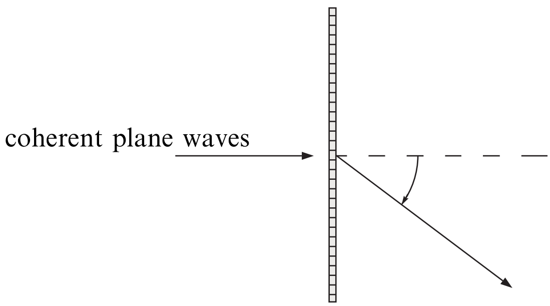
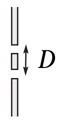
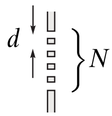
8.3 Fraunhofer diffraction

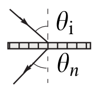
Gratings[a]

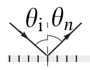


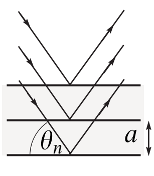
Young's double slits[b]	$I(s)=I_0\cos^2\frac{kDs}{2}$	(8.24)	$I(s)$ I_0 θ s D	diffracted intensity peak intensity diffraction angle $=\sin\theta$ slit separation
N equally spaced narrow slits	$I(s)=I_0\left[\frac{\sin(Nkds/2)}{N\sin(kds/2)}\right]^2$	(8.25)	λ N k d	wavelength number of slits wavenumber ($=2\pi/\lambda$) slit spacing
Infinite grating	$I(s)=I_0\sum_{n=-\infty}^{\infty}\delta\left(s-\frac{n\lambda}{d}\right)$	(8.26)	n δ	diffraction order Dirac delta function
Normal incidence	$\sin\theta_n=\frac{n\lambda}{d}$	(8.27)	θ_n	angle of diffracted maximum
Oblique incidence	$\sin\theta_n+\sin\theta_{\rm i}=\frac{n\lambda}{d}$	(8.28)	$\theta_{\rm i}$	angle of incident illumination
Reflection grating	$\sin\theta_n-\sin\theta_{\rm i}=\frac{n\lambda}{d}$	(8.29)		
Chromatic resolving power	$\frac{\lambda}{\delta\lambda}=Nn$	(8.30)	$\delta\lambda$	diffraction peak width
Grating dispersion	$\frac{\partial\theta}{\partial\lambda}=\frac{n}{d\cos\theta}$	(8.31)		
Bragg's law[c]	$2a\sin\theta_n=n\lambda$	(8.32)	a	atomic plane spacing





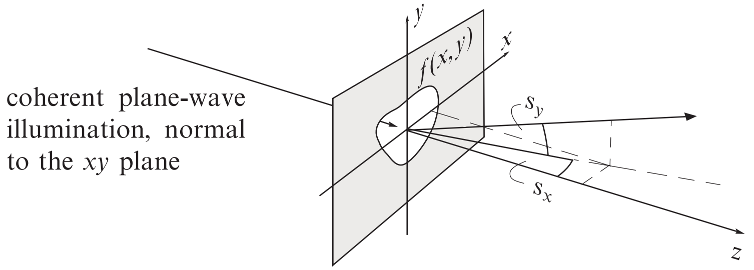






[a]Unless stated otherwise, the illumination is normal to the grating.
[b]Two narrow slits separated by D.
[c]The condition is for Bragg reflection, with $\theta_n=\theta_{\rm i}$.

Aperture diffraction



General 1-D aperture[a]	$\psi(s)\propto\int_{-\infty}^{\infty}f(x)\mathrm{e}^{-\mathbf{i}ksx}\,\mathrm{d}x$	(8.33)	ψ diffracted wavefunction; I diffracted intensity; θ diffraction angle; s $=\sin\theta$
	$I(s)\propto\psi\psi^*(s)$	(8.34)	
General 2-D aperture in (x,y) plane (small angles)	$\psi(s_x,s_y)\propto\iint_{\infty}f(x,y)\mathrm{e}^{-\mathbf{i}k(s_xx+s_yy)}\,\mathrm{d}x\,\mathrm{d}y$	(8.35)	f aperture amplitude transmission function; x,y distance across aperture; k wavenumber $(=2\pi/\lambda)$; s_x deflection $\parallel$ xz plane; s_y deflection $\perp$ xz plane
Broad 1-D slit[b]	$I(s)=I_0\frac{\sin^2(kas/2)}{(kas/2)^2}$	(8.36)	I_0 peak intensity; a slit width (in x); λ wavelength
	$\equiv I_0\,\mathrm{sinc}^2(as/\lambda)$	(8.37)	
Sidelobe intensity	$\frac{I_n}{I_0}=\left(\frac{2}{\pi}\right)^2\frac{1}{(2n+1)^2}\qquad(n>0)$	(8.38)	I_n nth sidelobe intensity
Rectangular aperture (small angles)	$I(s_x,s_y)=I_0\,\mathrm{sinc}^2\frac{as_x}{\lambda}\,\mathrm{sinc}^2\frac{bs_y}{\lambda}$	(8.39)	a aperture width in x; b aperture width in y
Circular aperture[c]	$I(s)=I_0\left[\frac{2J_1(kDs/2)}{kDs/2}\right]^2$	(8.40)	J_1 first-order Bessel function; D aperture diameter
First minimum[d]	$s=1.22\frac{\lambda}{D}$	(8.41)	λ wavelength
First subsid. maximum	$s=1.64\frac{\lambda}{D}$	(8.42)	
Weak 1-D phase object	$f(x)=\exp[\mathbf{i}\phi(x)]\simeq1+\mathbf{i}\phi(x)$	(8.43)	$\phi(x)$ phase distribution; $\mathbf{i}$ $\mathbf{i}^2=-1$
Fraunhofer limit[e]	$L\gg\frac{(\Delta x)^2}{\lambda}$	(8.44)	L distance of aperture from observation point; Δx aperture size

[a]The Fraunhofer integral.
[b]Note that $\mathrm{sinc}\,x=(\sin\pi x)/(\pi x)$.
[c]The central maximum is known as the "Airy disk."
[d]The "Rayleigh resolution criterion" states that two point sources of equal intensity can just be resolved with diffraction-limited optics if separated in angle by $1.22\lambda/D$.
[e]Plane-wave illumination.