7.9 Plasma physics

Warm plasmas

Landau length	$l_L = \dfrac{e^2}{4\pi\epsilon_0 k_B T_e}$ (7.248) $\simeq 1.67\times10^{-5}\,T_e^{-1}$ m (7.249)	l_L Landau length $-e$ electronic charge ϵ_0 permittivity of free space k_B Boltzmann constant T_e electron temperature (K)
Electron Debye length	$\lambda_{De} = \left(\dfrac{\epsilon_0 k_B T_e}{n_e e^2}\right)^{1/2}$ (7.250) $\simeq 69(T_e/n_e)^{1/2}$ m (7.251)	λ_{De} electron Debye length n_e electron number density (m^{-3})
Debye screening[a]	$\phi(r) = \dfrac{q\exp(-2^{1/2}r/\lambda_{De})}{4\pi\epsilon_0 r}$ (7.252)	ϕ effective potential q point charge r distance from q
Debye number	$N_{De} = \dfrac{4}{3}\pi n_e \lambda_{De}^3$ (7.253)	N_{De} electron Debye number
Relaxation times ($B=0$)[b]	$\tau_e = 3.44\times10^5 \dfrac{T_e^{3/2}}{n_e \ln\Lambda}$ s (7.254) $\tau_i = 2.09\times10^7 \dfrac{T_i^{3/2}}{n_e \ln\Lambda}\left(\dfrac{m_i}{m_p}\right)^{1/2}$ s (7.255)	τ_e electron relaxation time τ_i ion relaxation time T_i ion temperature (K) $\ln\Lambda$ Coulomb logarithm (typically 10 to 20) B magnetic flux density
Characteristic electron thermal speed[c]	$v_{te} = \left(\dfrac{2k_B T_e}{m_e}\right)^{1/2}$ (7.256) $\simeq 5.51\times10^3\,T_e^{1/2}$ $m\,s^{-1}$ (7.257)	v_{te} electron thermal speed m_e electron mass

[a]Effective (Yukawa) potential from a point charge q immersed in a plasma.
[b]Collision times for electrons and *singly* ionised ions with Maxwellian speed distributions, $T_i \lesssim T_e$. The Spitzer conductivity can be calculated from Equation (7.233).
[c]Defined so that the Maxwellian velocity distribution $\propto \exp(-v^2/v_{te}^2)$. There are other definitions (see *Maxwell–Boltzmann distribution* on page 112).

Electromagnetic propagation in cold plasmas[a]

Plasma frequency	$(2\pi\nu_{\rm p})^2 = \frac{n_{\rm e}e^2}{\epsilon_0 m_{\rm e}} = \omega_{\rm p}^2$ $\nu_{\rm p} \simeq 8.98 n_{\rm e}^{1/2}$ Hz	(7.258) (7.259)	$\nu_{\rm p}$ plasma frequency $\omega_{\rm p}$ plasma angular frequency $n_{\rm e}$ electron number density ($\rm m^{-3}$) $m_{\rm e}$ electron mass
Plasma refractive index ($B=0$)	$\eta = \left[1-(\nu_{\rm p}/\nu)^2\right]^{1/2}$	(7.260)	$-e$ electronic charge ϵ_0 permittivity of free space η refractive index ν frequency
Plasma dispersion relation ($B=0$)	$c^2k^2 = \omega^2 - \omega_{\rm p}^2$	(7.261)	k wavenumber ($=2\pi/\lambda$) ω angular frequency ($=2\pi/\nu$) c speed of light
Plasma phase velocity ($B=0$)	$v_\phi = c/\eta$	(7.262)	v_ϕ phase velocity
Plasma group velocity ($B=0$)	$v_{\rm g} = c\eta$ $v_\phi v_{\rm g} = c^2$	(7.263) (7.264)	$v_{\rm g}$ group velocity
Cyclotron (Larmor, or gyro-) frequency	$2\pi\nu_{\rm C} = \frac{qB}{m} = \omega_{\rm C}$ $\nu_{\rm Ce} \simeq 28\times10^9 B$ Hz $\nu_{\rm Cp} \simeq 15.2\times10^6 B$ Hz	(7.265) (7.266) (7.267)	$\nu_{\rm C}$ cyclotron frequency $\omega_{\rm C}$ cyclotron angular frequency $\nu_{\rm Ce}$ electron $\nu_{\rm C}$ $\nu_{\rm Cp}$ proton $\nu_{\rm C}$ q particle charge B magnetic flux density (T)
Larmor (cyclotron, or gyro-) radius	$r_{\rm L} = \frac{v_\perp}{\omega_{\rm C}} = v_\perp \frac{m}{qB}$ $r_{\rm Le} = 5.69\times10^{-12}\left(\frac{v_\perp}{B}\right)$ m $r_{\rm Lp} = 10.4\times10^{-9}\left(\frac{v_\perp}{B}\right)$ m	(7.268) (7.269) (7.270)	m particle mass (γm if relativistic) $r_{\rm L}$ Larmor radius $r_{\rm Le}$ electron $r_{\rm L}$ $r_{\rm Lp}$ proton $r_{\rm L}$ $v_\perp$ speed $\perp$ to $\boldsymbol{B}$ ($\rm m\,s^{-1}$)
Mixed propagation modes[b]	$\eta^2 = 1 - \frac{X(1-X)}{(1-X)-\frac{1}{2}Y^2\sin^2\theta_B \pm S}$, where $X=(\omega_{\rm p}/\omega)^2$, $Y=\omega_{\rm Ce}/\omega$, and $S^2 = \frac{1}{4}Y^4\sin^4\theta_B + Y^2(1-X)^2\cos^2\theta_B$	(7.271)	θ_B angle between wavefront normal ($\hat{\boldsymbol{k}}$) and $\boldsymbol{B}$
Faraday rotation[c]	$\Delta\psi = \underbrace{\frac{\mu_0 e^3}{8\pi^2 m_{\rm e}^2 c}}_{2.63\times10^{-13}} \lambda^2 \int_{\rm line} n_{\rm e}\boldsymbol{B}\cdot{\rm d}\boldsymbol{l}$ $= R\lambda^2$	(7.272) (7.273)	$\Delta\psi$ rotation angle λ wavelength ($=2\pi/k$) ${\rm d}\boldsymbol{l}$ line element in direction of wave propagation R rotation measure

[a] I.e., plasmas in which electromagnetic force terms dominate over thermal pressure terms. Also taking $\mu_{\rm r}=1$.

[b] In a collisionless electron plasma. The ordinary and extraordinary modes are the + and − roots of S^2 when $\theta_B=\pi/2$. When $\theta_B=0$, these roots are the right and left circularly polarised modes respectively, using the optical convention for handedness.

[c] In a tenuous plasma, SI units throughout. $\Delta\psi$ is taken positive if $\boldsymbol{B}$ is directed towards the observer.

Magnetohydrodynamics[a]

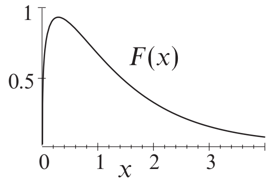
Sound speed	$v_s = \left(\frac{\gamma p}{\rho}\right)^{1/2} = \left(\frac{2\gamma k_B T}{m_p}\right)^{1/2}$	(7.274)	v_s γ p ρ k_B T m_p	sound (wave) speed ratio of heat capacities hydrostatic pressure plasma mass density Boltzmann constant temperature (K) proton mass
	$\simeq 166 T^{1/2}\,\mathrm{m\,s^{-1}}$	(7.275)		
Alfvén speed	$v_A = \frac{B}{(\mu_0 \rho)^{1/2}}$	(7.276)	v_A B μ_0	Alfvén speed magnetic flux density (T) permeability of free space
	$\simeq 2.18 \times 10^{16} B n_e^{-1/2} \quad \mathrm{m\,s^{-1}}$	(7.277)	n_e	electron number density ($\mathrm{m^{-3}}$)
Plasma beta	$\beta = \frac{2\mu_0 p}{B^2} = \frac{4\mu_0 n_e k_B T}{B^2} = \frac{2v_s^2}{\gamma v_A^2}$	(7.278)	β	plasma beta (ratio of hydrostatic to magnetic pressure)
Direct electrical conductivity	$\sigma_d = \frac{n_e^2 e^2 \sigma}{n_e^2 e^2 + \sigma^2 B^2}$	(7.279)	$-e$ σ_d σ	electronic charge direct conductivity conductivity ($B=0$)
Hall electrical conductivity	$\sigma_H = \frac{\sigma B}{n_e e}\sigma_d$	(7.280)	σ_H	Hall conductivity
Generalised Ohm's law	$\boldsymbol{J} = \sigma_d(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) + \sigma_H \hat{\boldsymbol{B}}\times(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$	(7.281)	$\boldsymbol{J}$ $\boldsymbol{E}$ $\boldsymbol{v}$ $\hat{\boldsymbol{B}}$	current density electric field plasma velocity field $=\boldsymbol{B}/\lvert\boldsymbol{B}\rvert$
Resistive MHD equations (single-fluid model)[b]				
	$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) + \eta\nabla^2\boldsymbol{B}$	(7.282)	μ_0 η	permeability of free space magnetic diffusivity $[=1/(\mu_0\sigma)]$
	$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho} + \frac{1}{\mu_0\rho}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} + \nu\nabla^2\boldsymbol{v} + \frac{1}{3}\nu\nabla(\nabla\cdot\boldsymbol{v}) + \boldsymbol{g}$	(7.283)	ν $\boldsymbol{g}$	kinematic viscosity gravitational field strength
Shear Alfvénic dispersion relation[c]	$\omega = k v_A \cos\theta_B$	(7.284)	ω $\boldsymbol{k}$ θ_B	angular frequency ($=2\pi\nu$) wavevector ($k=2\pi/\lambda$) angle between $\boldsymbol{k}$ and $\boldsymbol{B}$
Magnetosonic dispersion relation[d]	$\omega^2 k^2 (v_s^2 + v_A^2) - \omega^4 = v_s^2 v_A^2 k^4 \cos^2\theta_B$	(7.285)		

[a] For a warm, fully ionised, electrically neutral p^+/e^- plasma, $\mu_r = 1$. Relativistic and displacement current effects are assumed to be negligible and all oscillations are taken as being well below all resonance frequencies.
[b] Neglecting bulk (second) viscosity.
[c] Nonresistive, inviscid flow.
[d] Nonresistive, inviscid flow. The greater and lesser solutions for ω^2 are the fast and slow magnetosonic waves respectively.

Synchrotron radiation

Power radiated by a single electron[a]	$P_{\mathrm{tot}} = 2\sigma_{\mathrm{T}} c u_{\mathrm{mag}} \gamma^2 \left(\frac{v}{c}\right)^2 \sin^2\theta$	(7.286)
	$\simeq 1.59 \times 10^{-14} B^2 \gamma^2 \left(\frac{v}{c}\right)^2 \sin^2\theta \quad \mathrm{W}$	(7.287)
... averaged over pitch angles	$P_{\mathrm{tot}} = \frac{4}{3}\sigma_{\mathrm{T}} c u_{\mathrm{mag}} \gamma^2 \left(\frac{v}{c}\right)^2$	(7.288)
	$\simeq 1.06 \times 10^{-14} B^2 \gamma^2 \left(\frac{v}{c}\right)^2 \quad \mathrm{W}$	(7.289)
Single electron emission spectrum[b]	$P(\nu) = \frac{3^{1/2} e^3 B \sin\theta}{4\pi\epsilon_0 c m_{\mathrm{e}}} F(\nu/\nu_{\mathrm{ch}})$	(7.290)
	$\simeq 2.34 \times 10^{-25} B \sin\theta F(\nu/\nu_{\mathrm{ch}}) \quad \mathrm{W\,Hz^{-1}}$	(7.291)
Characteristic frequency	$\nu_{\mathrm{ch}} = \frac{3}{2}\gamma^2 \frac{eB}{2\pi m_{\mathrm{e}}} \sin\theta$	(7.292)
	$\simeq 4.2 \times 10^{10} \gamma^2 B \sin\theta \quad \mathrm{Hz}$	(7.293)
Spectral function	$F(x) = x \int_x^{\infty} K_{5/3}(y)\,\mathrm{d}y$	(7.294)
	$\simeq \begin{cases} 2.15 x^{1/3} & (x \ll 1) \\ 1.25 x^{1/2} \mathrm{e}^{-x} & (x \gg 1) \end{cases}$	(7.295)

P_{tot}	total radiated power
σ_{T}	Thomson cross section
u_{mag}	magnetic energy density $= B^2/(2\mu_0)$
$\boldsymbol{v}$	electron velocity ($\sim c$)
γ	Lorentz factor $= [1-(v/c)^2]^{-1/2}$
θ	pitch angle (angle between $\boldsymbol{v}$ and $\boldsymbol{B}$)
$\boldsymbol{B}$	magnetic flux density
c	speed of light
$P(\nu)$	emission spectrum
ν	frequency
ν_{ch}	characteristic frequency
$-e$	electronic charge
ϵ_0	free space permittivity
m_{e}	electronic (rest) mass
F	spectral function
$K_{5/3}$	modified Bessel fn. of the 2nd kind, order 5/3



[a]This expression also holds for cyclotron radiation ($v \ll c$).

[b]I.e., total radiated power per unit frequency interval.

Bremsstrahlung[a]

Single electron and ion[b]

$$\frac{\mathrm{d}W}{\mathrm{d}\omega} = \frac{Z^2e^6}{24\pi^4\epsilon_0^3c^3m_e^2}\frac{\omega^2}{\gamma^2v^4}\left[\frac{1}{\gamma^2}K_0^2\left(\frac{\omega b}{\gamma v}\right)+K_1^2\left(\frac{\omega b}{\gamma v}\right)\right] \quad (7.296)$$

$$\simeq \frac{Z^2e^6}{24\pi^4\epsilon_0^3c^3m_e^2b^2v^2} \quad (\omega b \ll \gamma v) \quad (7.297)$$

Thermal bremsstrahlung radiation ($v \ll c$; Maxwellian distribution)

$$\frac{\mathrm{d}P}{\mathrm{d}V\,\mathrm{d}\nu} = 6.8\times10^{-51}Z^2T^{-1/2}n_in_eg(\nu,T)\exp\left(\frac{-h\nu}{kT}\right) \quad \mathrm{W\,m^{-3}\,Hz^{-1}} \quad (7.298)$$

$$\text{where}\quad g(\nu,T)\simeq\begin{cases}0.28[\ln(4.4\times10^{16}T^3\nu^{-2}Z^{-2})-0.76] & (h\nu\ll kT\lesssim10^5kZ^2)\\ 0.55\ln(2.1\times10^{10}T\nu^{-1}) & (h\nu\ll10^5kZ^2\lesssim kT)\\ (2.1\times10^{10}T\nu^{-1})^{-1/2} & (h\nu\gg kT)\end{cases} \quad (7.299)$$

$$\frac{\mathrm{d}P}{\mathrm{d}V} \simeq 1.7\times10^{-40}Z^2T^{1/2}n_in_e \quad \mathrm{W\,m^{-3}} \quad (7.300)$$

ω	angular frequency ($=2\pi\nu$)	v	electron velocity	W	energy radiated
Ze	ionic charge	K_i	modified Bessel functions of order i (see page 47)	T	electron temperature (K)
$-e$	electronic charge	γ	Lorentz factor $=[1-(v/c)^2]^{-1/2}$	n_i	ion number density (m^{-3})
ϵ_0	permittivity of free space	P	power radiated	n_e	electron number density (m^{-3})
c	speed of light	V	volume	k	Boltzmann constant
m_e	electronic mass	ν	frequency (Hz)	h	Planck constant
b	collision parameter[c]			g	Gaunt factor

[a]Classical treatment. The ions are at rest, and all frequencies are above the plasma frequency.
[b]The spectrum is approximately flat at low frequencies and drops exponentially at frequencies $\gtrsim \gamma v/b$.
[c]Distance of closest approach.