

Chapter – 9

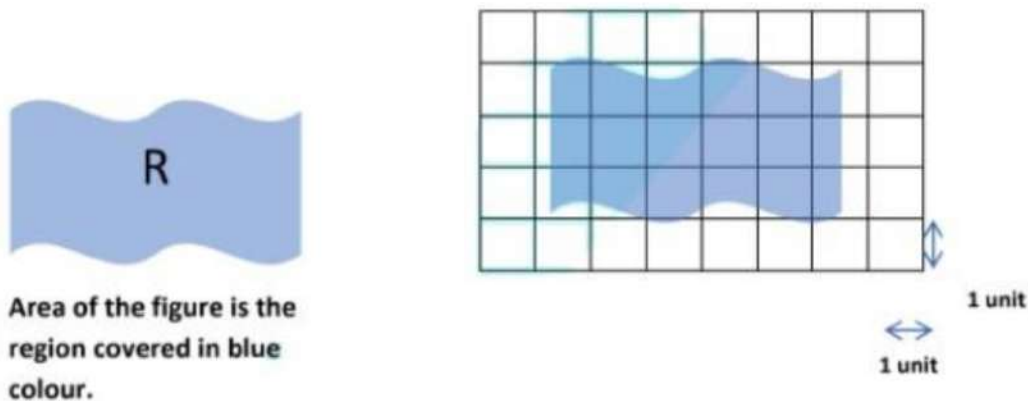
Areas of Parallelograms and Triangles

Introduction to Area between Parallelogram and Triangles

Introduction to Area

Area: Area can be defined as the space occupied by a closed two- dimensional shape or object.

The area of a closed figure is the number of unit-squares it covers.



Units of the area: Area is measured in square units such as square centimetres, square feet, square inches, etc.

The area of a closed figure/region R is denoted by $ar(R)$.

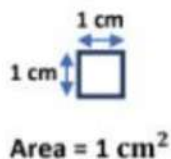
Suppose, this region R covers x number of unit squares, then its area will be represented as,

$$Ar(R) = x \text{ square units or } x \text{ unit}^2$$

How do we get the units of measurement of the area? Let us understand this.

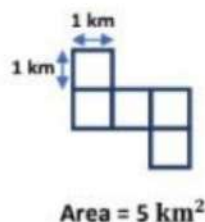
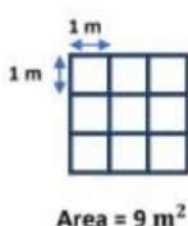
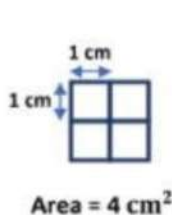
Let us find the area of a square with side 1 cm.

We know, the area of a square = side $\times$ side.



The area of the square with side 1 centimeter (cm) = 1 cm × 1 cm = 1 cm².

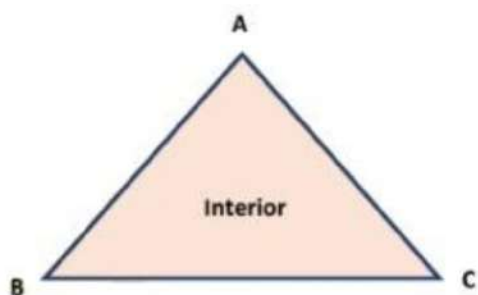
Similarly, the area of the figures below can be found by simply counting the number of unit squares.



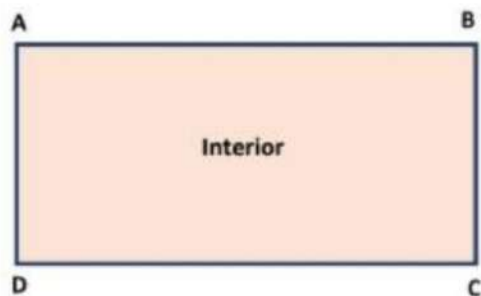
In this chapter, we will restrict our discussion to the area of the polygonal region. For example, the area of triangles, quadrilaterals, parallelograms.

We often find the area of the room floor to calculate the quantity of the carpet to be bought. Covering the floor with tiles, covering the wall with paint or wallpaper or building are other examples, wherein the computation of area is required.

A triangle is a closed, two- dimensional figure formed by three intersecting lines. The region enclosed by a triangle is its area.



The part of the plane enclosed by the triangle is called the interior region of the triangle. Hence, the area of a triangle is a triangular region formed by the union of a triangle and its interior.



Similarly, the area of a parallelogram ABCD is the union of the parallelogram and its interior.

We have already learned the formulae for finding the areas of different plane figures such as triangle, rectangle, parallelogram, square, etc.

In this chapter, we will study some relationships between the areas of these geometric figures under the condition like when they lie on the same base and between the same parallels.

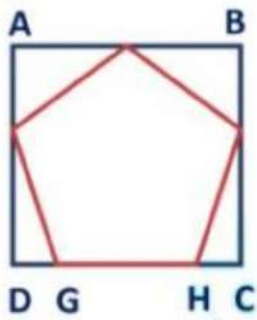
Figures on the same base and between the same parallels

Figures on the same base and between the same parallels

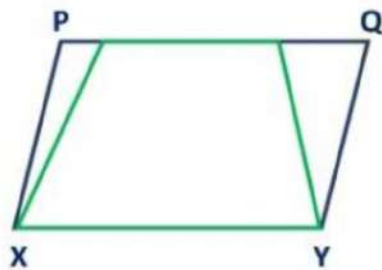
Two triangles with common base EF.



Two figures are between two parallels AB & CD but have different bases' length.

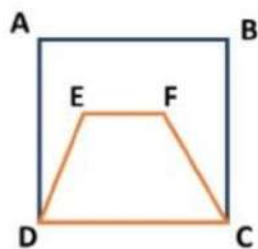


Two figures with common base XY and between two parallels XY & PQ.

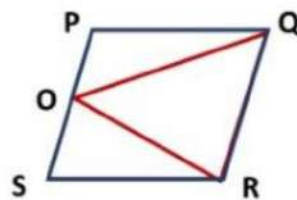


“So, two figures are said to be on the same base and between same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base”. [As shown in the figure above]

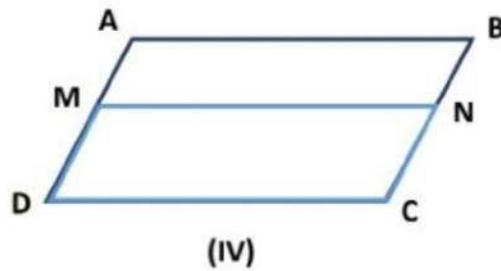
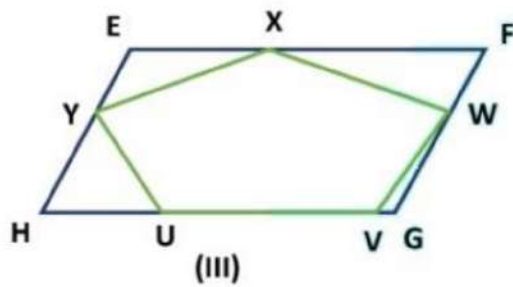
Example: Choose which of the following figures lie on the same base and between the same parallels. For such a figure, write the common base and the two parallels.



(I)



(II)



Solution:

I. Rectangle ABCD and trapezium EFCD are on the same base CD but they are not between the same parallels.

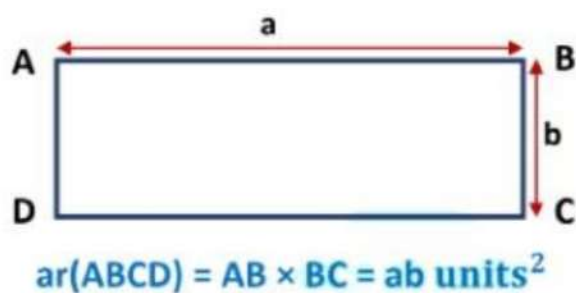
II. Parallelogram PQRS and ΔROQ are on the same base RQ and between the same parallel lines PS and QR.

III. Parallelogram EFGH and pentagon UVWXY are between the same parallel EF and HG but they are not on the same base.

IV. Parallelogram ABCD and parallelogram MNCD lie between the same parallels AB and CD and they have common base CD.

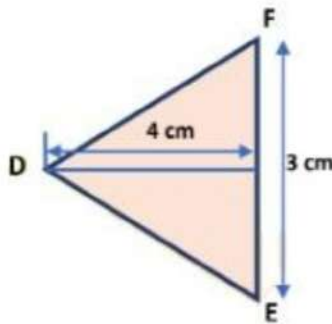
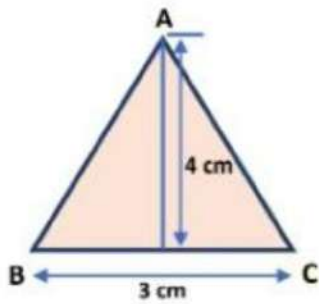
RECTANGLE AREA AXIOM

If ABCD is a rectangular region such that $AB = a$ units and $BC = b$ units in length, then



CONGRUENT AREA AXIOM

Consider two ΔABC and ΔDEF which are congruent to each other.



Now, we calculate the area of ΔABC , then we get

$$\text{ar}(\Delta ABC) = \frac{1}{2} \times \text{base} \times \text{height}.$$

$$\begin{aligned} &= \frac{1}{2} \times 3 \text{ cm} \times 4 \text{ cm}. \\ &= 3 \text{ cm} \times 2 \text{ cm}. \\ &= 6 \text{ cm}^2. \end{aligned}$$

Also, we calculate the area of ΔDEF , then we get

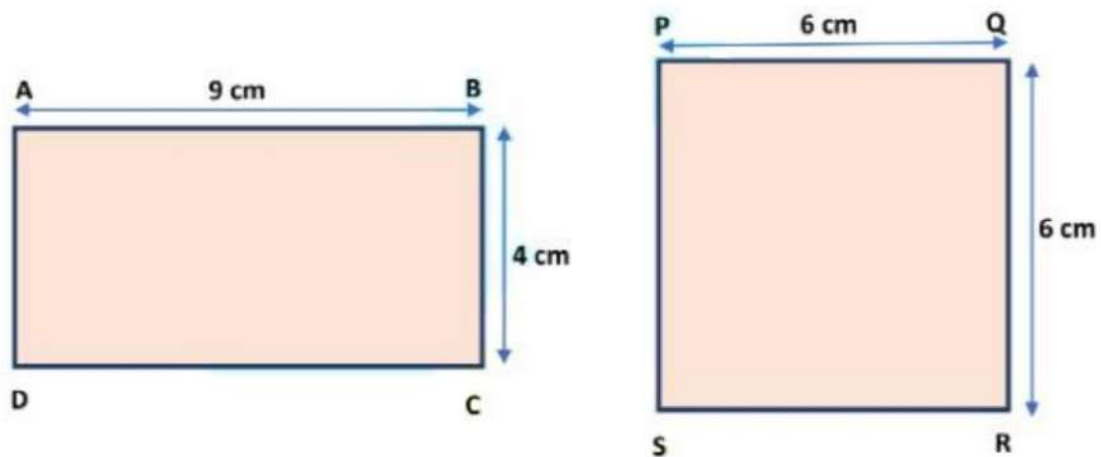
$$\text{ar}(\Delta DEF) = \frac{1}{2} \times \text{base} \times \text{height}.$$

$$\begin{aligned} &= \frac{1}{2} \times 3 \text{ cm} \times 4 \text{ cm}. \\ &= 3 \text{ cm} \times 2 \text{ cm}. \\ &= 6 \text{ cm}^2. \end{aligned}$$

Here, we also see that the area of triangles ABC and DEF are equal.

Therefore, we conclude that if two figures are congruent with each other, they must have equal areas.

Now, consider a rectangle ABCD and square PQRS which are equal in area.



We calculate the area of rectangle ABCD, then we get $\text{ar}(\text{ABCD}) = \text{Length} \times \text{Breadth}$.

$$= 9 \text{ cm} \times 4 \text{ cm}.$$

$$= 36 \text{ cm}^2$$

Also, we calculate the area of square PQRS, then we get
 $\text{ar}(\text{PQRS}) = \text{Side} \times \text{Side}$.
 $= 6 \text{ cm} \times 6 \text{ cm}$.
 $= 36 \text{ cm}^2$.

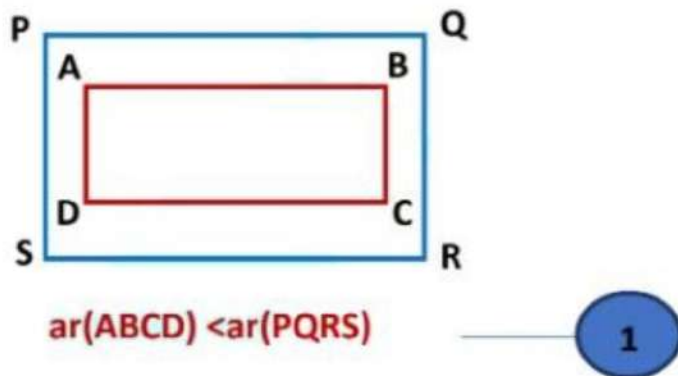
Here, we see that areas of triangles ABC and DEF are equal. Rectangles ABCD and square PQRS are not congruent, but still, they have equal areas. Therefore, we can conclude that if two figures are congruent, they will have equal areas whereas, if two figures have equal areas, they need not be congruent with each other.

AREA MONOTONE AXIOM:

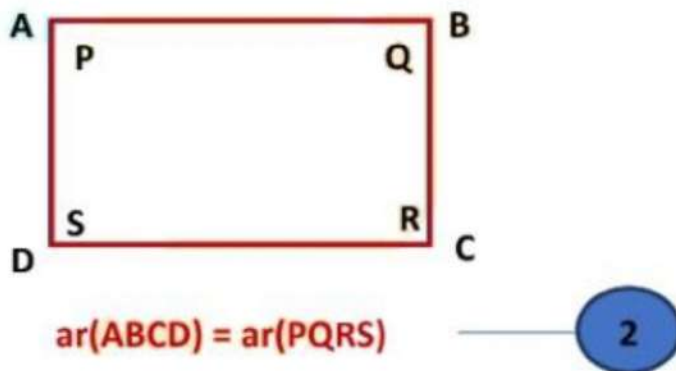
If R_1, R_2 are two polygonal regions such that R_1 is a part of R_2 , then

$$\text{ar}(R_1) \leq \text{ar}(R_2)$$

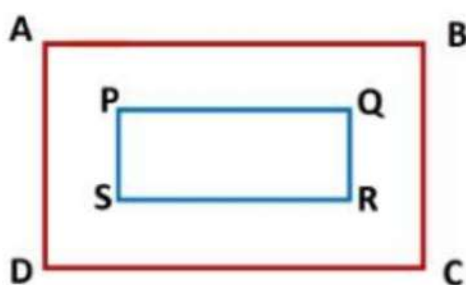
i.e. In the given figure region, ABCD is the part of region PQRS, that means



In the next figure (below) the region, the vertices A, B, C, D of parallelogram ABCD coincide with the vertices P, Q, R, S of the parallelogram PQRS.



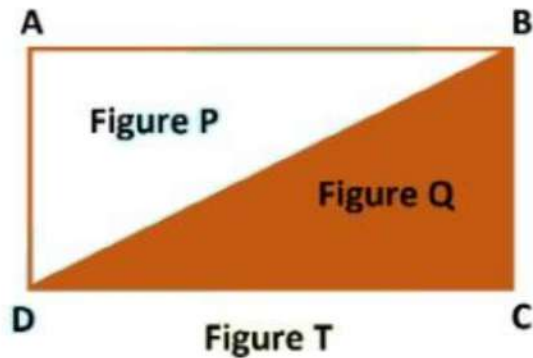
The third possibility for figure ABCD and PQRS is



But in this case, $\text{ar}(\text{ABCD}) > \text{ar}(\text{PQRS})$ and PQRS is the part of ABCD which is not held by the statement of the axiom. So, this case shall not be considered.

AREA ADDITION AXIOM:

If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q, then $\text{ar}(T) = \text{ar}(P) + \text{ar}(Q)$.



In the figure above, $\text{ar}(ABCD) = \text{ar}(\triangle ADB) + \text{ar}(\triangle BDC)$

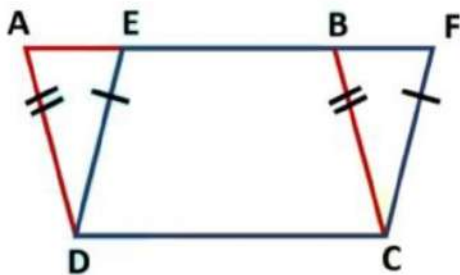
Parallelograms on the same base and between the same parallels

Parallelograms on the same base and between the same parallels.

Theorem 1: Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.

Given: Two parallelograms ABCD and ABPQ, which have the same base AB and which are between the same parallel lines AB and QC.

To prove: $\text{ar}(\parallel \text{gm } ABCD) = \text{ar}(\parallel \text{gm } EFCD)$



Proof: In $\triangle ADE$ and $\triangle BCF$

$\angle DAE = \angle CBF$ (Corresponding angles from $AD \parallel BC$ and transversal AF)

$\angle AED = \angle BFC$ (Corresponding angles from $ED \parallel FC$ and transversal AF)

Also, $AD = BC$ (Opposite sides of the parallelogram ABCD)

So, $\triangle ADE \cong \triangle BCF$ [By AAS criteria]

Therefore, $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$ (Congruent figures have equal areas)

Now, $\text{ar}(ABCD) = \text{ar}(\triangle ADE) + \text{ar}(EDCB)$

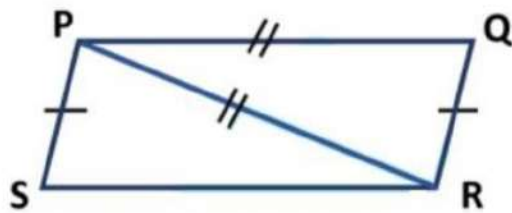
$= \text{ar}(\triangle BCF) + \text{ar}(EDCB)$ [From 1]

$= \text{ar}(EFCD)$

So, parallelograms ABCD and EFCD are equal in area.

Theorem 2: A diagonal of a parallelogram divides it into two triangles of equal area.

Given: A parallelogram PQRS in which PR is one of the diagonals.



To prove: $\text{ar}(\triangle PSR) = \text{ar}(\triangle RQP)$

Proof: In $\triangle PSR$ and $\triangle RQP$, we have

$PS = QR$ (Opposite side of the parallelogram PQRS)

$SR = QP$ (Opposite side of the parallelogram PQRS)

$PR = RP$ (Common side)

So, by SSS criterion of congruence, we have

$\triangle PSR \cong \triangle RQP$

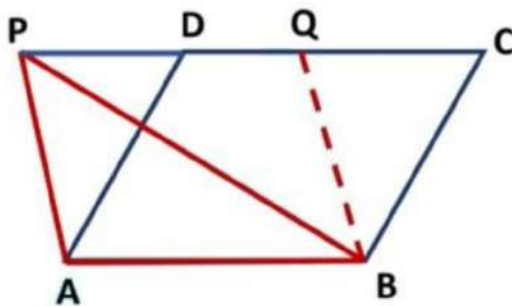
And we know that congruent figures have equal areas.

Hence, $\text{ar}(\triangle PSR) = \text{ar}(\triangle RQP)$.

Theorem 3: If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half of the area of the

parallelogram.

Given: A $\triangle ABP$ and $\parallel^{\text{gm}}ABCD$ on the same base AB and between the same parallels AB and PC .



To prove: $\text{ar}(\triangle PAB) = \frac{1}{2} \text{ar}(\parallel^{\text{gm}}ABCD)$

Construction: Draw $BQ \parallel AP$ to obtain another parallelogram $ABQP$.

Proof: Parallelogram $ABCD$ and $ABQP$ are on the same base AB and between the same parallels AB and PC .

Therefore, $\text{ar}(ABQP) = \text{ar}(ABCD)$ [By Theorem 1]

Now, in $\parallel^{\text{gm}}ABQP$, PB is a diagonal which divides it into two equal parts.

So, $\text{ar}(\triangle PAB) = \text{ar}(\triangle BQP)$ [By Theorem 2]

$\text{ar}(ABPQ) = \text{ar}(\triangle PAB) + \text{ar}(\triangle BQP)$

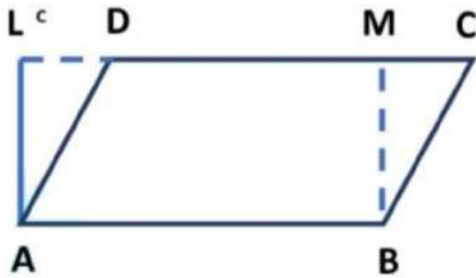
$\text{ar}(ABPQ) = \text{ar}(\triangle PAB) + \text{ar}(\triangle PAB)$ [From 2]

$\text{ar}(ABCD) = 2\text{ar}(\triangle PAB)$ [From 1]

$\text{ar}(\triangle PAB) = \frac{1}{2} \text{ar}(\parallel^{\text{gm}}ABCD)$

Theorem 4: The area of a parallelogram is the product of its base and the corresponding altitude.

Given: A parallelogram ABCD in which AB is the base and AL is the corresponding altitude.



To prove: $\text{ar}(\text{||gm ABCD}) = AB \times AL$

Construction: Complete the rectangle

ALMB by drawing $BM \perp CD$.

Proof: In $\triangle ADL$ and $\triangle BCM$

$\angle ALD = \angle BMC$ (Each is equal to 90°)

$\angle ADL = \angle BCM$

(Corresponding angles from $AD \parallel BC$ and transversal LC)

$AD = BC$ (Opposite side of the parallelogram ABCD)

So, $\triangle ADL \cong \triangle BCM$ [By AAS criterion of congruence]

And we know that congruence figures have equal areas

Therefore, $\text{ar}(\triangle ADL) = \text{ar}(\triangle BCM)$
 $\text{ar}(\text{||gm ABCD}) = \text{ar}(\text{ADMB}) + \text{ar}(\triangle BMC)$ [By area addition axiom]

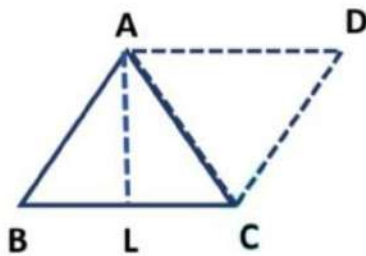
$\text{ar}(\text{||gm ABCD}) = \text{ar}(\text{ADMB}) + \text{ar}(\triangle ADL)$ [From 1]

$\text{ar}(\text{||gm ABCD}) = \text{ar}(\text{ABML})$ [$\text{ar}(\text{ABML}) = AB \times AL$]

$\text{ar}(\text{||gm ABCD}) = AB \times AL$

Hence, area of the $\parallel\text{gm}$ is the product of its side and the corresponding altitude
 $[\text{ar}(\parallel\text{gm}) = \text{Base} \times \text{Height}]$.

Theorem 5: The area of a triangle is half the product of any of its sides and the corresponding altitude.



Given: $\triangle ABC$ in which AL is the altitude to the side BC .

To prove: $\text{ar}(\triangle ABC) = \frac{1}{2} (BC \times AL)$

Construction: Through C and A draw $CD \parallel BA$ and $AD \parallel BC$ respectively, meeting each other at D .

Proof: We have,

$BA \parallel CD$ [By construction]

$AD \parallel BC$ [By construction]

Since a quadrilateral is a parallelogram if both pairs of opposite sides are parallel

$\therefore BCDA$ is a parallelogram.

As AC is a diagonal of $\parallel\text{gm } BCDA$

$\therefore \text{ar}(\triangle ABC) = \frac{1}{2} \text{ar}(\parallel\text{gm } BCDA)$

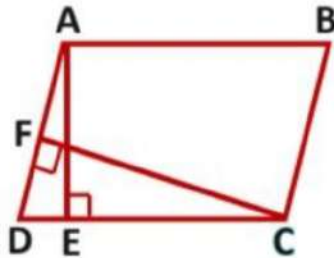
By theorem 4, the area of a $\parallel\text{gm}$ is the product of its base and the corresponding altitude.

$\text{ar}(\parallel\text{gm } BCDA) = BC \times AL$

$\Rightarrow \text{ar}(\triangle ABC) = \frac{1}{2} (BC \times AL)$

Example 1: In the figure, ABCD is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB=16$ cm, $AE=8$ cm and $CF=10$ cm, find AD.

(REFERENCE: NCERT)



In parallelogram ABCD, $CD=AB=16$ cm (Opposite sides of a parallelogram are equal)

We know that

Area of a parallelogram = Base $\times$ altitude.

For altitude AE with base CD

$$\text{ar}(\parallel^{\text{gm}}) = CD \times AE$$

For altitude CF with base AD

$$\text{ar}(\parallel^{\text{gm}}) = AD \times CF$$

Now, by using equation (1) and (2), we get

$$CD \times AE = AD \times CF$$

$$16 \times 8 = AD \times 10$$

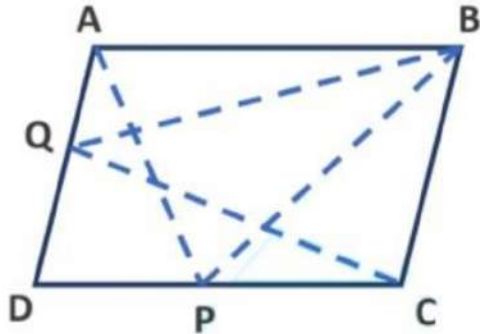
$$128 = AD \times 10$$

$$\frac{128}{10} = AD$$

$$AD = 12.8 \text{ cm}$$

Example 2: P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that $\text{ar}(\triangle APB) = \text{ar}(\triangle BQC)$.

(REFERENCE: NCERT)



Here, $\triangle APB$ and parallelogram ABCD are on the same base AB and between the same parallels AB and DC.

$$\therefore \text{ar}(\triangle APB) = \frac{1}{2} \text{ar}(\text{||}^{\text{gm}} \text{ABCD}) \quad [\text{Theorem 3}]$$

Similarly, $\triangle BQC$ and parallelogram ABCD are on the same base BC and between the same parallels BC and AD.

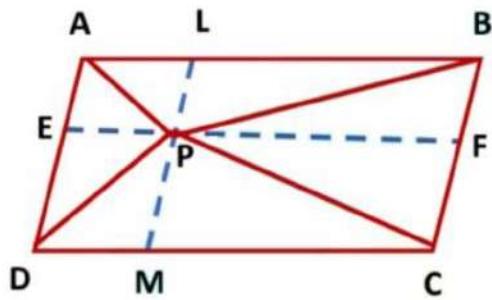
$$\therefore \text{ar}(\triangle BQC) = \frac{1}{2} \text{ar}(\text{||}^{\text{gm}} \text{ABCD}) \quad [\text{Theorem 3}]$$

Now, by using equation (1) and (2), we get

$$\text{ar}(\triangle APB) = \text{ar}(\triangle BQC)$$

Example 3: In figure, P is a point in the interior of a parallelogram ABCD. Show that

1. $\text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(\text{ABCD})$
2. $\text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \text{ar}(\triangle APB) + \text{ar}(\triangle PCD)$



(REFERENCE: NCERT)

Construction: Draw a line segment EF parallel to the line segment AB passing through P. Draw a line segment LM parallel to the line segment AD passing through P.

(1). In parallelogram ABCD,

$AB \parallel EF$ (By construction)

$AD \parallel BC$ (Opposite side of a parallelogram ABCD)

$\Rightarrow AE \parallel BF$

From equation (1) and (2), we obtain

$AB \parallel EF$ and $AE \parallel BF$

Therefore, quadrilateral ABFE is a parallelogram.

Since $\triangle APB$ and parallelogram ABFE are on the same base AB and between the same parallels AB and EF.

$$\therefore \text{ar}(\triangle APB) = \frac{1}{2} \text{ar}(\text{||gm ABFE}) \text{ [By Theorem 3]}$$

Similarly, $\triangle PCD$ and parallelogram EFCD are on the same base DC and between the same parallels DC and EF.

$$\therefore \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(\text{||gm EFCD}) \text{ [By Theorem 3]}$$

By adding (3) and (4), we get

$$\text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(\text{||gm ABFE}) + 12\text{ar}(\text{||gm EFCD}).$$

$$\text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(\text{||gm ABCD}).$$

(2). Quadrilateral ALMD is formed the parallelogram.

Since $\triangle APD$ and parallelogram ALMD are on the same base AB and between the same parallels AD and LM.

$$\therefore \text{ar}(\triangle APD) = \frac{1}{2} \text{ar}(\text{||gm ALMD}) \text{ [By Theorem 3]}$$

Similarly, $\triangle PBC$ and parallelogram BLMC are on the same base BC and between the same parallels BC and LM.

$$\therefore \text{ar}(\triangle PBC) = \frac{1}{2} \text{ar}(\text{||gm BLMC}) \text{ [By Theorem 3]}$$

By adding (6) and (7), we get

$$\text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \frac{1}{2} \text{ar}(\text{||gm ALMD}) + \frac{1}{2} \text{ar}(\text{||gm BLMC})$$

$$\text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \frac{1}{2} \text{ar}(\text{||gm ABCD})$$

By comparing equation (5) and (8), we get

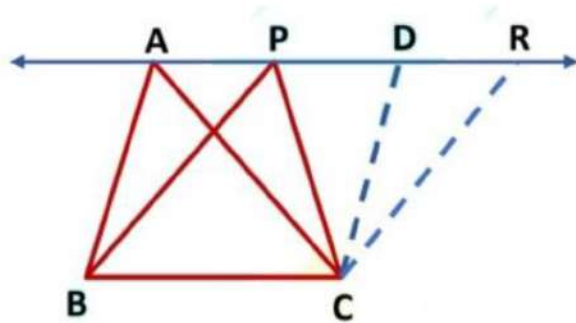
$$\text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \text{ar}(\triangle APB) + \text{ar}(\triangle PCD)$$

Triangles on the same base and between the same parallels

Triangles on the same base and between the same parallels.

Theorem 6: Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

Given: Two triangles ABC and PBC on the same base BC and between the same parallels BC and AP.



To prove: $\text{ar}(\triangle ABC) = \text{ar}(\triangle PBC)$

Construction: Draw $CD \parallel BA$ and $CR \parallel BP$ such that D and R lie on the same line AP.

Proof: By the construction, we get

Two parallelograms PBCR and ABCD on the same base BC and between the same parallels BC and AR.

$$\therefore \text{ar}(\parallel^{\text{gm}} ABCD) = \text{ar}(\parallel^{\text{gm}} PBCR) \quad [\text{Theorem 1}]$$

Now, we know that the diagonal of a parallelogram divides it into two triangles of equal area.

$$\therefore \text{ar}(\triangle PBC) = \frac{1}{2} \text{ar}(\parallel^{\text{gm}} PBCR) \quad (\parallel^{\text{gm}} PBCR \text{ with diagonal } PC)$$

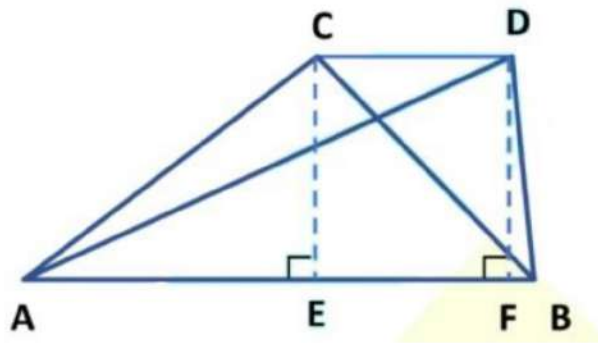
By multiplying both sides with $\frac{1}{2}$, we get

$$\Rightarrow \frac{1}{2} \text{ar}(\parallel^{\text{gm}} ABCD) = \frac{1}{2} \text{ar}(\parallel^{\text{gm}} PBCR)$$

From equation (2) and (3), we obtain

$$\text{ar}(\triangle ABC) = \text{ar}(\triangle PBC).$$

Theorem 7: Two triangles having the same base (or equal base) and equal areas lie between the same parallels.



Given: ΔABC and ΔADC both lie on the same base in a way that $\text{ar}(\Delta ABC) = \text{ar}(\Delta ADC)$.

To prove: ΔABC and ΔADB lie between the same parallel. i.e. $CD \parallel AB$
 Construction: Draws altitudes CE and DF of ΔACB & ΔADB on AB respectively.

Proof: Now according to question, it's said that ΔABC and ΔABD both lie on the same base and both have equal area.

Now, by construction, we have CE perpendicular to AB and DF perpendicular to AB .

Now, we know that lines perpendicular to the same line are parallel to each other, therefore, $CE \parallel DF$.

And it is given that, $\text{ar}(\Delta ABC) = \text{ar}(\Delta ABD)$

We know that,

Area of Triangle = $\frac{1}{2}$ Base x Height).

Area Of $\Delta ABC = \text{Area of } \Delta ABD$.

$$\Rightarrow \frac{1}{2} AB \times CE = \frac{1}{2} \times AB \times DF$$

$$\Rightarrow CE = DF.$$

From equation (1) & (2), we have

$CE = DF$ and $CE \parallel DF$.

So, we know that if in a quadrilateral if one pair of opposite sides are equal and parallel then the quadrilateral is a parallelogram.

Therefore, CDEF is a parallelogram.

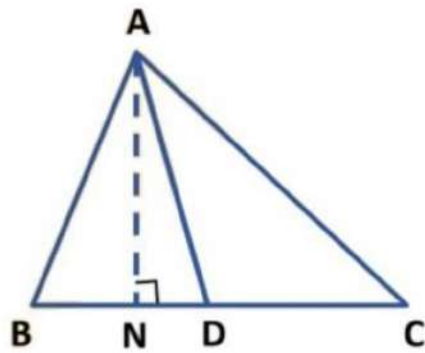
$CD \parallel EF$ (Opposite sides of the parallelogram)

Hence, $CD \parallel AB$.

Theorem 8: The median of a triangle divides it into two triangles of equal areas.

Given: $\triangle ABC$ with a median AD .

To prove: $\text{ar}(\triangle ABD) = \text{ar}(\triangle ACD)$ Construction: Draw $AN \perp BC$.



Proof: By theorem 5, we know that, area of a triangle is half the product of its base and corresponding altitude.

$$\text{ar}(\triangle ABD) = \frac{1}{2} \times \text{base} \times \text{altitude of } \triangle ABD.$$

$$\text{ar}(\triangle ABD) = \frac{1}{2} \times BD \times AN$$

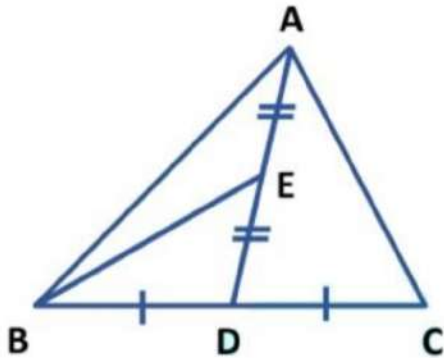
$$\text{Now, ar}(\triangle ACD) = \frac{1}{2} \times CD \times AN$$

$$\text{ar}(\triangle ACD) = \frac{1}{2} \times BD \times AN \quad (\because BD = CD)$$

From 1 & 2, $\text{ar}(\triangle ABD) = \text{ar}(\triangle ACD)$.

Examples 1: In a triangle ABC, E is the mid-point of median AD.

Show that $\text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$. [Reference: NCERT]



Given: $\triangle ABC$ with median AD and E is the mid-point of median to AD.

To prove: $\text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$.

Proof: By theorem 7, AD is a median of $\triangle ABC$ and median divides the triangle into two triangles of the equal area.

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ADC)$$

$$\Rightarrow \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC)$$

Similarly, in $(\triangle ABD)$, BE is the median

$$\therefore \text{ar}(\triangle BED) = \text{ar}(\triangle BAE)$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{2} \text{ar}(\triangle ABD)$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{2} \times \frac{1}{2} \text{ar}(\triangle ABC) \text{ [From (1)]}$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC).$$

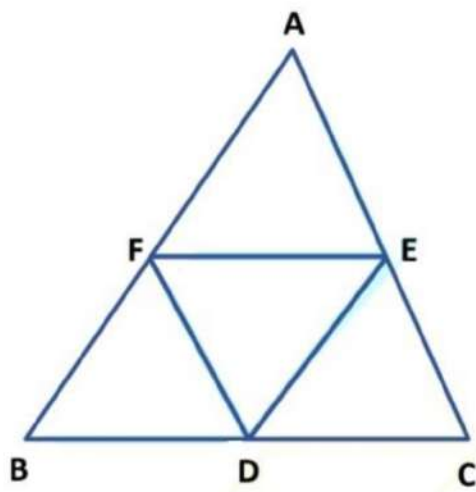
$$\text{Hence, } \text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC).$$

Example 2: D, E, and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC . Show that

(1) BDEF is a parallelogram

$$(2) \text{ar}(\Delta DEF) = \frac{1}{4} \text{ar}(\Delta ABC)$$

$$(3) \text{ar}(BDEF) = \frac{1}{2} \text{ar}(\Delta ABC) \quad [\text{Reference: NCERT}]$$



(1) In ΔABC , D and E are the mid-point of sides BC and AC respectively.

$$\therefore DE \parallel BA$$

$$\Rightarrow DE \parallel BF$$

[$\because$ Line joining the mid-point of two sides of a triangle is parallel to the third side and half of it.]

Similarly, $FE \parallel BC$

$$\Rightarrow FE \parallel BD$$

From (1) and (2), we obtain

BDEF is a parallelogram

(2). Now, DF is a diagonal of parallelogram BDEF.

By theorem 2, a diagonal of a parallelogram divides it into two triangles of equal area.

$$\therefore \text{ar}(\triangle BDF) = \text{ar}(\triangle DEF)$$

Similarly, DE is a diagonal of ||gm DCEF

$$\therefore \text{ar}(\triangle DCE) = \text{ar}(\triangle DEF)$$

And, FE is a diagonal of ||gm AFDE

$$\therefore \text{ar}(\triangle AFE) = \text{ar}(\triangle DEF)$$

From equation (3), (4) and (5), we get

$$\text{ar}(\triangle BDF) = \text{ar}(\triangle DCE) = \text{ar}(\triangle AFE) = \text{ar}(\triangle DEF)$$

$$\text{Now, ar}(\triangle ABC) = \text{ar}(\triangle BDF) + \text{ar}(\triangle DCE) + \text{ar}(\triangle AFE) + \text{ar}(\triangle DEF)$$

$$\text{ar}(\triangle ABC) = 4 \text{ ar}(\triangle DEF) \quad [\text{Using equation (6)}]$$

$$\frac{1}{4} \text{ ar}(\triangle ABC) = \text{ar}(\triangle DEF)$$

$$\text{ar}(\triangle DEF) = \frac{1}{4} \text{ ar}(\triangle ABC)$$

$$(3) \text{ ar}(\text{||gm BDEF}) = \text{ar}(\triangle BDF) + \text{ar}(\triangle DEF) \quad [\text{Using equation (3)}]$$

$$\text{ar}(\text{||gm BDEF}) = \text{ar}(\triangle DEF) + \text{ar}(\triangle DEF)$$

$$\text{ar}(\text{||gm BDEF}) = 2 \text{ ar}(\triangle DEF)$$

$$\text{ar}(\text{||gm BDEF}) = 2 \times \frac{1}{4}$$

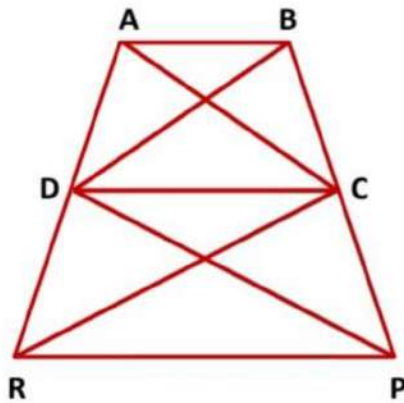
$$\text{ar}(\triangle ABC) \quad [\text{From equation (7)}]$$

$$\text{ar}(\text{||gm BDEF}) = \frac{1}{2} \text{ ar}(\triangle ABC)$$

Example 3: In figure, $\text{ar}(\triangle DRC) = \text{ar}(\triangle DPC)$ and $\text{ar}(\triangle BDP) = \text{ar}(\triangle ARC)$.

Show that both the quadrilateral ABCD and DCPR are trapeziums.

[Reference: NCERT]



It is given that, $\text{ar}(\Delta \text{DRC}) = \text{ar}(\Delta \text{DPC})$.

Here, ΔDRC and ΔDPC are on the same base CD and are equal in area. Therefore, they must be between the same parallels.

$\therefore \text{RP} \parallel \text{DC}$. Therefore, DCPR is a trapezium.

We also have, $\text{ar}(\Delta \text{BDP}) = \text{ar}(\Delta \text{ARC})$

Now, subtracting $\text{ar}(\Delta \text{DRC})$ from both sides, we get

$$\Rightarrow \text{ar}(\Delta \text{BDP}) - \text{ar}(\Delta \text{DRC}) = \text{ar}(\Delta \text{ARC}) - \text{ar}(\Delta \text{DRC}) \text{ [From equation (1)]}$$

$$\Rightarrow \text{ar}(\Delta \text{BDP}) - \text{ar}(\Delta \text{DPC}) = \text{ar}(\Delta \text{ARC}) - \text{ar}(\Delta \text{DRC})$$

$$\Rightarrow \text{ar}(\Delta \text{ADC}) = \text{ar}(\Delta \text{BDC})$$

$\therefore \Delta \text{ADC}$ and ΔBDC are on the same base DC and are equal in area.

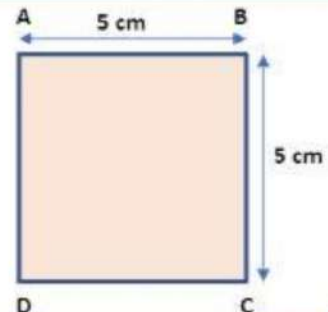
Therefore, they must be between the same parallel lines, i.e. $\text{AB} \parallel \text{DC}$.

Therefore, ABCD is a trapezium.

Summary of Area of Parallelograms and Triangles

Area

Area of figure ABCD is the number of unit square associated with the part of the plane enclosed by that figure. $\text{ar}(\text{ABCD}) = 25 \text{ cm}^2$

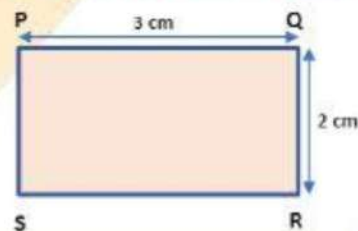
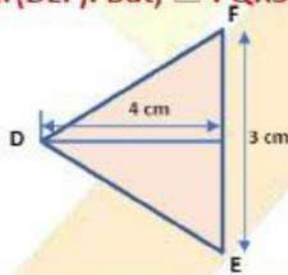
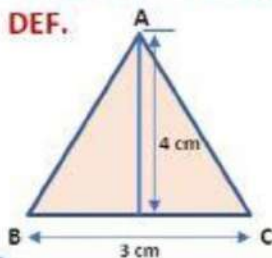


CONGRUENT AREA AXIOM

Two congruent figures have equal areas but the converse need not be always true.

$\triangle ABC \cong \triangle DEF$ & $\text{ar}(\text{ABC}) = \text{ar}(\text{DEF})$.

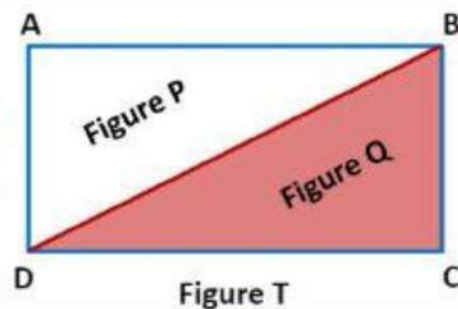
$\text{ar}(\text{PQRS}) = \text{ar}(\text{ABC}) = \text{ar}(\text{DEF})$. But, $\square \text{PQRS}$ is not congruent with $\triangle \text{ABC}$ & $\triangle \text{DEF}$.



AREA ADDITION AXIOM

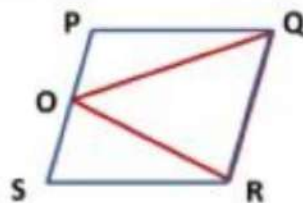
If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q, then $\text{ar}(\text{T}) = \text{ar}(\text{P}) + \text{ar}(\text{Q})$.

$\text{ar}(\text{ABCD}) = \text{ar}(\triangle \text{ADB}) + \text{ar}(\triangle \text{BCD})$.



Figures on the same base and between the same parallels

Two figures are said to be on the same base and between same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.

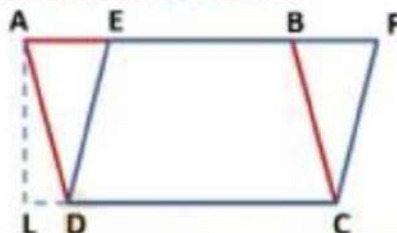


$\parallel^gm PQRS$ & $\triangle ROQ$ are on the same base RQ and between the same parallel's PS and QR.

Parallelograms on the same base and between the same parallels.

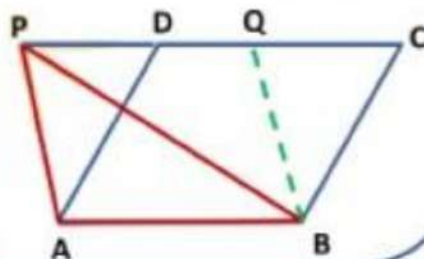
Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.

$$ar(\parallel^gm ABCD) = ar(\parallel^gm EFCD)$$



The area of a parallelogram is the product of its base and the corresponding altitude. $ar(\parallel^gm ABCD) = AB \times AL$.

If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half of the area of the parallelogram. $ar(\triangle PAB) = \frac{1}{2} ar(\parallel^gm ABCD)$



Triangles on the same base and between the same parallels.

Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

$$ar(\triangle ABC) = ar(\triangle PBC)$$

Two triangles having the same base (or equal base) and equal areas lie between the same parallels. $\triangle ABC$ and $\triangle PBC$ lie between the same parallel. i.e. $DE \parallel BC$.

