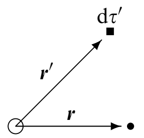
## 7.2 Static fields

### Electrostatics

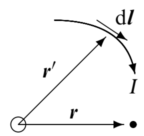
| | | | |
|---|---|---|---|
| Electrostatic potential | $\boldsymbol{E} = -\nabla\phi$ | (7.1) | $\boldsymbol{E}$ electric field; $\phi$ electrostatic potential |
| Potential difference[a] | $\phi_a - \phi_b = \int_a^b \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{l} = -\int_b^a \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{l}$ | (7.2) | $\phi_a$ potential at $a$; $\phi_b$ potential at $b$; $\mathrm{d}\boldsymbol{l}$ line element |
| Poisson's Equation (free space) | $\nabla^2\phi = -\dfrac{\rho}{\epsilon_0}$ | (7.3) | $\rho$ charge density; $\epsilon_0$ permittivity of free space |
| Point charge at $\boldsymbol{r}'$ | $\phi(\boldsymbol{r}) = \dfrac{q}{4\pi\epsilon_0\lvert\boldsymbol{r}-\boldsymbol{r}'\rvert}$ | (7.4) | $q$ point charge |
| | $\boldsymbol{E}(\boldsymbol{r}) = \dfrac{q(\boldsymbol{r}-\boldsymbol{r}')}{4\pi\epsilon_0\lvert\boldsymbol{r}-\boldsymbol{r}'\rvert^3}$ | (7.5) | |
| Field from a charge distribution (free space) | $\boldsymbol{E}(\boldsymbol{r}) = \dfrac{1}{4\pi\epsilon_0}\int_{\text{volume}} \dfrac{\rho(\boldsymbol{r}')(\boldsymbol{r}-\boldsymbol{r}')}{\lvert\boldsymbol{r}-\boldsymbol{r}'\rvert^3}\,\mathrm{d}\tau'$ | (7.6) | $\mathrm{d}\tau'$ volume element; $\boldsymbol{r}'$ position vector of $\mathrm{d}\tau'$ |




[a]Between points $a$ and $b$ along a path $\boldsymbol{l}$.

### Magnetostatics[a]

| | | | |
|---|---|---|---|
| Magnetic scalar potential | $\boldsymbol{B} = -\mu_0\nabla\phi_{\mathrm{m}}$ | (7.7) | $\phi_{\mathrm{m}}$ magnetic scalar potential; $\boldsymbol{B}$ magnetic flux density |
| $\phi_{\mathrm{m}}$ in terms of the solid angle of a generating current loop | $\phi_{\mathrm{m}} = \dfrac{I\Omega}{4\pi}$ | (7.8) | $\Omega$ loop solid angle; $I$ current |
| Biot–Savart law (the field from a line current) | $\boldsymbol{B}(\boldsymbol{r}) = \dfrac{\mu_0 I}{4\pi}\int_{\text{line}} \dfrac{\mathrm{d}\boldsymbol{l}\times(\boldsymbol{r}-\boldsymbol{r}')}{\lvert\boldsymbol{r}-\boldsymbol{r}'\rvert^3}$ | (7.9) | $\mathrm{d}\boldsymbol{l}$ line element in the direction of the current; $\boldsymbol{r}'$ position vector of $\mathrm{d}\boldsymbol{l}$ |
| Ampère's law (differential form) | $\nabla\times\boldsymbol{B} = \mu_0\boldsymbol{J}$ | (7.10) | $\boldsymbol{J}$ current density; $\mu_0$ permeability of free space |
| Ampère's law (integral form) | $\oint \boldsymbol{B}\cdot\mathrm{d}\boldsymbol{l} = \mu_0 I_{\mathrm{tot}}$ | (7.11) | $I_{\mathrm{tot}}$ total current through loop |




[a]In free space.

## Capacitance[a]

| | | |
|---|---|---|
| Of sphere, radius $a$ | $C=4\pi\epsilon_0\epsilon_{\mathrm{r}}a$ | (7.12) |
| Of circular disk, radius $a$ | $C=8\epsilon_0\epsilon_{\mathrm{r}}a$ | (7.13) |
| Of two spheres, radius $a$, in contact | $C=8\pi\epsilon_0\epsilon_{\mathrm{r}}a\ln 2$ | (7.14) |
| Of circular solid cylinder, radius $a$, length $l$ | $C\simeq[8+4.1(l/a)^{0.76}]\epsilon_0\epsilon_{\mathrm{r}}a$ | (7.15) |
| Of nearly spherical surface, area $S$ | $C\simeq 3.139\times 10^{-11}\epsilon_{\mathrm{r}}S^{1/2}$ | (7.16) |
| Of cube, side $a$ | $C\simeq 7.283\times 10^{-11}\epsilon_{\mathrm{r}}a$ | (7.17) |
| Between concentric spheres, radii $a<b$ | $C=4\pi\epsilon_0\epsilon_{\mathrm{r}}ab(b-a)^{-1}$ | (7.18) |
| Between coaxial cylinders, radii $a<b$ | $C=\dfrac{2\pi\epsilon_0\epsilon_{\mathrm{r}}}{\ln(b/a)}$ per unit length | (7.19) |
| Between parallel cylinders, separation $2d$, radii $a$ | $C=\dfrac{\pi\epsilon_0\epsilon_{\mathrm{r}}}{\operatorname{arcosh}(d/a)}$ per unit length<br>$\simeq\dfrac{\pi\epsilon_0\epsilon_{\mathrm{r}}}{\ln(2d/a)}$ $(d\gg a)$ | (7.20)<br>(7.21) |
| Between parallel, coaxial circular disks, separation $d$, radii $a$ | $C\simeq\dfrac{\epsilon_0\epsilon_{\mathrm{r}}\pi a^2}{d}+\epsilon_0\epsilon_{\mathrm{r}}a[\ln(16\pi a/d)-1]$ | (7.22) |

[a]For conductors, in an embedding medium of relative permittivity $\epsilon_{\mathrm{r}}$.

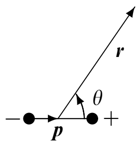
## Inductance[a]

| | | |
|---|---|---|
| Of $N$-turn solenoid (straight or toroidal), length $l$, area $A$ ($\ll l^2$) | $L=\mu_0 N^2 A/l$ | (7.23) |
| Of coaxial cylindrical tubes, radii $a$, $b$ ($a<b$) | $L=\dfrac{\mu_0}{2\pi}\ln\dfrac{b}{a}$ per unit length | (7.24) |
| Of parallel wires, radii $a$, separation $2d$ | $L\simeq\dfrac{\mu_0}{\pi}\ln\dfrac{2d}{a}$ per unit length, $(2d\gg a)$ | (7.25) |
| Of wire of radius $a$ bent in a loop of radius $b\gg a$ | $L\simeq\mu_0 b\left(\ln\dfrac{8b}{a}-2\right)$ | (7.26) |

[a]For currents confined to the surfaces of perfect conductors in free space.

## Electric fields[a]

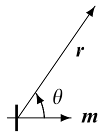
| | | |
|---|---|---|
| Uniformly charged sphere, radius $a$, charge $q$ | $\boldsymbol{E}(\boldsymbol{r}) = \begin{cases} \dfrac{q}{4\pi\epsilon_0 a^3}\boldsymbol{r} & (r<a) \\ \dfrac{q}{4\pi\epsilon_0 r^3}\boldsymbol{r} & (r \geq a) \end{cases}$ | (7.27) |
| Uniformly charged disk, radius $a$, charge $q$ (on axis, $z$) | $\boldsymbol{E}(z) = \dfrac{q}{2\pi\epsilon_0 a^2}\boldsymbol{z}\left(\dfrac{1}{\lvert z\rvert} - \dfrac{1}{\sqrt{z^2+a^2}}\right)$ | (7.28) |
| Line charge, charge density $\lambda$ per unit length | $\boldsymbol{E}(\boldsymbol{r}) = \dfrac{\lambda}{2\pi\epsilon_0 r^2}\boldsymbol{r}$ | (7.29) |
| Electric dipole, moment $\boldsymbol{p}$ (spherical polar coordinates, $\theta$ angle between $\boldsymbol{p}$ and $\boldsymbol{r}$) | $E_r = \dfrac{p\cos\theta}{2\pi\epsilon_0 r^3}$ | (7.30) |
| | $E_\theta = \dfrac{p\sin\theta}{4\pi\epsilon_0 r^3}$ | (7.31) |
| Charge sheet, surface density $\sigma$ | $E = \dfrac{\sigma}{2\epsilon_0}$ | (7.32) |

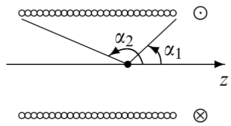



[a] For $\epsilon_{\mathrm{r}} = 1$ in the surrounding medium.

## Magnetic fields[a]

| | | |
|---|---|---|
| Uniform infinite solenoid, current $I$, $n$ turns per unit length | $B = \begin{cases} \mu_0 n I & \text{inside (axial)} \\ 0 & \text{outside} \end{cases}$ | (7.33) |
| Uniform cylinder of current $I$, radius $a$ | $B(r) = \begin{cases} \mu_0 I r/(2\pi a^2) & r<a \\ \mu_0 I/(2\pi r) & r \geq a \end{cases}$ | (7.34) |
| Magnetic dipole, moment $\boldsymbol{m}$ ($\theta$ angle between $\boldsymbol{m}$ and $\boldsymbol{r}$) | $B_r = \mu_0 \dfrac{m\cos\theta}{2\pi r^3}$ | (7.35) |
| | $B_\theta = \dfrac{\mu_0 m \sin\theta}{4\pi r^3}$ | (7.36) |
| Circular current loop of $N$ turns, radius $a$, along axis, $z$ | $B(z) = \dfrac{\mu_0 N I}{2}\dfrac{a^2}{(a^2+z^2)^{3/2}}$ | (7.37) |
| The axis, $z$, of a straight solenoid, $n$ turns per unit length, current $I$ | $B_{\mathrm{axis}} = \dfrac{\mu_0 n I}{2}(\cos\alpha_1 - \cos\alpha_2)$ | (7.38) |




[a] For $\mu_{\mathrm{r}} = 1$ in the surrounding medium.

## Image charges

| *Real charge, $+q$, at a distance:* | *image point* | *image charge* |
|---|---|---|
| $b$ from a conducting plane | $-b$ | $-q$ |
| $b$ from a conducting sphere, radius $a$ | $a^2/b$ | $-qa/b$ |
| $b$ from a plane dielectric boundary: | | |
| seen from free space | $-b$ | $-q(\epsilon_{\mathrm{r}}-1)/(\epsilon_{\mathrm{r}}+1)$ |
| seen from the dielectric | $b$ | $+2q/(\epsilon_{\mathrm{r}}+1)$ |