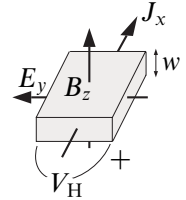


6.5 Electrons in solids

Free electron transport properties

Current density	$\mathbf{J} = -nev_d$	(6.60)	$\mathbf{J}$ current density n free electron number density $-e$ electronic charge v_d mean electron drift velocity τ mean time between collisions (relaxation time) m_e electronic mass
Mean electron drift velocity	$v_d = -\frac{e\tau}{m_e}\mathbf{E}$	(6.61)	$\mathbf{E}$ applied electric field σ_0 d.c. conductivity ($\mathbf{J} = \sigma\mathbf{E}$)
d.c. electrical conductivity	$\sigma_0 = \frac{ne^2\tau}{m_e}$	(6.62)	ω a.c. angular frequency $\sigma(\omega)$ a.c. conductivity
a.c. electrical conductivity ^a	$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$	(6.63)	C_V total electron heat capacity, V constant V volume $\langle c^2 \rangle$ mean square electron speed k_B Boltzmann constant T temperature T_F Fermi temperature
Thermal conductivity	$\lambda = \frac{1}{3} \frac{C_V}{V} \langle c^2 \rangle \tau$ $= \frac{\pi^2 nk_B^2 \tau T}{3m_e} \quad (T \ll T_F)$	(6.64) (6.65)	L Lorenz constant ($\simeq 2.45 \times 10^{-8} \text{ W } \Omega \text{ K}^{-2}$) λ thermal conductivity
Wiedemann–Franz law ^b	$\frac{\lambda}{\sigma T} = L = \frac{\pi^2 k_B^2}{3e^2}$	(6.66)	R_H Hall coefficient E_y Hall electric field J_x applied current density B_z magnetic flux density V_H Hall voltage I_x applied current ($= J_x \times \text{cross-sectional area}$) w strip thickness in z
Hall coefficient ^c	$R_H = -\frac{1}{ne} = \frac{E_y}{J_x B_z}$	(6.67)	
Hall voltage (rectangular strip)	$V_H = R_H \frac{B_z I_x}{w}$	(6.68)	



^aFor an electric field varying as $e^{-i\omega t}$.

^bHolds for an arbitrary band structure.

^cThe charge on an electron is $-e$, where e is the elementary charge (approximately $+1.6 \times 10^{-19} \text{ C}$). The Hall coefficient is therefore a negative number when the dominant charge carriers are electrons.

Fermi gas

Electron density of states ^a	$g(E) = \frac{V}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} E^{1/2} \quad (6.69)$ $g(E_F) = \frac{3}{2} \frac{nV}{E_F} \quad (6.70)$	E electron energy (>0) $g(E)$ density of states V “gas” volume m_e electronic mass $\hbar$ (Planck constant)/(2π)
Fermi wavenumber	$k_F = (3\pi^2 n)^{1/3} \quad (6.71)$	k_F Fermi wavenumber n number of electrons per unit volume
Fermi velocity	$v_F = \hbar k_F / m_e \quad (6.72)$	v_F Fermi velocity
Fermi energy ($T=0$)	$E_F = \frac{\hbar^2 k_F^2}{2m_e} = \frac{\hbar^2}{2m_e} (3\pi^2 n)^{2/3} \quad (6.73)$	E_F Fermi energy
Fermi temperature	$T_F = \frac{E_F}{k_B} \quad (6.74)$	T_F Fermi temperature k_B Boltzmann constant
Electron heat capacity ^b ($T \ll T_F$)	$C_{Ve} = \frac{\pi^2}{3} g(E_F) k_B^2 T \quad (6.75)$ $= \frac{\pi^2 k_B^2}{2E_F} T \quad (6.76)$	C_{Ve} heat capacity per electron T temperature
Total kinetic energy ($T=0$)	$U_0 = \frac{3}{5} nV E_F \quad (6.77)$	U_0 total kinetic energy
Pauli paramagnetism	$\mathbf{M} = \chi_{HP} \mathbf{H} \quad (6.78)$ $= \frac{3n}{2E_F} \mu_0 \mu_B^2 \mathbf{H} \quad (6.79)$	χ_{HP} Pauli magnetic susceptibility $\mathbf{H}$ magnetic field strength $\mathbf{M}$ magnetisation μ_0 permeability of free space μ_B Bohr magneton
Landau diamagnetism	$\chi_{HL} = -\frac{1}{3} \chi_{HP} \quad (6.80)$	χ_{HL} Landau magnetic susceptibility

^aThe density of states is often quoted per unit volume in real space (i.e., $g(E)/V$ here).

^bEquation (6.75) holds for any density of states.

Thermoelectricity

Thermopower ^a	$\mathcal{E} = \frac{J}{\sigma} + S_T \nabla T \quad (6.81)$	$\mathcal{E}$ electrochemical field ^b $\mathbf{J}$ current density σ electrical conductivity S_T thermopower
Peltier effect	$\mathbf{H} = \Pi \mathbf{J} - \lambda \nabla T \quad (6.82)$	T temperature $\mathbf{H}$ heat flux per unit area
Kelvin relation	$\Pi = T S_T \quad (6.83)$	Π Peltier coefficient λ thermal conductivity

^aOr “absolute thermoelectric power.”

^bThe electrochemical field is the gradient of $(\mu/e) - \phi$, where μ is the chemical potential, $-e$ the electronic charge, and ϕ the electrical potential.

Band theory and semiconductors

Bloch's theorem	$\Psi(\mathbf{r} + \mathbf{R}) = \exp(i\mathbf{k} \cdot \mathbf{R})\Psi(\mathbf{r})$	(6.84)	Ψ electron eigenstate $\mathbf{k}$ Bloch wavevector $\mathbf{R}$ lattice vector $\mathbf{r}$ position vector
Electron velocity	$\mathbf{v}_b(\mathbf{k}) = \frac{1}{\hbar} \nabla_{\mathbf{k}} E_b(\mathbf{k})$	(6.85)	$\mathbf{v}_b$ electron velocity (for wavevector $\mathbf{k}$) $\hbar$ (Planck constant)/ 2π b band index $E_b(\mathbf{k})$ energy band
Effective mass tensor	$m_{ij} = \hbar^2 \left[\frac{\partial^2 E_b(\mathbf{k})}{\partial k_i \partial k_j} \right]^{-1}$	(6.86)	m_{ij} effective mass tensor k_i components of $\mathbf{k}$
Scalar effective mass ^a	$m^* = \hbar^2 \left[\frac{\partial^2 E_b(k)}{\partial k^2} \right]^{-1}$	(6.87)	m^* scalar effective mass $k = \mathbf{k} $
Mobility	$\mu = \frac{ \mathbf{v}_d }{ \mathbf{E} } = \frac{eD}{k_B T}$	(6.88)	μ particle mobility v_d mean drift velocity $\mathbf{E}$ applied electric field $-e$ electronic charge D diffusion coefficient T temperature
Net current density	$\mathbf{J} = (n_e \mu_e + n_h \mu_h) e \mathbf{E}$	(6.89)	$\mathbf{J}$ current density $n_{e,h}$ electron, hole, number densities $\mu_{e,h}$ electron, hole, mobilities
Semiconductor equation	$n_e n_h = \frac{(k_B T)^3}{2(\pi \hbar^2)^3} (m_e^* m_h^*)^{3/2} e^{-E_g/(k_B T)}$	(6.90)	k_B Boltzmann constant E_g band gap $m_{e,h}^*$ electron, hole, effective masses
p-n junction	$I = I_0 \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right]$	(6.91)	I current I_0 saturation current V bias voltage (+ for forward)
	$I_0 = e n_i^2 A \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right)$	(6.92)	n_i intrinsic carrier concentration A area of junction $D_{e,h}$ electron, hole, diffusion coefficients
	$L_e = (D_e \tau_e)^{1/2}$	(6.93)	$L_{e,h}$ electron, hole, diffusion lengths
	$L_h = (D_h \tau_h)^{1/2}$	(6.94)	$\tau_{e,h}$ electron, hole, recombination times
			$N_{a,d}$ acceptor, donor, concentrations

^aValid for regions of k -space in which $E_b(\mathbf{k})$ can be taken as independent of the direction of $\mathbf{k}$.