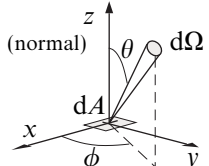


5.7 Radiation processes

Radiometry^a

Radiant energy ^b	$Q_e = \iiint L_e \cos \theta \, dA \, d\Omega \, dt \quad \text{J} \quad (5.145)$	Q_e radiant energy L_e radiance (generally a function of position and direction) θ angle between dir. of $d\Omega$ and normal to dA Ω solid angle
Radiant flux ("radiant power")	$\Phi_e = \frac{\partial Q_e}{\partial t} \quad \text{W} \quad (5.146)$ $= \iint L_e \cos \theta \, dA \, d\Omega \quad (5.147)$	A area t time Φ_e radiant flux
Radiant energy density ^c	$W_e = \frac{\partial Q_e}{\partial V} \quad \text{J m}^{-3} \quad (5.148)$	W_e radiant energy density dV differential volume of propagation medium
Radiant exitance ^d	$M_e = \frac{\partial \Phi_e}{\partial A} \quad \text{W m}^{-2} \quad (5.149)$ $= \int L_e \cos \theta \, d\Omega \quad (5.150)$	M_e radiant exitance
Irradiance ^e	$E_e = \frac{\partial \Phi_e}{\partial A} \quad \text{W m}^{-2} \quad (5.151)$ $= \int L_e \cos \theta \, d\Omega \quad (5.152)$	
Radiant intensity	$I_e = \frac{\partial \Phi_e}{\partial \Omega} \quad \text{W sr}^{-1} \quad (5.153)$ $= \int L_e \cos \theta \, dA \quad (5.154)$	E_e irradiance I_e radiant intensity
Radiance	$L_e = \frac{1}{\cos \theta} \frac{\partial^2 \Phi_e}{\partial A \, \partial \Omega} \quad \text{W m}^{-2} \text{sr}^{-1} \quad (5.155)$ $= \frac{1}{\cos \theta} \frac{\partial I_e}{\partial A} \quad (5.156)$	

^aRadiometry is concerned with the treatment of light as energy.

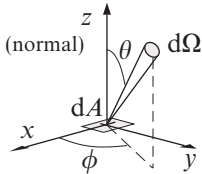
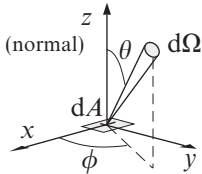
^bSometimes called "total energy." Note that we assume opaque radiant surfaces, so that $0 \leq \theta \leq \pi/2$.

^cThe instantaneous amount of radiant energy contained in a unit volume of propagation medium.

^dPower per unit area leaving a surface. For a perfectly diffusing surface, $M_e = \pi L_e$.

^ePower per unit area incident on a surface.

Photometry^a

Luminous energy ("total light")	$Q_v = \iiint L_v \cos \theta \, dA \, d\Omega \, dt \quad \text{lm s} \quad (5.157)$	Q_v luminous energy L_v luminance (generally a function of position and direction) θ angle between dir. of $d\Omega$ and normal to dA Ω solid angle
Luminous flux	$\Phi_v = \frac{\partial Q_v}{\partial t} \quad \text{lumen (lm)} \quad (5.158)$ $= \iint L_v \cos \theta \, dA \, d\Omega \quad (5.159)$	A area t time Φ_v luminous flux
Luminous density ^b	$W_v = \frac{\partial Q_v}{\partial V} \quad \text{lm m}^{-3} \quad (5.160)$	W_v luminous density V volume
Luminous exitance ^c	$M_v = \frac{\partial \Phi_v}{\partial A} \quad \text{lx} \quad (\text{lm m}^{-2}) \quad (5.161)$ $= \int L_v \cos \theta \, d\Omega \quad (5.162)$	M_v luminous exitance 
Illuminance ("illumination") ^d	$E_v = \frac{\partial \Phi_v}{\partial A} \quad \text{lm m}^{-2} \quad (5.163)$ $= \int L_v \cos \theta \, d\Omega \quad (5.164)$	
Luminous intensity ^e	$I_v = \frac{\partial \Phi_v}{\partial \Omega} \quad \text{cd} \quad (5.165)$ $= \int L_v \cos \theta \, dA \quad (5.166)$	E_v illuminance I_v luminous intensity
Luminance ("photometric brightness")	$L_v = \frac{1}{\cos \theta} \frac{\partial^2 \Phi_v}{\partial A \, d\Omega} \quad \text{cd m}^{-2} \quad (5.167)$ $= \frac{1}{\cos \theta} \frac{\partial I_v}{\partial A} \quad (5.168)$	
Luminous efficacy	$K = \frac{\Phi_v}{\Phi_e} = \frac{L_v}{L_e} = \frac{I_v}{I_e} \quad \text{lm W}^{-1} \quad (5.169)$	K luminous efficacy L_e radiance Φ_e radiant flux I_e radiant intensity
Luminous efficiency	$V(\lambda) = \frac{K(\lambda)}{K_{\max}} \quad (5.170)$	V luminous efficiency λ wavelength $K_{\max}$ spectral maximum of $K(\lambda)$

^aPhotometry is concerned with the treatment of light as seen by the human eye.

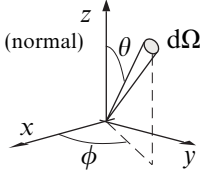
^bThe instantaneous amount of luminous energy contained in a unit volume of propagating medium.

^cLuminous emitted flux per unit area.

^dLuminous incident flux per unit area. The derived SI unit is the lux (lx). $1 \text{ lx} = 1 \text{ lm m}^{-2}$.

^eThe SI unit of luminous intensity is the candela (cd). $1 \text{ cd} = 1 \text{ lm sr}^{-1}$.

Radiative transfer^a

Flux density (through a plane)	$F_v = \int I_v(\theta, \phi) \cos \theta \, d\Omega \quad \text{W m}^{-2} \text{Hz}^{-1}$	(5.171)	 F_v flux density I_v specific intensity ($\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$) J_v mean intensity u_v spectral energy density Ω solid angle θ angle between normal and direction of Ω j_v specific emission coefficient ϵ_v emission coefficient ($\text{W m}^{-3} \text{Hz}^{-1} \text{sr}^{-1}$) ρ density α_v linear absorption coefficient n particle number density σ_v particle cross section l_v mean free path κ_v opacity τ_v optical depth, or optical thickness ds line element S_v source function
Mean intensity ^b	$J_v = \frac{1}{4\pi} \int I_v(\theta, \phi) \, d\Omega \quad \text{W m}^{-2} \text{Hz}^{-1}$	(5.172)	
Spectral energy density ^c	$u_v = \frac{1}{c} \int I_v(\theta, \phi) \, d\Omega \quad \text{J m}^{-3} \text{Hz}^{-1}$	(5.173)	
Specific emission coefficient	$j_v = \frac{\epsilon_v}{\rho} \quad \text{W kg}^{-1} \text{Hz}^{-1} \text{sr}^{-1}$	(5.174)	
Gas linear absorption coefficient ($\alpha_v \ll 1$)	$\alpha_v = n\sigma_v = \frac{1}{l_v} \quad \text{m}^{-1}$	(5.175)	
Opacity ^d	$\kappa_v = \frac{\alpha_v}{\rho} \quad \text{kg}^{-1} \text{m}^2$	(5.176)	
Optical depth	$\tau_v = \int \kappa_v \rho \, ds$	(5.177)	
Transfer equation ^e	$\frac{1}{\rho} \frac{dI_v}{ds} = -\kappa_v I_v + j_v \quad (5.178)$ or $\frac{dI_v}{ds} = -\alpha_v I_v + \epsilon_v \quad (5.179)$		
Kirchhoff's law ^f	$S_v \equiv \frac{j_v}{\kappa_v} = \frac{\epsilon_v}{\alpha_v}$	(5.180)	
Emission from a homogeneous medium	$I_v = S_v(1 - e^{-\tau_v})$	(5.181)	

^aThe definitions of these quantities vary in the literature. Those presented here are common in meteorology and astrophysics. Note particularly that the ambiguous term *specific* is taken to mean “per unit frequency interval” in the case of specific intensity and “per unit mass per unit frequency interval” in the case of specific emission coefficient.

^bIn radio astronomy, flux density is usually taken as $S = 4\pi J_v$.

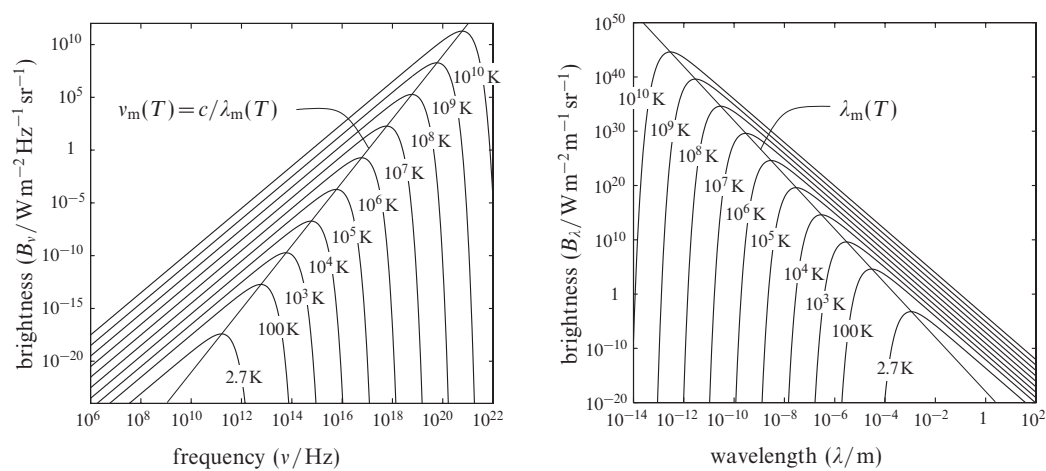
^cAssuming a refractive index of 1.

^dOr “mass absorption coefficient.”

^eOr “Schwarzschild's equation.”

^fUnder conditions of local thermal equilibrium (LTE), the source function, S_v , equals the Planck function, $B_v(T)$ [see Equation (5.182)].

Blackbody radiation

		
Planck function ^a	$B_v(T) = \frac{2hv^3}{c^2} \left[\exp\left(\frac{hv}{kT}\right) - 1 \right]^{-1} \quad (5.182)$	B_v surface brightness per unit frequency ($\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$)
	$B_\lambda(T) = B_v(T) \frac{dv}{d\lambda} \quad (5.183)$	B_λ surface brightness per unit wavelength ($\text{W m}^{-2} \text{m}^{-1} \text{sr}^{-1}$)
	$= \frac{2hc^2}{\lambda^5} \left[\exp\left(\frac{hc}{\lambda kT}\right) - 1 \right]^{-1} \quad (5.184)$	h Planck constant
Spectral energy density	$u_v(T) = \frac{4\pi}{c} B_v(T) \quad \text{J m}^{-3} \text{Hz}^{-1} \quad (5.185)$	c speed of light
	$u_\lambda(T) = \frac{4\pi}{c} B_\lambda(T) \quad \text{J m}^{-3} \text{m}^{-1} \quad (5.186)$	k Boltzmann constant T temperature $u_{v,\lambda}$ spectral energy density
Rayleigh–Jeans law ($hv \ll kT$)	$B_v(T) = \frac{2kT}{c^2} v^2 = \frac{2kT}{\lambda^2} \quad (5.187)$	
Wien's law ($hv \gg kT$)	$B_v(T) = \frac{2hv^3}{c^2} \exp\left(\frac{-hv}{kT}\right) \quad (5.188)$	
Wien's displacement law	$\lambda_m T = \begin{cases} 5.1 \times 10^{-3} \text{ m K} & \text{for } B_v \\ 2.9 \times 10^{-3} \text{ m K} & \text{for } B_\lambda \end{cases} \quad (5.189)$	λ_m wavelength of maximum brightness
Stefan–Boltzmann law ^b	$M = \pi \int_0^\infty B_v(T) dv \quad (5.190)$	M exitance
	$= \frac{2\pi^5 k^4}{15c^2 h^3} T^4 = \sigma T^4 \quad \text{W m}^{-2} \quad (5.191)$	σ Stefan–Boltzmann constant ($\simeq 5.67 \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4}$)
Energy density	$u(T) = \frac{4}{c} \sigma T^4 \quad \text{J m}^{-3} \quad (5.192)$	u energy density
Greybody	$M = \epsilon \sigma T^4 = (1 - A) \sigma T^4 \quad (5.193)$	ϵ mean emissivity A albedo

^aWith respect to the projected area of the surface. Surface brightness is also known simply as “brightness.” “Specific intensity” is used for reception.

^bSometimes “Stefan’s law.” Exitance is the total radiated energy from unit area of the body per unit time.