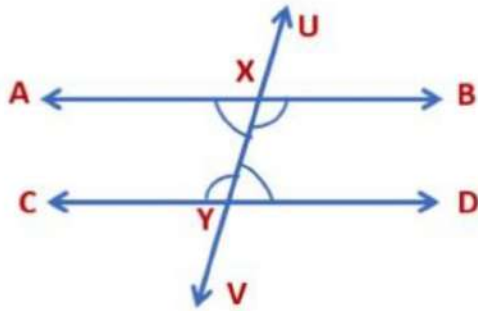
$$\angle BXY + \angle XYD = 180^\circ$$

Therefore,

$$\angle AXY + \angle CYX = 180^\circ$$

$$\angle BXY + \angle XYD = 180^\circ$$

Theorem 4 (Converse of theorem 4): If a transversal intersects two lines, in such a way that each pair of interior angles on the same side of the transversal is supplementary, then the lines are parallel.



Here, line UV intersects two parallel lines AB and CD at points X and Y respectively in such a way that two pairs of interior angles on the same side of the transversal are $\angle AXY$ and $\angle CYX$, $\angle BXY$ and $\angle XYD$.

We know, $\angle AXY + \angle CYX = 180^\circ$

$\angle BXY + \angle XYD = 180^\circ$

To prove: $AB \parallel CD$

Here, ray XB stands on line UV. Therefore, $\angle BXU$ and $\angle BXY$ are adjacent angles.

So, $\angle BXU + \angle BXY = 180^\circ$ (linear pair axiom) $\rightarrow$ Eq 3

(linear pair axiom states that if a ray stands on a line, then the sum of two adjacent angles so formed is 180°)

It is given that,

$\angle BXY + \angle XYD = 180^\circ \rightarrow$ Eq 4

Using Eq 3 and 4 we get,

$\angle BXU + \angle BXY = \angle BXY + \angle XYD$

On subtracting $\angle BXY$ from both sides we get,

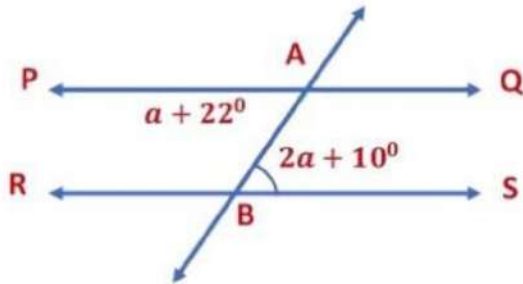
$\angle BXU + \angle BXY - \angle BXY = \angle BXY + \angle XYD - \angle BXY$

$\angle BXU + \angle BXY - \angle BXY = \angle BXY - \angle BXY + \angle XYD$

So, $\angle BXU = \angle XYD$

Now, $\angle BXU$ and $\angle XYD$ are corresponding angles.

According to the converse of corresponding angles axiom, if a transversal intersects two lines such that a pair of corresponding angles are equal, then the two lines are parallel to each other. So, $AB \parallel CD$.



Example: In the figure given below, $PQ \parallel RS$, find the value of a and $\angle PAB$ and $\angle ABR$.

Now, $\angle PAB = \angle ABS$ (Alternate interior angles)

$$a + 22^\circ = 2a + 10^\circ$$

$$22^\circ - 10^\circ = 2a - a$$

$$a = 12^\circ$$

$$\angle PAB = a + 22^\circ$$

$$\angle PAB = 12^\circ + 22^\circ (\because a = 12^\circ)$$

$$\angle PAB = 34^\circ$$

$$\text{We know, } \angle PAB = \angle ABS = 34^\circ$$

Ray AB stands on line RS. Therefore, $\angle ABR$ and $\angle ABS$ are adjacent angles.

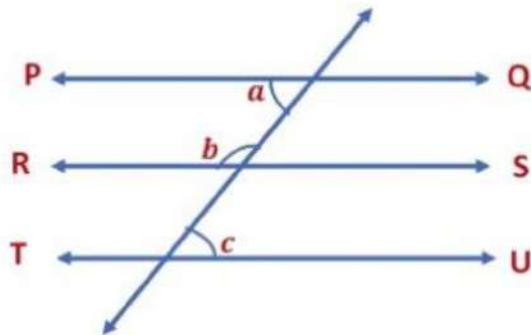
$$\text{So, } \angle ABR + \angle ABS = 180^\circ \text{ (linear pair axiom)}$$

$$\angle ABR + 34^\circ = 180^\circ (\because \angle PAB = 34^\circ)$$

$$\angle ABR = 180^\circ - 34^\circ = 126^\circ$$

$$\angle ABR = 126^\circ$$

Example: In the figure, $PQ \parallel RS \parallel TU$ and $a : b = 1 : 2$, find c .



Let, $a = 1x$ and $b = 2x$

Now, a and b are interior angles on the same side of the transversal.

If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

Therefore, $a + b = 180^\circ$

$$x + 2x = 180^\circ \quad (\because a = 1x, b = 2x)$$

$$3x = 180^\circ$$

$$x = \frac{180}{3}^\circ = 60^\circ$$

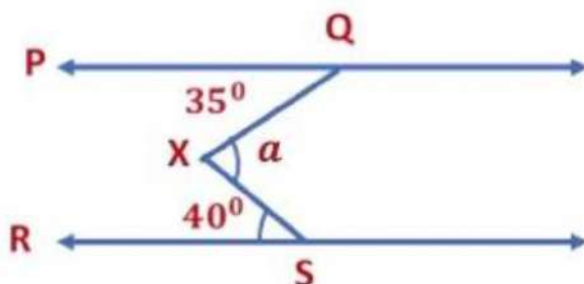
Now, $a = x = 60^\circ$

$$b = 2x = 2 \times 60^\circ = 120^\circ$$

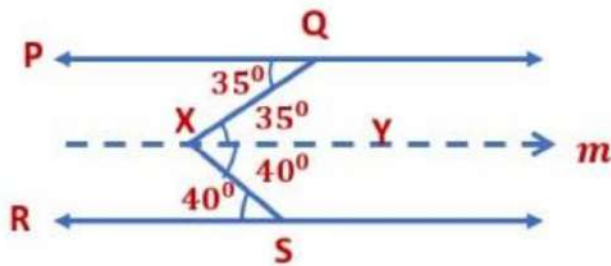
$a = c = 60^\circ$ (Alternate interior angle)

$c = 60^\circ$

Example: In the figure $PQ \parallel RS$, $\angle PQX = 35^\circ$ and $\angle RSX = 40^\circ$, find a .



Here we draw a line m parallel to PQ through X .



Here, $\angle PQX = \angle QXY = 35^\circ$ (Alternate interior angle)

Again, $\angle RSX = \angle YXS = 40^\circ$ (Alternate interior angle)

Now, $\angle QXS = \angle QXY + \angle YXS$

$\angle QXS = 35^\circ + 40^\circ (\because \angle QXY = 35^\circ \text{ and } \angle YXS = 40^\circ)$

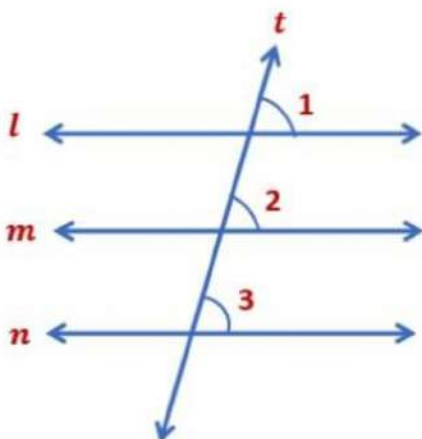
$\angle QXS = 75^\circ$

$\angle QXS = a = 75^\circ$

Therefore, $a = 75^\circ$

Lines parallel to the same line

If two lines are parallel to the same line then they are parallel to each other.



We draw a transversal t for the lines l , m and n , where line l is parallel to line n and line m is parallel to line n .

Now,

$$\angle 1 = \angle 3 (\text{corresponding angles axiom}) \rightarrow \text{Eq 1}$$

$$\text{Similarly, } \angle 2 = \angle 3 (\text{corresponding angles axiom}) \rightarrow \text{Eq 2}$$

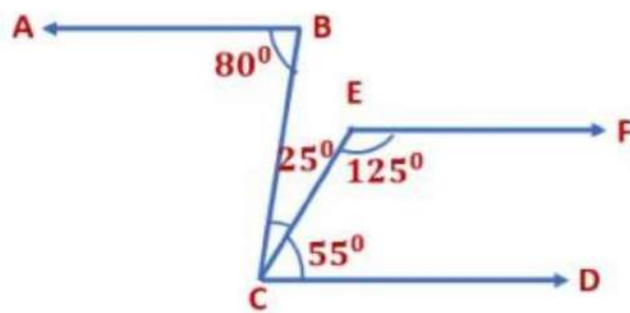
Using Eq 1 and 2 we get, $\angle 1 = \angle 2$

$\angle 1$ and $\angle 2$ are corresponding angles.

The converse of corresponding angles axiom states that if a transversal intersects two lines such that a pair of corresponding angles are equal, then the two lines are parallel to each other

Therefore, line l is parallel to line m ($l \parallel m$)

Example: In the given figure, show that $AB \parallel CD$.



$$\angle BCD = \angle BCE + \angle ECD$$

$$\angle BCD = 25^\circ + 55^\circ = 80^\circ$$

We see that $\angle ABC$ and $\angle BCD$ are alternate interior angles.

$$\angle ABC = \angle BCD = 80^\circ$$

If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel.

$$\therefore AB \parallel CD$$

OR

$$\angle FEC + \angle ECD = 125^\circ + 55^\circ = 180^\circ$$

If a transversal intersects two lines, in such a way that each pair of interior angles on the same side of the transversal is supplementary, then the lines are parallel.

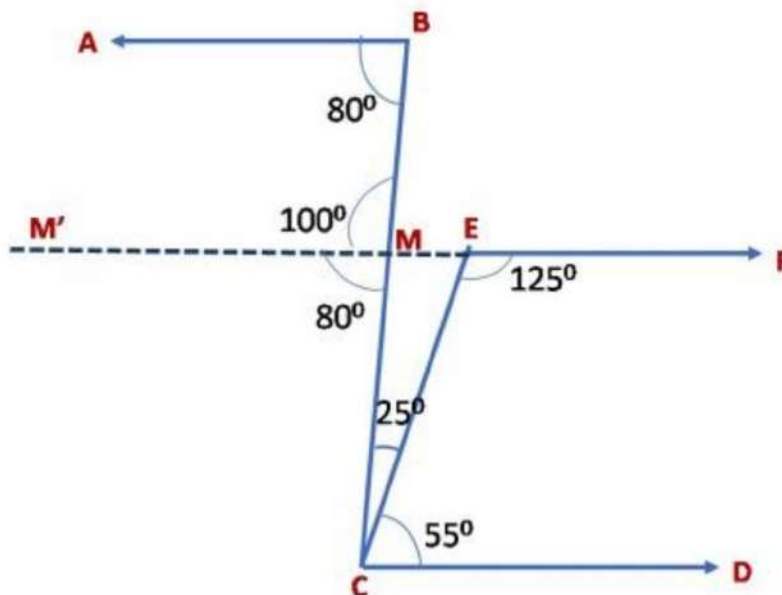
$\therefore CD \parallel EF \dots\dots\dots (1)$

Now let us extend EF till it intersects BC at M and extend it till M'.

Then since $CD \parallel EF$

We can find the remaining angles.

The remaining angles are as shown:



Now,

$$\angle CMM' = \angle DCM = 80^\circ - (\text{Interior alternate angles})$$

$$\angle BMM' + \angle CMM' = 180^\circ - \text{Linear Pair of angles}$$

$$\angle BMM' = 100^\circ$$

$$\angle BMM' + \angle MBA = 100^\circ + 80^\circ = 180^\circ$$

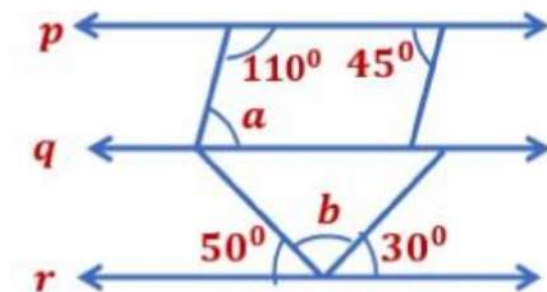
If a transversal intersects two lines, in such a way that each pair of interior angles on the same side of the transversal is supplementary, then the lines are parallel.

$$\therefore AB \parallel EF \dots\dots\dots (2)$$

From eq (1) and (2), we get

$$AB \parallel CD$$

Example: In the figure $p \parallel q \parallel r$. From the figure, find the ratio of $(a + b)$: $(b - a)$.



$$50^\circ + b + 30^\circ = 180^\circ$$

$$b + (50^\circ + 30^\circ) = 180^\circ$$

$$b + 80^\circ = 180^\circ$$

$$b = 180^\circ - 80^\circ$$

$$b = 100^\circ$$

We know that $p \parallel q$.

If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

$$\text{So, } 110^\circ + a = 180^\circ$$

$$a = 180^\circ - 110^\circ = 70^\circ$$

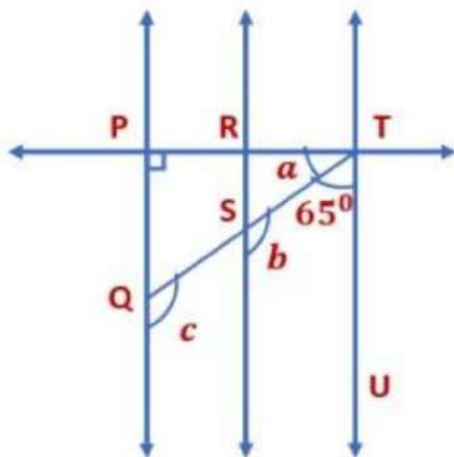
$$a = 70^\circ$$

$$a + b = 70^\circ + 100^\circ = 170^\circ$$

$$b - a = 100^\circ - 70^\circ = 30^\circ$$

$$(a + b) : (b - a) = 170^\circ : 30^\circ = 17^\circ : 3^\circ$$

Example: In the figure, $PQ \parallel RS$, $RS \parallel TU$, and $TP \perp PQ$. If $\angle STU = 65^\circ$, find the value of a , b and c .



We know, $PQ \parallel RS$, $RS \parallel TU$, and $TP \perp PQ$.

$$\angle STU = 65^\circ$$

$$b + 65^\circ = 180^\circ$$

(If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary)

$$b = 180^\circ - 65^\circ = 115^\circ$$

$$b = c = 115^\circ \text{ (corresponding angles)}$$

$$a + 65^\circ = 90^\circ (\because TP \perp PQ)$$

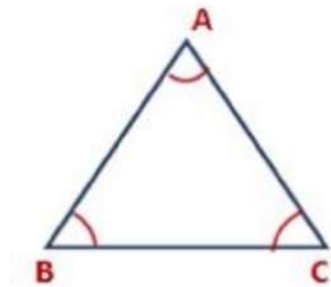
$$a = 90^\circ - 65^\circ = 25^\circ$$

$$\text{So, } a = 25^\circ, b = c = 115^\circ$$

Angle sum property of a triangle

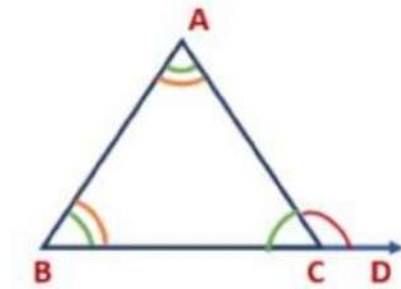
We know that the sum of all the angles of a triangle is 180° .

A triangle is a plane figure formed by three intersecting lines.



Sides	$\overline{AB}$, $\overline{BC}$, and $\overline{CA}$
Vertices	A, B, and C
Angles	$\angle A$, $\angle B$, and $\angle C$

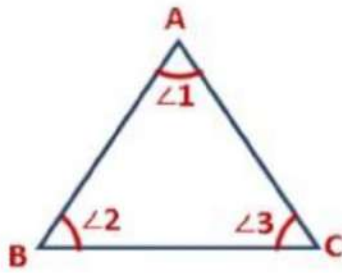
- $\angle A$, $\angle B$, and $\angle C$ are called interior angles of the triangle.
- When side BC is produced to D then we get an exterior angle, $\angle ACD$.
- $\angle BAC$ and $\angle ABC$ are called its interior opposite angles.



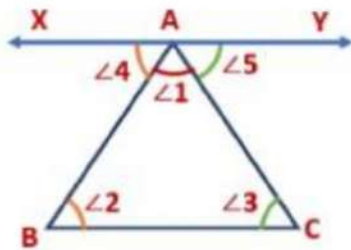
Theorem 1: The sum of the angles of a triangle is 180°

Here, $\angle 1$, $\angle 2$ and $\angle 3$ are the angles of $\triangle ABC$.

To prove: $\angle 1 + \angle 2 + \angle 3 = 180^\circ$



Construction: We draw a line XAY parallel to BC through the vertex A. (If we draw parallel lines then we can use the properties of parallel lines)



Here, XAY is a straight line.

$$\therefore \angle 4 + \angle 1 + \angle 5 = 180^\circ \rightarrow \text{Eq 1}$$

We see that XAY is parallel to BC, so, AB and AC are transversals.

So, $\angle 4 = \angle 2$ (Alternate Angles)

$\angle 3 = \angle 5$ (Alternate Angles)

Putting $\angle 4 = \angle 2$, and $\angle 3 = \angle 5$ in Eq 1 we get,

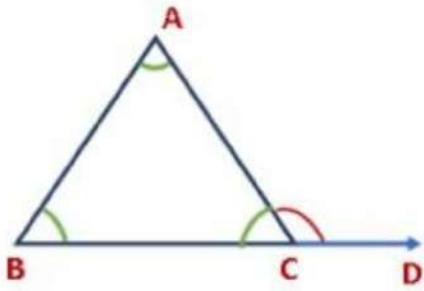
$$\angle 2 + \angle 1 + \angle 3 = 180^\circ \rightarrow \text{Eq 2}$$

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

Thus, the sum of three angles of a triangle is 180° .

Theorem 2: If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

In $\triangle ABC$, side BC is produced to D to form the exterior angle, $\angle ACD$.



To prove: $\angle A + \angle B = \angle ACD$

In $\triangle ABC$,

$\angle A + \angle B + \angle ACB = 180^\circ$ (Angle sum property of triangle) $\rightarrow$ Eq 1

Here, BCD is a straight line.

$\therefore \angle ACB + \angle ACD = 180^\circ \rightarrow$ Eq 2

From Eq 1 and 2 we get,

$$\angle A + \angle B + \angle ACB = \angle ACB + \angle ACD$$

Subtracting $\angle ACB$ from both sides we get,

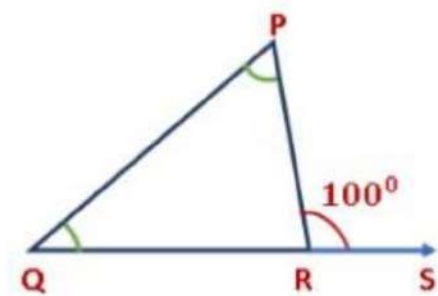
$$\angle A + \angle B + \angle ACB - \angle ACB = \angle ACB + \angle ACD - \angle ACB$$

$$\angle A + \angle B + \angle ACB - \angle ACB = \angle ACB - \angle ACB + \angle ACD$$

$$\angle A + \angle B = \angle ACD$$

Thus, the exterior angle is equal to the sum of the two interior opposite angles.

Example: The measure of the exterior angle, $\angle PRS$ of $\triangle PQR$ is 100° . If one of the interior angles is 20° , find the measure of the other two angles of $\triangle PQR$.



Exterior angle, $\angle PRS = 100^\circ$

One of the interior angles = 20°

Let $\angle Q = 20^\circ$

then the other interior opposite angle will be $\angle P$.

So, $\angle PRS = \angle Q + \angle P$ (exterior angle of a triangle is equal to the sum of the two interior opposite angles)

$$100^\circ = 20^\circ + \angle P$$

$$\angle P = 100^\circ - 20^\circ = 80^\circ$$

$$\text{Now, } \angle P + \angle Q + \angle PRQ = 180^\circ$$

(Angle sum property of triangle states that the sum of the angles of a triangle is 180°)

$$80^\circ + 20^\circ + \angle PRQ = 180^\circ$$

$$100^\circ + \angle PRQ = 180^\circ$$

$$\angle PRQ = 180^\circ - 100^\circ$$

$$\angle PRQ = 80^\circ$$

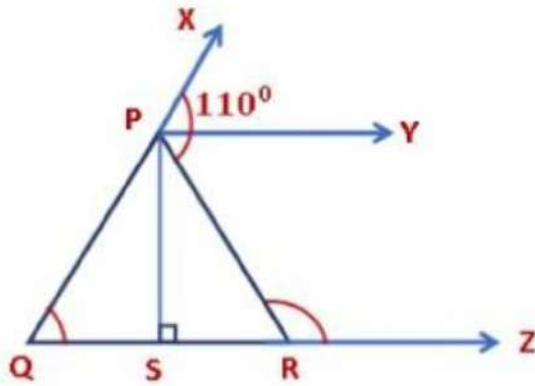
Therefore, the measures of the three angles of $\triangle PQR$ are

$$\angle P = 80^\circ$$

$$\angle Q = 20^\circ$$

$$\angle PRQ = 80^\circ$$

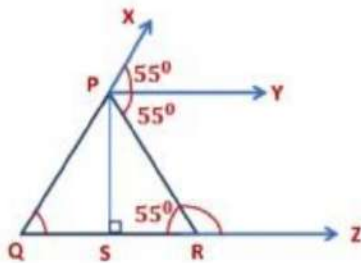
Example: In the figure given below, $PY \parallel SZ$, $PS \perp QR$, and PY bisect $\angle XPR$. If $\angle XPR = 110^\circ$ then find the measure of $\angle PQR$ and $\angle PRZ$.



Now, $\angle XPR = 110^\circ$

$\angle XPY = \angle YPR$ ($\because$ PY is the bisector which bisects $\angle XPR$)

$$\therefore \angle XPY = \angle YPR = \frac{110}{2}^\circ = 55^\circ$$



We know that PY is parallel to SZ .

$\angle YPR = \angle PRS = 55^\circ$ (alternate angles)

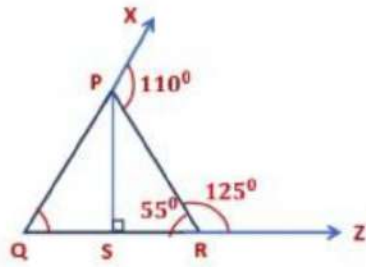
Here, SRZ is a straight line.

$$\therefore \angle PRS + \angle PRZ = 180^\circ$$

$$55^\circ + \angle PRZ = 180^\circ$$

$$\angle PRZ = 180^\circ - 55^\circ$$

$$\angle PRZ = 125^\circ$$



Now, $\angle XPR = \angle PRQ + \angle PQR$

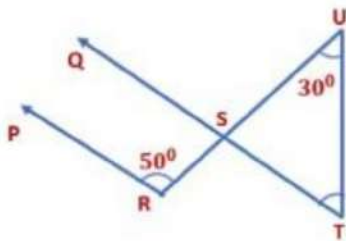
(Exterior angle of a triangle is equal
to the sum of the two interior opposite angles)

$$110^\circ = 55^\circ + \angle PQR$$

$$\angle PQR = 110^\circ - 55^\circ = 55^\circ$$

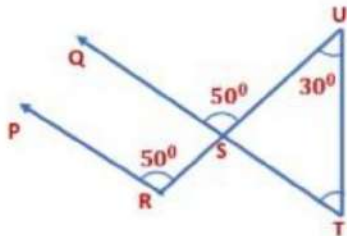
$$\angle PQR = 55^\circ \text{ and } \angle PRZ = 125^\circ$$

Example: In the figure given below, $PR \parallel QS$. If $\angle PRS = 50^\circ$ and $\angle SUT = 30^\circ$ then find $\angle STU$.



We know, $PR \parallel QS$ and so $\angle PRQ$ and $\angle QSU$ are a pair of corresponding angles.

$$\therefore \angle PRQ = \angle QSU = 50^\circ$$



We see that $\angle QSU$ is an exterior angle of the ΔSTU .

$\angle QSU = \angle SUT + \angle STU$ (exterior angle of a triangle is equal

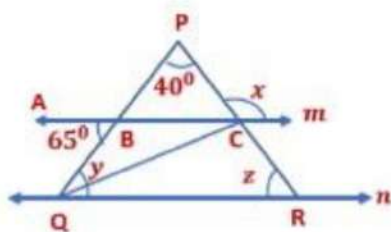
to the sum of the two interior opposite angles)

$$50^\circ = 30^\circ + \angle STU$$

$$\angle STU = 50^\circ - 30^\circ = 20^\circ$$

$$\angle STU = 20^\circ$$

Example: In the figure, $\angle BPC = 40^\circ$, $\angle ABQ = 65^\circ$ and lines m and n are parallel to each other. Find x , y , and z .

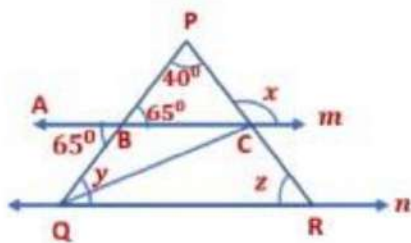


$$\angle ABQ = \angle PBC = 65^\circ \text{ (Vertically opposite angle)}$$

Here, x is an exterior angle of $\triangle PBC$.

$$\therefore x = \angle PBC + \angle BPC$$

$$x = 65^\circ + 40^\circ = 105^\circ$$



Line $m \parallel$ line n .

So, $\angle ABQ = \angle BQC = y = 65^\circ$ (Alternate angles)

In $\triangle PQR$,

$\angle P + \angle Q + \angle R = 180^\circ$ (Angle sum property of triangle states that the sum of the angles of a triangle is 180°)

$$40^\circ + y + z = 180^\circ$$

$$40^\circ + 65^\circ + z = 180^\circ$$

$$105^\circ + z = 180^\circ$$

$$z = 180^\circ - 105^\circ = 75^\circ$$

$$x = 105^\circ, y = 65^\circ \text{ and } z = 75^\circ$$