8.7 Coherence (scalar theory)

Mutual coherence function	$\boldsymbol{\Gamma}_{12}(\tau)=\langle\psi_1(t)\psi_2^*(t+\tau)\rangle$ (8.97)	$\boldsymbol{\Gamma}_{ij}$ mutual coherence function τ temporal interval ψ_i (complex) wave disturbance at spatial point i
Complex degree of coherence	$\gamma_{12}(\tau)=\frac{\langle\psi_1(t)\psi_2^*(t+\tau)\rangle}{[\langle\|\psi_1\|^2\rangle\langle\|\psi_2\|^2\rangle]^{1/2}}$ (8.98) $=\frac{\boldsymbol{\Gamma}_{12}(\tau)}{[\boldsymbol{\Gamma}_{11}(0)\boldsymbol{\Gamma}_{22}(0)]^{1/2}}$ (8.99)	t time $\langle\cdot\rangle$ mean over time γ_{ij} complex degree of coherence * complex conjugate
Combined intensity[a]	$I_{\text{tot}}=I_1+I_2+2(I_1I_2)^{1/2}\Re[\gamma_{12}(\tau)]$ (8.100)	I_{tot} combined intensity I_i intensity of disturbance at point i $\Re$ real part of
Fringe visibility	$V(\tau)=\frac{2(I_1I_2)^{1/2}}{I_1+I_2}\|\gamma_{12}(\tau)\|$ (8.101)	
if $\|\gamma_{12}(\tau)\|$ is a constant:	$V=\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}$ (8.102)	$I_{\max}$ max. combined intensity $I_{\min}$ min. combined intensity
if $I_1=I_2$:	$V(\tau)=\|\gamma_{12}(\tau)\|$ (8.103)	
Complex degree of temporal coherence[b]	$\gamma(\tau)=\frac{\langle\psi_1(t)\psi_1^*(t+\tau)\rangle}{\langle\|\psi_1(t)^2\|\rangle}$ (8.104) $=\frac{\int I(\omega)e^{-i\omega\tau}\,d\omega}{\int I(\omega)\,d\omega}$ (8.105)	$\gamma(\tau)$ degree of temporal coherence $I(\omega)$ specific intensity ω radiation angular frequency c speed of light
Coherence time and length	$\Delta\tau_c=\frac{\Delta l_c}{c}\sim\frac{1}{\Delta\nu}$ (8.106)	$\Delta\tau_c$ coherence time Δl_c coherence length $\Delta\nu$ spectral bandwidth
Complex degree of spatial coherence[c]	$\gamma(\boldsymbol{D})=\frac{\langle\psi_1\psi_2^*\rangle}{[\langle\|\psi_1\|^2\rangle\langle\|\psi_2\|^2\rangle]^{1/2}}$ (8.107) $=\frac{\int I(\hat{\boldsymbol{s}})e^{ik\boldsymbol{D}\cdot\hat{\boldsymbol{s}}}\,d\Omega}{\int I(\hat{\boldsymbol{s}})\,d\Omega}$ (8.108)	$\gamma(\boldsymbol{D})$ degree of spatial coherence $\boldsymbol{D}$ spatial separation of points 1 and 2 $I(\hat{\boldsymbol{s}})$ specific intensity of distant extended source in direction $\hat{\boldsymbol{s}}$ $d\Omega$ differential solid angle $\hat{\boldsymbol{s}}$ unit vector in the direction of $d\Omega$
Intensity correlation[d]	$\frac{\langle I_1I_2\rangle}{[\langle I_1\rangle^2\langle I_2\rangle^2]^{1/2}}=1+\gamma^2(\boldsymbol{D})$ (8.109)	k wavenumber
Speckle intensity distribution[e]	$\text{pr}(I)=\frac{1}{\langle I\rangle}e^{-I/\langle I\rangle}$ (8.110)	pr probability density
Speckle size (coherence width)	$\Delta w_c\simeq\frac{\lambda}{\alpha}$ (8.111)	Δw_c characteristic speckle size λ wavelength α source angular size as seen from the screen

[a] From interfering the disturbances at points 1 and 2 with a relative delay τ.
[b] Or "autocorrelation function."
[c] Between two points on a wavefront, separated by $\boldsymbol{D}$. The integral is over the entire extended source.
[d] For wave disturbances that have a Gaussian probability distribution in amplitude. This is "Gaussian light" such as from a thermal source.
[e] Also for Gaussian light.